Foundations
Probability: Univariate Models

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Exercise 2.7. [Deriving the inverse gamma density]
Let $X \sim Ga(a,b)$, and $Y = 1/X$. Derive the distribution of Y.

Solution.

$$Y = 1/X \qquad X = 1/Y$$

using the change of variables formula:

$$p(y) = p(x)\,|\frac{dx}{dy}|$$

$$p(y) = p(1/y)\,|\frac{d1/y}{dy}|$$

$$p(y) = \frac{b^a}{\Gamma(a)}(\frac{1}{y})^{a-1}\, e^{\frac{-b}{y}}(\frac{1}{y^2})$$

$$p(y) = \frac{b^a}{\Gamma(a)}y^{-(a+1)}\, e^{\frac{-b}{y}}$$

Which is the inverse gamma distribution.

$\square$