Natural Numbers

[1]

```
n = 0
while True:
  print(n)
  n = n+1
```

A recursive (inductive) definition of natural Numbers

1. $0 \in \mathbb{N}$
2. if $n \in \mathbb{N}$ then natural numbers $\in \mathbb{N}$
3. Nothing else is in $\mathbb{N}$

$S = \{-1, 0, 1, 2, \ldots\}$

Another recursive definition of $\mathbb{N}$ (natural numbers)

1. $0 \in \mathbb{N}$ and $i \in \mathbb{N}$
2. If $n \in \mathbb{N}$ then $n + 2 \in \mathbb{N}$

A recursive definition of $\mathbb{Z}$ (all integers)

1. $-1 \in \mathbb{Z}$ and $0 \in \mathbb{Z}$
2. If $x \in \mathbb{Z}$ and $x < 0$ then $x - 1 \in \mathbb{Z}$; If $x \in \mathbb{Z}$ and $x \geq 0$ then $x + 1 \in \mathbb{Z}$
3. Nothings else in $\mathbb{Z}$

A recursive definition of the set of fully paranthesized arithmetic *expressions*[1] on the real numbers, A

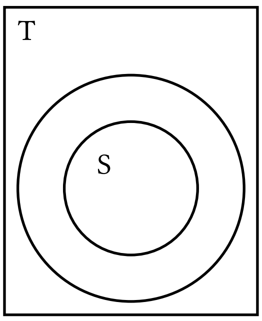
1. If $X \in \mathbb{R}$ then $x \in A$
2. If $x, y \in A$ then $(-x) \in A$; $(x + y) \in A$, $(x - y) \in A$, $(x * y) \in A$
3. Nothing else in A

$(-((9.5 - 2)x(100 - 4)))$

If S and T are sets, then $S = T$ exactly when S and T have the same elements

If S and T are sets then S is a subset of T exactly when each element of S is also an element of T

S "is a subset of" T is abbreviated as $S \subseteq T$



$S \subseteq T$

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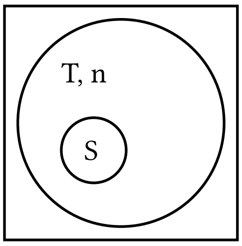
[1] *italicized* - bad handwriting from prof

S is a proper subset of T exactly when $S \subseteq T$ and $S \neq T$

"s is a proper subset of T" is abbreviated as $S \subset T$



$S \subset T$
