(anti)Reflexive Relations

Let $A = \{1, 2, 3, 4\}$

Consider the following binary relations on A

$R_0 = \{(1, 1), (2, 2), (3, 3), (4, 4)\}$ is reflexive

$R_3 = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (3, 3), (4, 1), (4, 4)\}$ is reflexive

$R_2 = \{(1, 1), (3, 3)\}$ is not reflexive

R_0	1	2	3	4
1	1	0	0	0
2	0	1	0	0
3	0	0	1	0
4	0	0	0	1

R_3	1	2	3	4
1	1	1	0	1
2	1	1	0	0
3	0	0	1	0
4	1	0	0	1

R_2	1	2	3	4
1	1	0	0	0
2	0	0	0	0
3	0	0	1	0
4	0	0	0	0

A binary relation R on the set A

is antireflexive exactly when

for each $a \in A$ it is not the case that $(a, a) \in R$

Example

The following binary relations on $\{1, 2, 3, 4\}$ are antireflexive

$\{\} = \emptyset$

$\{(1, 2), (1, 4), (2, 1), (4, 1)\}$

	1	2	3	4
1	1	1	0	0
2	0	0	1	0
3	0	0	0	0
4	0	0	0	0

(anti)Symmetric Relations

Symmetry

Let A be a set and 12 be a binary relation on A. R is symmetric exactly when for each $a \in A$ and $b \in A$, $R(a, b)$ if andy only if $R(b, a)$

Example

The follwing relations on $\{1, 2, 3, 4\}$

$R_0 = \{(1, 1), (1, 2), (2, 1)\}$ is symmetric

$R_1 = \{(1, 1), (4, 3)\}$ is not symmetric

R_0	1	2	3	4
1	**1**	1	0	0
2	1	**0**	0	0
3	0	0	**0**	0
4	0	0	0	**0**

R_1	1	2	3	4
1	**1**	0	0	0
2	0	**0**	0	0
3	0	0	**0**	0
4	0	0	1	**0**

Antisymmetry

Let A be a set and R be a binary relation on A. R is antisymmetric exactly when for each $a \in A$ and $b \in B$, if $R(a, b)$ and $R(b, a)$ then $a = b$

Example

Let $A = \{1, 2, 3, 4\}$

$\{\} = \emptyset$ is antisymmetric

$R_0 = \{(1, 1), (2, 1), (3, 1), (3, 2), (4, 1), (4, 2), (4, 3)\}$ is antisymmetric

$R_1 = \{(2, 3)\}$

R_0	1	2	3	4
1	**1**	0	0	0
2	1	**0**	0	0
3	1	1	**0**	0
4	1	1	1	**0**

R_1	1	2	3	4
1	**0**	0	0	0
2	0	**0**	1	0
3	0	0	**0**	0
4	0	0	0	**0**

Both Symmetric and Antisymmetric

If all elements are self loops $\{(a,a),(b,b)\}$

$\{1,2,3,4\}$

$\{\} = \emptyset$

$\{(1,1)\}$

$\{(1,1),(2,2)\}$

Neither Symmetric or AntiSymmetric

$\{(1,2),(3,4),(4,3)\}$

Transative Relations

Let A be a set and let R be a binary relation on A. R is transative exactly when for each $a,b,c \in A$, if $R(a,b)$ and $R(b,a)$ then $R(a,c)$

Examples

The follwing binary relations on {1,2,3,4} are transative

$R_0 = \{(2,1),(3,1),(3,2),(4,1),(4,2),(4,3)\}$