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Radiative Heat Transfer Through a Randomly Packed Bed of Spheres by the Monte Carlo Method

Radiative heat transfer through evacuated randomly packed beds of uniform-diameter spheres is considered. A Monte Carlo technique is used to simulate the energy bundle traveling through the voids of the bed. The randomly packed bed is assumed to be an absorbing-scattering medium with effective absorption and scattering coefficients. The packing pattern is modeled by a numerical simulation of rigid spheres slowly settling into a randomly packed assemblage. The Monte Carlo simulation of radiant energy transport through the packed beds generates the transmission curve as a function of bed height and sphere emissivity. The effective absorption and scattering coefficients of the randomly packed bed are evaluated by using the solution of the two-flux equations and Monte Carlo transmission results. Results show a strong dependence of the thermal radiative properties on the packing structure and the size and emissivity of constitutent spheres. Qualitative agreement is shown in comparison with other work which used regular cubic packing, and with existing experimental data.

Introduction

Radiative transfer through randomly packed beds of spheres is of importance due to its many industrial applications. Accurate prediction methods are particularly needed in high-performance cryogenic insulations [1, 2] and in the case of a loss of coolant accident for the pebble bed nuclear reactor. In general, two major transport mechanisms can occur in an evacuated randomly packed bed of spheres: conduction through the solid contact between spheres, and radiative transfer through the voids. However, these two processes can often be decoupled effectively and considered separately [3]. The main purpose of this study is to develop a methodology to examine the radiative heat transfer process through the randomly packed bed of spheres.

There are three commonly used models in the literature that describe the radiative heat transfer through packed beds. The first model common to a large number of analytical treatments is to approximate the heterogeneous random mixture of solid particles and voids by some regular geometrical arrangement of arbitrary solid and void bodies. Argo and Smith [4] and Chan and Tien [5] treated the solid and gas or void phases as alternating layers perpendicular to the direction of transfer.

The second type is based on a random walk process proposed by Rosseland [6]. It is assumed that when the mean free path of the photon in the packed beds is only a small fraction of the geometrical dimensions of the absorbing medium, the passage of a single quantum of radiant energy takes place along what may be regarded as a random path. This is a radiation diffusion process as is the diffusion of heat by gaseous conduction at ordinary pressures.

The third type of model is suggested by Van der Held [7], who considered the packed beds to be a pseudohomogeneous material, permitting description of the heat transfer processes by differential or integro-differential equations and boundary conditions. Hamaker [8] assumed that only two discrete fluxes exist inside the medium, one forward and the other backward, which leads to two coupled differential equations for radiation.

Brewster and Tien [9] have shown that dependent scattering effects must be considered when interparticle spacing is less than a few wavelengths. Thus, use of single scatter (independent scattering) properties in cases such as that studied here can introduce significant errors.

The objectives of the present study are to use a simulation of random packing of spheres to obtain information on the packed beds which affects the radiative transfer process and to perform Monte Carlo simulation of radiant energy transport through the packed beds by using the diffusion concept. The results of the transmission are used to predict the effective absorption coefficient, the scattering cross section, and the radiative conductivity.

General Considerations

The theoretical basis for this investigation is the two-flux model, which assumes only two discrete fluxes exist in the packed medium, one forward and one backward. This assumption simplifies the integro-differential equation of transfer into two coupled differential equations

$$\frac{di^+}{dx} = -(a+s)i^+ + si^- \qquad (1)$$

$$-\frac{di^-}{dx} = -(a+s)i^- + si^+ \qquad (2)$$

where x is the coordinate in the direction of radiative heat transfer and perpendicular to the boundary plane of the packed beds, i^+ and i^- are the radiation intensities in the positive and negative x-direction, and a and s are the effective absorption coefficient and the back-scattering coefficient. Parameters a and s have strong dependence on the geometric properties of the packing, such as the void fraction, size, and surface properties of the constituent spheres and the temperature of the system. Equations (1) and (2) can be solved readily with proper boundary conditions if the parameters a and s are known.

As shown by Chen and Churchill [10], the normalized transmitted flux defined as

$$S_n(L) = \frac{i^+(L)}{i^+(0)} = \frac{i^+(L)}{S_o} \qquad (3)$$

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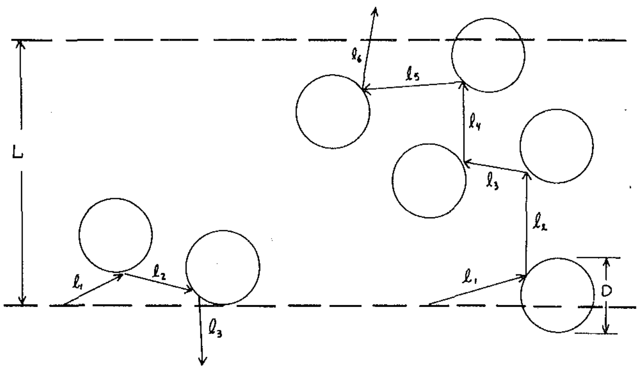

Fig. 1 Physical model of a randomly packed bed of equal-diameter spheres

can be found by solving the two-flux equations with the boundary conditions

$$i^+ = S_o \text{ at } x=0 \tag{4}$$

$$i^- = 0 \text{ at } x=L \tag{5}$$

The result is

$$S_n(L) = \frac{m}{m\cosh(mL) + a_n \sinh(mL)} \tag{6}$$

where $m^2 = a_n^2 - s^2$

$a_n = a + s$

and L is the bed height. Equations (4) and (5) represent a known input signal strength and no reflection. These boundary conditions also indicate that the boundary planes are black with emissivity equal to unity. Equation (6) is the analytical model used to fit experimental measurements or computer simulated results of S_n and L.

The functional dependence of S_n on L is affected by many factors, such as the packing structure of the bed, size and surface property of the constituent spheres, and the temperature of the system. This relationship can be obtained by either a direct measurement from an experiment or a computer simulation of the real transport process. Both of these methods encounter some difficulties. For instance, it is difficult to build a randomly packed bed of spheres, and the ordering effect due to the containing wall cannot be ignored, especially for thinner beds. The direct transmission in thin-layer samples contradicts the diffuse assumption of the two-flux model. If the bed is too thick, accurate measurement is difficult due to the weak intensity transmitted through the bed, which introduces greater deviation in the measurement data. One of the most important factors in determining the transmission curve of a packed bed may lie in the definition of the bed height. There is always a deviation of bed height within plus or minus one sphere diameter. Since the transmission ratios are plotted directly against the bed height, the bed height, L, plays an important role in the entire analysis. The scattered data reported on the value of emissivity of different materials, which must be used in the analytical predictions, also add to the uncertainty of the results.

The radiation transport through the randomly packed beds is simulated by a Monte Carlo technique using the known cumulative distribution function of the penetration distance. The radiative energy bundle is emitted from a reference plane at the higher temperature boundary of the bed. Each bundle path is traced, and all the interactions that occur in the medium before the bundle escapes from the bed are recorded.

In the analysis, the radiative properties are assumed gray and temperature-independent. The surface of the spheres is assumed specular, and the size of the bed is made large enough to make Rosseland's diffusion approximation valid. The packed beds are evacuated; i.e., there is no attenuation in the voids. The conduction mechanism is decoupled completely from the radiative process. The effects of diffraction are ignored in the present analysis. Such effects could well be important, and are a suitable area for further study.

The physical model consists of an infinite number of spheres of equal-diameter packed randomly in a slab of finite thickness as shown in Fig. 1. The randomness of the sphere packing is interpreted in the sense that the penetration distance for each energy bundle is determined from randomly packed spheres, and the center of each sphere being hit is determined randomly. Each path that a photon travels is independent of the others, and the history of the path is to be eliminated once its journey is over.

Random Packing of Spheres

Much effort has been expended on the subject of random packing of spheres in order to understand its special features and geometric properties. Knowledge of the packing structure within a randomly packed bed is essential for any rigorous analysis of the heat transfer within the bed. Among the approaches of solving the packing problem, simulation is attractive, since the construction of random assemblages of spheres is easier and the information about the packing structure needed for the heat transfer analysis is far more detailed than experimental results can offer. Moreover, the retention of the coordinates of the spheres in the randomly packed beds makes it possible to carry out later analysis for special purposes. The simulation used here is based on the work by Tory [11]. A computer code PACKUT is a modified version of the original work by Tory used to obtain information on the solid fraction distribution, as well as the extinction coefficient of the radiation in the randomly packed beds.

The simulation describes the slow sequential settling of

Nomenclature

- a = absorption coefficient
- a_n = $a + s$
- D = sphere diameter
- F = parameter for calculating radiant conductivity
- I_o = incident radiation intensity
- i^+ = forward radiant intensity
- i^- = backward radiant intensity
- K = extinction coefficient
- k_r = radiant conductivity
- L = bed thickness
- l = penetration distance
- l_m = mean penetration distance
- m = $(a_n^2 - s^2)^{1/2}$
- $\mathbf{n}$ = normal vector of the sphere surface
- n_i = components of the normal vector
- P = cumulative distribution function of the penetration distance
- p = probability density function of the penetration distance
- R_θ, R_γ = random numbers
- S_o = initial energy bundle strength
- s = scattering coefficient
- u_i = directional cosines
- w = size of the packing container
- x,y,z = positions in Cartesian coordinate system
- X,Y,Z = axes of Cartesian coordinate system
- γ = cone angle
- θ = circumferential angle
- ϵ = radiative surface emissivity

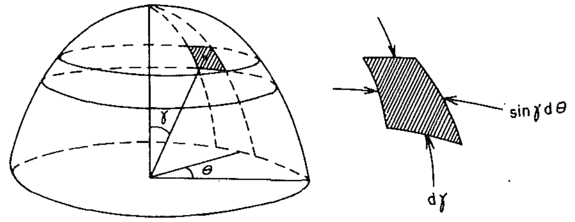

Fig. 2 Unit hemisphere used to obtain emission point and emission direction of a sphere

individual rigid spheres of equal diameter from a dilute suspension into a randomly packed assemblage. No bouncing and no bumping of precariously stable spheres to more stable positions is considered, and no consideration is made of spreading spheres apart to accommodate incoming spheres. It is assumed that each sphere in the bed must be supported by at least three others, and no sphere can overlap. Packing takes place in a semi-infinite box defined by $[x, y, z \mid 0<x<w, 0<y<w, z>0]$. The simulation can be used to describe the packing between flat walls, or between periodic vertical bounding surfaces to avoid the wall effects. The latter yields a packing which is effectively infinite in the xy-plane.

Once the initial coordinates of a sphere have been chosen, its path is completely determined. After dropping, each sphere seeks a position of minimum potential and, given the impenetrability of the spheres and floor, follows the constrained path along which the potential gradient is a maximum. With the initial position of each new sphere determined and the coordinates of spheres already in known position, the final position of each new sphere can be calculated. A sphere is stable when any roll would incresse its potential. Any sphere which touches the floor is considered to be stable. After the coordinates of all the sphere centers are stored in the computer memory, the packing properties of the bed can be computed.

The solid fraction of an array of spheres is defined as the ratio of the volume of the spheres to the total volume they occupy. Physically, it represents the density of the bed and is related to the probability that a radiant energy bundle can penetrate through the bed. There exists a range of solid fraction for a randomly packed bed of equal-diameter spheres [12]. The packing generated from this simulation represents a random loose packing and has a solid fraction of 0.58. As presently formulated, it is not possible to generate beds with other solid fractions using the simulation.

Within a randomly packed bed, spheres may have different numbers of neighbors in contact. When a radiant energy bundle is emitted from a base sphere, the immediate surrounding of the base sphere will have a great influence on the fate of the bundle. A search is done to find the clustering features of the packed beds. The contact-number frequency function is defined as the probability of the different possible contact numbers that can exist within a randomly packed bed. The simulation used in this study shows a strong peak at six contacts, and the average number of contacts is approximately six. Although it is realized that clustering in the vicinity of the base sphere does affect the radiation transport, in the following analysis, the extinction coefficient is obtained only for the most common situation when the contact number is six.

The Extinction Coefficient of Randomly Packed Beds of Equal Spheres

The radiation intensity through an evacuated randomly packed bed is attenuated by particle absorption and scattering. The change in intensity has been found experimentally to depend on the magnitude of the local intensity. If a coefficient of proportionality, K, which depends on the local properties of the medium is introduced, then the decrease is given by

$$di(l) = -K(l)\,i(l)\,dl \tag{7}$$

where i is the radiation intensity, and l is the coordinate along the direction of radiation. K is the extinction coefficient in the layer; it is a physical property of the material and has units of reciprocal length. For a gas medium, K is a function of the temperature, pressure, composition of the material, and wavelength of the incident radiation. For packed beds made up of uniform-diameter spheres of the same material, under the assumptions made in the previous section, the extinction coefficient is only a function of the packing arrangement and the size of the constituent spheres; it is independent of particle absorptivity.

A more appropriate interpretation can be achieved by reexamining equation (7) and noting that the fractional change in radiation intensity occurring over a distance, dl, is given by

$$K(l) = \frac{-\dfrac{di(l)}{i(l)}}{dl} \tag{8}$$

Hence, it is natural to interpret $K(l)$ as the probability per unit path length traveled that the radiant energy bundle will undergo a reaction with a sphere surface in the packed bed at the position l.

Equation (7) can also be solved by imposing an incident radiation intensity I_0 at $l = 0$, giving

$$i(l) = I_o \exp\left[-\int_o^l K(l^*)dl^*\right] \tag{9}$$

If we interpret this equation in the sense of probability, then

$\exp\left[-\int_o^l K(l^*)dl^*\right] \equiv$ probability that a radiant energy bundle moves a distance, dl, without any interaction

$K(l)\exp\left[-\int_o^l K(l^*)dl^*\right] \equiv$ probability that a radiant energy bundle has its first interaction in dl about l

With this interaction probability $P(l)$, we can calculate the average distance a photon travels before interacting with a sphere surface in the packed bed

$$l_m = \int_0^\infty K(l)\exp\left[-\int_o^l K(l^*)dl^*\right]dl \tag{10}$$

When $K(l)$ is constant, carrying out the integral gives

$$l_m = \int_0^\infty l \exp(-Kl)\,dl = \frac{1}{K} \tag{11}$$

demonstrating that the average penetration distance before absorption or scattering is the reciprocal of K when K does not vary along the path. Equation (11) provides a simple way of gaining some insight as to whether or not an absorbing-scattering medium is optically dense with regard to radiation traveling through it.

The value of K for different gas media under different physical conditions is a measurable quantity. For temperature-independent gray packed beds, the extinction coefficient is solely determined by the size of the constitutent spheres and the packing structure.

To obtain the variation of K as a function of distance from a base sphere, a Monte Carlo method is used with the known

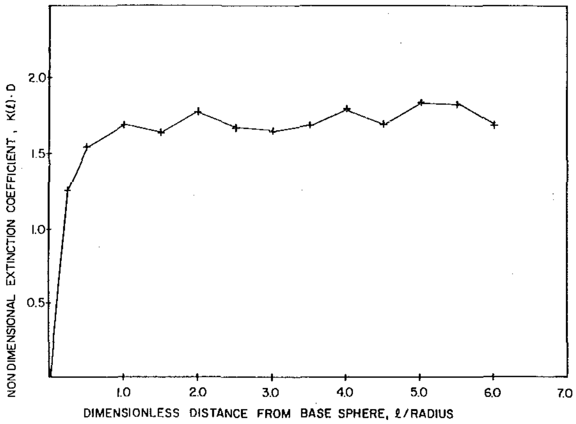

Fig. 3 Variation of extinction coefficient as a function of distance from the base sphere

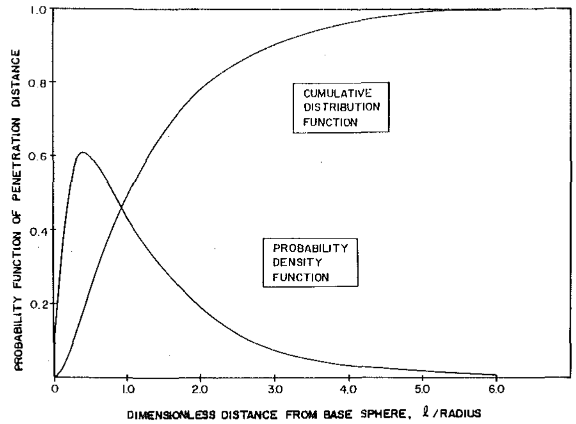

Fig. 4 Probability density function and cumulative distribution function of the penetration distance of radiant energy through a randomly packed bed of equal-diameter spheres

coordinates of all sphere centers in a randomly packed bed of spheres. A sphere with six contacts is first selected randomly as the starting base for the emission of radiant energy bundles. The emission point in the surface of the base sphere is also determined randomly by assuming that each area element or solid angle in the sphere surface has an equal probability of being selected. As shown in Fig. 2, the probability of selecting a surface point at cone angle γ, and circumferential angle θ, $p(\gamma,\theta)$, can be calculated as

$$p(\gamma,\theta)=\frac{D^2\sin\gamma\ d\gamma\ d\theta}{4\pi D^2}=\frac{\sin\gamma\ d\gamma\ d\theta}{4\pi} \tag{12}$$

Assuming no circumferential variation, the cumulative distribution function $P(\gamma)$ of $p(\gamma,\theta)$ is

$$P(\gamma)=\frac{\pi\int_0^\gamma \sin\gamma^* d\gamma^*}{4\pi}=\frac{1}{2}\ (1-\cos\gamma) \tag{13}$$

and $P(\gamma)$ is a number between 0 and 1. If we choose a number randomly from the range 0 to 1, then the corresponding γ angle can be calculated as

$$\gamma=\cos^{-1}(1-2R_\gamma) \tag{14}$$

Since no dependence on circumferential angle, θ, is assumed, it is fairly obvious from intuition that θ can be determined by

$$\theta=2\pi R_\theta \tag{15}$$

where R_θ is again a random number between 0 and 1.

Once the position of the emission point on a sphere surface is known, the direction of the energy bundle leaving that point can be determined following the same procedure. Again, γ and θ are determined randomly by choosing a point on the surface of the hemisphere whose base is the plane tangent to the base sphere at the emission point. Because of the constraint on the impenetrability of the sphere, the range for γ in this case is between 0 and $\pi/2$, which leds to the functional relation of $\gamma = \cos^{-1}(R_\gamma)$.

To find out whether this energy bundle emitted from that point in the direction determined above is able to penetrate a distance Δl about l, the coordinate of the midpoint of the line segment Δl is determined by

$$\begin{aligned} x&=\sin\gamma\cos\theta+x_o \\ y&=\sin\gamma\sin\theta+y_o \\ z&=\cos\gamma+z_o \end{aligned} \tag{16}$$

where (x_o, y_o, z_o) is the coordinate of the emission point. The two end points of the line segment are obtained by replacing l with $l+\Delta l/2$ and $l-\Delta l/2$. Since the coordinates of all the sphere centers are retained in the computer memory, a search can be done to check whether any of the spheres in the packed bed will intersect with the line segment identified above. The Δl chosen for this study is 1/10 of the sphere diameter. Mathematically, if any point on this line segment has a distance from the center of any sphere in the randomly packed bed of less than the sphere radius, this energy bundle is bound to be attenuated. It is assumed that the spheres are opaque, and only absorption and scattering are considered. Once the energy bundle hits the surface, there is an interaction between the bundle and the sphere surface. This process is repeated by emitting many bundles from different spheres. The fraction of the bundles that is intercepted by the spheres during travel through the distance Δl divided by Δl represents the extinction coefficient, $K(l)$, at position l. Different l are used to obtain the variation of the extinction coefficient as a function of l.

In performing the Monte Carlo calculation of $K(l)$, l up to three sphere diameters in length are examined. At each l value, 5000 energy bundles are emitted in order to obtain good statistical results. The $K(l)$ values approach a constant number in the statistical sense after l values greater than one sphere radius due to the homogeneity of the randomly packed spheres. The result of the $K(l)$ variation is shown in Fig. 3. It indicates that $K(l)\cdot D$ approaches approximately 1.75 after l greater than one sphere radius.

The product of $K(l)\cdot D$ approaching a constant reveals some interesting facts. The optical thickness of the packed beds is defined by $\int_0^l K(l^*)dl^*$, which in this case is approximately equal to $K(l)\cdot l$. If these two terms are written in nondimensional form, $(K(l)\cdot D)\cdot(l/D)$, it is clear that the optical thickness then depends directly on the extinction coefficients and the thickness of the absorbing-scattering medium. This is analogous to a gas that is of uniform composition, temperature, and pressure. $K(l)\cdot D$ equaling a constant indicates that the extinction coefficient is inversely proportional to the size of the constituent spheres in the packed bed, which implies that the effective thermal radiative properties have a strong dependence on the sphere size.

In order to calculate the radiant energy transport through the packed bed, the probability that the bundle has its first interaction in dl at different l, $p(l)dl$, must be known. With $K(l)$ at different l values known, $K(l)\ \exp[-\int_0^l K(l^*)dl^*]$ can be calculated by a numerical integration at each discrete point. The results are shown in Fig. 4. It shows that the most probable path length for a radiant energy bundle traveling through a randomly packed bed of equal-diameter spheres is approximately half of the sphere radius, and the probability that the bundle travels more than three sphere diameters before it hits a solid surface is nearly zero. The mean

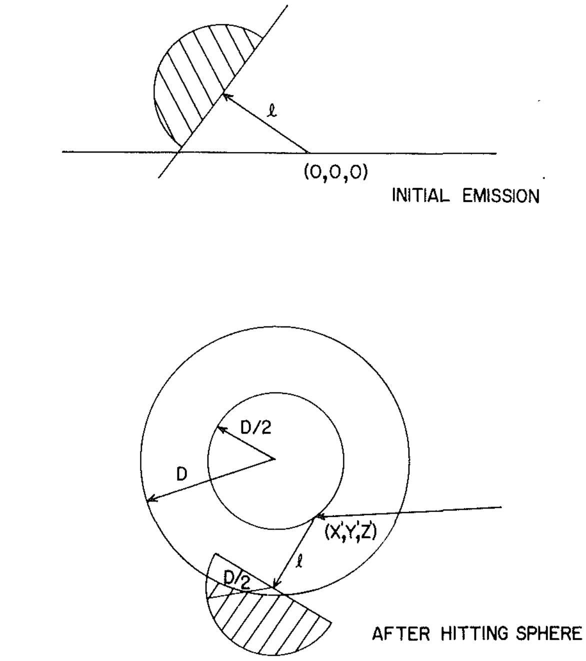

Fig. 5 Selection of sphere center coordinate after initial emission and after reflection

penetration distance of the radiation is obtained by multiplying the fraction absorbed at l by the distance l and then integrating over all path lengths from $l = 0$ to $l = \infty$ according to equation (10). The result shows that l_m of this system for the radiation is about 0.66 sphere diameters.

The cumulative distribution function of penetration distance, $P(l)$ is obtained by

$$P(l) = \int_0^l p(l^*)dl^* \tag{17}$$

The results are shown in Fig. 4. This curve is used to determine randomly the penetration distance of the radiation during each movement of the energy bundle.

Simulation of Radiant Energy Bundle Traveling Through a Randomly Packed Bed of Spheres

The main purpose of this simulation is to trace a sufficient number of radiant energy bundles to find out how much of the initial emitted energy is transmitted through the packed bed. The basic elements of the simulation consist of the determination of the penetration distance for each radiant energy bundle before it hits the solid surface, the random selection of the center coordinate of the sphere being hit, and the calculation of the reflected direction of the energy bundle by assuming that the sphere surface is specular.

In the present simulation, each radiant energy bundle is emitted from a reference origin. After emission, this energy bundle moves in a certain direction specified by the directional cosine (u_1, u_2, u_3) and penetrates a certain distance through the packed bed. This line segment with a direction can be viewed as a bound vector carrying a certain amount of energy, E_γ, in the three-dimensional space. The function relating a random number to the cone and circumferential angles for emission is given by [3]

$$\gamma = \sin^{-1}(R_\gamma^{1/2})$$

and (18)

$$\theta = 2\pi R_\theta$$

By choosing a random number between 0 and 1, the

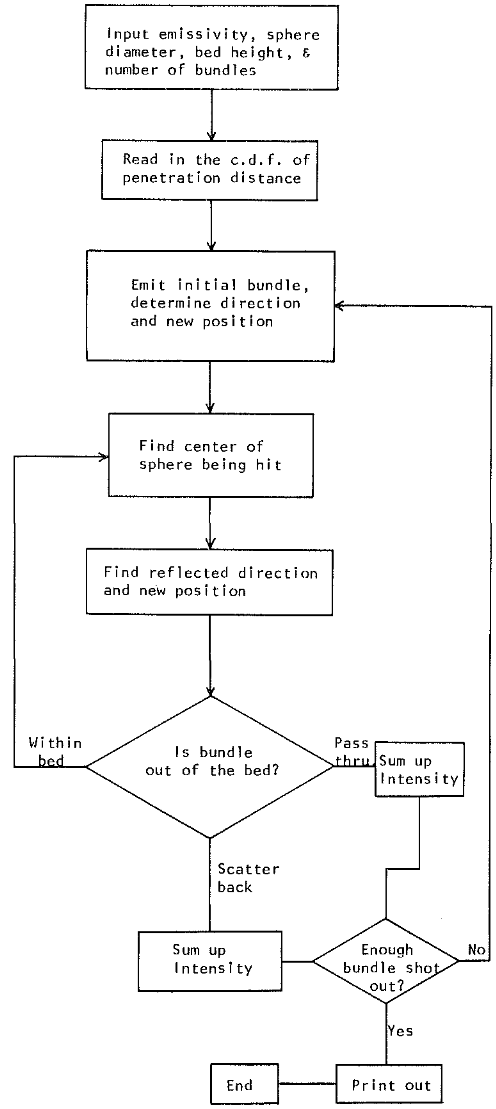

Fig. 6 Computer flow chart for radiant energy through a randomly packed bed of equal-diameter spheres

penetration distance can be obtained by mapping this random number to its corresponding function value according to Fig. 4. The end point of the radiative energy vector can now be determined. This point represents a location on the surface of the sphere that has an interaction with the incident radiation ray.

From the above given condition, an infinite number of possible choices is available for the center coordinate of the sphere being hit. To determine the center coordinate of the sphere being hit after the bundle carrying a unit of energy is first emitted, it is assumed there is no impenetrability constraint imposed on the selecting process. That is, the bounding plane of the packed bed has a wavy characteristic to eliminate the wall effect on the random packing of spheres. As shown in Fig. 5, the coordinate of the sphere center is determined by choosing randomly a point, as can be done by generating two random numbers and calculating the γ and θ angles according to the proper functional relationship, from the surface of a hemisphere whose base plane is perpendicular to the incident energy ray. The γ and θ angles are related to the random number, R_γ and R_θ, by

$$\gamma = \cos^{-1}(R_\gamma)$$

and

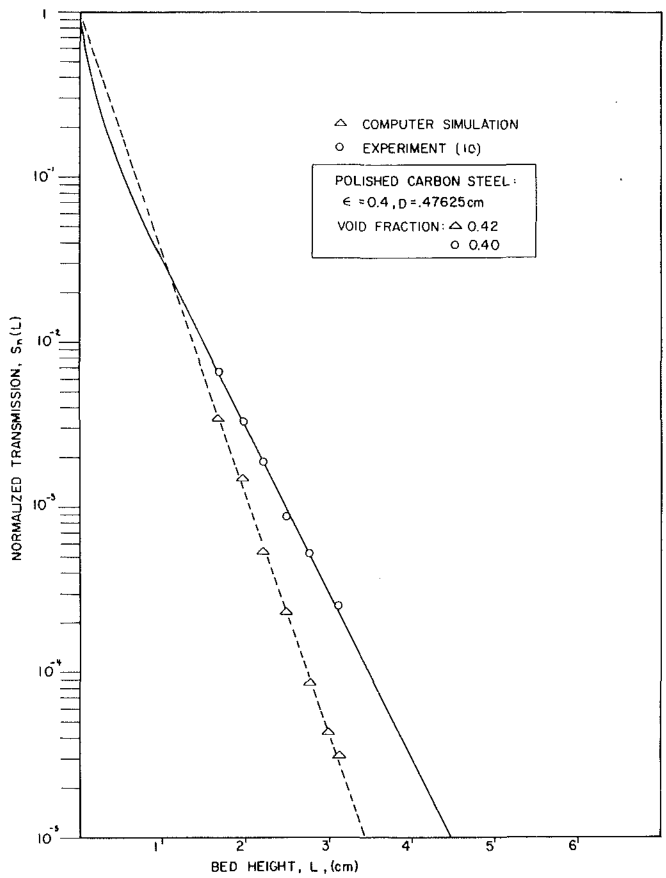

Fig. 7 Normalized transmission curves of radiant energy through a randomly packed bed of steel spheres

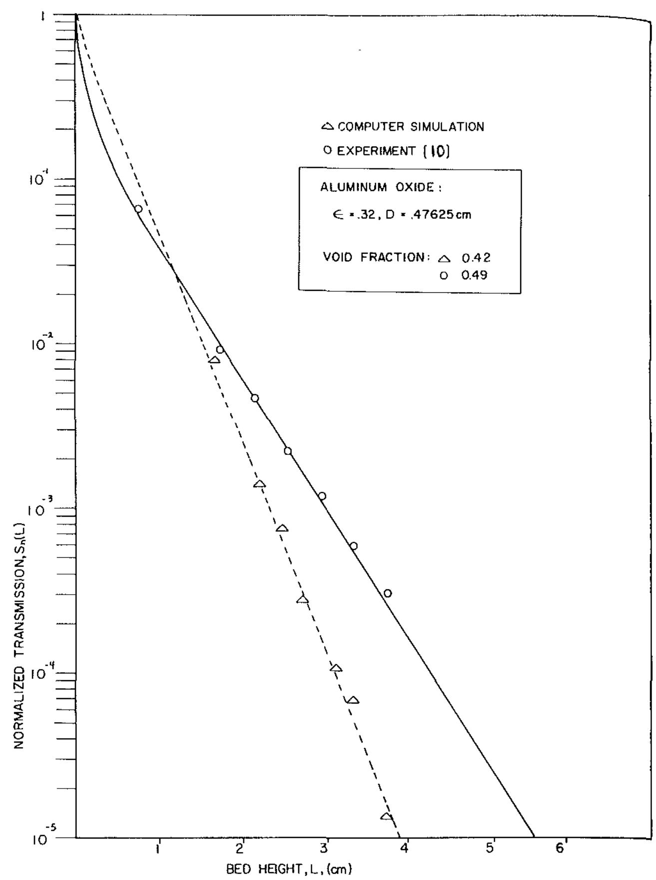

Fig. 8 Normalized transmission curves of radiant energy through a randomly packed bed of aluminum spheres

$$\theta = 2\pi R_\theta \tag{19}$$

The coordinate of the sphere center can now be calculated as

$$x' = (D/2)\sin\gamma\cos\theta$$
$$y' = (D/2)\sin\gamma\sin\theta$$
$$z' = (D/2)\cos\gamma$$

where x', y', z' are the coordinates of the sphere center at the X', Y', Z' coordinate system which uses the direction of the energy ray as the Z-axis. After a rotation and translation of the coordinate system, the sphere center coordinate according to the original coordinate system can be obtained. The transformation is needed due to the following involved reflection calculation.

After hitting a sphere surface, the energy bundle will suffer a decrease in the amount of energy it carries due to the absorption interaction. Out of the original E_γ, ϵE_γ is absorbed, and $(1-\epsilon)\ E_\gamma$ is scattered. The radiative transfer process continues with the bundle carrying less energy.

Because the surface of the sphere is assumed to be specular, the rays incident on a sphere surface are reflected according to the following fundamental laws of geometrical optics: (*a*) the angle of reflection is equal to the angle of incidence; and (*b*) the incident ray, the surface normal at the intersection point, and the reflected ray all lie in the same plane. These two laws are sufficient to determine the direction cosine $u_1{}'$, $u_2{}'$, $u_3{}'$ of the reflected ray, given as

$$u_i = u_i - 2n_i \sum_{k=1}^{3} n_k u_k \qquad i=1,2,3 \tag{20}$$

where u_i is the direction cosine of the incident ray, and n_i is the component of the unit vector **n** at the point of intersection; **n** is given by

$$\mathbf{n} = \frac{\operatorname{grad} h_j(x',y',z')}{|\operatorname{grad} h_j(x',y',z')|} \tag{21}$$

where (x', y', z') is the initial point of the reflected vector, and h_j is a continuous function of the sphere surface.

Once the direction cosines of the reflected ray are determined, another random number is drawn to determine the path length for this bundle of energy $(1-\epsilon)E$. With this penetration length and the directional cosines $u_1{}'$, $u_2{}'$, $u_3{}'$, the end point of the reflected ray vector can be obtained. At this point, again, interaction occurs between the energy bundle and the sphere surface whose center coordinate is yet to be determined.

In the present simulation, only one sphere reflects the ray at any moment. That sphere will impose another constraint on selecting the center coordinate of the sphere to be hit by the reflected ray due to the impenetrability of the solid sphere, which requires that two sphere centers be separated by at least a sphere diameter distance to avoid overlapping. As shown in Fig. 5, due to this extra constraint, only part of the hemisphere surface is not eligible for the selection of the sphere center. It may require several repetitions of the selecting process to obtain a satisfactory sphere center, especially when the end point of the ray vector is too close to the previously hit sphere. The process is repeated until either the ray has lost most of its energy by multiple reflections or it passes through the bounding planes.

On passing through a randomly packed bed of equal-diameter spheres, the diffuse incident flux will be absorbed and reflected repeatedly. By performing the above transport simulation, the transmittance of packed beds of different thickness can be obtained by summing all the energy passing through the bounding plane and dividing it by the total energy emitted. The flow chart of the Monte Carlo simulation program is shown in Fig. 6. Figures 7 and 8 show the result of simulation for ϵ values of 0.32 and 0.40, which represent the emissivities of aluminum oxide and polished carbon steel as reported in [10]. The diameter of the sphere is 0.476 cm. Also shown in Figs. 7 and 8 are the experimental results of [10].

The present simulation cannot exactly duplicate the experiment in [10]. The result of the transmission in this study is done in an ideally randomly packed bed of spheres. The differences in transmission results between this study and [10] stem from many factors. First, a substantial deviation exists in the emissivity data reported in the literature, which directly affects the prediction of energy transmitted through packed beds. The assumptions made in the simulation, such as the specular surface, temperature, and wavelength independence of the radiative properties may also change the transmission. The characteristics of the packed beds in the experiment are affected by the wall effect, and the deviation of the solid fraction from random packing of equal-diameter spheres makes important differences in the extinction coefficient. Different definitions of the bed height for these two approaches also contribute to the differences in the transmission curves. However, the functional dependence of the transmission on the emissivity indicates the same trend for both studies as shown in Figs. 7 and 8.

The Effective Thermal Radiative Properties of a Randomly Packed Bed of Spheres from the Transmission Simulation

Computer simulation of radiative transport through randomly packed beds supplies values of S_n as a function of l. It is also desired to obtain values of a and s by regression on equation (6). As the fractional deviation (rather than absolute deviation) is approximately equal for all data points in the simulation, the least square error requirement specified is that

$$\Phi = \sum_{j=1}^{n} \left[\frac{S_n(l_j) - S_n'(l_j)}{S_n(l_j)} \right]^2 \tag{22}$$

be minimized. In equation (22), n denotes the number of data points, $S_n(l_j)$ is the simulation value of S_n for the jth data point, and $S_n{}'(l_j)$ is the correlation value of S_n at l.

Since equation (6) is a nonlinear model, analytic regression is not possible, and it is necessary to resort to iterative methods. The numerical method used in this study was proposed and derived by Chen and Churchill [10]. Equation (6) is expanded into a Taylor series with higher terms truncated. Then the first derivatives of Φ, with respect to the variables a and m, are set to be zero. The resulting two equations are solved simultaneously to obtain the values of m and a_n. This iterative process repeats until m and a_n converge to as many significant figures as desired. The values of radiation parameters a and s can then be calculated from these converged values by

$$s = (a_n^2 - m^2)^{1/2} \tag{23}$$

$$a = a_n - s \tag{24}$$

The results of a and s for ϵ values of 0.32 and 0.4 are shown in Table 1. These values have a large discrepancy when compared with the values shown in [10]. But the predicted trend for the dependence of a and s on emissivity appears to agree well with that experiment. In general, the predicted absorption coefficient is much higher while the back-scattering cross section is lower. The sum of a and s for both cases, which represents the extinction coefficient of the system, centers around a constant value statistically. For D = 0.476 cm, and using the previous conclusion that $K \cdot D$ = 1.75, K can be calculated to be 3.67 cm^{-1}. This agreement verifies that the extinction coefficient in this kind of system is a function of the diameter but not of the emissivity of the constituent sphere.

Table 1 Effective absorption and back-scattering coefficients of a randomly packed bed of equal-diameter spheres

Emissivity	Absorption coefficient	Scattering coefficient	Extinction coefficient
	(cm^{-1})	(cm^{-1})	(cm^{-1})
0.32	1.325	2.477	3.80
0.40	1.847	2.073	3.92

The radiant conductivity defined by

$$k_r = -(i^+ - i^-)/(dT/dx) \tag{25}$$

can be calculated according to the following relationship derived by [10]

$$k_r = \frac{8\sigma T_o^3}{a+2s} \tag{26}$$

where σ is the Stefan-Boltzmann constant, and T_o represents the bulk temperature of the packed bed. From Rosseland's diffusion approximation [3], the radiant conductivity can be expressed as

$$k_r = \frac{16\sigma T_o^3}{3(a+s)} \tag{27}$$

Both of these models can be rewritten as

$$k_r = 4\bar{F}\sigma D T_o^3 \tag{28}$$

where $\bar{F} = 2/(a+2s)D$ for equation (26) and $\bar{F} = 4/3(a+s)D$ for equation (27).

Equation (28) shows the radiant conductivity dependence on particle size or transmissivity, bulk bed temperature, and effective thermal radiative properties. By substituting the obtained values of a and s, the $\bar{F}$ values for the above two models, when ϵ is equal to 0.40, are 0.70 and 0.71.

Discussion and Conclusion

The transmission curves for different emissivities are obtained by a Monte Carlo simulation of radiative transport through randomly packed beds. The results of transmission are correlated in terms of a two-flux model to obtain the effective absorption coefficient and scattering cross section. The radiant conductivity can also be calculated from these quantities either by a two-flux model or by Rosseland's diffusion model.

The transmission results are compared with a similar experiment [10]. Work by Chan and Tien [5] is also compared with the results of the present study. Reference [5] studied radiative transfer through simple cubic packing by determining the scattering diagram of a unit cell, the optical properties of a series of thin microsphere layers, and the solution of the two-flux equations. No numerical results on transmission data or effective radiative properties were reported by their study; only a discussion of the qualitative variation of K, a, and s with some system parameters was presented. The fact that the extinction coefficient is inversely proportional to the sphere diameter agrees well in all three investigations and with rough extimates based on the Mie theory of single scattering [5]. The dependence of a and s on emissivity also agrees well. The difference of the transmission results between this study and the experiment was discussed in the section titled, "Simulation of Radiant Energy Bundle Traveling Through a Randomly Packed Bed of Spheres." It is also pointed out in [5] that this discrepancy may be attributed to the fact that in the experimental system, due to the finite thickness of the packed beds, a considerable portion of energy may pass through the bed without encountering the absorption and scattering process. In the present simulation, the wavy wall boundaries eliminate the nonrandomness near the wall to resemble a totally random system, which in turn reduces the transmission of radiation through the bed. The a and s are direct results of the transmission data and two-flux model. The existence of a difference between [10] and the present work is expected.

The results of the present study lead to a better understanding of the radiative heat transfer mechanism in a randomly packed bed of spheres. The Monte Carlo simulation allows a much clearer picture of the physical phenomena involved.

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