Lecture 10 Homework by Aj. Suppawong:

Ex	Size X	Color X	Shape X	Class (Y) Y
1	small	red	circle	positive
2	large	red	circle	positive
3	small	red	triangle	negative
4	large	blue	circle	negative

attributes

class

$$\hat{P}(x_i \mid c_j) = \frac{N(X_i = x_i, C = c_j) + 1}{N(C = c_j) + k}$$

# of values of X_i → N of attributes

Ex : $P(\text{small} \mid Y) = \frac{1+1}{2+3} = \frac{2}{5} = 0.4$

จำนวนของ Small ที่เป็น Pos / จำนวนของ Pos ทั้งหมด

X มีสามอัน

Calculate P(Y) and P(X_i|Y) with smoothing for conditional probability.

Probability (Smoothed)	Y= positive	Y= negative
P(*Y*)	2/4 = 0.5 มี Pos 2 จาก 4	2/4 = 0.5
P(small | *Y*)	(1+1)/(2+3) = 2/5 = 0.4	(1+1)/(2+3) = 2/5 = 0.4
P(medium | *Y*)	(0+1)/(2+3) = 1/5 = 0.2	(0+1)/(2+3) = 1/5 = 0.2
P(large | *Y*)	(1+1)/(2+3) = 2/5 = 0.4	(1+1)/(2+3) = 2/5 = 0.4
P(red | *Y*)	(2+1)/(2+3) = 3/5 = 0.6	(1+1)/(2+3) = 2/5 = 0.4
P(blue | *Y*)	(0+1)/(2+3) = 1/5 = 0.2	(1+1)/(2+3) = 2/5 = 0.4
P(green | *Y*)	(0+1)/(2+3) = 1/5 = 0.2	(0+1)/(2+3) = 1/5 = 0.2
P(square | *Y*)	(0+1)/(2+3) = 1/5 = 0.2	(0+1)/(2+3) = 1/5 = 0.2
P(triangle | *Y*)	(0+1)/(2+3) = 1/5 = 0.2	(1+1)/(2+3) = 2/5 = 0.4
P(circle | *Y*)	(2+1)/(2+3) = 3/5 = 0.6	(1+1)/(2+3) = 2/5 = 0.4

จะทำนายมาให้

Given x = <medium ,red, circle>, calculate P(Y=positive|x) and P(Y=negative|x)

P(+|x) = P(medium|+) · P(red|+) · P(circle|+) · P(+)/P(x)

= 0.2 · 0.6 · 0.6 · 0.5 / P(x)

= 0.036 / P(x)

P(-|x) = P(medium|-) x P(red|-) x P(circle|-) x P(-)/P(x)

= 0.2 · 0.4 · 0.4 · 0.5 / P(x)

= 0.016 / P(x)

P(+|x) + P(-|x) = 1 ➔ 0.036+0.016 = P(x) ➔ P(x) = 0.052 $= \frac{0.036}{P(x)} + \frac{0.016}{P(x)} = 1$

$\frac{0.052}{P(x)} = 1$

$P(x) = 0.052$

∴ P(+|x) = 0.036/0.052 = 0.69
P(-|x) = 0.016/0.052 = 0.31

เทียบค่า Pos กับ Neg มากกว่าตอบ

Which class should x be classified into?

⇨ Positive

Probability Estimation Example

Ex	Size	Color	Shape	Class
1	small	red	circle	positive
2	large	red	circle	positive
3	small	red	triangle	negative
4	large	blue	circle	negative

Probability	Y= positive	Y= negative
$P(Y)$	0.5	0.5
$P(\text{small} \mid Y)$	0.5	0.5
$P(\text{medium} \mid Y)$	0.0	0.0
$P(\text{large} \mid Y)$	0.5	0.5
$P(\text{red} \mid Y)$	1.0	0.5
$P(\text{blue} \mid Y)$	0.0	0.5
$P(\text{green} \mid Y)$	0.0	0.0
$P(\text{square} \mid Y)$	0.0	0.0
$P(\text{triangle} \mid Y)$	0.0	0.5
$P(\text{circle} \mid Y)$	1.0	0.5

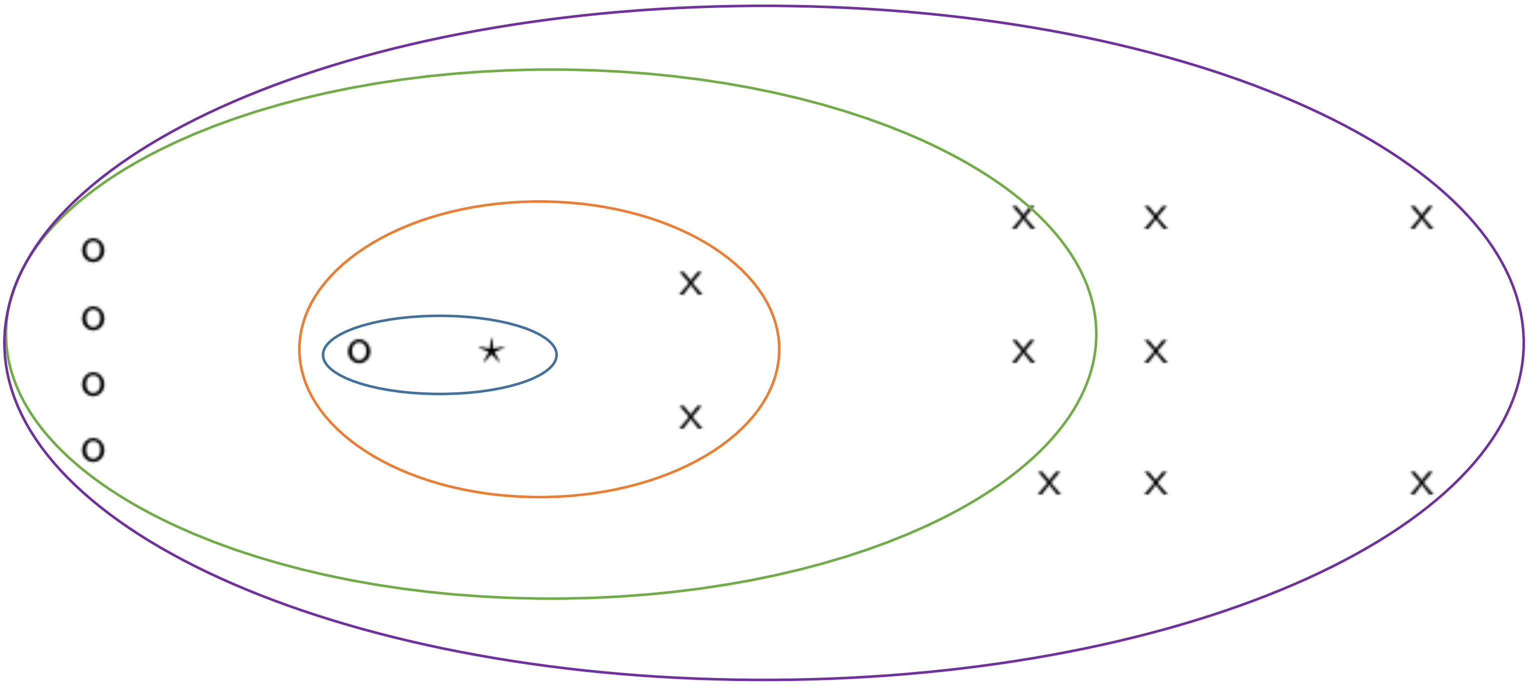
* No smoothing has been applied yet.

จะไม่เอาสูตรนี้มาคิด

$$\hat{P}(x_i \mid c_j) = \frac{N(X_i = x_i, C = c_j) + 1}{N(C = c_j) + \underbrace{k}_{\#\text{ of values of } X_i}}$$

Ex : $P(\text{small} \mid Y) = \dfrac{1}{2} = \dfrac{1}{2} = 0.5$

$\dfrac{\text{จำนวนของ small ที่เป็น POS}}{\text{จำนวนของ POS ทั้งหมด}}$



How is star classified by:

(i) 1-NN → O
(ii) 3-NN → X
(iii) 9-NN → O
(iv) 15-NN → X

Lecture 11 Homework Solution by Aj. Suppawong

Data

Subject	X	Y
1	1.0	1.0
2	1.5	2.0
3	3.0	4.0
4	5.0	7.0
5	3.5	5.0
6	4.5	5.0
7	3.5	4.5

Initial Centroids

Group	Subjects	Centroid
A	1	(1.0, 1.0)
B	4	(5.0, 7.0)

With the settings above, run K-Means until it converges (cluster members do not changes). After each iteration, list the members of each group and the new centroids. In case of tie, choose group A. Each iteration includes the assignment and recomputation steps.

$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ ค่าน้อยสุด

X_1 X_2

	Iteration 0	Iteration 1			Iteration 2			Iteration 3		
Point	Data table	Dist uA	Dist uB	Assigned u	Dist uA	Dist uB	Assigned u	Dist uA	Dist uB	Assigned u
1	(1,1)	0.00	7.21	A	1.57	5.38	A	0.56	5.02	A
2	(1.5,2)	1.12	6.10	A	0.47	4.28	A	0.56	3.92	A
3	(3,4)	3.61	3.61	A (tie)	2.03	1.78	B	3.05	1.42	B
4	(5,7)	7.21	0.00	B	5.64	1.85	B	6.66	2.20	B
5	(3.5,5)	4.72	2.50	B	3.14	0.73	B	4.16	0.41	B
6	(4.5,5)	5.32	2.06	B	3.77	0.53	B	4.78	0.61	B
7	(3.5,4.5)	4.30	2.92	B	2.73	1.08	B	3.75	0.72	B
Centroids		$\frac{1+1.5+3}{3}$	$\frac{1+2+4}{3}$							
uA	(1,1)	1.83	2.33		1.25	1.50		1.25	1.50	
uB	(5,7)	4.13	5.38		3.90	5.10		3.90	5.10	

เอาตาราง Data ใส่ สูตร = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

	Iteration 0	Iteration 1			Iteration 2			Iteration 3		
Point		Dist uA	Dist uB	Assigned u	Dist uA	Dist uB	Assigned u	Dist uA	Dist uB	Assigned u
1	(1,1)	0.00	7.21	A	1.57	5.38	A	0.56	5.02	A
2	(1.5,2)	1.12	6.10	A	0.47	4.28	A	0.56	3.92	A
3	(3,4)	3.61	3.61	A (tie)	2.03	1.78	B	3.05	1.42	B
4	(5,7)	7.21	0.00	B	5.64	1.85	B	6.66	2.20	B
5	(3.5,5)	4.72	2.50	B	3.14	0.73	B	4.16	0.41	B
6	(4.5,5)	5.32	2.06	B	3.77	0.53	B	4.78	0.61	B
7	(3.5,4.5)	4.30	2.92	B	2.73	1.08	B	3.75	0.72	B
เอา Inti Centroids ใส่										
Centroids										
uA	(1,1)	1.83	2.33		1.25	1.50		1.25	1.50	
uB	(5,7)	4.13	5.38		3.90	5.10		3.90	5.10	

Iteration 1

P_1 $U_A = \sqrt{(1-1)^2+(1-1)^2} = 0.00$ P_1 $U_B = \sqrt{(1-5)^2+(1-7)^2} = 7.21$ เลือกค่าน้อยใส่ Assigned U

ทำไปเรื่อยๆ จนครบ จากนั้นดูจำนวนแต่ละตัวของ A และ B มาคิดใน Centroids แบบนี้

นับ A ได้ 3 ในช่องของ UA = X ของตัวที่ได้ค่า Assigned U รวมกันหาร N

$U_A = \frac{1+1.5+3}{3} = 1.83$ $U_A = \frac{1+2+4}{3} = 2.33$

นับ B ได้ 4 $\frac{5+3.5+4.5+3.5}{4} = 4.13$ $\frac{7+5+5+4.5}{4} = 5.38$

Iteration 2 ทำเหมือนครั้งแรกแต่ x_1, y_1 เป็น Centroids ของ Iteration 1

P_1 $U_A = \sqrt{(1-1.83)^2+(1-2.33)^2} = 1.57$ P_1 $U_B = \sqrt{(1-4.13)^2+(1-5.38)^2} = 5.38$ เลือกค่าน้อยใส่ Assigned U

ทำไปเรื่อยๆ จนครบ จากนั้นดูจำนวนแต่ละตัวของ A และ B มาคิดใน Centroids แบบนี้

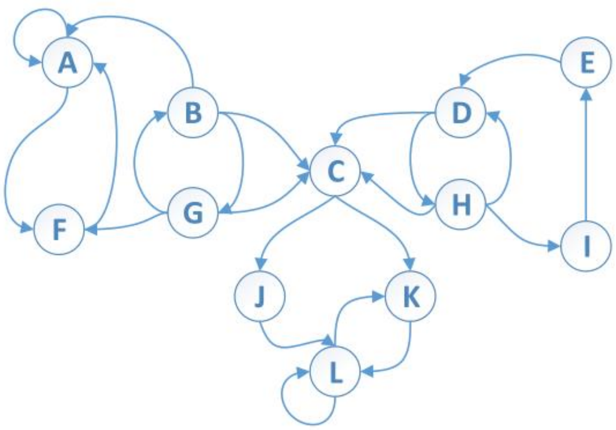
นับ A ได้ 2 ในช่องของ UA = X ของตัวที่ได้ค่า Assigned U รวมกันหาร N

$U_A = \frac{1+1.5}{2} = 1.25$ $U_A = \frac{1+2}{2} = 1.5$

นับ B ได้ 5 $\frac{3+5+3.5+4.5+3.5}{5} = 3.90$ $\frac{4+7+5+5+4.5}{5} = 5.10$

Lecture 12 Homework Solution by Aj. Suppawong

Assume that we are working with the following graph with 12 nodes.



- Initially, each node is assigned a PageRank score of $PR_0 = \frac{1}{12} \approx 0.083333$. Run PageRank for **2** iterations. *At the end* of each iteration, report the PageRank score of each page. The PageRank score is calculated using the following equation:

$$PR(p) = \sum_{v \in B_p} \frac{PR(v)}{N_v}$$

- Where p is a page, B_p is the set of p's backlinks, N_v is the number of v's out-going links.

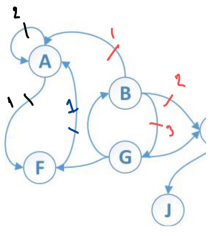
Page		Iteration 0 (Initialization)	Iteration 1	Iteration 2
PageRank Score	A	0.083333	0.15	0.16
	B	0.083333	0.03	0.01
	C	0.083333	0.13	0.09
	D	0.083333	0.11	0.10
	E	0.083333	0.08	0.03
	F	0.083333	0.07	0.09
	G	0.083333	0.03	0.01
	H	0.083333	0.04	0.06
	I	0.083333	0.03	0.01
	J	0.083333	0.04	0.06
	K	0.083333	0.08	0.17
	L	0.083333	0.21	0.23

Iteration 1

(A) ดูว่าอะไร Point ถึง A บ้าง $\left\langle \begin{matrix} A \\ B \\ F \end{matrix} \right\}$ ดูว่าตัวพวกนี้ Point ออกจากตัวเองกี่เส้น

ตามสูตร

$$\boxed{PR_{t+1}(P_i) = \sum_{P_j} \frac{PR_t(P_j)}{C(P_j)}}$$



$$= \frac{0.0833}{\underset{\underset{A}{\downarrow}}{2}} + \frac{0.0833}{\underset{\underset{B}{\downarrow}}{3}} + \frac{0.0833}{\underset{\underset{F}{\downarrow}}{1}} = 0.15 \ \#$$

Iteration 2

(A) ทำเหมือนเดิม แต่เอาค่าจาก iteration 1

$$= \frac{0.15}{\underset{\underset{A}{\downarrow}}{2}} + \frac{0.03}{\underset{\underset{B}{\downarrow}}{3}} + \frac{0.07}{\underset{\underset{F}{\downarrow}}{1}} = 0.155 \approx 0.16 \ \#$$

$$\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

Lec13: LSI Homework

1. Find the Eigenvalues of the given matrix. Order the Eigenvalues from largest to smallest when answering.

$$\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} \overset{a}{1-\lambda} & \overset{b}{4} \\ \overset{c}{2} & \overset{d}{3-\lambda} \end{bmatrix} = 0$$

$ad - bc$

$(1-\lambda)(3-\lambda) - 4 \times 2 = 0$

$: 3 - x - 3x + x^2 - \cancel{4 \times 2}\ 8$

$: -5 - x - 3x + x^2$

$: -5 - 4x + x^2$

$: x^2 - 4x - 5$

$(x+1)(x-5)$

$: [-5, 1]$

Ans: 5, -1

2. Find the Eigenvalues of the given matrix. Order the Eigenvalues from largest to smallest when answering.

$$\begin{bmatrix} 7.5148 & 0 & 0 \\ 0 & 5.3215 & 0 \\ 0 & 0 & 1.9921 \end{bmatrix}$$

Ans: 7.5148, 5.3215, 1.9921

3. The given matrix A can be decomposed into $A = U\Sigma V^T$, using Singular Value Decomposition. Compute Σ, where Σ has the same dimension as A, containing singular values ordered in the descending order.

$$A = \begin{bmatrix} 1 & 4 & 2 \\ 2 & 3 & 1 \end{bmatrix}$$

Ans:

$$\begin{bmatrix} 5.8205 & 0 & 0 \\ 0 & 1.0591 & 0 \end{bmatrix}$$

Solving for Eigenvalues and Eigenvectors

$$A - \lambda I = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = \begin{bmatrix} 1-\lambda & 1 \\ 4 & 1-\lambda \end{bmatrix}$$

$$\begin{vmatrix} 1-\lambda & 1 \\ 4 & 1-\lambda \end{vmatrix} = 0$$

set det = 0

$$(1-\lambda)(1-\lambda) - (4)(1) = 0$$
$$1 - 2\lambda + \lambda^2 - 4 = 0$$
$$\lambda^2 - 2\lambda - 3 = 0$$
$$(\lambda - 3)(\lambda + 1) = 0$$

Eigenvalue — $\lambda = 3, \lambda = -1$

Solving for Eigenvalues and Eigenvectors

$A = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix}$ $\quad \lambda = 3$, $\lambda = -1$ $\quad (A - \lambda I)\vec{x} = \vec{0}$

$$A - 3I = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 4 & -2 \end{bmatrix} \xrightarrow{R_2 + 2R_1} \begin{bmatrix} -2 & 1 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$-2x_1 + x_2 = 0$

$x_2 = 2$

choose $x_1 = 1$

making $x_2 = 2$

$$\vec{x} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

Solving for Eigenvalues and Eigenvectors

$A = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix}$ $\quad \lambda = 3$, $\lambda = -1$ $\quad (A - \lambda I)\vec{x} = \vec{0}$

$$A - (-1)I = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix} \longrightarrow \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$2x_1 + x_2 = 0$

$x_2 = -2\cancel{x_1}\ 1$

choose $x_1 = 1$

$$\vec{x} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$|A| = \begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

$$\det(A) = a * \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b * \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c * \begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

Finally det(A) is: $\det(A)$: $aei - afh - bdi + bfg + cdh - ceg$

$$\det(A) = aei - afh - bdi + bfg + cdh - ceg$$

Solving for Eigenvalues and Eigenvectors

$$A - \lambda I = \begin{bmatrix} 1-\lambda & 0 & 0 \\ 3 & -2-\lambda & 0 \\ 2 & 3 & 4-\lambda \end{bmatrix}$$

$$\begin{vmatrix} 1-\lambda & 0 & 0 \\ 3 & -2-\lambda & 0 \\ 2 & 3 & 4-\lambda \end{vmatrix}$$

นำ det จากข้างบน

$$= (1-\lambda)[(-2-\lambda)(4-\lambda) - (0)(3)] - 0 + 0$$

ใช้สูตร

$$(1-\lambda)(-2-\lambda)(4-\lambda) = 0$$

$\lambda = 1 \qquad \lambda = -2 \qquad \lambda = 4$

Solving for Eigenvalues and Eigenvectors

$A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & -2 & 0 \\ 2 & 3 & 4 \end{bmatrix}$ $\quad \lambda = 1$, $\lambda = -2$, $\lambda = 4$ $\quad (A - \lambda I)\vec{x} = \vec{0}$

$$A - (1)I = \begin{bmatrix} 1 & 0 & 0 \\ 3 & -2 & 0 \\ 2 & 3 & 4 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 3 & -3 & 0 \\ 2 & 3 & 3 \end{bmatrix}$$

แถวสอง

$3x_1 - 3x_2 = 0$

choose $x_2 = 1$

$x_1 = x_2 = 1$

แทนในสมการ

แถวสาม

$2\cancel{x_1} + 3\cancel{x_2} + 3x_3 = 0$ (1, 1)

$x_3 = -\frac{5}{3}$

$$\vec{x} = \begin{bmatrix} 1 \\ 1 \\ -\frac{5}{3} \end{bmatrix}$$

Solving for Eigenvalues and Eigenvectors

$A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & -2 & 0 \\ 2 & 3 & 4 \end{bmatrix}$ $\quad \lambda = 1$, $\lambda = -2$, $\lambda = 4$ $\quad (A - \lambda I)\vec{x} = \vec{0}$

$$A - (-2)I = \begin{bmatrix} 1 & 0 & 0 \\ 3 & -2 & 0 \\ 2 & 3 & 4 \end{bmatrix} - \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 3 & 0 & 0 \\ 2 & 3 & 6 \end{bmatrix}$$

$x_1 = 0$ $\qquad 2\cancel{x_1} + 3x_2 + 6x_3 = 0$ (0)

$x_1 = 0$

$3x_1 - 0x_2 - 0x_3 = 0$

$3x_2 + 6x_3 = 0$ $\qquad x_3 = 1$

$x_2 = -2x_3$ $\qquad x_2 = -2$

$x_2 = -2(1)$

$$\vec{x} = \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix}$$

Solving for Eigenvalues and Eigenvectors

$A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & -2 & 0 \\ 2 & 3 & 4 \end{bmatrix}$ $\quad \lambda = 1$, $\lambda = -2$, $\lambda = 4$ $\quad (A - \lambda I)\vec{x} = \vec{0}$

$$A - (4)I = \begin{bmatrix} 1 & 0 & 0 \\ 3 & -2 & 0 \\ 2 & 3 & 4 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} = \begin{bmatrix} -3 & 0 & 0 \\ 3 & -6 & 0 \\ 2 & 3 & 0 \end{bmatrix}$$

$3x_1 = 0$

$3\cancel{x_1} - 6x_2 = 0$ (0)

$x_1 = 0$ $\qquad 2\cancel{x_1} + 3\cancel{x_2} = 0$ (0, 0)

$x_2 = 0$

$x_3 \rightarrow$ free variable

$$\vec{x} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$