Recursive Definition Review

$\mathbb{N} = \{0, 1, 2, 3, ...\}$

1. $O \in \mathbb{N}$
2. If $n \in \mathbb{N}$ then ntrl $\in \mathbb{N}$
3. Nothing else is in $\mathbb{N}$

$F = \{5, 55, 555, ...\}$

1. ...
2. If $n \in F$ then n5 $\in F$
3. ...

Functions

Let A and B be non-empty sets

A function f from A to B assigns each element of A to exactly one element of B

$f : A \to B$ means "f is a function from A to B"

$f(a)$ denotes (represents) the member of B assigned to a $(a \in A)$ by f

When $f : A \to B$

- f <u>maps</u> A to B
- A is the <u>domain</u> of f
- B is the <u>co-domain</u> of f

When $f(a) = b$

- b is the <u>image</u> of a under f
- a is the <u>pre-image</u> of b under f

When $f : A \to B$
- The <u>range of f</u> is the set: $\{f(a) \mid a \in A\}$

Let $f : \{1, 2\} \to \{1, 2\}$ where $f(1) = 1$ and $f(2) = 1$

range of f is $\{1\}$

Specifying Functions

1. Formula
   - $f : \mathbb{N} \to \mathbb{N}$
   - $f(x) = x + 5$
   - $g : \mathbb{Z} \to \mathbb{R}$
     - $g(x) = \frac{\sin(x)}{2}$
2. Rules
   - $f : \mathbb{Z} \to \mathbb{Z}$
   - $f(x) = \{-1 \text{ if } x < 0; 0 \text{ if } x = 0; 1 \text{ if } x > 1\}$
3. Ordered Pairs
   - $f : \mathbb{N} \to \mathbb{N}$
   - $f(x) = x^2$
   - The <u>graph of</u> f: $\{(0,0), (1,1), (2,4), (3,9), ...\} = \{(a,b) \mid f(a) = b\}$

The and ceil functions

floor/ceil $: \mathbb{R} \to \mathbb{Z}$

floor$(x) =$ the largest integer less than or equal to x