Non-Paramatric Statistics Exercise 4

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Exercise 2.3

Implement the histogram rule $h_{D,s}$ in an algorithm that only uses $O(n)$ spaces, where n is the number of samples. Visualize the effect of different widths on the data sets of Exercise 2.2.

Solution:

Here we describe our implementation:

Input is the data set D of n samples, a given point as origin for cell generation (x_0, y_0) and the width of cells s.

Our algorithm identifies each cubic cell A with its center c_A and uses a dictionary to store c_A as keys and the respective histogram values of each cell (as values of the dictionary). This data structure enables storage complexity to stay within $O(n)$.

Step 1: For each cell A, the algorithm calculates $|\{i \in \mathbb{N} : x_i \in A\}|$.

It means, that for each point $d \in D$ our algorithm determines $A(x)$ by a simple calculation and sees whether $c_{A(x)}$ is already a key in the dictionary. If $c_{A(x)}$ already exists in the dictionary, the value of $c_{A(x)}$ will increase 1; else the key $c_{A(x)}$ will be created and receives the value 1.

Step 2: For each key $c_{A(x)}$ from the dictionary, the algorithm devides its value $|\{i \in \mathbb{N} : x_i \in A\}|$ by $n * s^2$, so that the histogram values $h_{D,s}$ are generated.

Step 3: The algorithm plots $c_{A(x)}$ as scatters and uses colours to represent different histogram values. The module *matplotlib.cm* is deployed for the colour scheme.

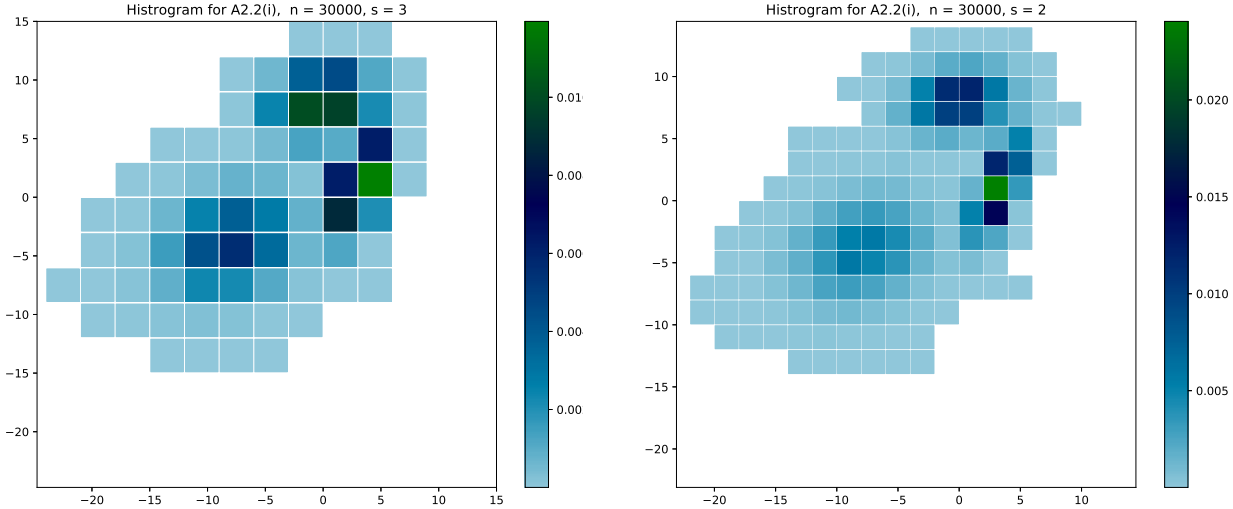
Now we present our graphical results for Exercise 2.2 i**):**

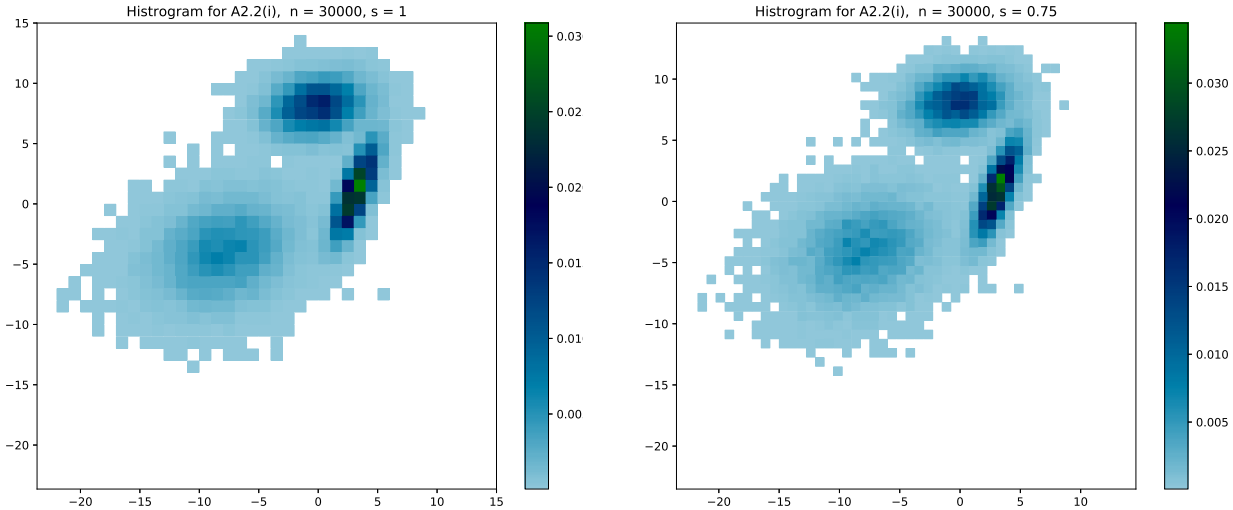
We draw 30,000 samples from the distribution **P** with

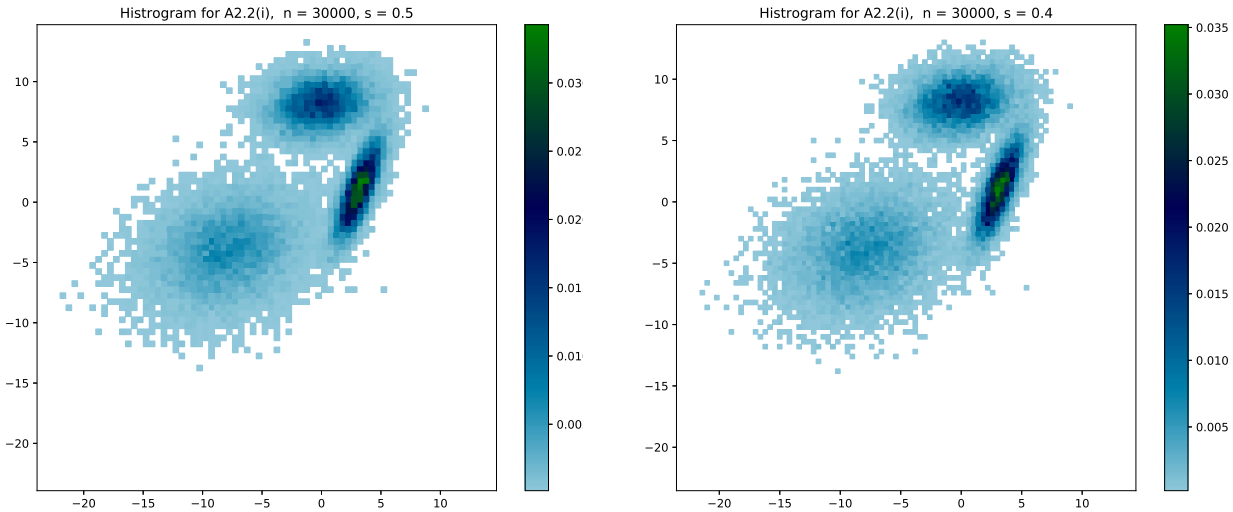
$$\mathbf{P} \sim \tfrac{1}{3}N(\left(\begin{smallmatrix}0\\3e\end{smallmatrix}\right), \left(\begin{smallmatrix}5 & 0.5\\0.5 & 2\end{smallmatrix}\right)) + \tfrac{1}{3}N(\left(\begin{smallmatrix}\pi\\1\end{smallmatrix}\right), \left(\begin{smallmatrix}ln3 & 1.8\\1.8 & 5\end{smallmatrix}\right)) + \tfrac{1}{3}N(\left(\begin{smallmatrix}-8\\-4\end{smallmatrix}\right), \left(\begin{smallmatrix}ln3+5ln10 & 2\\2 & 6.6666\end{smallmatrix}\right)).$$

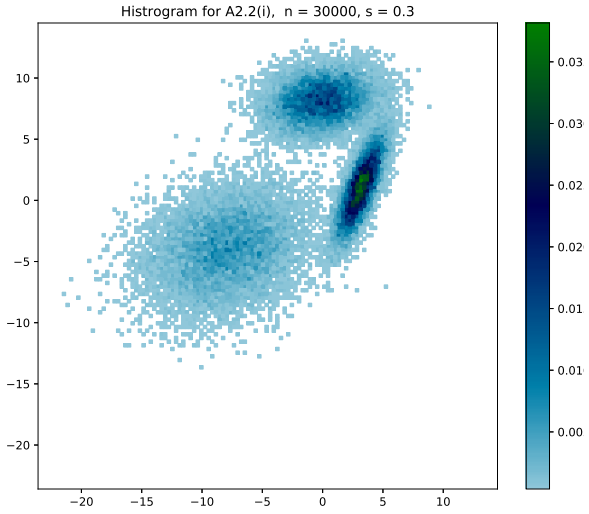
On the next page, we will finally show our results for Exercise 2.2 i).

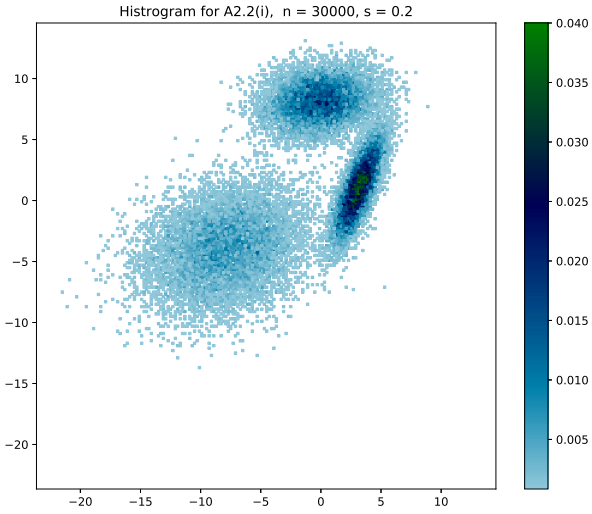
For different choice of s, we obtained following different histograms:

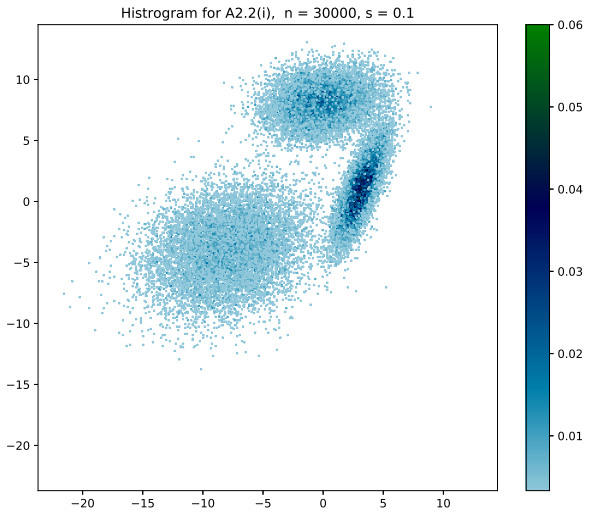


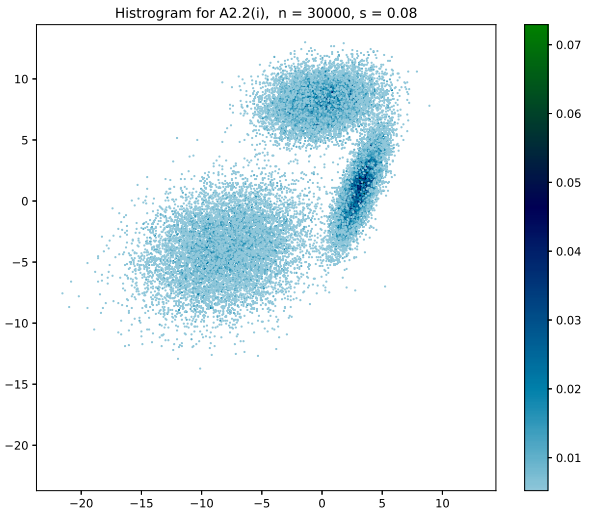




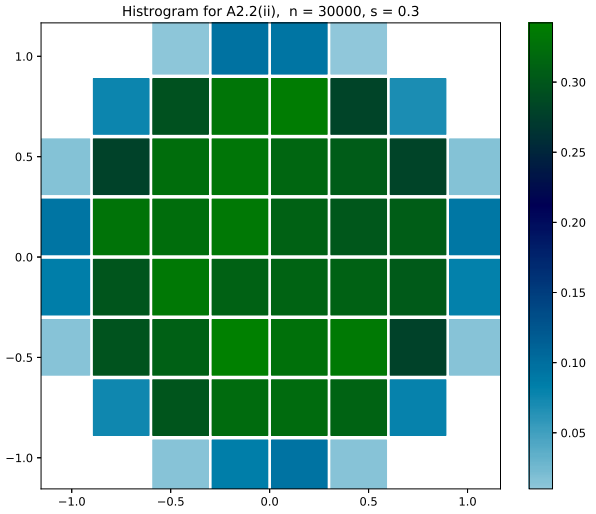

Histrogram for A2.2(i), n = 30000, s = 0.3

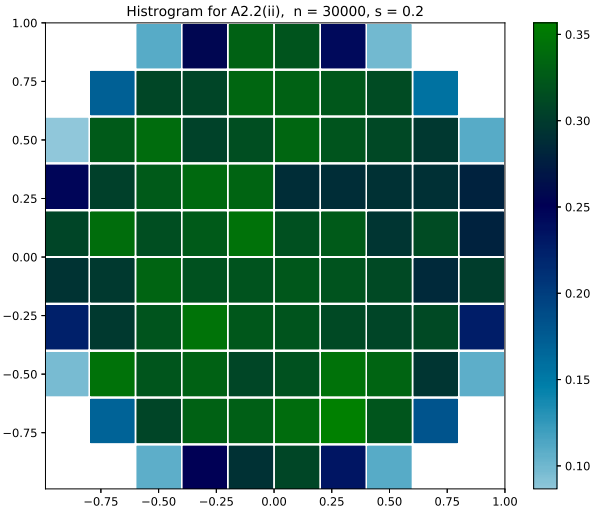

Histrogram for A2.2(i), n = 30000, s = 0.2

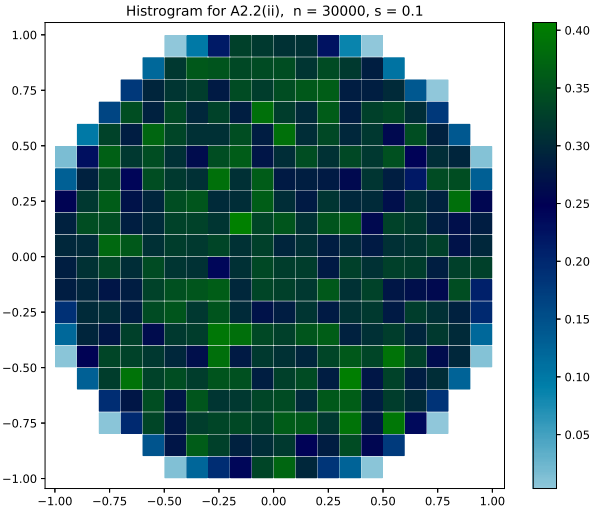

Histrogram for A2.2(i), n = 30000, s = 0.1

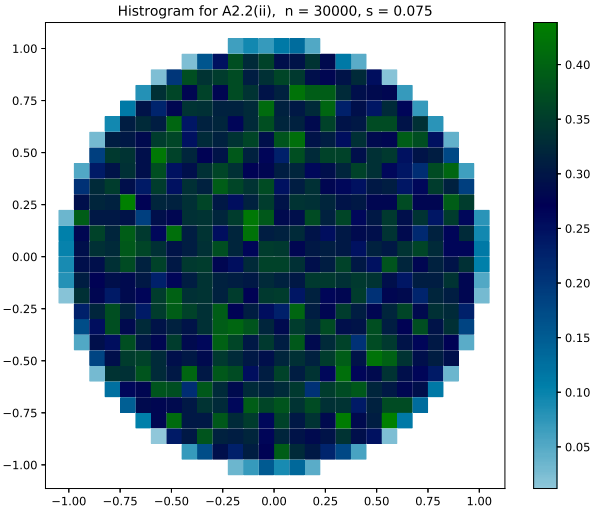

Histrogram for A2.2(i), n = 30000, s = 0.08

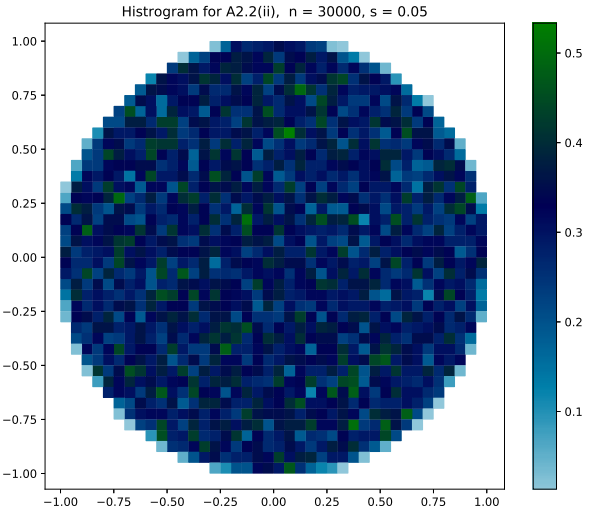
Also we present our graphical results for Exercise 2.2 ii**):**

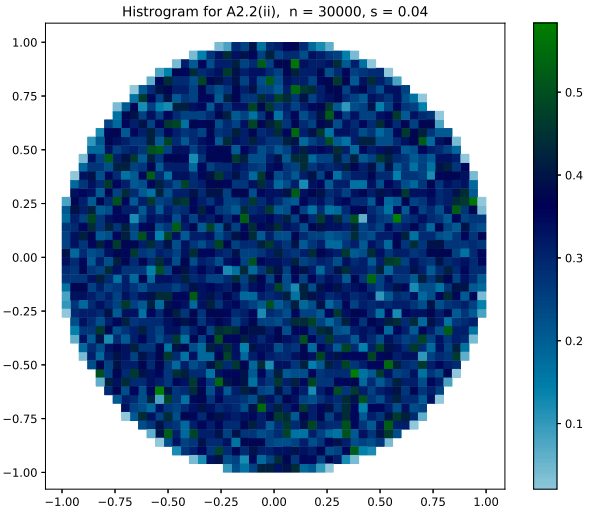


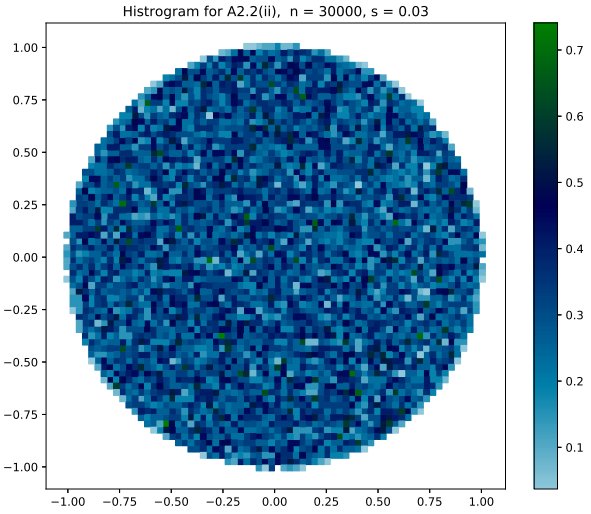


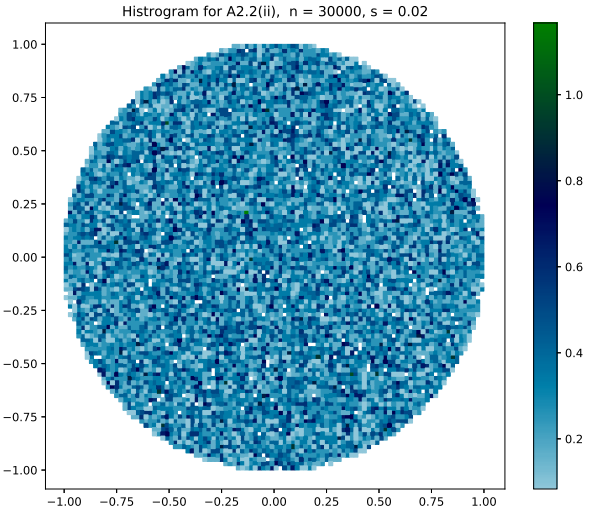


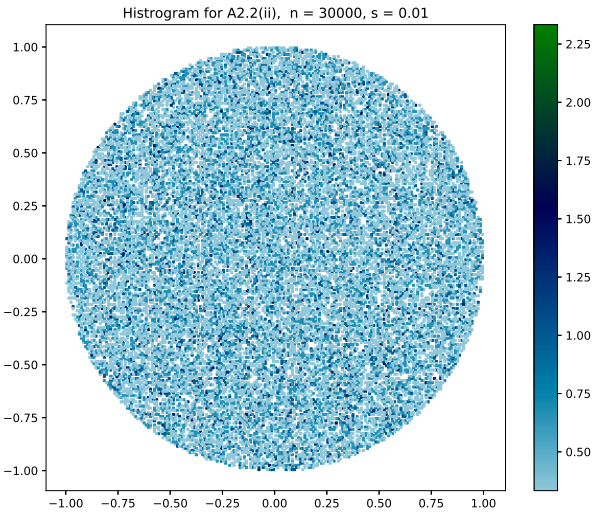


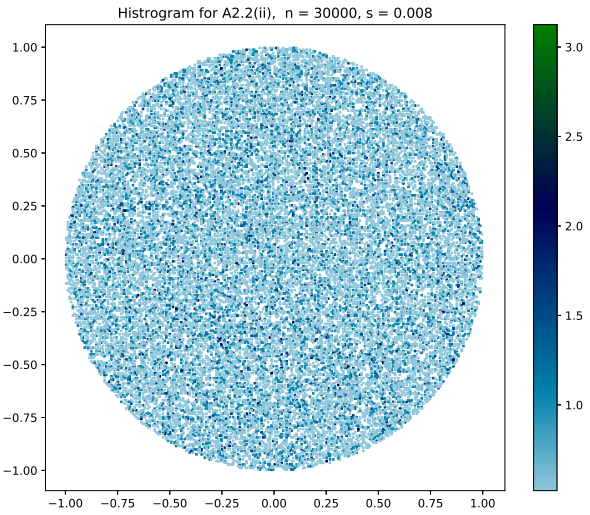





Histrogram for A2.2(ii), n = 30000, s = 0.03


Histrogram for A2.2(ii), n = 30000, s = 0.02


Histrogram for A2.2(ii), n = 30000, s = 0.01


Histrogram for A2.2(ii), n = 30000, s = 0.008