Foundations
Probability: Univariate Models

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Exercise 2.9. [Bayes rule for medical diagnosis]
After your yearly checkup, the doctor has bad news and good news. The bad news is that you tested positive for a serious disease, and that the test is 99% accurate (i.e., the probability of testing positive given that you have the disease is 0.99, as is the probability of testing negative given that you don't have the disease). The good news is that this is a rare disease, striking only one in 10,000 people. What are the chances that you actually have the disease? (Show your calculations as well as giving the final result.)

Solution. Let X represent having the disease and Y represent testing positive.

Using Bayes rule:

$$P(X|Y) = \frac{P(X)\,P(Y|X)}{P(Y)}$$

$$= \frac{P(X)\,P(Y|X)}{P(X)\,P(Y|X) + P(X`)\,P(Y|X`)}$$

$P(X)$ = the probability of having the disease = $\frac{1}{10{,}000} = 0.0001$

$P(X`)$ = the probability of not having the disease $= 1 - 0.0001 = 0.9999$

$P(Y|X)$ = the probability of testing positive when having the disease $= 0.99$

$P(Y|X`)$ = the probability of testing positive when not having the disease $= 1 - 0.99 = 0.01$

$P(X|Y)$ = the probability of having the disease when testing positive

$$P(X|Y) = \frac{0.0001 * 0.99}{0.0001 * 0.99 + 0.9999 * 0.01} = 0.0098$$

The chances of having the disease when testing positive are 0.98%. □