1. $GL(n,\mathbf{F}_q)$

1.1. Conjugacy classes in $GL(2,\mathbf{F}_q)$.

- Fourier analysis on finite groups and applications, Chapter 21.

Table 1. Conjugacy classes in $GL(2,\mathbf{F}_q)$, q odd, $\delta \in \mathbf{F}_{q^2}^\times \setminus \mathbf{F}_q^\times$

Representatives	# Elements in class	# Classes
$\begin{pmatrix} r & 0 \\ 0 & r \end{pmatrix}$, Central	1	$q-1$
$\begin{pmatrix} r & 1 \\ 0 & r \end{pmatrix}$, Parabolic	q^2-1	$q-1$
$\begin{pmatrix} r & 0 \\ 0 & s \end{pmatrix}$, $r \neq s$, Hyperbolic	q^2+q	$\frac{(q-1)(q-2)}{2}$
$\begin{pmatrix} r & s\delta \\ s & r \end{pmatrix}$, $s \neq 0$, Elliptic,	q^2-q	$\frac{q(q-1)}{2}$

1.2. Representations of $GL(2,\mathbf{F}_q)$ from inductions.

- Fourier Analysis on finite groups and representations, Chapter 21.

Example 1.1 (1-dimensional representations α)**.** For every multiplicative character α of $\mathbf{F}_q^\times$, we have a 1-dimensional representation given by

$$\alpha(g) = \alpha(\det(g)).$$

Example 1.2 (Principle series representations I, π_α)**.** Every multiplicative character α of $\mathbf{F}_q^\times$ can also be viewed as a character of the Borel subgroup. Then we have

$$\operatorname{Ind}_B^G \alpha = \alpha \otimes \operatorname{Ind}_B^G \mathbb{1} = \alpha \oplus \pi_\alpha.$$

Example 1.3 (Principle series representations II, $\rho_{\alpha,\beta}$)**.** Suppose we have two multiplicative character α, β of $\mathbf{F}_q^\times$, they give a character of the Borel subgroup

$$\mu_{\alpha,\beta} \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} = \alpha(a)\beta(b).$$

By induction, we get irreducible representation of G is and only if $\alpha \neq \beta$

$$\rho_{\alpha,\beta} = \operatorname{Ind}_B^G \mu_{\alpha,\beta}.$$

When $\alpha = \beta$, $\rho_{\alpha,\alpha} = \operatorname{Ind}_B^G \alpha = \alpha \oplus \pi_\alpha$.

Example 1.4 (Discrete series/cuspidal representations, σ_ν)**.** Every **nondecomposible** character ν (i.e. $\nu \neq \chi \circ \mathrm{Norm}$) of the maximal torus $K = \left\{ \begin{pmatrix} x & y\delta \\ y & x \end{pmatrix} | y \neq 0 \right\} \cong \mathbf{F}_{q^2}^\times$ gives a **reducible** representation $\operatorname{Ind}_K^G \nu$ by induction. The following virtual representation is actually our discrete series representation

$$\sigma_\nu = \pi_1 \otimes \rho_{\alpha,1} - \rho_{\alpha,1} - \operatorname{Ind}_K^G \nu.$$

1.3. Representation of $GL(2,\mathbf{F}_q)$ from Drinfeld curve.

Table 2. Character table of $GL(2,\mathbf{F}_q)$

	$\begin{pmatrix} r & 0 \\ 0 & r \end{pmatrix}$	$\begin{pmatrix} r & 1 \\ 0 & r \end{pmatrix}$	$\begin{pmatrix} r & 0 \\ 0 & s \end{pmatrix}$, $r \neq s$	$\begin{pmatrix} r & s\delta \\ s & r \end{pmatrix}$, $s \neq 0$
α	$\alpha(r)$	$\alpha(r)^2$	$\alpha(rs)$	$\alpha(\mathrm{N}(r+s\sqrt{\delta}))$
π_α	$q\alpha(r)^2$	0	$\alpha(rs)$	$-\alpha(\mathrm{N}(r+s\sqrt{\delta})$
$\rho_{\alpha,\beta}$, $\alpha \neq \beta$	$(q+1)\alpha(r)\beta(r)$	$\alpha(r)\beta(r)$	$\alpha(r)\beta(s) + \alpha(s)\beta(r)$	$-\alpha(\mathrm{N}(r+s\sqrt{\delta}))$
σ_ν, $\nu \neq \nu^q$	$(q-1)\nu(r)$	$-\nu(r)$	0	$-\nu(z)-\nu(\overline{z})$