## 1. Selberg trace formula for $GL(2, \mathbf{F}_q)$

• Fourier Analysis on finite groups and applications, Chapter 23.

**Theorem 1.1** (Selberg trace formula for finite symmetric spaces G/K). Suppose that G/K is a finite symmetric space and  $\Gamma$  is a subgroup of G. If f is a K-bi-invariant function on G and  $\rho = \operatorname{Ind}_{\Gamma}^G \mathbb{1}$ , then

$$\sum_{\pi \in \widehat{G}^K} m(\pi, \rho) (\mathscr{F} f)(\pi) = \sum_{\{\gamma\} \in \Xi_{\Gamma}} \frac{|G_{\gamma}|}{|\Gamma_{\gamma}|} I_G(f, \gamma).$$

**Definition 1.2** ( $\{\gamma\}$ ,  $G_{\gamma}$ ,  $\Gamma_{\gamma}$ ,  $\Xi_{\gamma}$ ).

**Definition 1.3** (Spherical transform  $\mathscr{F}f$  and  $\widehat{G}^K$ ). The G-representations that appear in  $\mathrm{Ind}_K^G\mathbbm{1}$  (or  $L^2(G/K)$ ) is denoted by  $\widehat{G}^K$ .

**Definition 1.4** (Orbital sum  $I_G(f,\gamma)$ ). The orbital sum of f at  $\gamma$  is

$$I_G(f,\gamma) = \sum_{x \in G_{\gamma} \setminus G} f(x^{-1}\gamma x).$$

## 2. Selberg trace formula examples

- Fourier analysis on finite groups with applications, Chapter 23.
- Harmonic analysis database,  $GL(2, \mathbf{F}_q)$  conjugacy classes.

2.1. 
$$G = GL(2, \mathbf{F}_q), \Gamma = GL(2, \mathbf{F}_q), K = \begin{pmatrix} x & y\delta \\ y & x \end{pmatrix}$$
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