MACM 101 — Discrete Mathematics I

Exercises on Combinatorics and Probability. Due: Friday, November 21st (at the beginning of the class)

Reminder: the work you submit must be your own. Any collaboration and consulting outside resources must be explicitly mentioned on your submission.
Please, use a pen. 30 points will be taken off for pencil written work.

1. The letters ABCDEFGH are to be used to form strings of length 5. How many strings contain the letter A if repetitions are allowed?

2. There are five distinct computer science books, three distinct mathematics books, and two distinct art books. In how many ways can these books be arranged on a shelf if no two of the three mathematics books are together?

3. There are five distinct computer science books, three distinct mathematics books, and two distinct art books. In how many ways can these books be arranged on a shelf if one of the art books is on the left of all the mathematics books, and the other art book is on the right of all the mathematics books?

4. How many 5-cards poker hands are there containing all 4 suits? (Recall that a full deck of cards contains 52 cards, 13 distinct cards of each suit.)

5. Let n and r be positive integers with $n \geq r$. Prove that
$$\binom{r}{r} + \binom{r+1}{r} + \cdots + \binom{n}{r} = \binom{n+1}{r+1}$$

6. How many integers between 1 and 1,000,000 have the sum of the digits equal to 15?

7. How many strings can be formed by ordering the letters SALESPERSONS if no two S's are consecutive?

8. A 3×7 rectangle is divided into 21 squares each of which is colored red or black. Prove that the board contains a nontrivial rectangle (not $1 \times k$ or k) whose four corner squares are all black or all red.

9. An unprepared student takes a 10-question multiple choice quiz (4 given answers for each question, of which only one is correct) and guesses at the answer to every question. What is the probability that the student answers exactly five questions correctly?

10. A pair of dice is loaded. The probability that a 2 appears on the first die is 3/13 and the probability that a 4 appears on the second die is 3/13. Other outcomes for each die appear with probability 2/13. What is the probability of 6 appearing as the sum of the numbers when the two dice are rolled?