Foundations
Probability: Univariate Models

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Exercise 2.11. [Probabilities are sensitive to the form of the question that was used to generate the answer]
(Source: Minka.) My neighbor has two children. Assuming that the gender of a child is like a coin flip, it is most likely, a priori, that my neighbor has one boy and one girl, with probability 1/2. The other possibilities—two boys or two girls—have probabilities 1/4 and 1/4.

a. Suppose I ask him whether he has any boys, and he says yes. What is the probability that one child is a girl?

b. Suppose instead that I happen to see one of his children run by, and it is a boy. What is the probability that the other child is a girl?

Solution. let G = girl and B = boy and the two children are 1 and 2.

Assuming the gender of a child is like a coin flip, we have 4 possible outcomes: $\{B_1B_2, B_1G_2, G_1G_2, G_1B_2\}$ each with a probability of 1/4.

a. Given the event that neighbor has answered yes to having a boy, the possible outcomes are: $\{B_1B_2, B_1G_2, G_1B_2\}$ since having two girls is no longer a possibility. In this case, the probability that one child is a girl (doesn't matter which child) is:

$$= \frac{P(G_1B_2) + P(B_1G_2)}{P(G_1B_2) + P(B_1G_2) + P(B_1B_2)}$$
$$= \frac{2}{3}$$

b. In the event of seeing one child and it is a boy, let's assume that you saw child 1 (the same holds if you saw child 2, just flip the numbers), the possible outcomes are: $\{B_1B_2, B_1G_2\}$. In this case, the probability that the other child is a girl is:

$$= \frac{P(B_1G2)}{P(B_1G_2) + p(B_1B_2)} = \frac{1}{2}$$

$\square$