Learning Bayesian Networks

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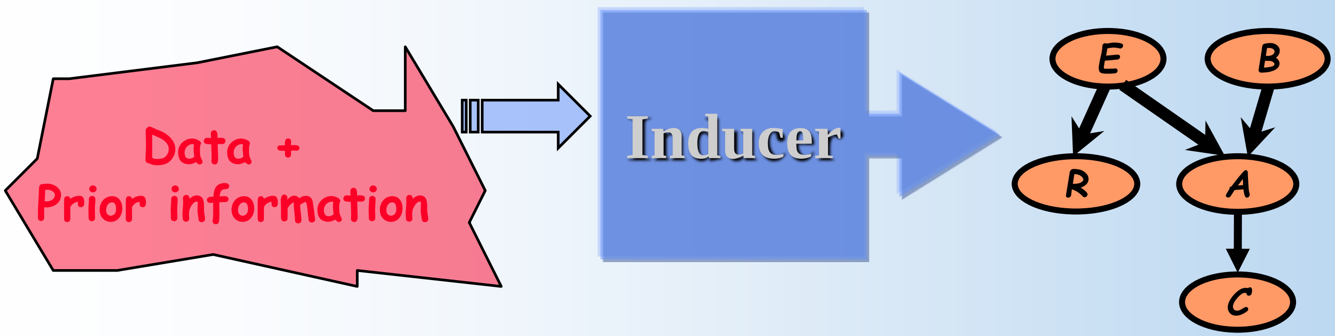
Some material adopted from slides by Nir Friedman, Jack Breese, and Daphne Koller.

Learning Bayesian networks



E	B	P(A | E,B)	
e	b	.9	.1
e	$\bar{b}$	.7	.3
$\bar{e}$	b	.8	.2
$\bar{e}$	$\bar{b}$	.99	.01

The Learning Problem

	Known Structure	**Unknown Structure**
Complete Data	Statistical parametric estimation (closed-form eq.)	Discrete optimization over structures (discrete search)
Incomplete Data	Parametric optimization (EM, gradient descent...)	Combined (Structural EM, mixture models…)

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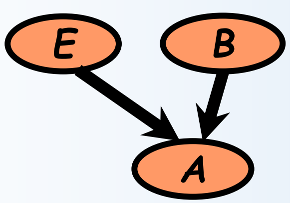
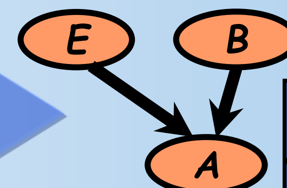
$\bar{e}$	b	?	?
$\bar{e}$	$\bar{b}$	?	?

E, B, A
<Y,N,N>
<Y,Y,Y>
<N,N,Y>
<N,Y,Y>
.
.
<N,Y,Y>





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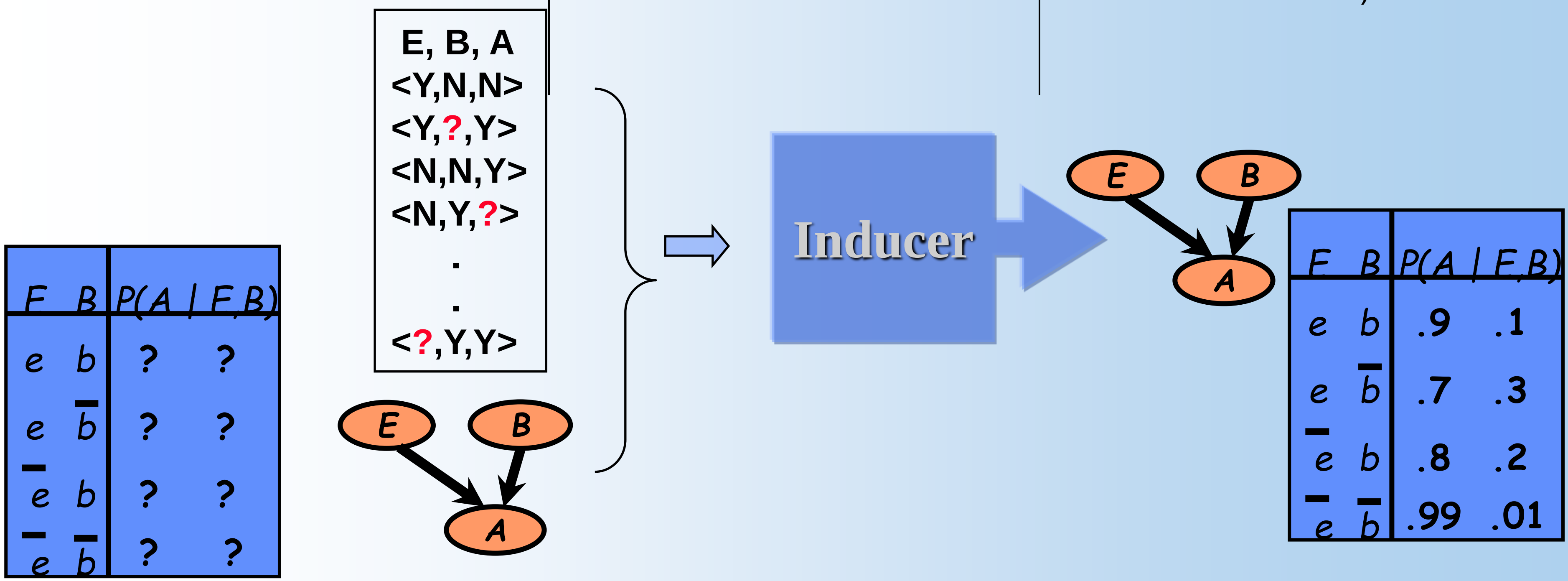

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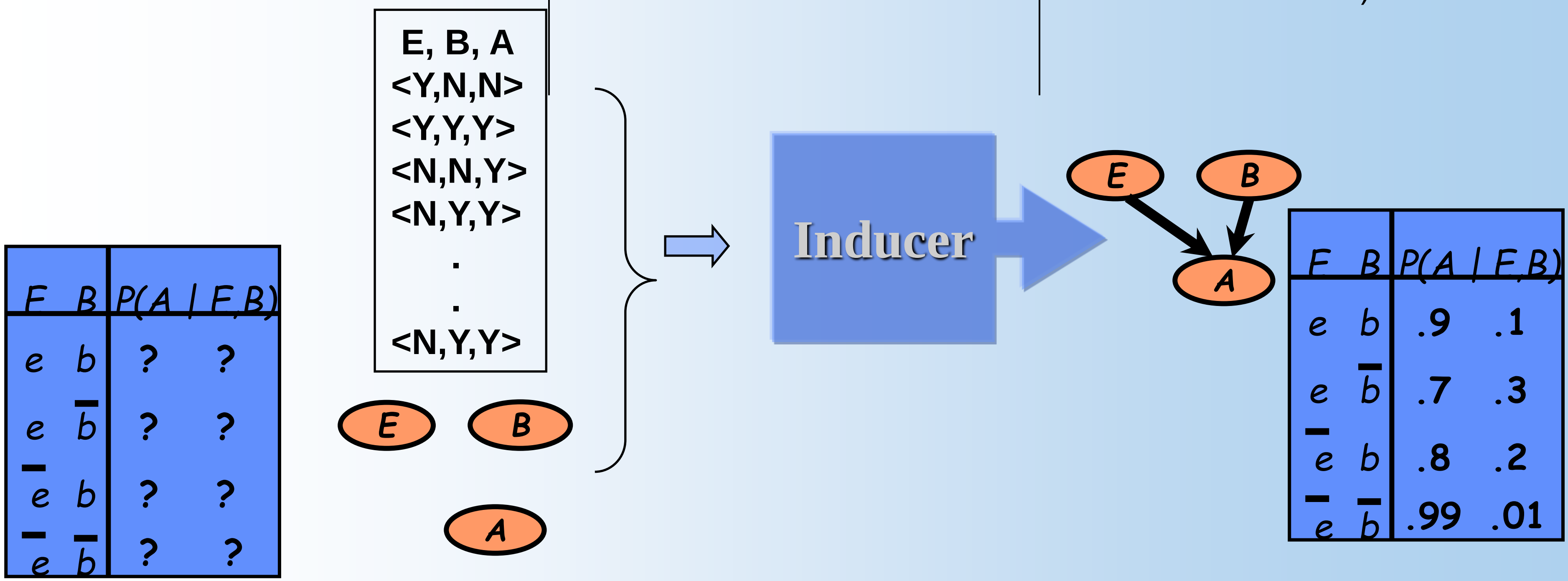

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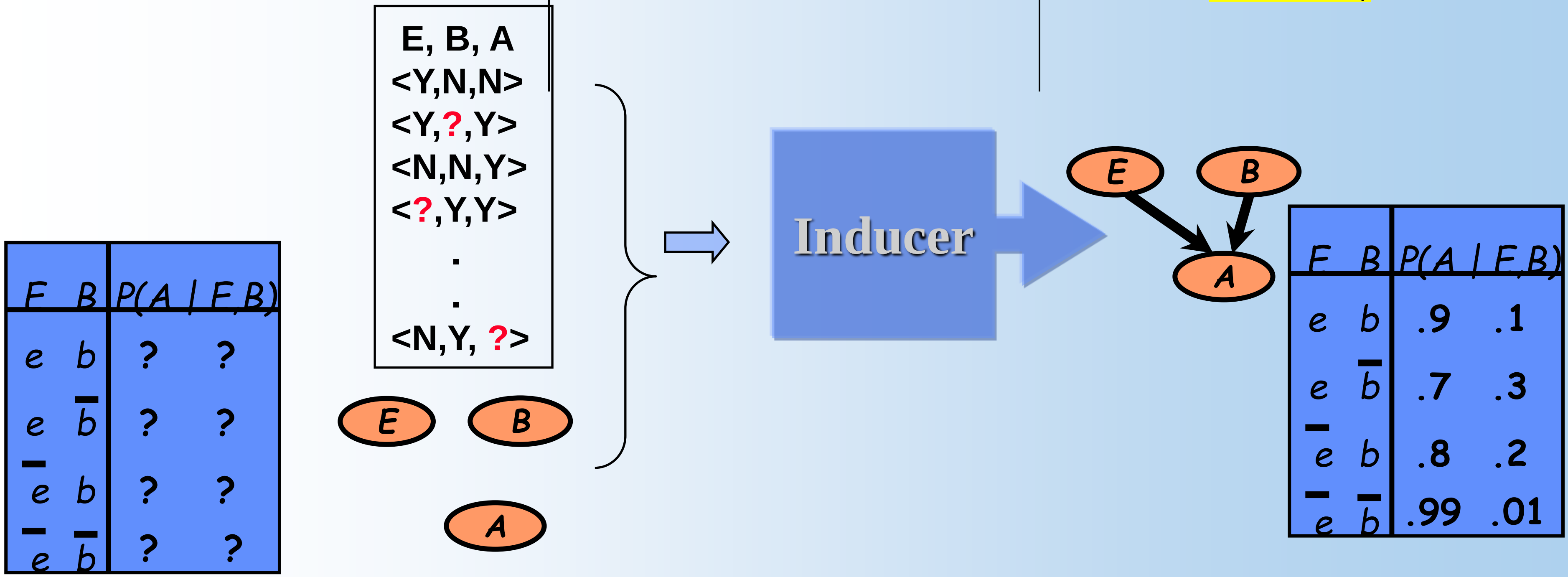
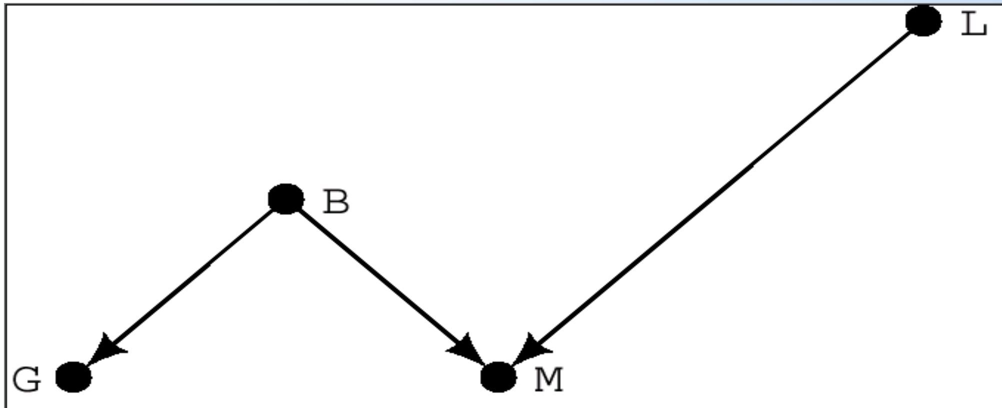


A Network and Training Data



G	M	B	L	Number of instances
True	*True*	*True*	*True*	54
True	*True*	*True*	*False*	1
True	*False*	*True*	*True*	7
True	*False*	*True*	*False*	27
False	*True*	*True*	*True*	3
False	*False*	*True*	*False*	2
False	*False*	*False*	*True*	4
False	*False*	*False*	*False*	2
				100

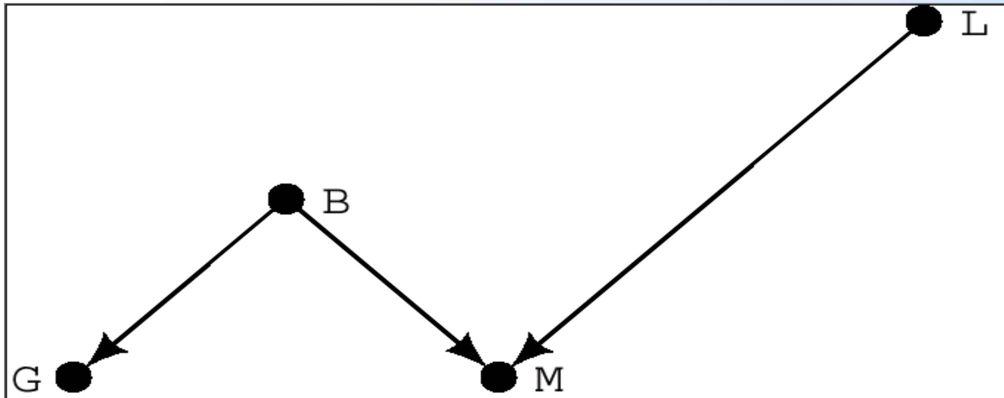
No Missing Data

- If we have an ample number of training samples, we have only to compute sample statistics for each node and its parents.
- CPT for some node *V* given its parents *Pa*(*V*)
  - The sample statistics for *V* and Pa(V):

$$\hat{p}(V = v_i \mid \mathrm{Pa} = \mathrm{p}_j) = \frac{n(V = v_i \wedge Pa = p_j)}{n(Pa = p_j)}$$

  - Given by the number of samples in D having $V = v_i$ and Pa(V)=p_j divided by the number of samples having Pa(V)=p_j

An Example for No Missing Data



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				100

$$\hat{p}(B = True) = 0.94$$

$$\hat{p}(L = True) = 0.68$$

$$\hat{p}(M = True \mid B = True, L = False)$$

$$= \frac{1}{30} = 0.03$$

Laplace Smoothing

It is often useful to be able to combine expert opinion with data, particularly when data is scarce. This can be done if we can assign a virtual sample size to the expert's opinion.

$$\hat{P}(A_j = a_{jk} \mid C = c_i) = \frac{n_c + mp}{n + m}$$

n_c : number of training examples for which $A_j = a_{jk}$ and $\mathrm{C} = c_i$

n : number of training examples for which $C = c_i$

p : prior estimate (usually, $p = 1/t$ for t possible values of A_j)

m : weight to prior (number of "virtual" examples, $m \geq 1$)

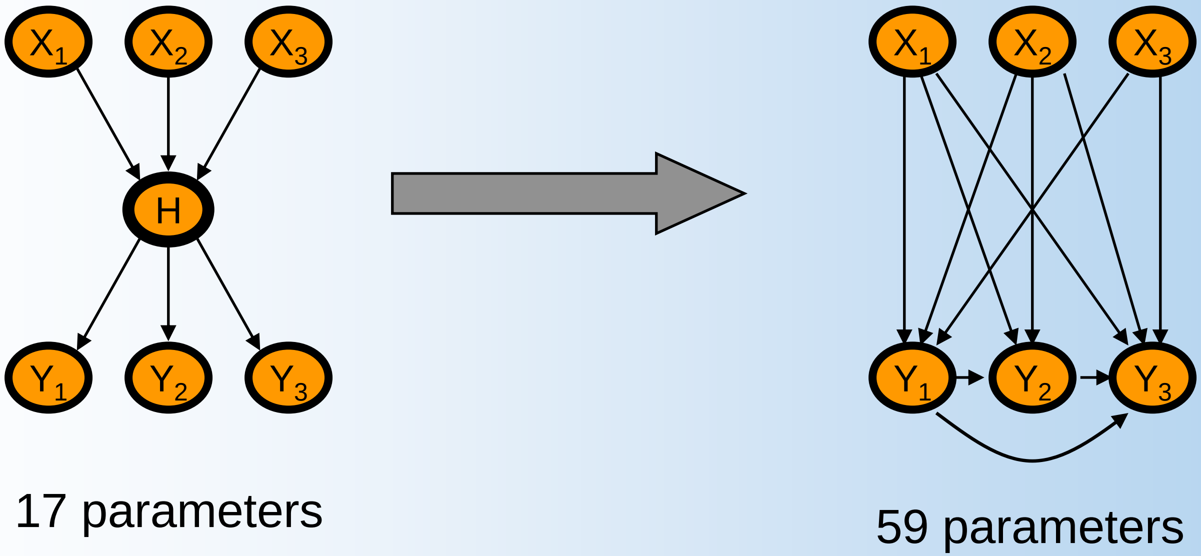
Incomplete Data

- Data is often **incomplete**
  - Some variables of interest are not assigned value

- This phenomena may happen when we have
  - Missing values
  - Hidden variables

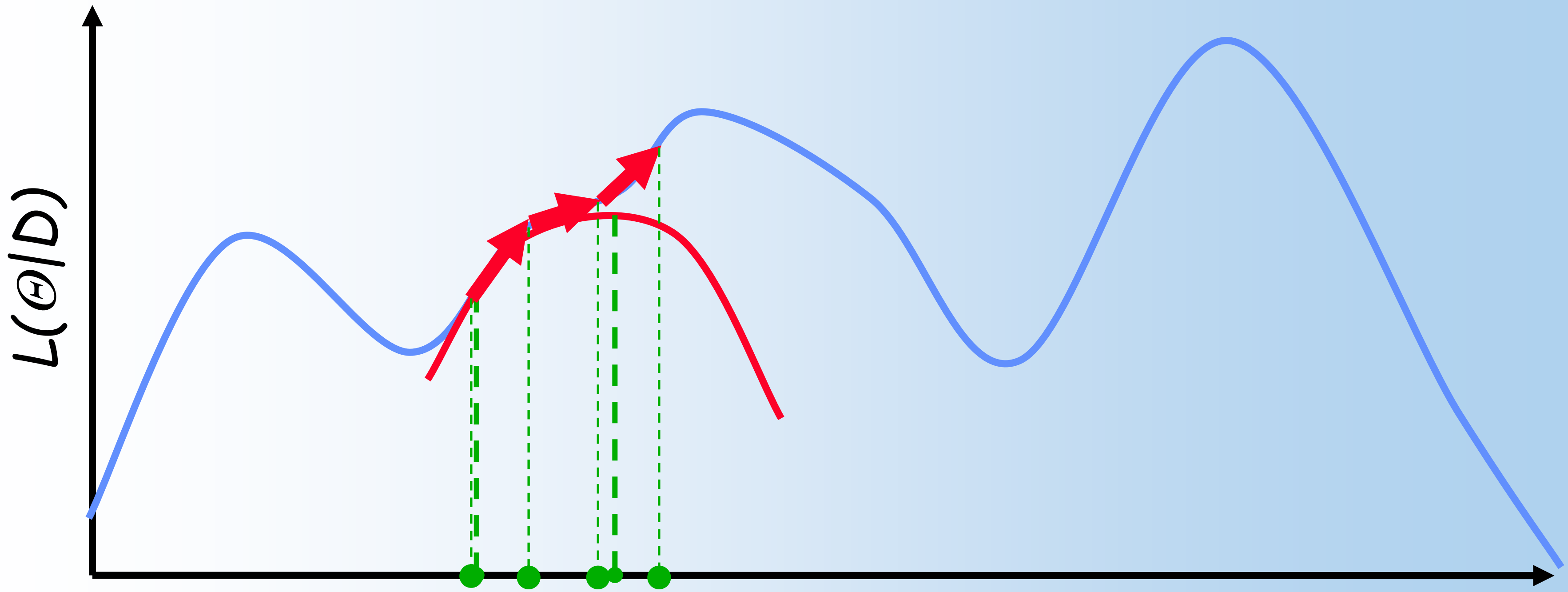
Hidden (Latent) Variables

- Attempt to learn a model with variables we never observe
- Why should we care about unobserved variables?
  - Limited data
  - Overfitting



MLE from Incomplete Data

- Finding MLE parameters: **nonlinear optimization** problem



Gradient Ascent:

Follow gradient of likelihood w.r.t. to parameters

Find local maxima

Require multiple restarts to find approx. to the global maximum

Expectation Maximization (EM):

Use "current point" to construct alternative function (which is "nice")

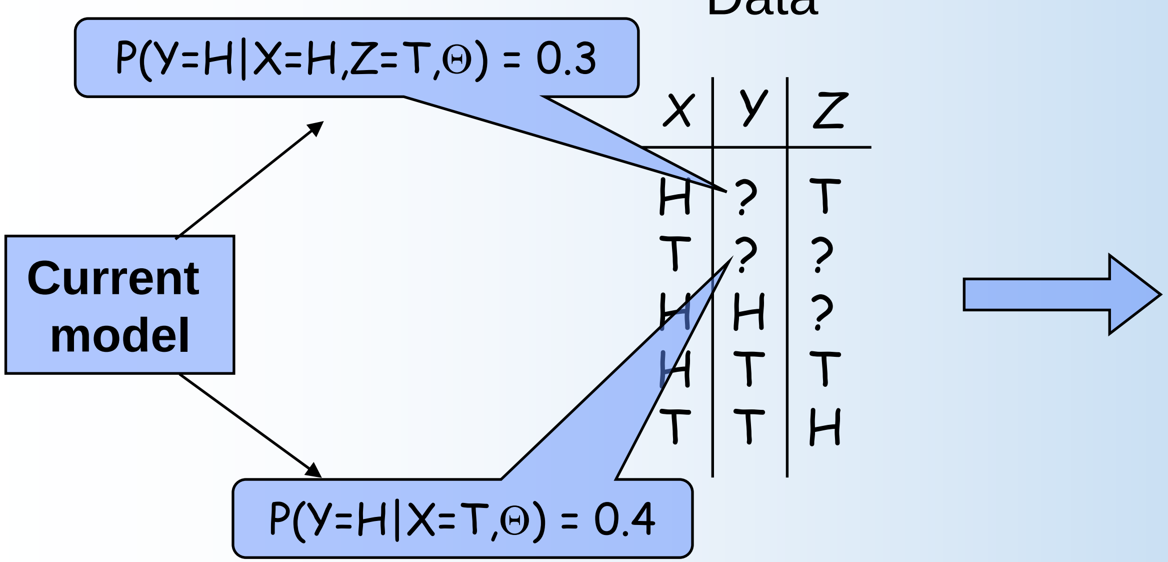
Guaranty: maximum of new function is better scoring than current point

Expectation Maximization (EM)

- A general purpose method for learning from incomplete data

Intuition:

- If we had access to counts, then we can estimate parameters
- However, missing values do not allow to perform counts
- “Complete” counts using current parameter assignment



Data

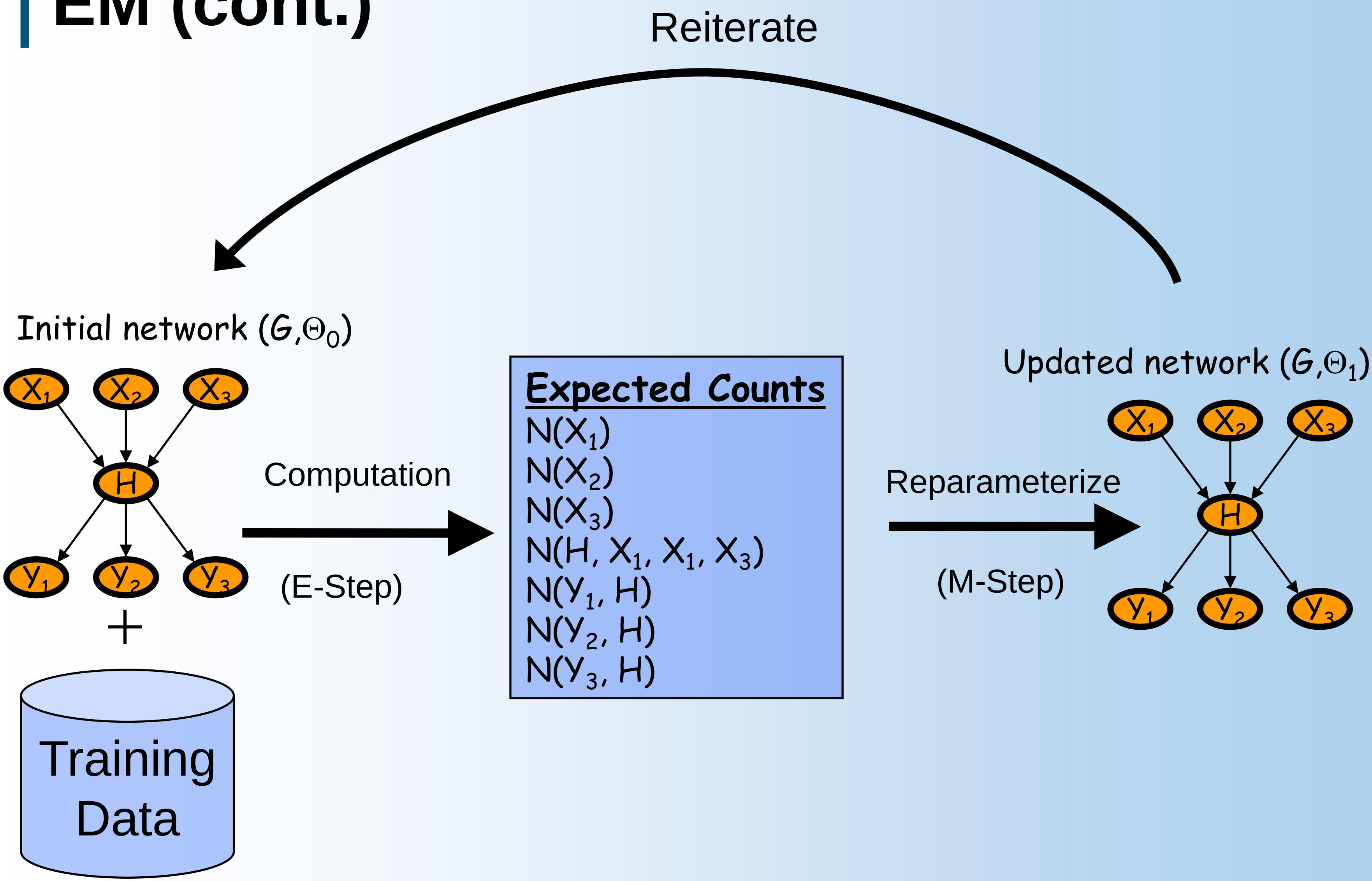
X	Y	Z
H	?	T
T	?	?
H	H	?
H	T	T
T	T	H

Expected Counts

N (X,Y)

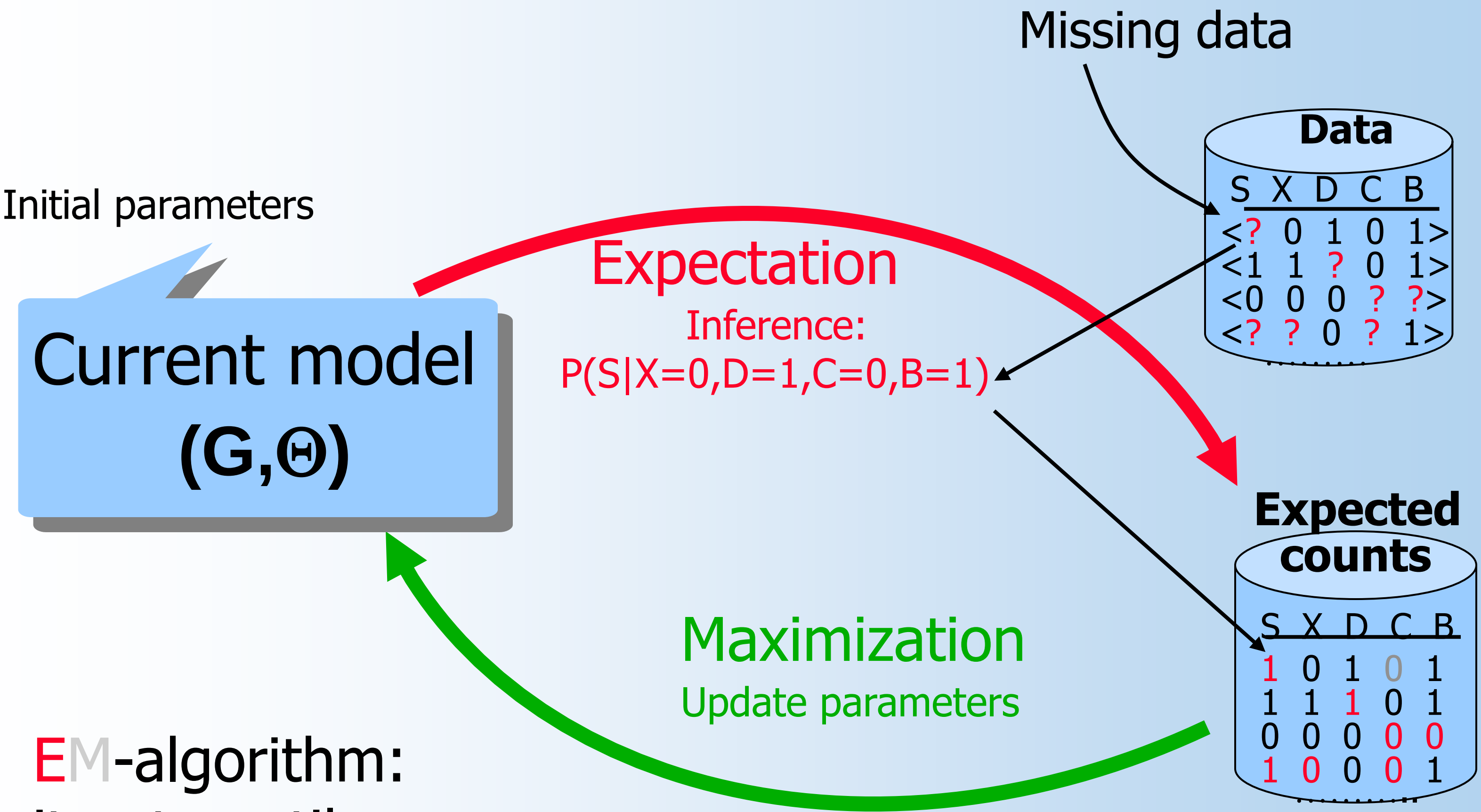
X	Y	#
H	H	1.3
T	H	0.4
H	T	1.7
T	T	1.6

EM (cont.)



Reiterate
Initial network (G,Θ_0)
X_1 X_2 X_3
H
Y_1 Y_2 Y_3
+
Training Data
Computation
(E-Step)
Expected Counts
$N(X_1)$
$N(X_2)$
$N(X_3)$
$N(H, X_1, X_1, X_3)$
$N(Y_1, H)$
$N(Y_2, H)$
$N(Y_3, H)$
Reparameterize
(M-Step)
Updated network (G,Θ_1)
X_1 X_2 X_3
H
Y_1 Y_2 Y_3

EM (cont.)



EM-algorithm:
iterate until convergence

The EM Algorithm for Bayes Nets

- First, random values are selected for the parameters in the CPTs for the entire network.
- Secondly, the needed weights are computed by using the Bayes net.
- Thirdly, these weights are in turn used to estimate new CPTs.
- Then, the second step and the third step are iterated until the CPTs converge.

EM in Practice

Initial parameters:
- Random parameter settings
- “Best” guess from other source

Stopping criteria:
- Small change in likelihood of data
- Small change in parameter values

Avoiding bad local maxima:
- Multiple restarts

Difficulties:
- More missing data $\Rightarrow$ many more local maxima
- Many hidden variables $\Rightarrow$ can result in over fitting the data