Project Euler

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Problem 1

Naive solution: Iterate through all numbers from and check if they are multiples of 3 or 5 and sum them. This solution is $\mathcal{O}(n)$, which seems fine but we can do better.

Cool solution: Gauss's formula we can find an $\mathcal{O}(1)$ solution. Recall Gauss's formula for the sum of the first n natural numbers.

$$f(n) = \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \tag{1}$$

Now we try to find a formula for the sum of the first n natural numbers that are multiples of a natural number k. Such series can be represented by the following sum:

k+2k+3k+4k+....+tk = k(1+2+3+...+t), where tk is the biggest natural number less than n and a multiple of k.

Using Gauss's formula in (1), we arrive at the following formula:

$$g(n) = \frac{k(t)(t+1)}{2} \tag{2}$$

To calculate numbers that are multiples of either 3 or 5, we use the simple formula:

sum of multiples of 3 or 5 = sum of multiples of 3+sum of multipeles of 5−sum of multiples of 15

Putting our formula into action:

$$\text{sum of multiples of 3 or 5} = \frac{3(333)(334)}{2} + \frac{5(199)(200)}{2} - \frac{15(66)(67)}{2} = 166833+99500-33165 = 233168 \tag{3}$$