Problems to Week 12 Tutorial — MACM 101 (Fall 2014)

1. Show that if any 14 numbers are selected from the set $S = \{1, 2, 3, \ldots, 25\}$, there are at least two whose sum is 26.

2. Let triangle ABC be equilateral, with $AB = 1$. Show that if we select 10 points in the interior of this triangle, there must be at least two whose distance apart is less than $1/3$.

3. During the first six weeks of his senior year in college, Brace sends out at least one resume each day but no more than 60 resumes in total. Show that there is a period of consecutive days during which he sends out exactly 23 resumes.

4. Joshua draws two ping-pong balls from a bowl of twenty ping-pong balls numbered 1 to 20. Provide a sample space for this experiment if

   (a) the first ball drawn is replaced before the second ball is drawn;

   (b) the first ball drawn is not replaced before the second ball is drawn.

5. Twenty five slips of paper, numbered $1, 2, 3, \ldots, 25$, are placed in a box. If Amy draws six of these slips, without replacement, what is the probability that (a) the second smallest number drawn is 5? (b) the fourth largest number drawn is 15? (c) the second smallest number drawn is 5 and the fourth largest number drawn is 15?

6. Let S be the sample space for an experiment, and let A, B be events from S such that $Pr(A) = 0.4$, $Pr(B) = 0.3$, and $Pr(A \cap B) = 0.2$. Determine $Pr(\overline{A})$, $Pr(\overline{B})$, $Pr(A \cup B)$, $Pr(\overline{A \cup B})$, $Pr(A \cap \overline{B})$, $Pr(A \cup \overline{B})$.

7. Three missiles are fired at an enemy arsenal. The probabilities the individual missile will hit the arsenal are 0.75, 0.85, and 0.9. Find the probability that at least two of the missiles hit the arsenal.