

**Problem 1.**

$$\{a \mid a \in \mathbb{Z}, -3 \leq a \leq 11\}$$

**Problem 2.**

$$\{\emptyset, \{7\}, \{8\}, \{9\}, \{7, 8\}, \{7, 9\}, \{8, 9\}, \{2, 8, 9\}\}$$

**Problem 3.**

- a. The set of even natural numbers:  $E = \{0, 2, 4, 6, \dots\}$ 
  1.  $0 \in E$
  2. If  $n \in \mathbb{N}$  and  $\text{mod}(n, 2) = 0$  then  $n \in E$
  3. Nothing else is in  $E$
- b. The set of natural numbers that when divided by 7 have a remainder of 3 :  $R = \{3, 10, 17, 24, \dots\}$ 
  1.  $3 \in R$
  2.  $n \in \mathbb{N}$  and  $\text{mod}(n, 7) = 3$  then  $n \in E$
  3. Nothing else is in  $E$
- c. The set of positive integers whose decimal representations contain only the digit 5 :  $F = \{5, 55, 555, 5555, \dots\}$ 
  1.  $5 \in F$
  2. If  $n \in F$  then  $10n + 5 \in F$
  3. Nothing else is in  $F$
- d. The set of positive integers whose decimal representations contain only the digits 5 and 7:  $S = \{5, 7, 55, 57, 75, 77, \dots\}$ 
  1.  $5 \in S$  and  $7 \in S$
  2. If  $n \in S$  then  $10n + 5$  and  $10n + 7 \in F$
  3. Nothing else is in  $F$

**Problem 4.**

- a. The set of natural numbers that contain only the repeated digit sequence 345 :  $A = \{345, 345345, 345345345, \dots\}$ 
  1.  $345 \in A$
  2. If  $n \in A$  then  $1000n + 345 \in A$
  3. Nothing else is in  $A$
- b. The set of natural numbers that are not multiples of 4 :  $B = \{1, 2, 3, 5, 6, 7, 9, 10, 11, \dots\}$ 
  1.  $n \in \mathbb{N}$  and  $\text{mod}(n, 4) \neq 0$  then  $n \in B$
  2. Nothing else is in  $B$
- c. The set of natural numbers that are powers of 2 :  $C = \{1, 2, 4, 8, 16, \dots\}$ 
  1.  $1 \in C$
  2. If  $n \in \mathbb{N}$  then  $2n \in C$
  3. Nothing else is in  $C$
- d. The set of natural numbers that contain an odd number of occurrences of the digit 4 :  $D = \{4, 444, 44444, \dots\}$ 
  1.  $4 \in D$
  2. If  $n \in D$  then  $100n + 44 \in D$
  3. Nothing else is in  $D$
- e. The set of integers that are both negative and odd.  $E = \{-1, -3, -5, \dots\}$ 
  1.  $-1 \in E$
  2. If  $n \in E$  then  $n - 2 \in E$
  3. Nothing else is in  $E$