Aircraft Sizing Project

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The aircraft sizing process will be used to analyze the request for proposal for the 2020 aircraft sizing requirements stated in the AIAA undergraduate competition. The method will be described below and result in an aircraft with a wing area greater then 200 ft^2, an aspect ratio larger then 6, empty weight of at least 7100 lbs, fuel weight of at least 3800 lbs, and a minimum take off weight of 14800 lbs.

I. Nomenclature

R	=	Range [nmi]
E	=	Endurance [hr]
η_p	=	Propeller Efficiency
C_{bhp}	=	Power Specific Fuel Consumption
$\frac{W}{S}$	=	Wing Loading [$\frac{lb}{ft^2}$]
hp	=	Engine Power [hp]
σ	=	Desnsity Ratio at Altitude
AR	=	Aspect Ratio
e	=	Wing Span Efficiency
M	=	Mach Number
q	=	Dynamic Pressure [$\frac{lb}{ft^2}$]
C_L	=	Coefficient of Lift
C_D	=	Coefficient of Drag
S_{wet}	=	Wetted Area [ft^2]
S_{ref}, S	=	Wing Area [ft^2]

II. Introduction

A. Requirements Review

The requirements for the mission are listed within the request for proposal (RFP), some requirements are optional while others are required. The main ones that will effect this design process are the mission requirement, the ceiling requirement (30,000 ft), the runway requirement (4,000 ft), and the payload requirement (6,000 lbs).

B. Mission Review

There are two missions that the aircraft should be able to complete, a design mission that would involve combat, and a ferry mission which a lighter payload and a longer distance.

Table 1 Sections of Design Mission

Section	Mission	Description	$\frac{w_i}{w_{i-1}}$
1	Warm Up / Taxi	5 Minutes	0.97
2	Take Off	Austere field, 50 ft obstacle, ≤ 4,000 ft	
3	Climb	To cruise altitude, ≥ 10,000 ft; with range credit	0.98
4	Cruise	100 nmi	$\exp \frac{-RC_{bhp}}{550\eta_p {}^L/_D}$
5	Descent	To 3,000 ft; no range credit; completed within 20 minutes of the initial climb	0.995
6	Loiter	On station, four hours, no stores drops	$\exp \frac{-EC_{bhp}V}{550\eta_p {}^L/_D}$
7	Climb	To cruise altitude, ≥ 10,000 ft; with range credit	0.98
8	Cruise	100 nmi	$\exp \frac{-RC_{bhp}}{550\eta_p {}^L/_D}$
9	Descent / Landing	To austere field over 50 ft obstacle in ≤ 4000 ft	0.995
10	Taxi / Shutdown	5 minutes	
11	Reserves	Sufficient for climb to 3,000 ft and loiter for 45 minutes	$\exp \frac{-EC_{bhp}V}{550\eta_p {}^L/_D}$

Table 2 Sections of Ferry Mission

Section	Mission	Description	$\frac{w_i}{w_{i-1}}$
1	Warm Up / Taxi	5 Minutes	0.97
2	Take Off	Austere field, 50 ft obstacle, ≤ 4,000 ft	
3	Climb	To cruise altitude, ≥ 10,000 ft; with range credit	0.98
4	Cruise	900 nmi	$\exp \frac{-RC_{bhp}}{550\eta_p {}^L/_D}$
5	Descent / Landing	To austere field over 50 ft obstacle in ≤ 4000 ft	0.995
6	Taxi / Shutdown	5 minutes	
7	Reserves	Sufficient for climb to 3,000 ft and loiter for 45 minutes	$\exp \frac{-EC_{bhp}V}{550\eta_p {}^L/_D}$

III. Similarity Analysis

A similarity analysis was conducted in order to seed what aircraft in this field looked like while also seeking potential candidates for a seed aircraft. The search for the aircraft started broad with looking at a wide range of attack aircraft and then once some general parameters were collected, top level design choices could be made. A table with these vehicles are shown below, this data was pulled form Doc8643, a document used by air traffic service with basic aircraft dimensions[1].

Table 3 Similarity Analysis of Attack Aircraft

Plane Name	Range [nmi]	TOFL [ft]	LFL [ft]	AR	Wingspan [ft]	Power [hp]	Wing Area [ft^2]	Empty Weight [lbs]
Embraer Super Tucano	720	1804	2821	6.67	37.33	1,604	209	7,055
Beechcraft T-6 Texan II	900	2165	3379	6.29	33.42	1,100	177.5	4,707
Douglas A-1 Skyraider	1,144	1969	1969	6.25	50.02	2,700	400.33	11,968
Fairchild Republic A-10	2,240	3937	2001	6.53	57.50	9065*	506	24,959
Republic P-47 Thunderbolt	900	886	1378	5.54	40.78	2,000	300	10,000

Using table 3 of aircraft data while keeping in mind the specific requirements of this craft some patterns can be found such as most aircraft in this field use propellers meaning that a turboprop will also most likely be the best suited for the RFP requirements. Using this comparison the two seed aircraft that will be used for further analysis are the Embraer Super Tucano and Beechcraft T-6 Texan II.

IV. Constraint Diagram

A constraint diagram is typically constrained by 3 curves, take off, landing, and cruise. Each section of the mission has it's own function that will plot it's constraint on a wing loading to specific power graph. The relations can be puled from various equations within Raymer [2].

$$\frac{W}{S}_{TO} = \text{TOP}\sigma C_{L_{TO}}\frac{hp}{W} \qquad \text{Raymer 5.8} \tag{1}$$

$$S_{\text{land}} = 80\frac{W}{S}\frac{1}{\sigma C_{L_{max}}} + S_a \qquad \text{Raymer 5.11} \tag{2}$$

$$\frac{hp}{W}_{TO} = \frac{v_{\text{cruise}}}{550\eta_p}\frac{1}{L/D}\frac{W_{\text{cruise}}}{W_{TO}}\frac{hp_{TO}}{hp_{\text{cruise}}} \qquad \text{Raymer 5.4} \tag{3}$$

For these equations parameters must be either assumed or calculated in order to numerically obtain these relations. Starting with the take off equation 1, the take off parameter or TOP can be estimated by using an empirical chart relating take off distance and TOP. The sigma for both take off and landing are given as the highest rated altitude the vehicle is supposed to land at which in this case is 6000 ft. The $C_{L_{TO}}$ and a $C_{l_{max}}$ can be assumed to be 1.45 and 1.6 respectively but will be further analyzed later. Next for the landing equation 2, S_a is an empirical approach parameter that varies between 450-1000 ft but for GA aircraft is about 600. Finally for the cruise constraint equation 3 there are a lot of parameters that must found. Starting with the cruise velocity because the RFP requirements say that 100 nmi must be covered in 20 minutes the velocity must be at least 300 knots. Next the propeller efficiency it typically $\eta_p = 0.8$, the weight ratio is estimated at 0.875 and the power lapse at cruise is estimated to be 0.704 by assuming cruise at 10,000 ft and a linear power lapse curve is related to σ.

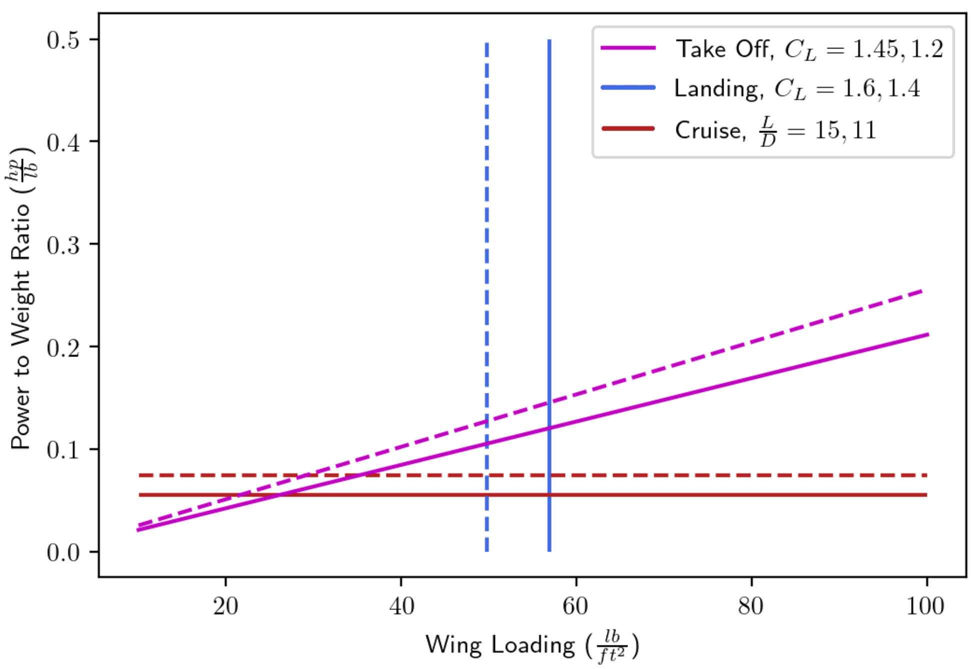

Fig. 1 Constraint Diagram

Using this diagram for each parameter there are two estimated values, the solid line correlates to the first value in the legend and the dashed line correlates to the second value in the legend. Using this diagram it can be seen that the take off and cruise constraints have a lot less change then the landing constraint does. This means that the coefficient of lift for landing should be further investigated in order to get a better answer.

V. Dimensional Diagrams

A dimensional diagram is a useful tool where you have n variables you constrain and then you optimize the weight of the aircraft to attempt to satisfy n requirements. Depending on the seed aircraft, estimation equations, and requirement difficulty the requirement may not be able to be satisfied. For the given requirements none seem to unreasonable looking at the constraint diagrams. For this analysis the aspect ratio and wing area will be changed and the weight will be used to satisfy the take off requirement and the mission requirement.

A. Estimation

There are lots of parameters that must be estimated in the analysis, the more detail that has went into the estimate the more that parameter has a change on the final answer. Starting with the estimation for calculations on the seed aircraft. The areas of the wing and stabilizer can be estimated graphically and the wetted area can be estimated using the side profile area and top profile area using the equation below.

$$S_{wet} \approx 3.4 \left(\frac{A_{TOP} + A_{SIDE}}{2} \right) \qquad \text{Raymer 7.12} \tag{4}$$

One note to make is that the values for the empty weight was replaced with the basic empty weight pulled from the USAF website [3], the reason this was replaced was because of fact that the basic empty weight adds the weight of optional equipment along with the empty weight. The weight seems more fitting of the type of aircraft being designed in this scenario.

All of the parameters for the seed aircraft have been estimated using the methods described above, next the process for finding the weights given the aspect ratio and wing area. One way this can be approached is by guessing a weight and then using that guess to find the calculated weight. The guess is refined until the guess weight is the same as the calculated weight.

The first estimations will involve the aerodynamics of the vehicle. Starting with approximating that the area of the horizontal stabilizer is $\frac{1}{4}$ the area of the wing and the vertical stabilizer is $\frac{1}{8}$ the size of the wing. Next assumption is that span efficiency only depends on aspect ratio and C_f only depends on Reynolds number and Mach number. The drag coefficient constant K can be calculated using span efficiency allowing C_L and C_D assuming steady flight and drag is only effected by zero lift drag and induced drag, these relations can be seen below.

$$e = 1.78(1 - 0.045AR^{0.68}) - 0.64 \qquad \text{Raymer 12.49} \tag{5}$$

$$C_{f\,turb} = \frac{0.455}{(\log_{10} R)^{2.58}(1 + 0.144M^2)^{0.65}} \qquad \text{Raymer 12.27} \tag{6}$$

$$K = \frac{1}{\pi ARe} \tag{7}$$

$$C_L = \frac{W}{q \cdot S_{ref}} \tag{8}$$

$$C_D = C_f \frac{S_{wet}}{S_{ref}} + K{C_L}^2 \tag{9}$$

The next set of estimations needed involve the mission fuel fraction, the segments of these estimations can be seen in the rightmost column in table 1 and table 2. Multiplying these ratios together gives the total weight fraction and can be used to find the fuel needed for the mission. After the mission analysis is complete the take of requirement must be satisfied, this can be done by using equation 1. This horsepower can be compared to the horsepower needed within the mission for cruise and loiter, the maximum of the two will be chosen.

Once an required horsepower is found the delta weights can be estimated, the delta weights is the change in weight that occurs for every parameter selected. To find the change in weight for the wing and stabilizer surfaces the empirical table can be used. The fighters category was selected for finding the delta weight although one could consider taking a weighted average between the fighter and general aviation aircraft because this is an analysis of a *light* attack aircraft. Another delta weight that must be considered is the change in weight of the engine which was found by referencing a chart of turboprop weights vs shaft horsepower [4] and a multiplier of $0.451\frac{lb}{hp}$ was found by a trendline fit of the engines.

Table 4 Empirical Weight Relations (Raymer 15.2)

Item	Fighters	Transports	General Aviation	Multiplier
Wing	9.0	10.0	2.5	S_{exp} [ft^2]
Horizontal Tail	4.0	5.5	2.0	S_{exp} [ft^2]
Vertical Tail	5.3	5.50	2.0	S_{exp} [ft^2]
Fuselage	4.8	5.0	1.4	S_{wet} [ft^2]

After the delta weights were calculated the empty weight could be estimated by summing the seed aircraft empty weight with all of the delta weights and the fuel can be estimated by using the $\frac{W_e}{W_0}$ and the take off weight.

VI. Additional Parameter Consideration

A. Altitude Lapse of Engine

A relation of thrust depending on altitude for turboprops can be shown below [5].

$$T = P_{sl}\left(A_p \frac{\rho}{\rho_{sl}} - B_p\right)\frac{\eta_p}{V_\infty} \tag{10}$$

Where $\eta_p \approx 0.8$ and $A_p \approx 1.132, B_p = A_p - 1$. The relation can be incorporated by substituting the equivalent horsepower whenever the power is reverenced. This also means that the delta weight for the engine will be evaluated using maximum horsepower at sea level. Also because the design mission varies at differing altitudes the equivalent horsepower must be evaluated at each mission segment.

B. Landing

The same landing equation 2 can be used to find the landing distance for the aircraft once the weights are calculated. The landing distance can now be relative to engine performance at maximum runway altitude and also have the reserve weight incorporated into landing distance length.

C. Ceiling

The ceiling was evaluated using the following equation

$$\frac{T}{W} - \frac{D}{W} = G \qquad \text{(Raymer 5.27)} \tag{11}$$

Where G is the climb gradient of the aircraft at altitude and can be multiplied by the horizontal velocity to get the vertical velocity. This parameter was evaluated right after take off because that is where the aircraft would be the heaviest and therefore the hardest spot for the aircraft to reach it's ceiling.

D. G-Loading

To sustain a g load of n while maintaining level flight the following equation can be used

$$T = qSC_{D0} + \frac{n^2W^2K}{qS} \qquad \text{(Raymer 5.23)} \tag{12}$$

$$n = \sqrt{\frac{qS}{KW^2}(T - qSC_{D0})} \tag{13}$$

This parameter was only evaluated during the loiter section during the design mission because that is where combat is going to happen and therefore where level g-loading is most useful.

VII. Diagram Generation

Now combining all of these methods together the weights can be found, because there are two missions this aircraft is designed for the greater of the two weights were selected along with the additional parameters from that mission. A plot of the empty weight, fuel weight, and take off weight for each seed aircraft can be shown below.

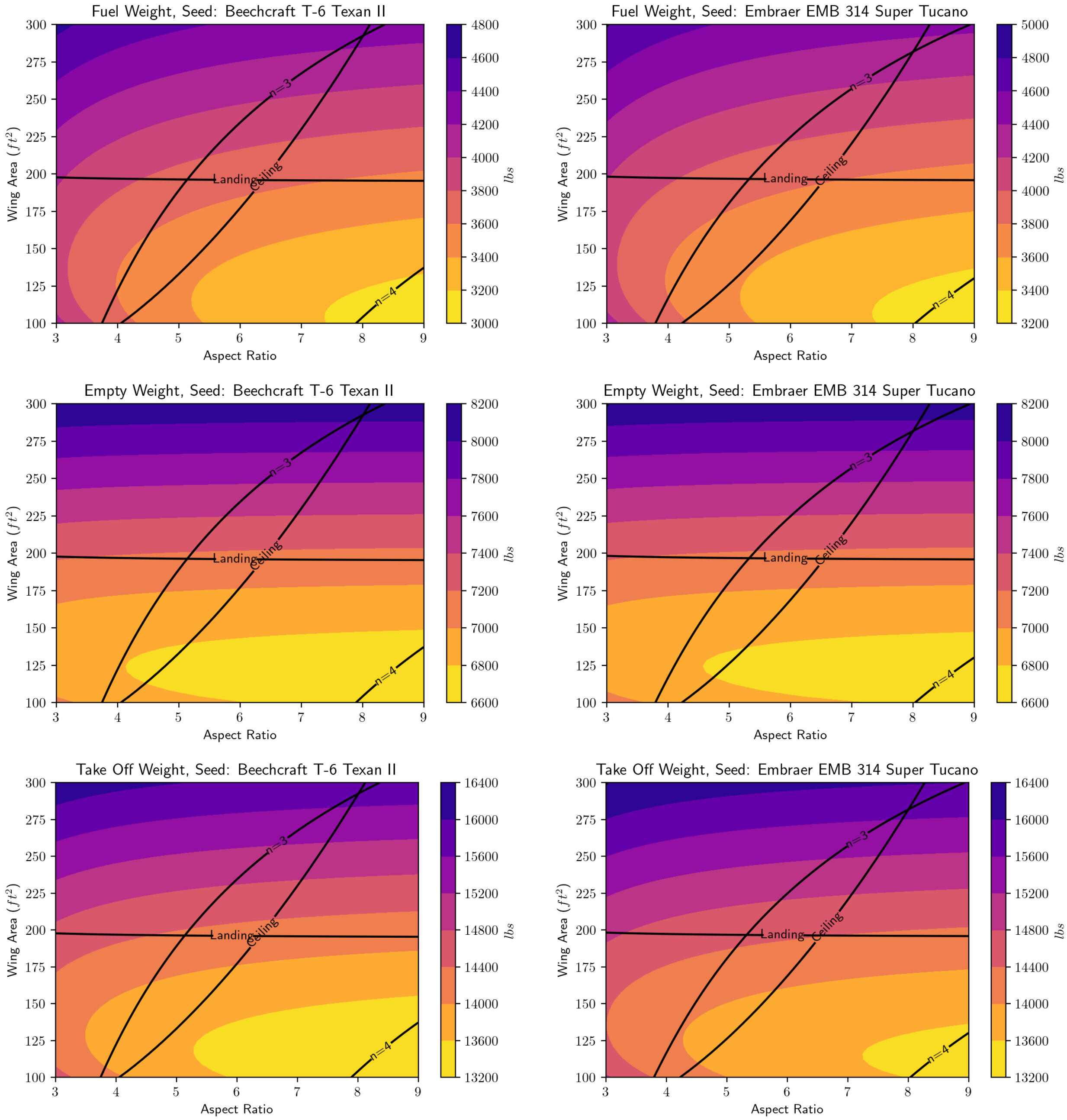

Fig. 2 Generated Dimensional Diagrams

VIII. Sizing Selection

Using these diagrams a size can be selected, the constraint lines show that the landing distance does not depend too much on aspect ratio meaning the system is constrained at a wing area about about 200 ft^2. Next the ceiling constraint constrains the system more then a level load factor of 3 and means the aspect ratio needs to be at least 6. For the range of constrained part of the graph the weight does not seem to increase too much as aspect ratio increase meaning to minimize weight it would be more important to try to minimize wing area.

IX. Conclusion

You can see by both see aircraft fitting the constraint so well that there is not additional confidence because that means two separate aircraft give similar answers even though the do not have relation to each other. Also these seeds give the ability to compare values with historical values from the aircraft making verifying the estimations easy. Some things to be further investigated in future analysis are take off and landing C_L along with the survivability requirement which was not used in the analysis but may have a large effect on the results.

References

[1] "Aircraft Database," , ???? URL https://doc8643.com/aircrafts.

[2] Raymer, D., of Aeronautics, A. I., and Astronautics, *Aircraft Design: A Conceptual Approach*, AIAA Textbook Series, American Institute of Aeronautics and Astronautics, 1989. URL https://books.google.com/books?id=Q9QeAQAAIAAJ.

[3] "T-6A Texan II," , Mar 2003. URL https://www.af.mil/About-Us/Fact-Sheets/Display/Article/104548/t-6a-texan-ii/.

[4] Nicolai, L., and Carichner, G., *Fundamentals of Aircraft and Airship Design*, No. v. 1 in AIAA education series, American Institute of Aeronautics and Astronautics, 2010. URL https://books.google.com/books?id=qA0oAQAAMAAJ.

[5] Lutze, F. H., "Performance Thrust Models," *Virginia Tech AOE 3104*, 2012, p. 1.