Problems to Week 13 Tutorial — MACM 101 (Fall 2014)

1. Determine the quotient q and the remainder r for each of the following, where a is the dividend and b is the divisor.
   (a) $a = 123$, $b = 17$; (b) $a = -115$, $b = 12$;
   (c) $a = 0$, $b = 42$; (d) $a = 434$, $b = 31$.

2. Write each of the following 10-base numbers in base 2 and 16
   (a) 137; (b) 6243; (c) 12345.

3. Convert each of the following binary numbers to base 10
   (a) 11001110; (b) 00110001.

4. For positive integers a, b and $d = \gcd(a, b)$, prove that
$$\gcd\left(\frac{a}{d}, \frac{b}{d}\right) = 1.$$

5. Let n be a positive integer. Prove that $\gcd(n, n + 2)$ equals 1 or 2.

6. Find the greatest common divisor of 168 and 456, and also numbers u, v such that $gcd(168, 456) = 168u + 456v$.

7. Find the greatest common divisor and the least common multiple of 630 and 40452.

8. Find the prime factorization of
   (a) 148500; (b) 7114800; (c) 7882875.

9. How many positive divisors are there for
$$n = 2^{14}3^9 5^8 7^{10} 11^3 13^5 37^{10}.$$

10. A perfect square is a number n such that $n = k^2$ for some integer k. Determine the smallest perfect square that is divisible by 7!.