Problems to Week 3 Tutorial — MACM101 (Fall 2014)

1. Use truth tables to verify absorption law.
2. Show that $(p \to q) \wedge (q \to r) \to (p \to r)$ is a tautology (use equivalences).
3. Show that the following compound statements are logically equivalent:
   - $(p \to r) \wedge (q \to r)$ and $(p \vee q) \to r$;
   - $\neg p \to (q \to r)$ and $q \to (p \vee r)$.
4. Is $(p \vee q) \to (q \to (p \wedge q)$ a contradiction?
5. Verify that
$$(p \leftrightarrow q) \wedge (q \leftrightarrow r) \wedge (r \leftrightarrow p) \Leftrightarrow (p \to q) \wedge (q \to r) \wedge (r \to p).$$
6. Negate the following statement and simplify the result $p \vee q \vee (\neg p \wedge \neg q \wedge r)$.
7. Let "Nand" be the logic connective defined by $p \uparrow q \iff \neg(p \wedge q)$. Express $\neg, \vee, \wedge$ using only Nand. (difficult problem, not for everyone)
8. Verify that the Rule of Disjunctive Syllogism is a valid argument. (Use the corresponding tautology.)
9. Verify that the following is a tautology by showing that it is impossible for the conclusion to have truth value 0 while the premises have truth value 1:
$$((p \to q) \wedge (r \to s) \wedge (p \vee r)) \to (q \vee s).$$