# MACM 101 — Discrete Mathematics I

## Exercises on Sets and Relations. Due: Friday, October 24th (at the beginning of the class)

**Reminder: the work you submit must be your own. Any collaboration and consulting outside resources must be explicitly mentioned on your submission.**
**Please, use a pen. 30 points will be taken off for pencil written work.**

1. Using laws of set theory show that

$$\overline{(A \cup B) \cap C} = (\overline{A} \cup \overline{C}) \cap (\overline{B} \cup \overline{C}).$$

2. Let $A$, $B$, and $C$ be sets. Show that

$$(\overline{A} \cup B) \cap (\overline{C} - A) = \overline{C} - A.$$

   Draw Venn diagrams for the expressions on both sides.

3. What can you say about the sets $A$ and $B$ if we know that $\overline{A \cap B} = \overline{B}$?

4. Show that for any sets $A$, $B$, and $C$

$$(A \triangle B) \triangle C = A \triangle (B \triangle C).$$

5. Make a list of pairs, construct the matrix, and draw the graph of the relation $R$ from the set $A = \{0, 1, 2, 3, 4\}$ to the set $B = \{0, 1, 2, 3\}$ such that $(a, b) \in R$ if and only if $a - b < 1$.

6. Prove that

$$C \times B \times (A \cup C) = (C \times B \times A) \cup (C \times B \times C).$$

7. Let $R$ be the relation on $\mathbb{Z} \times \mathbb{Z}$, that is elements of this relation are pairs of pairs of integers, such that $((a, b), (c, d)) \in R$ if and only if $a - d = c - b$. Show that $R$ is an equivalence relation.

8. Relation $R$ is given by matrix

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}.$$

   Is $R$ an order? If yes, what its minimal, maximal, least, and greatest elements are?

9. Let $A = \{1, 2, 3, 4\}$, and let $R$ be a binary relation on $A \times A$ given by: $((a, b), (c, d)) \in R$ if and only if $a$ divides $c$ and $b$ divides $d$. Show that $R$ is an order and draw its diagram.

10. Give an example of a relation that is not reflexive, symmetric, not transitive, and not antisymmetric.