Problems to Week 11 Tutorial — MACM 101 (Fall 2014)

1. The college food plan allows a student to choose three pieces of fruit every day. The fruits available are apples, bananas, peaches, pears, and plums. For how many days can a student make a different selection?

2. (a) How many ways can a student choose 8 out of 10 questions to answer on an exam?
   (b) How many ways can a student choose 8 out of 10 questions to answer on an exam if the first three questions must be answered?

3. Three fair six-sided dice are tossed and the numbers showing on top are recorded.
   (a) How many different record sequences are possible?
   (b) How many of the records contain exactly one six?
   (c) How many of the records contain exactly 2 fours?

4. How many ways can you choose three of seven fiction books and two of six nonfiction books to take with you on your vacation?

5. In how many ways can 15 (identical) candy bars be distributed among five children? with the restriction that the youngest gets only one or two?

6. In how many ways can Lisa toss 100 (identical) dice so that at least three of each type of face will be showing?

7. Determine the coefficient of x^9y^3 in the expansion of (a) $(x+y)^{12}$, (b) $(x+2y)^{12}$, (c) $(2x-3y)^{12}$, (d) $(x-y)^{14}$.

8. Show that for all positive integers m and n,
$$n \cdot \left(\begin{array}{c} m+n \\ m \end{array} \right) = (m+1) \cdot \left(\begin{array}{c} m+n \\ m+1 \end{array} \right).$$

9. With n a positive integer, evaluate the sum
$$\left(\begin{array}{c} n \\ 0 \end{array} \right) - 3 \left(\begin{array}{c} n \\ 1 \end{array} \right) + 3^2 \left(\begin{array}{c} n \\ 2 \end{array} \right) + \ldots + (-1)^n 3^n \left(\begin{array}{c} n \\ n \end{array} \right).$$

10. For every positive number n, show that
$$\left(\begin{array}{c} n \\ 0 \end{array} \right) + \left(\begin{array}{c} n \\ 2 \end{array} \right) + \left(\begin{array}{c} n \\ 4 \end{array} \right) + \ldots = \left(\begin{array}{c} n \\ 1 \end{array} \right) + \left(\begin{array}{c} n \\ 3 \end{array} \right) + \left(\begin{array}{c} n \\ 5 \end{array} \right) + \ldots$$