Numerical Methods

WAFFLE'S CRAZY PEANUT

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1 Solution of Equations:

1.1 Fixed-point Iteration:

Iterate within the limits of roots.

Procedure:

(i) Identify whether the given $f(x)$ is linear algebraic, non-linear algebraic, or transcendental equation.

(ii) Start finding $f(0)$, $f(1)$, ... until there's a change of sign in the value, which corresponds to the limit (a, b) within which the root lies.

(iii) Write the function in the form $x = \phi(x)$

(iv) Check the condition $|\phi'(a)| < 1$ and $|\phi'(b)| < 1$

(v) Now, find x_0, $x_1 = \phi(x_0)$, $x_2 = \phi(x_1)$, ... for values lying in the limit (a, b) and stop when the repetition of rounded values occurs.

Note: For infinite series, as there's no specific interval, find the roots directly by taking $x = f(x)$, neglecting higher powers, and iterating using step (v).

Keep in mind Root-finding will be easier if iterations begin with the value nearer to a or b, based on whether $|f(a)|$ or $|f(b)|$ is closer to zero.

If $f(a)$ is closer to zero, then the root is closer to a, and vice versa.

1.2 Newton-Raphson Method:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Procedure:

(i) Identify whether the given $f(x)$ is linear algebraic, non-linear algebraic, or transcendental equation.

(ii) Start finding $f(0)$, $f(1)$, ... until there's a change of sign in the value, which corresponds to the limit (a, b) within which the root lies.

(iii) Now, find x_0, x_1, x_2... using the iterative formula, and proceed until repetition occurs.

Some formulas (can be derived):

- If $x = \frac{1}{N}$,

$$x_{n+1} = x_n(2 - Nx_n)$$

- If $x = \sqrt{N}$,

$$x_{n+1} = \frac{1}{2}\left(x_n + \frac{N}{x_n}\right)$$

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$$x_{n+1} = \frac{1}{2}\left(x_n + \frac{1}{Nx_n}\right)$$

- If $x = N^{1/k}$,

$$x_{n+1} = \frac{1}{k}\left((k-1)x_n + \frac{N}{x_n^{k-1}}\right)$$

1.3 Solution of linear system of equations: