Rockets & Missiles

WAFFLE'S CRAZY PEANUT

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Multi-stage rockets:

$$\text{Mass ratio, } \Lambda_i = \frac{M_{0i}}{M_{0i} - M_i}$$

$$\text{Payload ratio, } \lambda_i = \frac{M_{0i+1}}{M_{0i}}$$

$$\text{Propellant ratio, } \psi_i = \frac{M_{pi}}{M_{0i}}$$

$$\text{Structural efficiency, } \epsilon_i = \frac{M_{ci}}{M_{ci} + M_{pi}}$$

$$\psi_i = (1 - \epsilon_i)(1 - \lambda_i), \quad \psi_{total} = \frac{\sum M_{pi}}{M_{0i}}$$

$$\Lambda_i = \frac{1}{1 - \psi_i} = \frac{1}{\epsilon_i(1 - \lambda_i) + \lambda_i}$$

$$\lambda_{total} = \frac{M_{uN}}{M_{0i}}, \quad G = \frac{M_0}{M_u}$$

Basic stuff:

$$F = \dot{m} g \ I_{sp}$$

$$\Lambda = \frac{M_0}{M_0 - \dot{m} t}$$

(for constant thrust),

$$P(t) = 1 - \frac{1}{\Lambda}(\ln\Lambda + 1)$$

$$\psi_0 = \frac{F}{M_0 g}, \ t_b = \frac{I_{sp}}{\psi_0}\left(1 - \frac{1}{\Lambda}\right)$$

(for constant specific thrust),

$$\beta_0 = \frac{F}{Mg}, \ \dot{m} = \frac{M_0 \beta_0}{I_{sp}\Lambda}, \ t_b = \frac{I_{sp}}{\beta_0} \ln\Lambda$$

1 Vertical flight:

$$v = g \ I_{sp} \ln\Lambda - gt, \quad t_c = I_{sp} \ln\Lambda$$

$$h(t) = g \ \frac{I_{sp}^2}{\psi_0} \ P(t) - \frac{1}{2} \ gt^2$$

$$h(t_c) = g \ \frac{I_{sp}^2}{\psi_0} \left(\frac{1}{2} \ \psi_0 \ (\ln\Lambda)^2 - \ln\Lambda + 1 - \frac{1}{\Lambda}\right)$$

$$h = h_b + v_b(t - t_b) - \frac{1}{2} \ g \ (t - t_b)^2$$

$$(h_b)_{max} = g \ \frac{I_{sp}^2}{2} \frac{P_b^2}{(1 - \frac{1}{\Lambda})^2}$$

2 Constant pitch angle:

$$v_x = g\ I_{sp}\ \ln\Lambda\ cos\theta,\ \ v_z = g\ I_{sp}\ \ln\Lambda\ sin\theta - gt$$

$$v_b = \Delta v_{ideal}\sqrt{1 - \frac{2t_b\ sin\theta}{I_{sp}\ \ln\Lambda} - \frac{t_b^2}{I_{sp}^2(\ln\Lambda)^2}}$$

$$tan\alpha = \frac{t\ cos\theta}{I_{sp}\ \ln\Lambda - t\ sin\theta}$$

(for constant thrust),

$$tan\alpha = \frac{cos\theta}{\psi_0 - sin\theta}$$

(for constant specific thrust),

$$tan\alpha = \frac{cos\theta}{\beta_0 - sin\theta}$$

$$M = M_0\ \exp\Big(\frac{-\beta_0}{I_{sp}}\ t\Big)$$

(relation to vertical flight)

$$t'_c = t_c\ sin\theta,\ x(t) = h\ cos\theta\ (\text{no g!}),\ z(t) = h\ sin\theta$$

(same relation goes for v_x & v_z and x_c & z_c)

for impulse shot, $(x, z) = g\ I_{sp}\ \ln\Lambda\ t$ ... sine & cosine

3 Gravity turn:

$$\frac{dv}{dt} = \frac{F}{m} - g\ sin\gamma = g(\beta_0 - sin\gamma)$$

$$\frac{d\gamma}{dt} = \frac{-g}{v} cos\gamma$$

$$\Gamma = \frac{1}{cos\gamma}\big(\frac{1 - sin\gamma}{1 + sin\gamma}\big)^{\beta_0/2}$$

$$\frac{dv}{dt} = \big(\frac{\beta_0 - \sin\gamma}{cos\gamma}\big)\ d\gamma$$

$$t = \frac{v\ (\beta_0 + sin\gamma)}{g(\beta_0^2 - 1)}$$