Learning Symmetry with Tangible Robots

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Abstract. Robots bring a new potential for embodied learning in classrooms. With our project, we aim to ease the task for teachers and to show the worth of tangible manipulation of robots in educational contexts. In this article, we present the design and the evaluation of two pedagogical activities prepared for a primary school teacher and targeting common misconceptions when learning reflective symmetry. The evaluation consisted of a comparison of remedial actions using haptic-enabled tangible robots with using regular geometrical tools in practical sessions. Sixteen 10 y.o. students participated in a between-subject experiment in a public school. We show that this training with the tangible robots helped the remediation of parallelism and perpendicularity related mistakes commonly made by students. Our findings also suggest that the haptic modality of interaction is well suited to promote children's abstraction of geometrical concepts from spatial representations.

Keywords: Robot for Learning, Tangible, Haptic, Education, Child-Robot Interaction

1 Introduction

Children acquire knowledge in their everyday life and build mental representations through experiences, and the building of prior knowledge is particularly true for the domain of geometry. Didactisists in geometry differentiate *spatial knowledge*, linked with perceptible space, and *geometrical knowledge*, conceptualized and abstracted [1,2]. Hence, solving a geometrical problem goes beyond visual recognition of spatial relations between shapes or ability to use geometrical tools; it requires the learner to analyze these spatial relations and the properties of the shapes using mathematics and logic. In a way, geometrical knowledge provides a model of spatial knowledge. As pointed by Oberdorf et al.[3], practitioners in early childhood mathematics have observed that many children have numerous misconceptions in geometry. To avoid misconceptions when teaching about geometry, Laborde et al. [4] recommends: (1) the distinction between spacial graphical relations and geometrical relations, (2) the use of movement between spacial representations, (3) the recognition of geometrical relations and (4) finally the ability to imagine possible geometrical relation. In order to provide teaching tools to follow these recommendations, research in computer-assisted learning have investigated geometrical tools since decades. Several solutions have been proposed to use computer simulation to render geometrical transformations so that learners could understand properties of isometries [5,6]. Yet, computers are underused in geometry teaching, partially due to logistic issues. But, besides the logistical difficulties but also because several studies showed that tangible manipulations and manual interactions were important for development and can benefit learning [7,8].

Nowadays, robots are being used in broader contexts, among which of course is education. Recent research in Human-Robot Interaction reports multiple topics taught using robots [9,10,11]. In this article, we describe the design of two symmetry activities using tangible robots and a study that aimed to compare our activities to traditional practical activities in geometry. Our activities use the haptic-enabled Cellulo robots [12] that let the children feel geometrical shapes without seeing them on paper, providing embodied interaction with a simulation.

2 Related Work

2.1 Robots in Education

In education, the logo turtle entered schools nearly 40 years ago. However, if the field of interest is not new, the robots are. Indeed, they have changed a lot over time; sequentially or event-based programmable, they also integrate a wide spectrum of sensors and actuators in an increasingly minimal package. More recently, new robotic systems are designed and built to be brought into schools to teach subjects other than STEM in the curricula. Social robots (e.g. Nao and DragonBot) have been used in previous studies to teach languages [9], handwriting [13] and nutrition [11]. We also find new robotics tools that aim to be used broadly in schools. For instance, Ozgur et al. proposed bringing tangible robot interaction experience to learners [12,14].

In the present article, we propose to include teachers in a participatory design process and to specifically investigate the use of manipulation and haptic feedback that could benefit learners. In our approach, a school teacher was involved at every stage of the design of the learning tasks, starting from the design of concepts to the execution of the tests.

2.2 Tangibles and Haptics for Learning

The use of Tangible User Interfaces (TUIs) for education has been widely researched in order to foster learning [15,16,17,18]. However, these tangibles for the most part are passive, e.g. they are tokens moved manually by the user. The use of active tangibles in education have found recent developments with the Cellulo robots [12]. In their study [14], authors have started to explore the use of such active tangibles in a learning activity on wind. They specifically showed that certain types of haptic feedback (repelling forces) were less perceived than others (attracting forces). They also showed that the underlying graphics of a map of Europe could disturb the abstraction of concepts such as atmospheric pressure, leading children to focus on spatial references (e.g. cities, mountains) rather than what they were feeling with the robotic device. This leads us to understand that the use of haptics and graphics should be well balanced in the interaction design in order to allow the learner to build efficient mental representations.

An interesting dimension in learning with tangible devices is the use of shared resources to trigger collaboration among learners. It is established in the literature that collaboration is a lever of learning [19] which amounts to saying that one learns better with several collaborators, rather than being all alone. Collaborative Learning usually takes the form of an instruction in which students work in groups towards a common academic goal. Collaborative setups have also the advantage to make learners explicity express their reasoning in order to engage with their peers.

Zacharia et al. gives a literature review of touch sensory feedback and its effect on learning through experimentation [20]. They reported several studies in which haptic feedback was used for learners (forces and fields, gears, biochemistry); showing positive relations between haptics and learning sciences. This review however concludes that touch sensory feedback can be beneficial in some cases for learners but under the condition of existing prior knowledge; i.e. having already encountered the notion before, the learners could then build a multi-modal representation of the concept, constituting a more solid ground for future learning. Another conclusion of Zacharia et al. is that the touch sensory channel seems mostly relevant for abstract concepts.

In this paper, we present a participatory design with a primary school teacher for using haptic-enabled robots in classrooms to teach geometry. We present results from learning outcomes in using the robots compared to the classical practical sessions.

3 System Description

The system is composed of tangible and haptic-enabled Cellulo robots, paper posters used for localisation of the robots and an Android tablet application used to manage the activity logic and for the user to select the activities.

Cellulo (see Figure 3) is a small tangible robot that was designed to be used in education [12]. It is 7.5cm in diameter with a white mouse-like appearance. It has a three wheel omni-directional drive system (3DOFs) allowing back-drivibility - the robot can move and can be moved without being damaged [21]. The on-board localization system uses a dotted pattern printed on paper[22] to calculate the absolute position and orientation of each robot with sub-millimeter accuracy. A QtQuick API allows to develop PC and tablet compatible activities featuring many robots. This application logs the robots' position and touch sensor data.

Cellulo allows various types of interactions. It can display RGB colors through its 6 top LEDs. 6 touch capacitive sensors allow to determine if the robot is grasped. The robot has also been used to render various types of haptic feedback [23], including to render direction and intensities of forces, a point, a line or a closed shape in a 2D space. As this tangible robot works only on dotted sheet of paper, the implementation uses extensively spatial zones in the 2D space to trigger events. One can for instance define a zone on paper that will change the LEDs color of the robot or make it vibrate (in an oscillating motion).

4 Design of the Activities

In order to test how tangible robots could be used by the teachers, we decided to co-design a practical session with a mathematics school teacher.

A first meeting session with the teacher aimed to present the capabilities of the robots. We showed a demonstration of the platform and then discussed what difficulties she might have in her class. One of the difficulties pointed out by the teacher was that children cannot easily abstract geometrical concepts from spatial perception [1,2]. Pens leaving a mark on paper when solving the symmetrical problem can lead children to think only in terms of spatial relations and not in terms of geometrical relations [4]. Children tend to stay at the perceptive level and this lack of abstraction to the geometrical concepts leads to misconceptions.

After several meetings discussing the abilities of the robots and potential needs of children in the teacher's classroom, we decided to focus on isometries; and in particular, on reflective symmetry (also known as orthogonal or axial symmetry). Several weeks of developments with a regular contact with the teacher enabled us to develop a suitable activity for the teacher.

A first pilot study was conducted with a classroom of twenty 8 y.o. children in Switzerland. The goal of this pilot was to test the understanding and usability of the system with robots by children, as well as to train the teacher to use the system. By the end of the pilot study, the teacher was able to run all activities by herself without the help of the researchers.

In practical geometry teaching, paper, pen and ruler are usually used. Children experience paper on the symmetrical axis and draw reflected objects accordingly to it. This drawing allows to keep the folding spatial reference of the parts of the object that is being transformed. However, axial or reflectional symmetries can be achieved through geometrical transformations (similarly to rotation and translation) but this transformation can be hard to notice or understand when the original shape remains in place.

Drawing on paper does not allow the rendering of the transformation as a process of isometries. Quite often, because of this limitation in the rendering properties of symmetries, children form misconceptions when taught symmetry. These misconceptions often only disappear much later on during the child's academic years [24]. Previous works in educational sciences presented that some of these misconceptions come from prior knowledge or are formed when manipulating paper for symmetry understanding [25,4].

In our work, we chose to focus on the following two misconceptions:
(1) Parallelism and orientation This misconception comes from the fact that children tend to do a translation of the object when asked to perform an axial symmetry. We can often observe that the child draws the symmetry with parallel edges to the original shape. As for an example of a mistake due to this misconception, the child would performed a translation instead of the symmetry. Paper with grid could enforce the parallelism misconception making the learner to follow the vertical or horizontal lines of the grid to draw the reflective shape [4].
(2) Perpendicularity of axis This misconception is linked with the fact that children are often presented with vertical or horizontal axis of symmetry. Because of this, they tend to treat symmetries as left-right of up-down transformations and fail when the

axis is oblique. An example of this type of mistake due to this misconception would be that the child treats the symmetry as if the axis was vertical.

4.1 Paper posters

We prepared with the school teacher a poster sheet for each activity with the robots (800x600 mm) and several copies of the same sheets for the traditional tools (A3). The poster map in Figure 1 is used in a first activity that aims to address the misconception 1 on parallelism and orientation. This activity is meant to focus on the reflective symmetry of a blue triangle with regard to a vertical axis. In addition to the vertical axis, we added a grid to help the learners to measure the distances from the vertices of the shape to the symmetry axis.

For the second misconception on perpendicularity, we designed a sheet for the reflective symmetry of a line segment (see Figure 2). In the robot condition, the two vertices of the line segment are represented by two robots. The vertices of the symmetrical axis can be manipulated through two robots (the orange and blue robots).

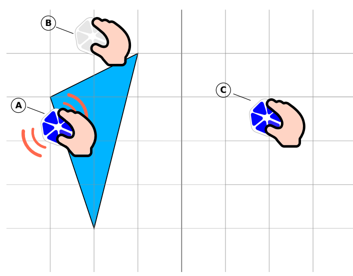


Fig. 1: Interaction design for Activity 1. (A) : Border rendering as a vibration and LEDs lighted. (B): Rendering of empty space with LEDs white. (C): Rendering of inner of the symmetric shape with still LEDs blue color.

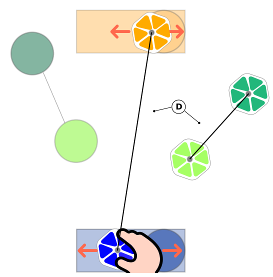


Fig. 2: Interaction design for Activity 2. (D) Black elastic string representing the segments.

4.2 Interaction Design

For the first activity, the border of the shape was rendered with haptic feedback (vibration) and the interior by lighting the robot in blue (see Figure 1). Each pair of children would start by scanning the left side of the poster sheet on which the original shape was drawn. If the child was scanning outside of the shape the robot's LEDs would turn white. The reflective shape would similarly be rendered but only if the teacher activated this option through the tablet. The option of feeling the reflected shape was at first disabled, allowing time for children to make a hypothesis before they could check by scanning the sheet of paper.

Before starting the second activity, a 3D printed colored "hat" was attached to each robot to be able to differentiate them (see Figure 2). These colored hats also had a nib with an elastic thread symbolizing the segment between two robot points. This way, the orange and blue robots were attached and symbolized the symmetry axis, and the green robots the shape to be reflected. Two areas printed on paper determined where the orange and the blue robot could be. By sliding the blue or orange robot in their respective areas, the symmetry segment symbolized by the green robots would adjust their position in real-time to keep the symmetry correct.

5 Research Hypothesis

In the following study, we propose to evaluate the influence of tangible manipulation of robots on correction of two types of common misconceptions in reflective symmetry compared to the use of classical geometric tools.

Practical sessions using paper to teach about orthogonal symmetry can lead to misconceptions [25,26]. Without the drawing of the symmetry on paper, children were forced to build a mental representation of the geometry. We believe that this process will favor the acquisition of the skill and correct the misconceptions in the haptic conditions. We believe that pairs of learners that easily used the system would be able to better collaborate and perform the tasks in a more efficient way.

As the haptic modality provides the ability to render edges of a shape without imprint on paper, we believe that it would help children to abstract the geometric properties. We first hypothesize that our system is usable and that children can solve the task with the robot as they would with their regular geometry tools. Because of the above statements, we also expect more children in the experimental condition to correct their misconceptions.

6 Case Study

The experiment was conducted by a school teacher with her classroom following the five steps below :
Introduction: This first session with children aimed to clarify the vocabulary. It was done as a short intervention two days before the experiment. The teacher would show natural pictures of axial symmetry (a butterfly, a snowflake and the reflection of a tree on a lake). The teacher would ask children about particularities of the 3 images and ask to place one (or more) symmetry axes on the images. Then the teacher drew the axes and explain the terms "axial symmetry" and "axis of symmetry" to the whole class.
Pre-test: The pretest was taken by all the children in order to evaluate the presence of misconceptions and was taken with the regular pen and ruler tools [4]. Another

[4] We provide the material used in the experiment here: https://github.com/WafaJohal/Cellulo-Symmetry-Material

school teacher of the same grade was in charge of the grading which consisted simply in denoting the presence or absence of the two misconceptions we focused on.
Pairing and Grouping: Children were then be paired according to their misconceptions. The goal was to have as many homogeneous pairs as possible with children sharing the same misconceptions. The pairs where then assigned to the control or experimental conditions. The control group would perform the same tasks but manipulating only pen, paper and ruler, whereas, the experimental group would use the robots on a printed map.
Problem Solving Activities: About one week after the pre-test, each pair had to solve two problems in a semi-guided practical activity (Section 6.2 describes the two activities). The teacher could intervene to ask the children to question their proposed solution. The interventions were scripted and were the same for the two conditions (with robots vs with paper).
Post-test : A post-test two weeks after the experiment concluded the experiment. The post-test material was a variation of the pretest (e.g. original image on the other side of the axis).

6.1 Participants and Apparatus

A classroom of 16 children of ages between 11 and 13 (M=12, SD=1), participated to a between subjects experiment in a school of Switzerland. The pre-test and post-test were individual but the practical sessions were performed in pairs (in order to force children to express their reasoning processes in solving the geometrical problem). We recorded the interaction with two video cameras.

6.2 Activities

A1: Finding the symmetrical figure In this task, the figure used for the symmetry is a blue triangle. None of the edges of the triangle follow the grid underneath. The goal of this activity is for the children to hypothesize the position of the symmetrical figure and verify it through manipulation. This overall aim of this task is to correct the misconception 1 (parallelism and translation).

The control group (see Figure 3) used regular tools such as a ruler and a pen. They drew the symmetry and then could check their hypothesis by folding the paper on the symmetry axis and placing the paper on the windows. Using the paper transparency, they could see if their hypothesis was correct or not. They could then refine their drawing and verify again until they were convinced that their answer was correct.

The experimental group used a robot (see Figure 4). First, the children were invited to familiarize themselves with the interaction with the robot. The teacher asked them to grab a robot and to move it on the table. Children would then be asked to observe the behavior of the robot: When the robot was on the border of the geometrical shape, it would vibrate, giving a haptic rendering of the border; and when the robot would be inside the figure, the top LEDs of the robot would replicate the color of the geometrical figure (i.e. blue). The child would thus explore the blue triangle and be asked to notice the haptic and visual feedback. They would then be asked to hypothesize on the position of the symmetry by pointing at the potential

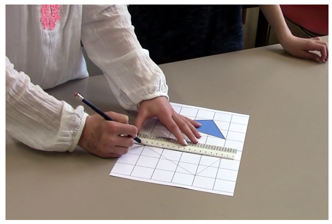


Fig. 3: Activity 1 - Control condition. With pen and ruler as tools.

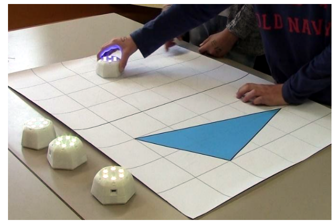


Fig. 4: Activity 1 - Experimental condition. Using a robot as a sensor with haptic feedback.

place of the symmetrical triangle on the grid. In order to check their hypothesis, they would use the robot to feel the symmetric shape. As for the control group, they could refine their hypothesis freely until they were satisfied with their answer.

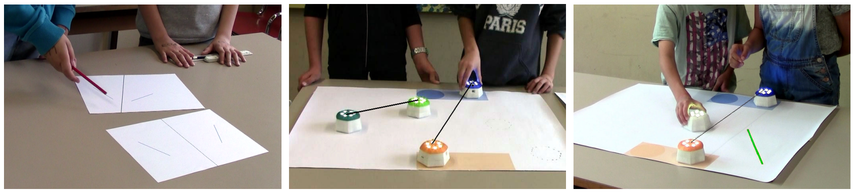
A2: Manipulation of the orientation of the symmetry axis In the second activity, the goal was to correct the misconception of children that the image of the axial symmetry is always either vertical or horizontal. In the activity, they were facing an oblique axis and had to estimate or observe the symmetry of a line segment. Children were asked to hypothesize on the symmetry and to check their answer by observing the symmetry. In the control condition, the children had 2 paper sheets, one with a vertical axis, one with an oblique axis. They had to draw the symmetric segment (Figure 5a).

This implementation of the experimental condition made use of 4 robots. The robots were mounted with 3D printed colored tops that were used to visually group the robots into segments. Two robots (blue and orange) where linked with an elastic rope and acted as the vertices of the axis of symmetry. The two other robots, mounted with green tops, where also linked and displayed the position of the symmetric segment. The activity will start with the two green robots being in the original position of the segment, and the axis in a vertical position. The children were then asked to point at the position of the symmetrical segment. The teacher then announced that the symmetry would occur. The two green robots would then move to their symmetric positions. The children could then change the slope of the axis by moving the blue and orange robots like sliders within their area of action (respectively the blue and orange boxes).

7 Results

Both the control and the experimental groups took the same pretest and post-test. These tests aimed to measure the disappearance or persistence of misconceptions after manipulating either traditional geometry tools such as paper and ruler (control group) or the robots (experimental group).

Remediation of Misconception 1 The first misconception was concerning mistakes dealing with Parallelism and Orientation of the shape (i.e. doing a translation

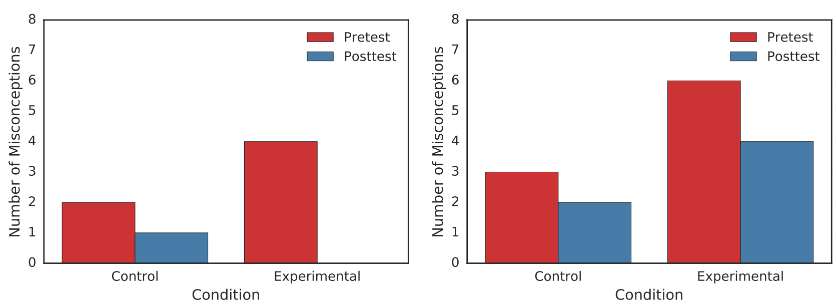


(a) Control condition. Two sheets of papers, with the same original shape being a segment. On one sheet the axis is vertical and on the other the axis is oblique

(b) Experimental condition without haptics. The blue and orange robots are linked with a rope and act as vertexes of the symmetrical axis. They can be moved within the blue and orange area respectively. The green robots act as vertexes of the symmetrical segment. As the axis changes orientation and move; they move accordingly to keep the symmetry correct.

(c) Bonus Activity - Experimental condition with haptics. The blue and orange robots are linked with a rope and act as the vertexes of the symmetrical axis. They can be moved within the blue and orange area respectively. The third robot is used as an haptic device to feel the symmetrical segment.

Fig. 5: Setup of Activity 2 on manipulation of the axis from vertical to oblique orientation



(a) Misconception 1 **(b) Misconception 2**

Fig. 6: Sum of misconceptions in each group for the pretest and post-test

instead of a reflection). For the pretest, two pairs in the experimental condition and one pair in the control condition presented this misconception. After the practical session, the misconception was maintained for one member of the pair in control, and disappeared for all the students in the experimental condition. Even-though the remediation (number of disappearing misconceptions of Type 1) seems better in the robot condition, a Pearson Chi2 test shows that it is not significantly different compared to the traditional one ($\chi-squared=1.56, df=1, p-value=0.21$).

Remediation of Misconception 2 The misconception disappeared for some learners between the pretest and the post-test but several students in both control and experimental group still had the misconception after the practical activities. We could explain the persistence of the second misconceptions for certain children who associated spatial relations rather than geometrical abstractions. A Pearson Chi2 test did not show significant difference between traditional and robot activities for the correction of the second misconception ($\chi-squared=1.1, df=2, p-value=0.58$). These results tend to demonstrate that the robot activity is as effective as the teacher's traditional method to correct common misconceptions.

7.1 Assessment of Collaboration

In order to evaluate the quality of the interaction within the group, we proposed to the teacher who participated to the design to annotate video recordings of the practical sessions. We selected two stereotypical groups: one for which the activity seemed to work in correcting the misconceptions, one for which it didn't work as well. For these two groups we asked the teacher to comment the videos of interaction according to two dimensions: 1) nature of collaboration and 2) topic of the exchange. In the nature of collaboration we distinguished: conflicts (expressing disagreement) and arguments (arguing for a thesis). The topic of exchange could be either on the use of the robot (" you should mover it like that") or on the notion (" see, the axis is here") These assessments on collaboration for these two stereotypical pairs showed more arguments and conflicts in the pair that was able to repair successfully their misconception. These results are in line with the literature in collaborative learning stating that conflicts are positively correlated with learning gain in collaborative tasks. About the topics of intra-group communications, both groups were referring more to the notion than to the usage of the robotic tool. This argues in favor of our hypothesis claiming the ease of use of the device, even though it was novel for students.

7.2 Bonus Activity

As the haptic modality to render the shape seemed to give good results for the activity 1, we decided with the teacher to implement another instance of the activity 2 using haptic feedback. Children still had to manipulate the symmetry axis by moving the blue and orange robots (see Figure 5c) and to observe its effect on the symmetric segment. But this time the symmetric segment was rendered with haptic and light feedbacks (similar to the blue triangle in the first activity). The pairs of children performed this bonus activity three weeks after the post-test and another test (a variation of the two first) was administrated two days after the bonus practical activity. The control group also took the bonus test but without any practical activity in between. Figure 7 shows that the bonus activity remediated for the misconception 2 for all students in the experimental condition. Concerning the control group, we observe that the retention was good. Only one student kept the misconception.

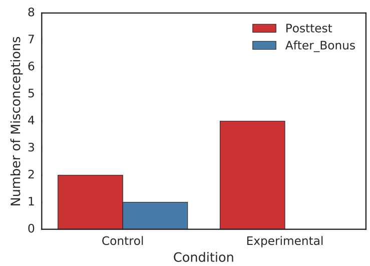


Fig. 7: Sum of misconceptions in each condition for the post-test and the after bonus test.

8 Discussion

The primary goal of this experiment was to evaluate our design in an ecological scenario involving the teacher. The results from the user study show that overall the robots led to similar solving of misconceptions compared to the traditional tools. Yet the haptic modality seemed to play an important role in the abstraction and solving of the tasks.

Children had high expectations after the teacher first mentioned an activity with robots. Some children felt that the robots were precious and fragile and hence were holding back from manipulating them freely. We believe that this novelty effect could disappear after some time and would let children adopt this new tool.

Another major contribution of our work is the co-design of ecologically tested tangible robots activities for geometry. The teacher was able to use the system alone in her classroom. Very few technical issues occurred during the experiment and most of them were due to Bluetooth connection (the teacher was able to fix and handle them without our intervention). The school teacher reported that she enjoyed this experience and proposed to continue our collaboration to develop new applications for other mathematical tasks.

Some limitations should be stated. First of all, we chose to keep the scenario identical for all the pairs of learner, starting with activity 1 then 2. The post-test were taken by the students only after the two activities and there could have been an order effect influencing the learning between activity. Further investigation with another experimental design and with more students could evaluate such effect. While the Bonus activity seems to repair the misunderstanding for all the students, it could have also been the case for a traditional practical session using the pen and ruler. Here again, our hypothesis concerning the impact of haptics could be tested by comparing haptic and non-haptic interactions in the same task, in a counterbalanced experimental design.

9 Conclusions and Future works

This article's main contributions are two-fold: First, we present the participatory design and implementation of two learning activities using tangible haptic-enabled robotic devices. Our design process was driven by the teacher's constraints and needs in terms of misconceptions of her students and led to the implementation of a standalone tablet application with activities on reflective symmetry. Second, we present results from an ecological study that showed the feasibility for teachers to use robots in pedagogical scenarios. Children were able to experience various types of interaction with the same robotics device and within the same conceptual tasks; e.g. used as a scan device, tangible points. They contained very different sets of interaction affordances utilizing both active and passive robot behaviors, and were well received by the children. The fact that the symmetrical shape was not drawn on paper also forced children to develop mental representations. In this particular case, our results are in line with some previous works [27,28] showing that 2D shapes could be perceived with haptic feedback. However, further studies should be run to confirm these results that were obtained on a relatively small sample size.

Our future work will focus in studying the use of haptics in curricular activity in more depth, in order to better understand how this new learning modality with robots can help learners acquire abstract concepts. We also plan to investigate several configurations of collaborative setups (N robots and M children), in order to explore the effect of shared resources on collaboration. This investigation will aim to design smart behaviors for the robots to reallocate themselves to enhance collaboration among learners. For these future experiments, we plan to include more participants in our study to guarantee soundness of the results.

Acknowledgements

We would like to thank the Swiss National Science Foundation for supporting this project through the National Centre of Competence in Research Robotics.

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