Problems to Week 4 Tutorial — MACM 101 (Fall 2014)

1. For each of the following pairs of statements use Modus Ponens or Modus Tollens to make a valid argument.

   - "If Janice has trouble starting her car, then her daughter Angela will check Janice's spark plugs.
     Janice had trouble starting her car."
   - "If Brady solved the first problem correctly, then the answer he obtained is 137.
     Brady's answer to the first problem is not 137."

2. Write the following argument in symbolic form. Then establish the validity of the argument or give a counterexample to show that it is invalid.

   > If Dominic goes to the racetrack, then Helen will be mad. If Ralph plays cards all night, then Carmela will be mad. If either Helen or Carmela gets mad, then Veronica (their attorney) will be notified. Veronica has not heard from either of these two clients. Consequently, Dominic didn't make it to the racetrack and Ralph didn't play cards all night.

3. Give the reasons for each step needed to show that the following argument is valid
   Premises: $p, p \to q, s \vee r, r \to \neg q$
   Conclusion: s.

Steps	Reasons
1. p	
2. $p \to q$	
3. q	
4. $r \to \neg q$	
5. $q \to \neg r$	
6. $\neg r$	
7. $s \vee r$	
8. s	

4. Consider the universe of all polygons with three or four sides, and define the following predicates for this universe:

   $A(x)$: all interior angles of x are equal;
   $E(x)$: x is a equilateral triangle;

$H(x)$: all sides of x are equal;
$P(x)$: x has an interior angle that exceeds $180°$;
$Q(x)$: x is a quadrilateral;
$R(x)$: x is a rectangle;
$S(x)$: x is a square;
$T(x)$: x is a triangle.

Translate each of the following statements into an English sentence, and determine whether the statement is true or false.

(a) $\forall x\ (Q(x) \oplus T(x))$;
(b) $\exists x\ (T(x) \wedge P(x))$;
(c) $\exists x\ (Q(x) \wedge \neg R(x))$;
(d) $\forall x\ (H(x) \to E(x))$;
(e) $\forall x\ (S(x) \leftrightarrow (A(x) \wedge H(x)))$.

5. Let $P(x,y), Q(x,y)$ denote the following predicates

$$P(x,y): \quad x^2 \geq y \qquad\qquad Q(x,y): \quad x+2 < y.$$

If the universe for each of x, y consists of all real numbers, determine the truth value for each of the following statements

(a) $P(-3,8) \wedge Q(1,3)$;
(b) $P(\frac{1}{2},\frac{1}{3}) \vee \neg Q(-2,-3)$;
(c) $P(1,2) \leftrightarrow \neg Q(1,2)$.

6. Let $P(x), Q(x)$, and $R(x)$ denote the following predicates

$$P(x): \quad x^2 - 8x + 15 = 0 \qquad Q(x): \quad x \text{ is odd} \qquad R(x): \quad x > 0.$$

If the universe for x consists of all integers, determine the truth value for each of the following statements. If a statement is false, give a counterexample.

(a) $\forall x\ (P(x) \to Q(x))$;
(b) $\exists x\ (P(x) \to Q(x))$;
(c) $\exists x\ (R(x) \to P(x))$;
(d) $\forall x\ ((P(x) \vee Q(x)) \to R(x))$.

7. Write the negation of each of the following statements as an English sentence — without symbolic notation. (Here the universe consists of all the students at the university where Professor Lenhart teaches.)

(a) Every student in Professor Lenhart's C++ class is majoring in computer science or mathematics.
(b) At least one student in Professor Lenhart's C++ class is a history major.