Problems to Week 7 Tutorial — MACM 101 (Fall 2014)

1. Investigate the truth or falsity of the following using 3 methods: Venn diagrams, laws of set theory, and proving that the left side is a subset of the right one and vice versa.
$$A - (B \cup C) = (A - B) \cap (A - C).$$

2. Use the laws of set theory to establish that
$$\overline{(A \cap B) \cup (\overline{A} \cap C)} = (A \cap \overline{B}) \cup (\overline{A} \cap \overline{C}).$$

3. Using the laws of set theory, simplify each of the following
   (a) $(A \cap B) \cup (A \cap B \cap \overline{C} \cap D) \cup (\overline{A} \cap B)$,
   (b) $(A - B) \cup (A \cap B)$.

4. Construct the power set of the set $\{\emptyset, \{1\}, \{\{a\}\}\}$.

5. Let A, B, C, D be nonempty sets. Prove that $A \times B \subseteq C \times D$ if and only if $A \subseteq C$ and $B \subseteq D$.

6. Prove that $A \times (B - C) \subseteq (A \times B) - (A \times C)$. Does the equality holds?

7. Determine which of the following relations R on the set A are reflexive, symmetric, transitive, and anti-symmetric.
   (a) $A = \{1, 2, 3\}$ and $R = \{(1, 1), (1, 2), (2, 1), (2, 2)\}$. Draw the graph and the matrix of this relation.
   (b) A is the set of all students at SFU, and $(x, y) \in R$ means that the height of x differs from the hight of y by no more than one inch.
   (c) A is the set of ordered pairs of real numbers, that is, $A = \mathbb{R} \times \mathbb{R}$, and $((x_1, x_2), (y_1, y_2)) \in R$ if and only if $x_1 = y_1$ and $x_2 \leq y_2$.

8. Check that the following relations R on the set A are equivalence relations, find their equivalence classes, the number of equivalence classes, and determine which equivalence class the element z belongs to.
   (a) Let A be the set of all possible strings of 3 or 4 letters in alphabet $\{A, B, C, D\}$, let $z = BCAD$, and let $(x, y) \in R$ if and only if x and y have the same first letter and the same third letter.
   (b) Let A be the power set of $\{1, 2, 3, 4, 5\}$, let $z = \{1, 2, 3\}$, and let $(x, y) \in R$ if and only if $x \cap \{1, 3, 5\} = y \cap \{1, 3, 5\}$.