Problems to Week 6 Tutorial — MACM 101 (Fall 2013)

1. Let $A = \{1, \{1\}, \{2\}\}$. Which of the following statements are true?

a)	$1 \in A$	*b)*	$\{1\} \in A$
c)	$\{1\} \subseteq A$	*d)*	$\{\{1\}\} \subseteq A$
e)	$\{2\} \in A$	*f)*	$\{2\} \subseteq A$
g)	$\{\{2\}\} \subseteq A$		

2. For $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ let $A = \{1, 2, 3, 4, 5\}$, $B = \{1, 2, 4, 8\}$, $C = \{1, 2, 3, 5, 7\}$, and $D = \{2, 4, 6, 8\}$. Determine each of the following:

a)	$(A \cup B) \cap C$	*b)*	$A \cup (B \cap C)$
c)	$\overline{C} \cup \overline{D}$	*d)*	$\overline{C \cap D}$
e)	$(A \cup B) - C$	*f)*	$A \cup (B - C)$
g)	$(B - C) - D$	*h)*	$B - (C - D)$
i)	$(A \cup B) - (C \cap D)$		

   Draw Venn diagrams for each of the expressions.

3. Prove each of the following using Venn diagrams. (Assume a universe U.)

   (a) If $A \subseteq B$ and $C \subseteq D$, then $A \cap C \subseteq B \cap D$ and $A \cup C \subseteq B \cup D$.

   (b) $A \subseteq B$ if and only if $A \cap \overline{B} = \emptyset$.

   (c) $A \subseteq B$ if and only if $\overline{A} \cup B = U$.

4. Prove or disprove each of the following:

   (a) For sets $A, B, C \subseteq U$, if $A \cup C = B \cup C$, then $A = B$.

   (b) For sets $A, B, C \subseteq U$, if $A \cup C = B \cup C$ and $A \cap C = B \cap C$, then $A = B$.

5. Prove that $A - B = A \cap \overline{B}$.

6. Prove that $A \Delta B = (\overline{A} \cap B) \cup (A \cap \overline{B})$.