Logical Equivalence

Propositions A and B are logically equivalent, $A = B$ exactly when $A \leftrightarrow B$ is a totalogy.

A recursive definition ofor the set of logical propositions, P

1. $T \in P, F \in P, p \in P$ where p is a propostional variable.
2. If $x, y \in P$, then $\neg \in P, x \wedge y \in P, x \vee y \in P, x \rightarrow y \in P, x \leftrightarrow \in P$
3. Nothing else is in P

$\equiv$ is a binary relation on P

$\equiv \{(A, B) | A, B \in P \text{ and } A \leftrightarrow B\}$ is a totalogy

$\equiv$ is reflexive, symetric, transitive

De Morgan's Laws

$\neg(A \wedge B) \equiv (\neg A) \vee \neg B$

$\neg(A \vee B) \equiv (\neg A) \wedge (\neg B)$

A	B	$\neg A$	$\neg B$	$A \wedge B$	$\neg(A \wedge B)$	$(\neg A) \vee \neg B$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

Logical Equivalence	Name
$A \wedge T \equiv A$ $A \vee F \equiv A$	Identity Laws
$A \vee T \equiv T$ $A \wedge F \equiv F$	Domination Laws
$A \vee A \equiv A$ $A \wedge A \equiv A$	Idempotent Laws
$\neg\neg A \equiv A$	Double Negative Law
$A \vee B \equiv B \vee A$ $A \wedge B \equiv B \wedge A$	Commutative Laws
$(A \vee B) \vee C \equiv A \vee (B \vee C)$ $(A \wedge B) \wedge C \equiv A \wedge (B \wedge C)$	Associative Laws
$A \vee (B \wedge C) \equiv (A \vee B) \wedge (A \vee C)$ $A \wedge (B \vee C) \equiv (A \wedge B) \vee (A \wedge C)$	Distributive Laws
$\neg(A \wedge B) \equiv (\neg A) \vee \neg B$ $\neg(A \vee B) \equiv (\neg A) \wedge (\neg B)$	De Morgan's Laws
$A \vee (A \wedge B) \equiv A$ $A \wedge (A \vee B) \equiv A$	Absorption Laws
$A \vee \neg A \equiv T$ $A \wedge \neg A \equiv F$	Complement Laws
$A \rightarrow B \equiv (\neg A) \vee B$ $A \leftrightarrow B \equiv (A \rightarrow B) \wedge (B \rightarrow A)$	Conditional Identities