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# Questions one:

## Part A:

Barbara's mark in mathematics must be 1. This starts with the alphabetical grading where Alan has the highest mark (24), then Barbara, Charles, David, and Ellen. This tells us that Allan must have four 5's and one four due to knowing he must have a total mark of 24. We know that Ellen has a 5 in mathematics telling us that Alan must have a 4 in mathematics. Therefore, Barbara must have either a 3 or 2 or 1 in mathematics but at the same time the second highest overall grade. To narrow it down further we must use the alphabetical mark rule (where Charles must have a lower total grade than Barbara) as well as the rule that Charles must have 4 grades that are the same.

So far, the mark table looks like this: With Charles showing he must have x repeated 4 times with one y or one x and y repeated 4 times.

| | English | History | French | Mathematics | Science | Total |
|---|---|---|---|---|---|---|
| Alan | 5 | 5 | 5 | 4 | 5 | 24 |
| Barbara | | | | 3 or 2 or 1 | | a |
| Charles | X or y | X or y | X or y | X or y | Y or x | b (a > b) |
| David | | | | | | |
| Ellen | | | | 5 | 3 | |

By using these two last rules we can deduce that Barbara must have a 1 in mathematics. Say for the first scenario Barbara is given a 2 in Math and Charles a 3 in math. Charles would have to repeat this score 3 more times in English, History, and French or repeat the mark he got in Science in these same three classes (due to him having the same grade in 4 subjects). Due to the grades already in Science Charles can either have a 4, 2, 1. If Charles has a 4 here and repeated the 4 in 3 other classes, his total mark would be greater than Barbara's therefore that is not an option. If Charles took a 4 and repeated his potential 3 in mathematics 3 more times this would result in a total grade of 16. This would cause Barbara to have a 4 in English, History, and French and then a 2 in Mathematics and Science to compete with Charles grade. But this would only result in a total mark of 16 therefore not an option since this equals Charles. If Charles takes a 1 in Science and repeats his grade of 3 in Mathematics across his other classes this obeys the Hierarchy rule but forces David and Ellen to have the same total marks, if they are given better grades then the hierarchy structure for Alan, Barbara, and Charles will fail therefore this is not an option. Looks like this:

| | English | History | French | Mathematics | Science | Total | |
|---|---|---|---|---|---|---|---|
| Alan | 5 | 5 | 5 | 4 | 5 | 24 | |
| Barbara | 4 | 4 | 4 | 2 | 2 | x | 16 |
| Charles | 3 | 3 | 3 | 3 | 1 | y (x > y) | 13 |
| David | 2 | 2 | 2 | 1 | 4 | | 11 |
| Ellen | 1 | 1 | 1 | 5 | 3 | | 11 |

Now the other options is to give Charles a 2 in Science and a 3 in all his other subjects giving him a total of 15. This would allow Barbara to have a 4 in all her subjects but Math where she would have a 2 giving her a total of 18. But this cannot also work because it forces Ellen to have a greater total mark than David which is not allowed. Even with giving David the greatest marks possible while maintaining Alan's, Barbara's, and Charles hierarchy system Ellen still has a greater total mark than

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David.
This looks like this:

| | English | History | French | Mathematics | Science | Total |
|---|---|---|---|---|---|---|
| Alan | 5 | 5 | 5 | 4 | 5 | 24 |
| Barbara | 2 | 4 | 4 | 2 | 4 | 16 |
| Charles | 3 | 3 | 3 | 3 | 2 | 15 |
| David | 4 | 2 | 2 | 1 | 1 | 10 |
| Ellen | 1 | 1 | 1 | 5 | 3 | 11 |

Now if we give Barbara a 3 in Mathematics and Charles a 2, we run into similar problems where all the different arrangements will break the total mark hierarchy system. More simply Because Ellen has lowest possible score of 11 given her known grade of 3 in Science and 5 in Math, 5 + 8 + 3(1) = 11. The three ones coming from giving her the lowest grade in her remaining subjects. So therefore, David needs a grade of at least 12. Now with Charles having a 2 in math he must either repeat this or repeat the given grade in Science, this leads to the problem where to make Charles have a higher total grade than David and Ellen he must have a 4 in Science (since 3 is unavailable) and repeat this in all classes but Math but this will force a grade higher than Barbara's therefore not an option. But if Charles repeats his 2 from math 4 times and to make him possible competitive with David and Ellen he must take a 4 in one of his subjects. But this only leads to a total mark of 12 meaning that Charles, David, and Ellen can't obey the mark hierarchy therefore not an option.

Therefore, the only viable options that obey all the rules can exist with Barbara having a 1.

Therefore, Barbara has a mark of 1 in Mathematics.

## Part B:

Knowing that the correct answer must be with Barbara having a 1 mathematics, through trial and error this table forms:

| | English | History | French | Mathematics | Science | Total |
|---|---|---|---|---|---|---|
| Alan | 5 | 5 | 5 | 4 | 5 | 24 |
| Barbara | 4 | 4 | 4 | 1 | 2 | 15 |
| Charles | 3 | 3 | 3 | 3 | 1 | 13 |
| David | 2 | 2 | 2 | 2 | 4 | 12 |
| Ellen | 1 | 1 | 1 | 5 | 3 | 11 |

This is because given that the grades for Charles and David are the only unknown grades in Math and either a 2 or 3 is left, the hierarchy rule can be used. If Charles is given a 2 and David a 3 in Math then the highest-grade Charles can get without having a higher grade than Barbara is a 12 which then breaks the hierarchy rule for David and Ellen (2 in four classes and a 4 in one gives Charles a 12). But if Charles is given a 3 and a 2 to David then this allows Charles, Charles can't have a 4 In Science because then Barbara cannot have a higher grade than him since the next highest available is a 2 and with giving Barbara the highest mark possible for her she only gets 15 with Charles having a 16. Now if Charles is given a 2 in Science this will leave him with a total mark of 14 meaning Barbara has to take a 4 in Science to have a higher mark than him. This forces David to take a 1 in Science leaving no possible combination that allows David to have a higher grade than Ellen, only combinations that give Ellen and David the same total mark. Therefore, the only correct table lies

with Charles having a one in Science and then David taking the next highest mark in Science to have a greater grade than Ellen (this being a 4) leaving a 2 for Barbara to take. This leaves the rest of the table to be filled out as the one showed above.

This shows that Alan, Charles, and David obtained the same mark in 4 of their classes.

# Question 2:

| Team | Played | Won | Lost | Drawn | Goals for | Goals against | Points |
|---|---|---|---|---|---|---|---|
| A | | | | | | 1 | 4 |
| B | 1 | | | | | | |
| C | | | | | 5 | 0 | 6 |
| D | | | | | | 4 | |
| E | 4 | | | 2 | | 2 | 2 |

## Part 1:

We know team E has drawn twice and has two points and played 4 games therefore they have 2 losses and 0 wins.

Since E has played 4, each team must of played at least once against E therefore each team has at least one played game and we know B has only played on game therefore there one game was against E.

Team E must of lost both its games 1-0 to have 2 losses and 2 goals against and 2 draws of 0-0.

We know team B has only played team E because team E has played all their games and B has only played one, therefore any goals scored by B must be against E. Team E has only 2 goals against and has two losses therefore B could have only scored a max of 1 goal against E too allow E to have 2 losses. So, if B scored 1 or 0 goals this makes the goals against total at 8 or 7 respectively. Since there can only be as many goals for as goals against that tells us that there can be only 8 or 7 goals for in total. Now team C have scored 5. Also based on the above fact that team E must have 2 losses of 1 goal to 0 to fit their 2 goals against criteria and two draws of 0-0 this tells us that they must have 0 goals for because any goals for to get a draw would result in more goals against which is not allowed. This tells us that team C has played team D since if we give all of team C's goals for to the teams other than team D to fill their goals against requirements, team C will have goals for left to allocate and the only team left is team D. We know this because team B must have 0 goals against because they have only played against team E and team E have 0 goals for as previously proven. Therefore, there have been 7 goals against in total and 7 goals for in total.

At this point the table worked off above proven rules so far looks like this:

| Team | Played | Won | Lost | Drawn | Goals for | Goals against | Points |
|---|---|---|---|---|---|---|---|
| A | 1<E> | | | | | 1 | 4 |
| B | 1 = E | | | | 0 or 1 | 0 | |
| C | 1<E> | | | | 5 | 0 | 6 |
| D | 1<E> | | | | | 4 | |
| E | 4 | 0 | 2 | 2 | 0 | 2 | 2 |
| | | | | | 7 | 7 | |

1<E> means that this team has played at least one game and that team is team E.
1 = E means that this team has played one game and that team was against team E.

## Part 2:

Since we know there are 7 goals for and that team E must have two loses of 1-0, either 1 of those goals against for team E to lose came from team C and another from either team A, B, or D or two of teams A, B, or D scored one goal each against team E too give team E their two losses. But knowing that team C has 6 points means that they had to score against team E. This is because to have 6 points you need either 3 wins or 2 wins and 2 draws. If team E drew against team C then team C would have to draw against another team and beat two other teams but since team B has only played against team E, team C don't have enough available games to get 6 points with two draws and 2 wins therefore they must have obtained 6 points through 3 wins therefore they must have beat team E. Therefore, team C must have beaten team, A, D, and E. Considering we know that team A has only had one goal against and team C could have only scored one goal against team E we now know that team C must have scored the remainder of their goals against team D. This being 5 – (1+1) = 3.

Therefore, team C beat team D 3-0.