Problem 1

Let $A = \{1, 4, 8, 16\}$ and $B = \{2, 4, 16, 32, 64\}$

$A \cup B$

$A \cup B = \{1, 2, 4, 8, 16, 32, 64\}$

$A \cap B$

$A \cap B = \{4, 16\}$

$A - B$

$A - B = \{1, 8\}$

$B - A$

$B - A = \{2, 32, 64\}$

$|\mathcal{P}(A)|$

$|\mathcal{P}(A)| = 16$

$|\mathcal{P}(S)| = 2^{|S|}$

Problem 2

Prove each of the following by building a membership table

$(A \cap B) \cup \left(A \cap \overline{B}\right) = A$

A	B	$\overline{B}$	$A \cap B$	$A \cap \overline{B}$	$(A \cap B) \cup \left(A \cap \overline{B}\right)$
1	1	0	1	0	1
1	0	1	0	1	1
0	1	0	0	0	0
0	0	1	1	0	0

$\overline{A - (B \cup C)} = \overline{A} \cup (B \cup C)$

A	B	C	$\overline{A}$	$(B \cup C)$	$A - (B \cup C)$	$\overline{A - (B \cup C)}$	$\overline{A} \cup (B \cup C)$
1	1	1	0	1	0	1	1
1	1	0	0	1	0	1	1
1	0	1	0	1	0	1	1
1	0	0	0	0	1	0	0
0	1	1	1	1	0	1	1
0	1	0	1	1	0	1	1
0	0	1	1	1	0	1	1
0	0	0	1	0	0	1	1

$A \cap (B - C) = (A \cap B) - C$

A	B	C	$(B-C)$	$(A \cap B)$	$A \cap (B-C)$	$(A \cap B) - C$
1	1	1	0	1	0	0
1	1	0	1	1	1	1
1	0	1	0	0	0	0
1	0	0	0	0	0	0
0	1	1	0	0	0	0
0	1	0	1	0	0	0
0	0	1	0	0	0	0
0	0	0	0	0	0	0

$(A \cup B) - C = (A - C) \cup (B - C)$

A	B	C	$(A \cup B)$	$(A-C)$	$(B-C)$	$(A \cup B) - C$	$(A-C) \cup (B-C)$
1	1	1	1	0	0	0	0
1	1	0	1	1	1	1	1
1	0	1	1	0	0	0	0
1	0	0	1	1	0	1	1
0	1	1	1	0	0	0	0
0	1	0	1	0	1	1	1
0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0

Problem 3→ DNE?

Problem 4

Complete the recursive definition of each of the following sets

$A = \{(x, y, z) | x \in \mathbb{N}, y \in \mathbb{N}, z \in \mathbb{N} \text{ and } x = y = z\}$

1. $(0, 0, 0) \in A$
2. If $(n, n, n) \in A$ then $(n + 1, n + 1, n + 1) \in A$
3. Nothing else in A

$B = \{(x, y) | x \in \mathbb{N}, y \in \mathbb{N} \text{ and } y = x^2\}$

Do not use multiplication or exponentiation in the definition. Hint: note that $(x + 1)^2 = x^2 + x + x + 1$

1. $(0, 0) \in B$
2. If $(x, y) \in B$ then $(x + 1, x^2 + x + x + 1) \in B$
3. Nothing else in B

$C = \{(x, y) | x \in \mathbb{N}, y \in \mathbb{N} \text{ and } y \text{ is a multiple of } x\}$

1. $(x, 0) \in C$ for each $x \in \mathbb{N}$
2. If $(x, y) \in C$ then $(x, y + x) \in C$
3. Nothing else in C

Problem 5

Give the recursive definition of the following sets

$A = \{(x, y) | x \in \mathbb{N}, y \in \mathbb{N}, x \text{ is a multiple of } 2 \text{ and } y \text{ is a multiple of } 3\}$

1. $(0, 0) \in A$
2. If $\frac{x}{2} \in \mathbb{N}$ and $\frac{y}{3} \in \mathbb{N}$ then $(x + 2, y + 3) \in A$
3. Nothing else in A

$B = \{(a, b) | a, b \in \mathbb{N} \text{ and } b - a = 11\}$

1. $(11, 0) \in B$
2. If $(x, y) \in B$ then $(x + 1, y + 1) \in B$
3. Nothing else in B