Problems to Week 5 Tutorial — MACM 101 (Fall 2014)

1. Negate and simplify each of the following.

   (a) $\forall x\ (P(x) \to Q(x))$;

   (b) $\exists x\ ((P(x) \vee Q(x)) \to R(x))$.

2. Let the universe for the variables in the following statements consists of all real numbers. In each case negate and simplify the given statement.

   (a) $\forall x \forall y\ ((x < y) \to \exists z\ (x < z < y))$;

   (b) $(\forall x \forall y\ ((x > 0) \wedge (y > 0))) \to (\exists z\ (xz > y))$.

3. Determine which of the following arguments are valid and which are invalid. Provide an explanation for each answer. (Let the universe consist of all people presently residing in Canada.)

   (a) All mail carriers carry a can of mace.
   Mrs. Bacon is a mail carrier.
   Therefore Mrs. Bacon carries a can of mace.

   (b) All law-abiding citizens pay their taxes.
   Mr. Pelosi pays his taxes.
   Therefore Mr. Pelosi is a law-abiding citizen.

   (c) All people who are concerned about the environment recycle their plastic containers.
   Margarite is not concerned about the environment.
   Therefore Margarite does not recycle her plastic containers.

4. For a prescribed universe and any open statements $P(x), Q(x)$, prove that
$$\forall x\ (P(x) \wedge Q(x)) \iff (\forall x\ P(x)) \wedge (\forall x\ Q(x)) .$$

5. Provide the reasons for the steps verifying the following argument. (Here a denotes a specific but arbitrary chosen element from the given universe.) (For Step 9 refer to Table 2.19 in the textbook.)
   Premises: $\forall x\ (P(x) \to (Q(x) \wedge R(x)))$, $\forall x\ (P(x) \wedge S(x))$.
   Conclusion: $\forall x\ (R(x) \wedge S(x))$.

Steps	Reasons
1. $\forall x\ (P(x) \to (Q(x) \wedge R(x)))$	
2. $\forall x\ (P(x) \wedge S(x))$	

3. $P(a) \to (Q(a) \wedge R(a))$
4. $P(x) \wedge S(a)$
5. $P(a)$
6. $Q(a) \wedge R(a)$
7. $R(a)$
8. $S(a)$
9. $R(a) \wedge S(a)$
10. $\forall x\ (R(x) \wedge S(x))$.

6. Give a direct proof that, for all integers k and ℓ, if k, ℓ are both even, then $k + \ell$ is even.

7. Give a proof by contraposition that, for all integers k and ℓ, if $k\ell$ is odd, then k, ℓ are both odd.