

## Recursive Definition Review

$$\mathbb{N} = \{0, 1, 2, 3, \dots\}$$

1.  $0 \in \mathbb{N}$
2. If  $n \in \mathbb{N}$  then  $n+1 \in \mathbb{N}$
3. Nothing else is in  $\mathbb{N}$

$$F = \{5, 55, 555, \dots\}$$

1. ...
2. If  $n \in F$  then  $n5 \in F$
3. ...

## Functions

Let  $A$  and  $B$  be non-empty sets

A function  $f$  from  $A$  to  $B$  assigns each element of  $A$  to exactly one element of  $B$

$f : A \rightarrow B$  means “ $f$  is a function from  $A$  to  $B$ ”

$f(a)$  denotes (represents) the member of  $B$  assigned to a ( $a \in A$ ) by  $f$

When  $f : A \rightarrow B$

- $f$  maps  $A$  to  $B$
- $A$  is the domain of  $f$
- $B$  is the co-domain of  $f$

When  $f(a) = b$

- $b$  is the image of  $a$  under  $f$
- $a$  is the pre-image of  $b$  under  $f$

When  $f : A \rightarrow B$

- The range of  $f$  is the set:  $\{f(a) \mid a \in A\}$

Let  $f : \{1, 2\} \rightarrow \{1, 2\}$  where  $f(1) = 1$  and  $f(2) = 1$

range of  $f$  is  $\{1\}$

## Specifying Functions

1. Formula

- $f : \mathbb{N} \rightarrow \mathbb{N}$
- $f(x) = x + 5$
- $g : \mathbb{Z} \rightarrow \mathbb{R}$ 
  - $g(x) = \frac{\sin(x)}{2}$

2. Rules

- $f : \mathbb{Z} \rightarrow \mathbb{Z}$
- $f(x) = \{-1 \text{ if } x < 0; 0 \text{ if } x = 0; 1 \text{ if } x > 0\}$

3. Ordered Pairs

- $f : \mathbb{N} \rightarrow \mathbb{N}$
- $f(x) = x^2$
- The graph of  $f$ :  $\{(0, 0), (1, 1), (2, 4), (3, 9), \dots\} = \{(a, b) \mid f(a) = b\}$

## **The and ceil functions**

floor/ceil :  $\mathbb{R} \rightarrow \mathbb{Z}$

floor( $x$ ) = the largest integer less than or equal to  $x$