Estruturas de Dados / Programação 2
Grafos

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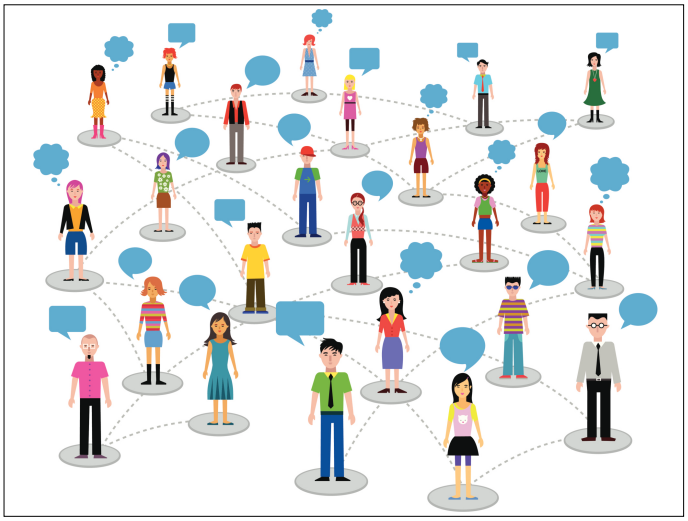


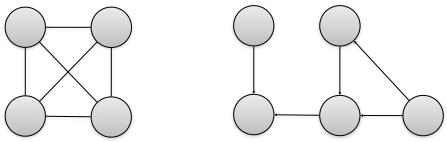
How would you implement one?

Not linear, right?!

Graphs

Graph

- "A graph is an ordered pair $G = (V, E)$ comprising a set V of vertices or nodes together with a set E of edges or lines"
- An edge is related to two vertices
- We may have undirected and directed graphs

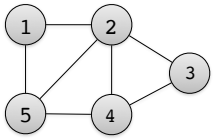


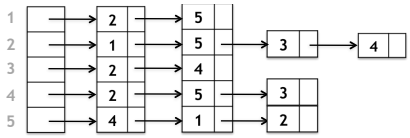
From http://en.wikipedia.org/wiki/Graph_(mathematics)

Representations

Adjacency lists

- Consists of an array *Adj* of $|V|$ lists
- *For each $u \in V$, the adjacency list Adj[u]* contains all vertices v such that there is an edge $(u,v) \in E$

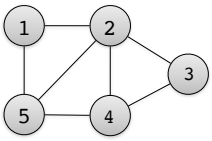




Adjacency matrixes

- Consists of a |V| x |V| matrix A = (a_{ij}) such that

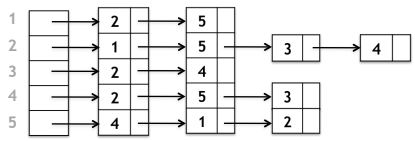
$$a_{ij} = \begin{cases} 1, \text{ if } (i, j) \in E \\ 0, \text{ otherwise} \end{cases}$$



	1	2	3	4	5
1	0	1	0	0	1
2	1	0	1	1	1
3	0	1	0	1	0
4	0	1	1	0	1
5	1	1	0	1	0

Adjacency lists versus Adjacency matrixes

- Lists
  - *No quicker way to determine $(u,v) \in E$*

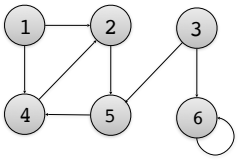


- Matrixes
  - Solve $(u,v) \in E$ *quickly*
  - But there is a tradeoff: more memory

	1	2	3	4	5
1	0	1	0	0	1
2	1	0	1	1	1
3	0	1	0	1	0
4	0	1	1	0	1
5	1	1	0	1	0

Exercise

- Write the adjacency list and the adjacency matrix for the following directed graph



Abstract Data Type: Graph

Graph ADT

```
graph* create_graph();

void add_edge(graph *graph, int vertex1, int vertex2);

void dfs(graph *graph, int source);

void bfs(graph *graph, int source);

void print_graph(graph *graph);
```

Structs

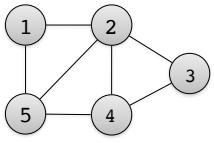
```
struct adj_list {
  int item;
  adj_list *next;
};


struct graph {
  adj_list *vertices[MAX_SIZE];
  short visited[MAX_SIZE];
};
```

Function to create Graphs…

```
graph* create_graph()
{
  graph *graph = (graph*) malloc(sizeof(graph));
  int i;
  for (i = 1; i <= MAX_SIZE - 1; i++) {
    graph->vertices[i] = NULL;
    graph->visited[i] = 0;
  }
  return graph;
}
```

Creating a Graph

```
graph *undirected_graph = create_graph();

add_edge(undirected_graph, 1, 2);
add_edge(undirected_graph, 1, 5);
add_edge(undirected_graph, 2, 5);
add_edge(undirected_graph, 2, 3);
add_edge(undirected_graph, 2, 4);
add_edge(undirected_graph, 3, 4);
add_edge(undirected_graph, 4, 5);
```



Adding edges…

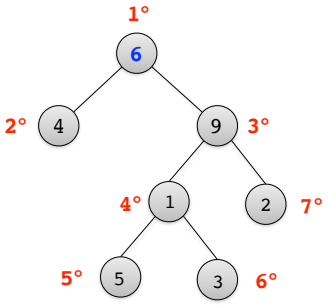
```
void add_edge(graph *graph, int vertex1, int vertex2)
{
  adj_list *vertex = create_adj_list(vertex2);
  vertex->next = graph->vertices[vertex1];
  graph->vertices[vertex1] = vertex;

  //Undirected graph. Edge to the other direction as well.
  vertex = create_adj_list(vertex1);
  vertex->next = graph->vertices[vertex2];
  graph->vertices[vertex2] = vertex;
}

adj_list* create_adj_list(int item)
{
  adj_list *new_adj_list = (adj_list*) malloc(sizeof(adj_list));
  new_adj_list->item = item;
  new_adj_list->next = NULL;
  return new_adj_list;
}
```

Graph Traversals

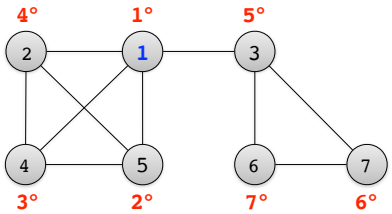
Depth-first Search (DFS)

- We mark a vertex as soon as we visit it
- Then we try to move to an unmarked adjacent vertex
- If there are no unmarked vertices at our current position, we backtrack along the vertices we already visited until we come to a vertex that is adjacent to one we haven't visited, visit that one, and continue the process

DFS Example



Another DFS Example

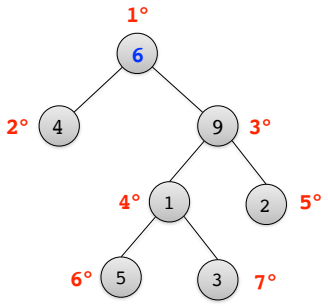


```
void dfs(graph *graph, int source)
{
  graph->visited[source] = 1;
  printf("%d\n", source);

  adj_list *adj_list = graph->vertices[source];

  while (adj_list != NULL) {
    if (!graph->visited[adj_list->item]) {
      dfs(graph, adj_list->item);
    }
    adj_list = adj_list->next;
  }
}
```

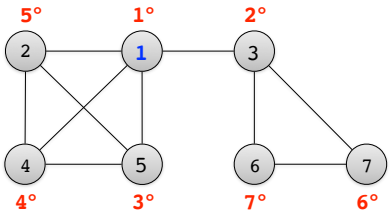
Breath-first Search (BFS)

- From a source, it explores every vertex reachable from this source
- This way, it explores the neighbor nodes first, before moving to the next level neighbors

BFS Example



Another BFS Example

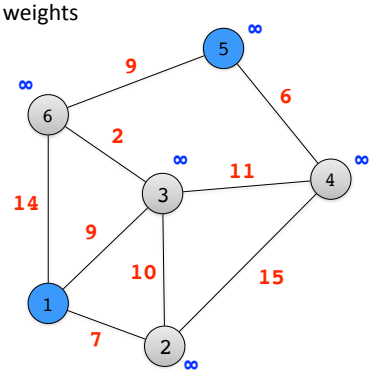


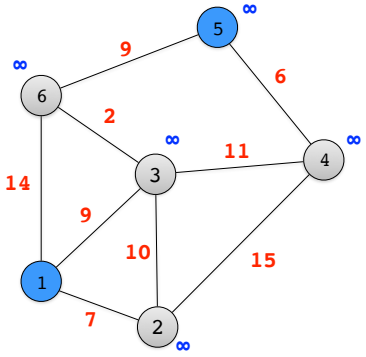

```
void bfs(graph *graph, int source)
{
  queue *queue = create_queue();
  int dequeued;
  adj_list * adj_list = graph->vertices[source];
  graph->visited[source] = 1;
  enqueue(queue, source);

  while (!is_empty(queue)) {
    dequeued = dequeue(queue);
    adj_list = graph->vertices[dequeued];
    while (adj_list != NULL) {
      if (!graph->visited[adj_list->item]) {
        printf("%d\n", adj_list->item);
        graph->visited[adj_list->item] = 1;
        enqueue(queue, adj_list->item);
      }
      adj_list = adj_list->next;
    }
  }
}
```

Shortest Paths

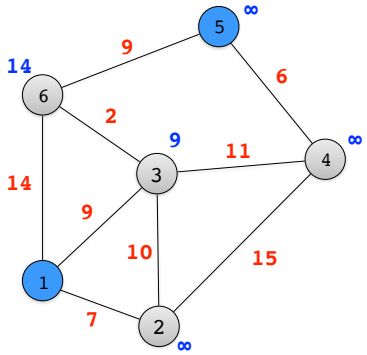
Dijkstra algorithm

- Edges with weights





Path_Length(1 —> 6) = 14
Path_Length(1 —> 3 —> 6) = 11

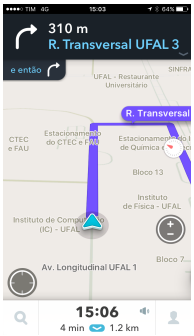


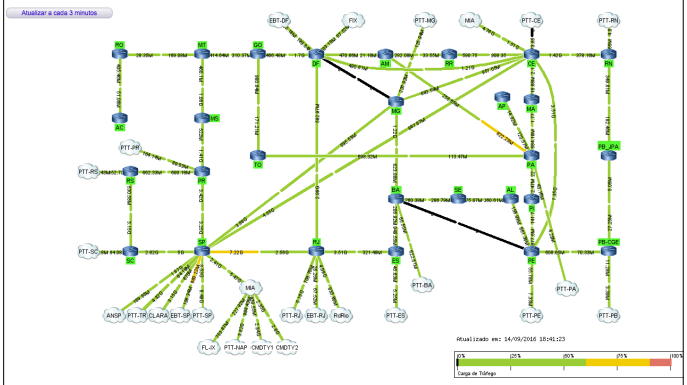
Let's execute when
u = 3; dist[u] = 9
v = 6; dist[v] = 14

v = 6; dist[v] = 11

```
for each neighbor v of u
  alt ← dist[u] + length(u, v)
  if alt < dist[v]
    dist[v] ← alt
    prev[v] ← u
```

Applications



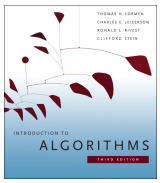


That's it?!

No!

- But don't worry!
- Union-Find and Kruskal soon!
- Graph is a very big subject in mathematics and computer science
- There will be specific classes about graphs in the future! 😃

References



Chapters 22, 23, and 24

Chapter 8