Introduction

Discrete Mathematics

Andrei Bulatov

Course Info

- **Instructor: Andrei Bulatov**
  - Email: abulatov@sfu.ca
  - Room: TASC 8013
  - Office hours (tentative):
    Wednesday 2:00 – 2:00 (from Sept 1th)
    Monday 2:00 – 4:00 (from Sept 22th)
- **Teaching Assistants:**
  - Yu Yang, email: yya119@sfu.ca
  - Shengdong Zhang, email: sza75@sfu.ca
  - Maryam Hayati, email: maryam.hayati@gmail.com
- **Course webpage**
  - http://www.cs.sfu.ca/CourseCentral/101.MACM/abulatov

Course Info

- **Course objective:**

  To introduce basic concepts and applications of discrete mathematics.

- **Syllabus:**
  - Logic and Formal Reasoning
  - Set Theory, Functions and Relations
  - Mathematical Induction
  - Combinatorics
  - Number Theory
  - Graphs and Trees (if time permits)

Course Info

- **Textbook:**

  R. P. Grimaldi, *Discrete and Combinatorial Mathematics (an Applied Introduction)*, Addison-Wesley, 2004.

  - It is impossible to finish studying all the contents of the textbook in one semester. The contents not covered in lectures/slides are not required.
  - The content and order of topics, as presented in the class, do not one-to-one correspond to any part of the book. Use of Subject Index is advised.
  - In few cases the notation and terminology in the class differs from that in the book

Course Info

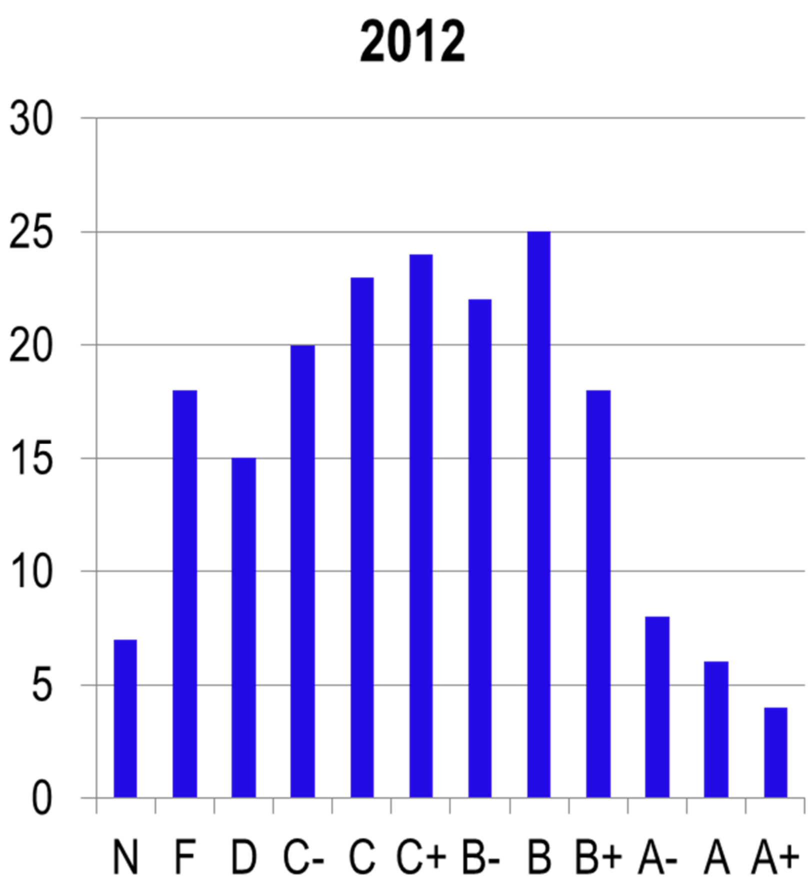
- **References:**
  - H. Rosen, *Discrete Mathematics and Its Applications*, 7/E, McGraw-Hill, 2012.
  - R. L. Graham; D. E. Knuth; and O. Patashnik, *Concrete Mathematics*, Addison-Wesley, Reading, MA, 1994
  - T. H. Cormen, C. E. Leiserson, R. L. Rivest, and C. Stein, *Introduction to Algorithms*, 2nd Edition, MIT Press, Cambridge, MA, 2001.
  - G. Andrews, *Number theory*, Saunders or Dover Publications, Inc.
  - H. Enderton, *A Mathematical Introduction to Logic*, Harcourt/Academic Press, 2001

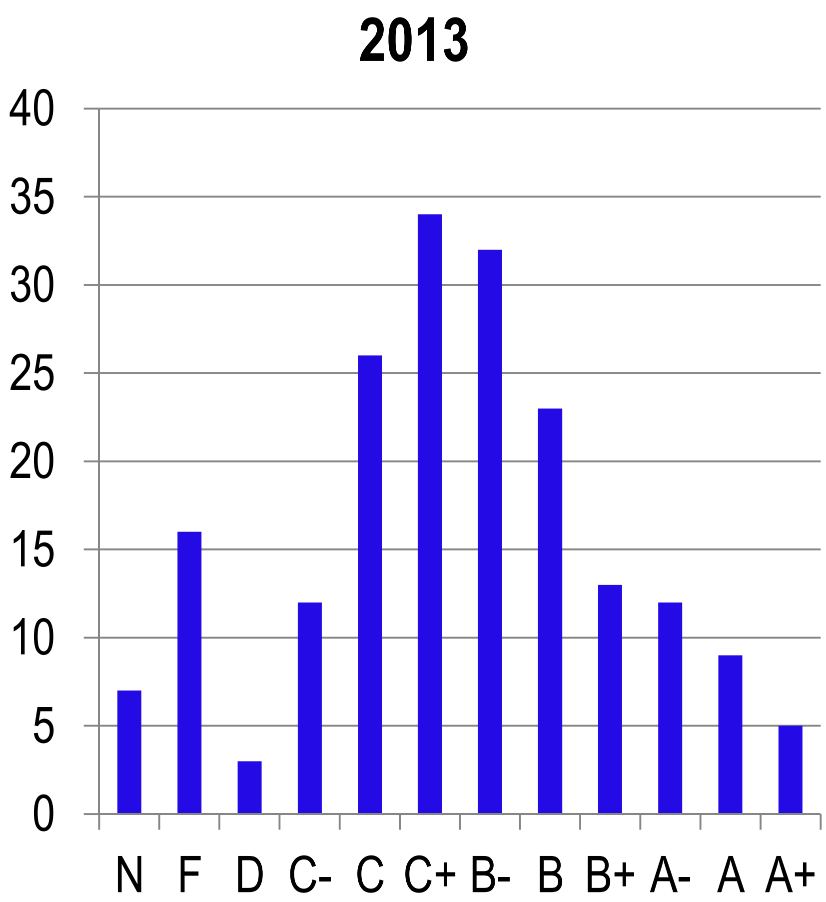
Course Info

- **Grading:**
  - 10 Tutorials attendance (10 $\times$ 1%)
  - 5 Assignments (5 $\times$ 5%)
  - 2 Midterms (2 $\times$ 15%)
  - 1 Final Exam 35%

Course Info

- **Previous results:**





Prerequisites

- Not much of specific knowledge
- Some general knowledge is needed, as there will be examples
- Modest math erudition (e.g., 5th Euclid's postulate, see next slide)
- Basics

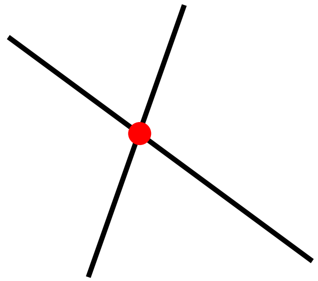
$$2 \times 2 = 4$$

$$7 \times 8 = \cancel{55}\ 56$$

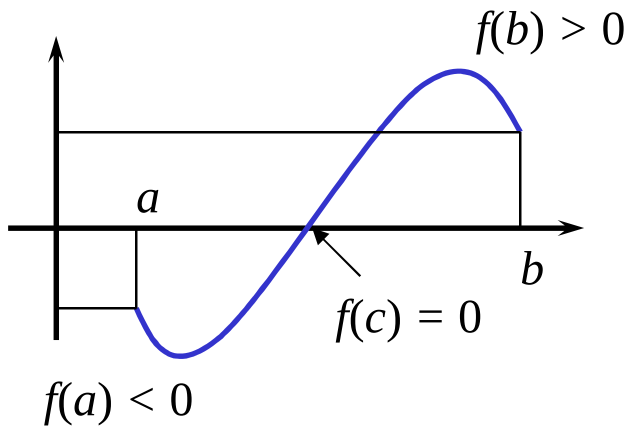
Two Mathematics

Continuous Mathematics

- Fifth Euclid's Postulate



- Intermediate value theorem



Continuous Mathematics (cntd)

- Laws of Physics

$$\vec{F} = m\frac{d^2X}{dt^2}$$ Newton's second law of motion

$$\nabla \times \mathbf{E} = \frac{\partial \mathbf{B}}{\partial t}$$ Maxwell's law of electromagnetism

- Disciplines: geometry, calculus, differential equations, topology, …

- Applications: physics, engineering, astronomy, …

Discrete Mathematics

- Discrete Mathematics comprises all branches of mathematics that do not use the idea of continuity.

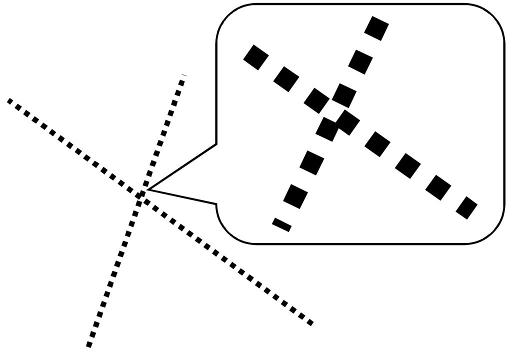
- `Formal' definition (Wikipedia):

  Discrete mathematics, sometimes called **finite mathematics**, is the study of mathematical structures that are fundamentally discrete, in the sense of not supporting or requiring the notion of continuity. Most, if not all, of the objects studied in finite mathematics are countable sets, such as the integers.

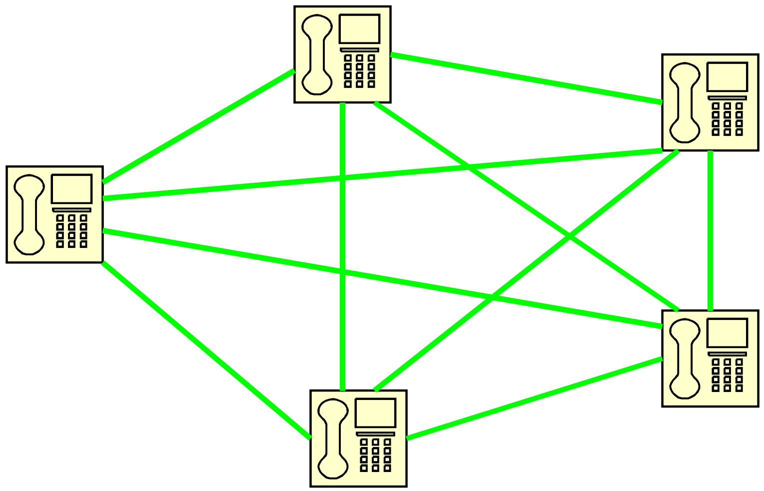
  For contrast, see continuum, topology, and mathematical analysis

Discrete Mathematics (cntd)

- Removing continuity
  - Discrete fifth Euclid's Postulate (???)

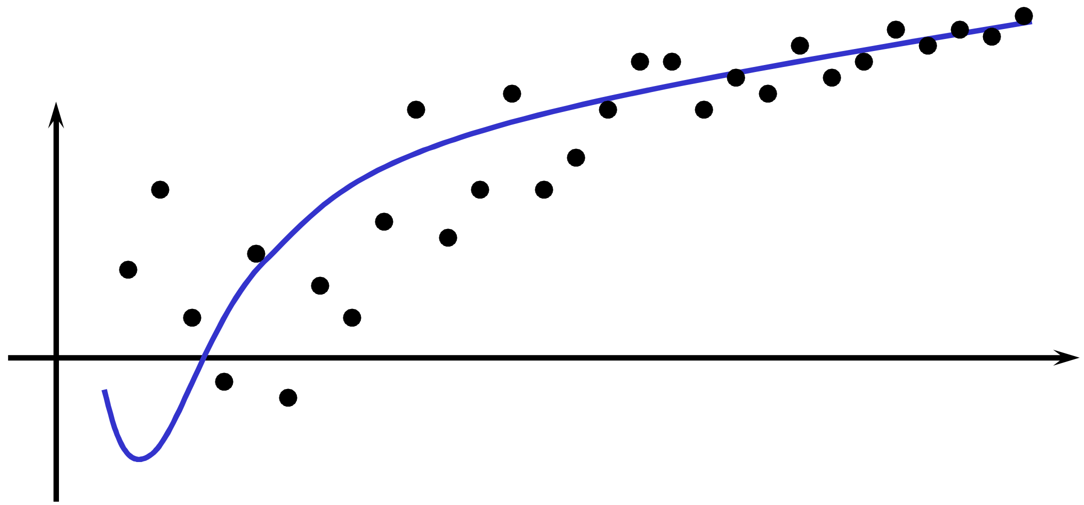


  - Graphs



Discrete Mathematics (cntd)

- Asymptotics



Properties of a discrete objects are `approximated' using a continuous function

- Laws of discrete mathematics:

$$((\forall x\, F(X) \to G(X)) \wedge F(a)) \to G(a)$$

The rule of universal specification

$$X^n + Y^n = Z^n$$ does not hold for any $n > 2$ and integer X, Y, Z

Great Fermat's Theorem

Topics in DM

- Wikipedia says that Discrete mathematics usually includes:
  - logic - a study of reasoning
  - set theory - a study of collections of elements
  - number theory
  - combinatorics - a study of counting
  - graph theory
  - algorithmics - a study of methods of calculation
  - information theory
  - the theory of computability and complexity - a study on theoretical limitations on algorithms …
- algebra – a study of algebraic systems (Bulatov)
- discrete probability theory (Grimaldi)

This is too much for us !!

CS at SFU:

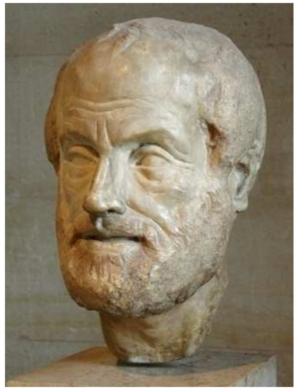
MATH-232	Elementary linear algebra
CMPT-413	Computational linguistics
CMPT-379	Principles of compiler design
CMPT-384	Symbolic computing
CMPT-307	Data structures and algorithms
CMPT-308	Computability and complexity
CMPT-405	Design and analysis of computing algorithms
CMPT-406	Computational geometry
CMPT-407	Computational complexity
CMPT-408	Theory of computer networks / communications
MACM-300	Introduction to formal languages and automata
MACM-401	Symbolic computation
CMPT-477	Introduction to formal verification
CMPT-705	Design and analysis of algorithms
CMPT-706	Parallel algorithms
CMPT-710	Computational complexity
CMPT-725	Logical methods in computational intelligence
CMPT-813	Computational geometry
CMPT-815	Algorithms of optimization
CMPT-816	Theory of communication networks
CMPT-721	Knowledge representation and reasoning
CMPT-814	Algorithmic graph theory

Our goal

is to learn basic concepts and terminology that provide basis and common language for those and many other courses.

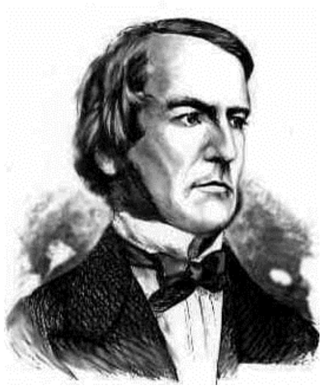
Logic

- Formal logic, syllogisms



Aristotle
384 - 322 B.C.

- Mathematical logic, formal reasoning



George Boole
1815 - 1864

- Computational logic, formal verification

Pentium FDIV bug
1994

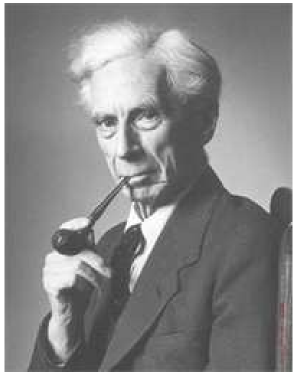
- Other applications: artificial intelligence, robotics, software verification, automated theorem proving, …

Set theory

- Naïve set theory

Georg Cantor
1845 - 1918

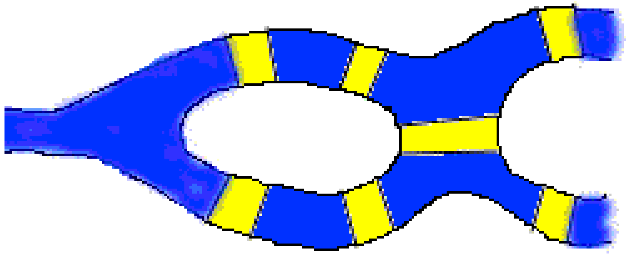
- Axiomatic set theory

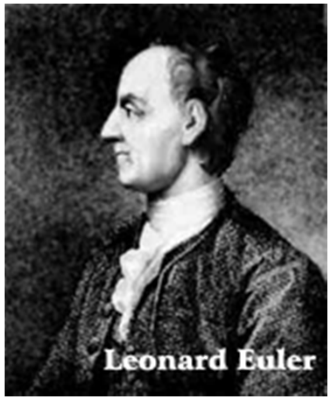


Bertrand Russell
1872 - 1970

Graphs

- Toy graph theory
  Konigsberg 7-bridge problem





Leonard Euler
1707 - 1783

- Other applications: modeling of nearly everything,
  electric circuits, networking, linguistics, data storage, coding theory, games, scheduling, combinatorial algorithms, …

- One more face

Paul Erdös
1913 - 1996

Number Theory

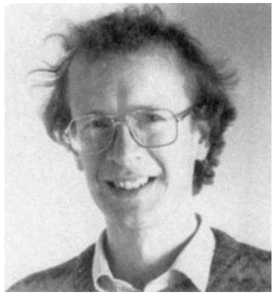
- Arithmetic (*Arithmetica*)

?

Diophantus
200 - 284

- Number theory

Pierre de Fermat
1608 - 1672



- Algebraic geometry

Andrew Wiles
1953 - ?

- Other applications: cryptography