Aircraft Structures - I

WAFFLE'S CRAZY PEANUT

(Last updated: 26/4/14)

1 Static Determinacy of Structures

Mechanics Recall:

Coplanar forces - confined to a common plane, can be parallel or concurrent. Their general resolution is,

$$\sum F_x = 0,\ \sum F_y = 0,\ \sum M_P = 0$$

Sum of forces along the "mutually perpendicular" directions is zero, and the sum of moments (including those contributed by the forces) about any point is **zero**.

(i) For *concurrent forces*, the equilibrium conditions $\sum F_x = 0$ and $\sum F_y = 0$ are enough.

(ii) For *parallel forces* the conditions $\sum F = 0$ along direction of the forces, and $\sum M_P = 0$ are suffice to resolve them.

1.1 Plane Truss Analysis

Static Indeterminacy:

- If the number of unknown reactions developed > number of static equilibrium equations, then it is a **statically indeterminate** structure.

  (While truss works can make such cases statically determinate "externally", the trusses themselves need to be statically determinate "internally").

- The stability of a truss work is governed by

$$n = 2j - 3$$

  n = number of members, j = number of joints

  (a) LHS = RHS: Structure is statically determinate, and it's a fail-safe design (i.e) it can survive on the event of failure (other members can still be able to hold the structure).

  (b) LHS > RHS: Structure is statically indeterminate "internally", and it corresponds to a safe-life design (i.e.) they can live a long time without any repairing requirements.

  (c) LHS < RHS: It's no longer a structure. It's a mechanism, which collapses as a whole.

Assumptions made:

- Members are pin-joined.
- Members resist the point loads in the form of tension and compression.
- Loads are applied only at joints.

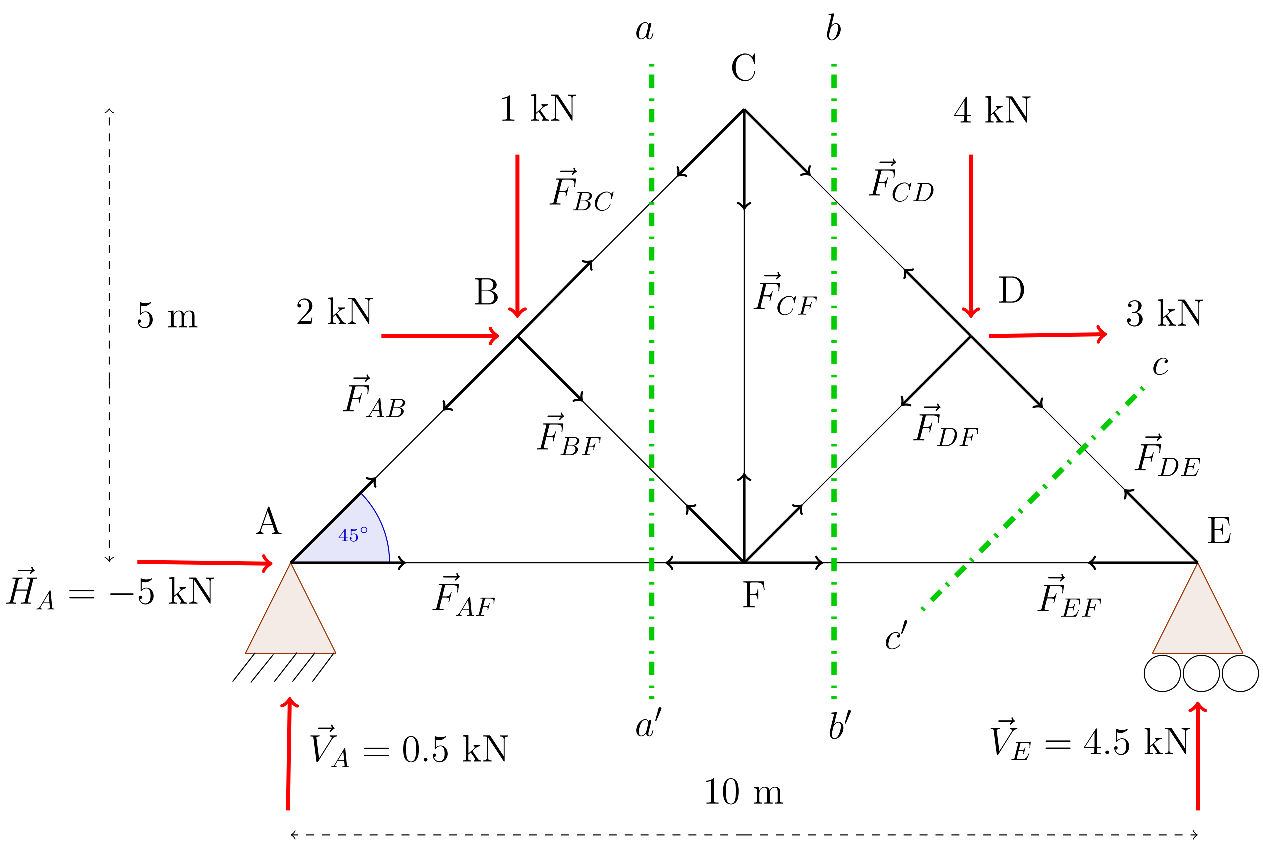


Figure 1: Plane truss

Methods:

(i) Method of Joints

(ii) Method of Section

(iii) Method of Shear

1.2 Method of Joints

Resolve the forces joint by joint.

Procedure:

(a) Find the reaction at the supports, and check whether the truss work is statically determinate externally and internally.

(b) Choose a "resolvable" joint* (where the number of unknown reactions $\leq$ the number of equilibrium equations) and find the tension on the members.

(c) As the forces are concurrent, the equilibrium equations $F_x = 0$ and $F_y = 0$ are suffice to resolve.

*__Note:__ Some joints may have a greater number of unknowns. For e.g., joint F in the above truss has five tensile forces acting on it. They can be used to check the calculations.

Keep in mind:

1) Forces are *assumed* to be tensile at all joints (i.e) they act away from the joints.

2) The mutually perpendicular directions can be in any orientation. Even a tangent and normal to a given joint can do the resolution of forces.

3) Don't confuse yourselves with **Newton's third law**. This convention does not mean that.

   For e.g., in the above truss, F_{AB} is the tension between A and B. At joint A, $\vec{F}_{AB}$ acts from A $\rightarrow$ B, whereas at the joint B, it acts from B $\rightarrow$ A, according to our assumption (see **1st point**).

   But, it's true. Because, tension is similar to reaction. You resolve it once, and get the direction, then that direction remains the same throughout the problem. So, $\vec{F}_{AB} = \vec{F}_{BA}$

1.3 Method of Section:

Slice the structure into sections. As the truss needs tensile and compressive forces to be stable, you apply the tensile forces manually at the loose ends of the section, and resolve them.

(It's very similar to D'Alembert's principle, used to analyze an accelerating object, wherein you provide an opposing force to balance the acceleration of the object, and thereby resolve it using static equilibrium equations. This analysis is common in classical mechanics)

Slicing is shown in Fig.1 by aa', bb' and cc'.

Procedure:

(a) Slice the structure appropriately between joints. No more than 3 unknown forces should appear, for the sliced part to be statically determinate "internally".

(b) Use the moment equilibrium condition (i.e.) moment about any point, $\vec{M}_P = 0$.

Keep in mind: Moment is simply force times the "perpendicular distance" ($\vec{M} = \vec{r} \times \vec{F}$). So, the forces that pass through the point do not contribute any moment. And, while resolving, stick to a particular direction (clockwise/anticlockwise).

For example, taking left slice of aa' in Fig.1, moments $\vec{M}_A$, $\vec{M}_F$ and $\vec{M}_B$ can be equated to zero.

Regarding the perpendicular distance, (for e.g.) while using $\vec{M}_F = 0$, 1 kN is at $r = 2.5$ m (horizontal), whereas 2 kN is at $r = 2.5$ m (vertical).

1.4 Space Truss

Analyze the forces on members from the orthographically projected truss work.

Moment recall:

Moment is always taken about a line. So, the moment of a force about a line parallel to its direction is **zero!**

$$\text{Moment} = \text{Force} \times \text{Perpendicular distance}$$

Moment is a vector. Its direction is perpendicular to the plane containing $\vec{r}$ and $\vec{F}$

$$\vec{M} = \vec{r} \times \vec{F}$$

Resolution of moments, and magnitude of net moment are similar to that of the forces,

$$M = \sqrt{{M_x}^2 + {M_y}^2 + {M_z}^2}$$

Note:

- When a force is transferred from one point to another point, it goes as a force with same magnitude and direction, but also with a moment.
- Just as forces are indicated by arrows ($\rightarrow$), moments are indicated by double-headed arrows ($\twoheadrightarrow$). Their rotation is given by one of the thumb rules, left/right depends on the question.

Procedure:

(a) If the given members are in 3D space, then project them orthographically in 2D (front and side views).

(b) Tabulate the distances of members relative to the vertical or horizontal (for each D, S, and V).

(c) Find the magnitude of the net distance using,

$$|L| = \sqrt{D^2 + S^2 + V^2}$$

(d) Find $\frac{D}{L}, \frac{S}{L}$, and $\frac{V}{L}$.

(e) Now that the direction cosines are found, apply the force equilibrium to find the forces on members,

$$\sum F_i \cdot \frac{D}{L} = 0, \sum F_i \cdot \frac{S}{L} = 0, \sum F_i \cdot \frac{V}{L} = 0$$

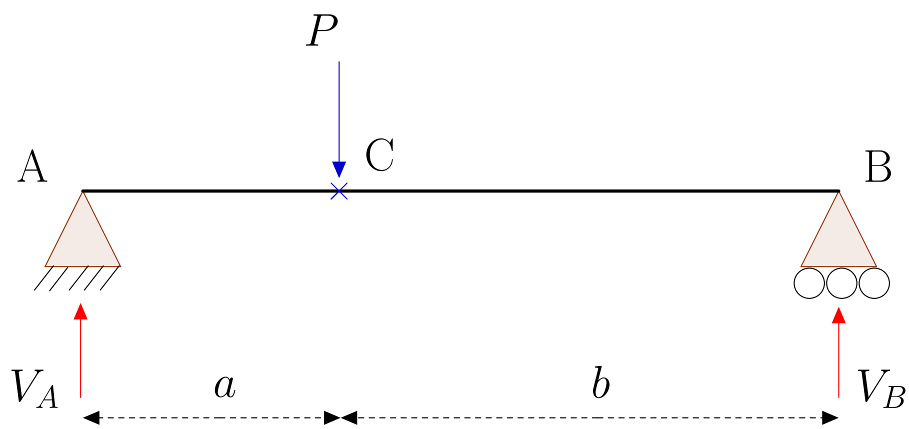
Note:

- All the forces should be accounted for equilibrium (including the externally applied ones, with appropriate SI units).
- D, S and V *can* be negative, when the members extend from origin to the negative region of the coordinate system

Formulas Recall:

Simply-supported beams:

- For a point load P at a distance a from the support,



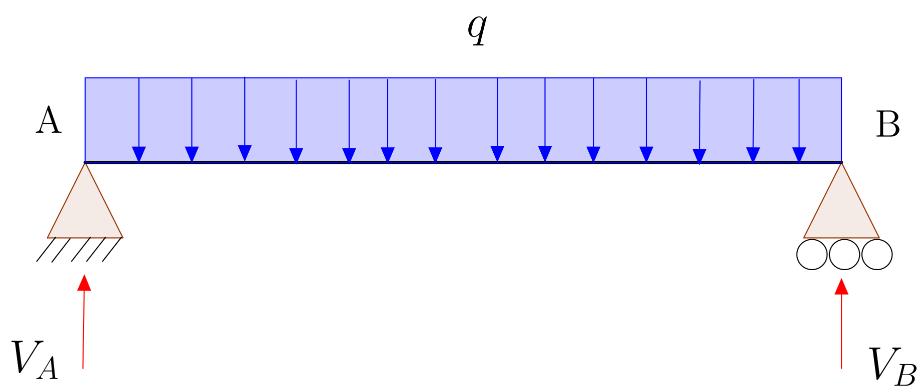
(i) Maximum Bending Moment:

$$M_C = \frac{Pab}{l}$$

(ii) Maximum deflection:

$$\delta_C = -\frac{Pa^2b^2}{3EIl}$$

- For a distributed load of intensity q throughout the length of the beam,



(i) Maximum Bending Moment:

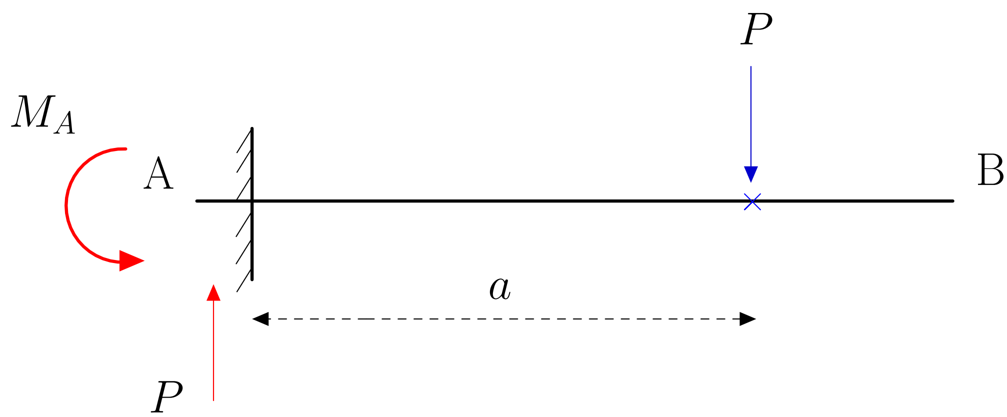
$$M_{max} = \frac{ql^2}{8}$$

(ii) Maximum deflection:

$$\delta_C = -\frac{5ql^4}{384EI}$$

Cantilever beams:

- For a point load P acting at a distance a from the fixed end,



(i) Maximum Bending Moment:

$$M_A = -Pa$$

(ii) Slope at B:

$$\theta_B = -\frac{Pa^2}{2EI}$$

(iii) Maximum deflection:

$$\delta_{max} = -\frac{Pl^2}{6EI}\left(3l - a\right)$$

1.5 Clapeyron's 3-moment equation:

Procedure:

(a) Split the given indeterminate beam into spans, and use Clapeyron's equation for the spans. Each span should be "statically determinate".

$$M_1L_1 + 2M_2(L_1{+}L_2) + M_3L_2 = -\frac{6A_1a_1}{L_1} - \frac{6A_2b_2}{L_2}$$

$L_1,\ L_2 =$ Lengths of spans considered
$M_1,\ M_2,\ M_3 =$ Support moments
$A_1,\ A_2 =$ Area of Bending moment diagrams
$a_1,\ b_2 =$ Centroidal distance from moment diagrams

(b) Apply the formula for two spans at a time, to obtain a solvable set of equations.

(c) Solve the equations to get the moments.

(d) Take moments at the right/left of a support from a certain joint, to get the respective reactions.

1.6 Moment Distribution method:

- Stiffness factor : Factor (number) multiplied to the magnitude of couple required to produce unit rotation at that end of span.

$$\left(\frac{4EI}{L},\ \frac{3EI}{L}\right)$$

- Distribution factor : Fraction of chosen stiffness factor with respect to the total stiffness factor at the chosen joint.

$$\left(\frac{sf_1}{sf_1+sf_2}, \frac{sf_2}{sf_1+sf_2}\right)$$

- Carry – over factor : When an external moment is applied, 50 % of the moment is carried over to the near end of the span.
- Fixed – end moments : Moments in the member when it's fixed against rotation.

$$\left(\frac{Pa^2b}{L^2}, \frac{Pab^2}{L^2}, \frac{qL^2}{12}\right)$$

Procedure:

(a) Write down the factors and calculate the fixed-end moments.

(b) Apply counter moments at the extreme ends of the beam, which is then carried over to the near end of the span.

(c) Any unbalanced moment at a particular joint is now shared on either side as per the distribution factor. (carry-over if any, is also taken into account)

(d) Sum up the values of moments and iterate (c) till there's a repetition in the moment values.

2 Failure Theories

2.1 Maximum Principal Stress Theory:

$$\tau_{\max} = \frac{T r_{\max}}{J}$$
$$\sigma_{\max} = \frac{M_{\max}\, y}{I}$$

Principal stresses,

$$\sigma_{1,2} = \frac{\sigma_{xx} + \sigma_{yy}}{2} \pm \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \tau_{xy}^2}$$

$$\text{Max}\ (\sigma_1, \sigma_2) < \sigma_{yp}$$

Theory of Rankine & Mohr: (Procedure)

(i) Positive axes correspond to tensile σ_{yp}, whereas negative axes correspond to compressive σ_{yp}. Form a rectangle from the values.

(ii) For Rankine, check whether Max (σ_1, σ_2) lies within the rectangle.

(iii) For Mohr, join σ_t and σ_c to form a trapezium and finally check the same.

2.2 Maximum Principal Strain Theory:

$$\epsilon_1 = \frac{1}{E}\,\big(\sigma_1 - \nu(\sigma_2 + \sigma_3)\big)$$

$$\forall\ \sigma_1 > \sigma_2 > \sigma_3$$

Condition for safety,

$$\sigma_1 - \nu(\sigma_2 + \sigma_3) < \sigma_{yp}$$

For a **thin-walled cylinder**,

$$\left(\frac{r}{t} > 10 \quad \text{and} \quad \frac{d}{t} > 20\right)$$

$$\sigma_c = \frac{pr}{t} \quad \sigma_t = \frac{pr}{2t}$$

$$\sigma_c < \sigma_{yp}$$

2.3 Maximum Shear Stress Theory:

$$\tau_{\max_1} = \frac{\sigma_1 - \sigma_2}{2},\ \ \tau_{\max_2} = \frac{\sigma_2 - \sigma_3}{2},\ \ \tau_{\max_3} = \frac{\sigma_3 - \sigma_1}{2}$$

$$\text{Max}\ \big(|\tau_1|, |\tau_2|, |\tau_3|\big) < \frac{\sigma_{yp}}{2}$$

2.4 Octahedral Shear Stress Theory:

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 < 2\sigma_{yp}^2$$

2.5 Maximum Strain Energy Theory:

$$\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\gamma(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1) < \sigma_{yp}^2$$