# Natural Numbers

[1]

```python
n = 0
while True:
  print(n)
  n = n+1
```

# A recursive (inductive) definition of natural Numbers

1. $O \in \mathbb{N}$
2. if $n \in \mathbb{N}$ then natural numbers $\in \mathbb{N}$
3. Nothing else is in $\mathbb{N}$

$S = \{-1, 0, 1, 2, ...\}$

# Another recursive definition of $\mathbb{N}$ (natural numbers)

1. $O \in \mathbb{N}$ and $i \in \mathbb{N}$
2. If $n \in \mathbb{N}$ then $n + 2 \in \mathbb{N}$

# A recursive definition of $\mathbb{Z}$ (all integers)

1. $-1 \in \mathbb{Z}$ and $0 \in \mathbb{Z}$
2. If $x \in \mathbb{Z}$ and $x < 0$ then $x - 1 \in \mathbb{Z}$; If $x \in \mathbb{Z}$ and $x \geq 0$ then $x + 1 \in \mathbb{Z}$
3. Nothings else in $\mathbb{Z}$

# A recursive definition of the set of fully paranthesized arithmetic *expressions*[1] on the real numbers, $A$

1. If $X \in \mathbb{R}$ then $x \in A$
2. If $x, y \in A$ then $(-x) \in A$; $(x + y) \in A$, $(x - y) \in A$, $(x * y) \in A$
3. Nothing else in $A$

$(-((9.5 - 2)x(100 - 4)))$

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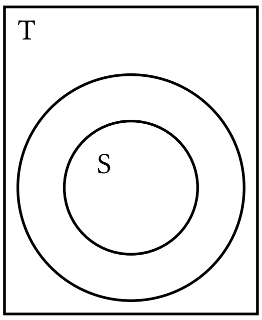
If $S$ and $T$ are sets, then $S = T$ exactly when $S$ and $T$ have the same elements

If $S$ and $T$ are sets then $S$ is a subset of $T$ exactly when each element of $S$ is also an element of $T$

$S$ "is a subset of" $T$ is abbreviated as $S \subseteq T$




$S \subseteq T$

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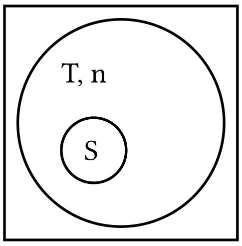
[1] *italicized* - bad handwriting from prof

$S$ is a proper subset of $T$ exactly when $S \subseteq T$ and $S \neq T$

"$s$ is a proper subset of $T$" is abbreviated as $S \subset T$




$S \subset T$

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