Composition of Functions

Let $f : A \to B$ and $g : B \to C$

Create a new function $h : A \to C$

$h(a) = g(f(a))$

h is the composition of f and g

$h = g \circ f$

Example

$A = \{a, b, c\}$

$B = \{1, 2, 3, 4\}$

$C = \{\text{bear}, \text{cat}, \text{dog}\}$

placeholder for image

$f : A \to B$ $g : B \to C$

$g \circ f : A \to C$

$g \circ f(a) = \text{bear}$

$g \circ f(b) = \text{cat}$

$g \circ f(c) = \text{dog}$

Example

$f : \mathbb{Z} \to \mathbb{Z}$ $g : \mathbb{Z} \to \mathbb{Z}$

$f(x) = 2x + 3$ $g(x) = 3x + = 2$

$g \circ f(x) = g(f(x)) = 3f(x) + 2$

$= 3(2x + 3) + 2$

$= 6x + 9 + 2$

$= 6x + 11$

$f \circ g(x) = f(g(x))$

$= 2g(x) + 3$

$2(3x + 2) + 3$

$= 6x + 4 + 3$

$= 6x + 7$

Inverse of Function Compositions

Let $f : A \to B$ and $g : B \to C$ eachs be invertible

$(g \circ f)^{-1}(y) = x$ when $g \circ f(x) = y$

$g(f(x)) = y$

$g^{-1}(g(f(x))) = g^{-1}(y)$

$f(x) = g^{-1}(y)$

$f^{-1}(f(x)) = f^{-1}(g^{-1}(y))$

$x = f^{-1}(g^{-1}(x))$

$(g \circ f)^{-1} = f^{-1} \circ g^{-1}$

Binary Relations

A binary relation from set A to set B is a subset of $A \times B$

If R is a binary relation from set A to set B, the following are synonyms of $(a, b) \in R$

aRb

$R(a, b)$

"a is related to b by R"

Example

Let A be a set of students and B be a set of courses

$R = \{(a, b) | a \in A, b \in B, \text{and student } a \text{ is enrolled in course } b\}$

Binary relations can be displayed using a table

Example

Let $A = \{0, 1, 2\}$, $B = \{a, b\}$

and $R = \{(0, a), (0, b), (1, a), (2, b)\}$

	A		
	R	a	b
	0	1	1
B	1	1	0
	2	0	1