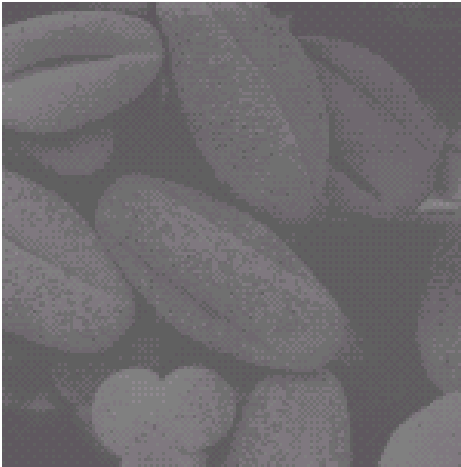


Digital Image Processing

Lecture # 4 **Image Enhancement - II**

Histogram: Example



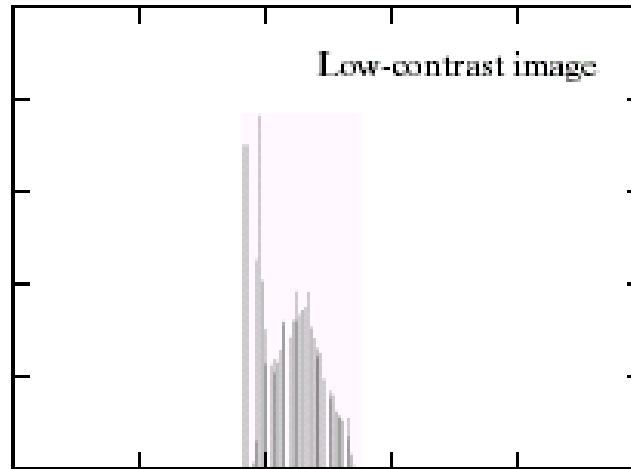
Low Contrast Image

How would the
histograms of these
images look like?



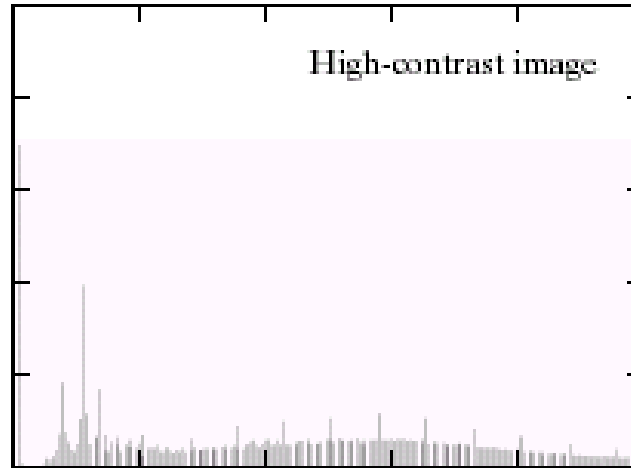
High Contrast Image

Histogram: Example



Low contrast image

Histogram is narrow and centered toward the middle of the gray scale

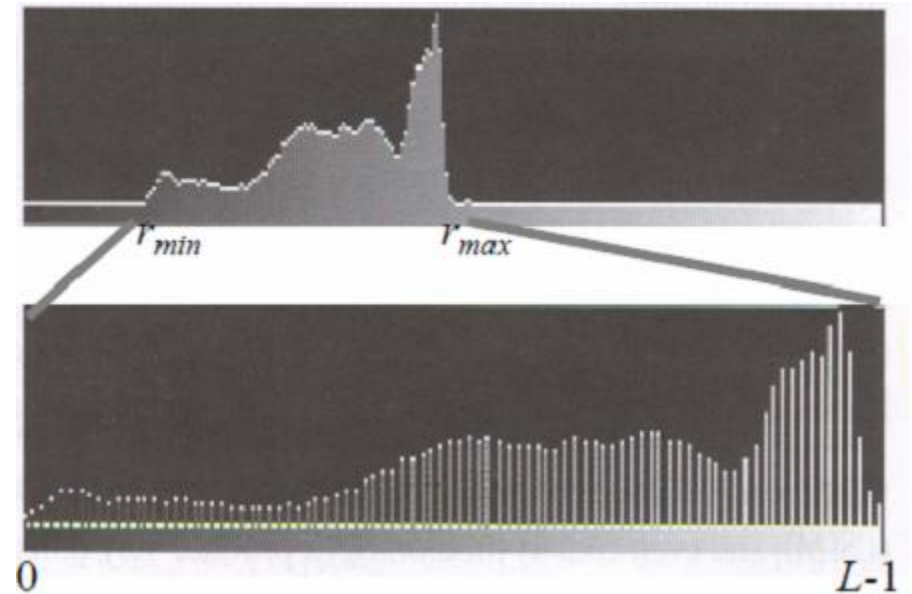


High contrast image

Histogram covers broad range of the gray scale and the distribution of pixels is not too far from uniform with very few vertical lines being much higher than the others

Contrast Stretching

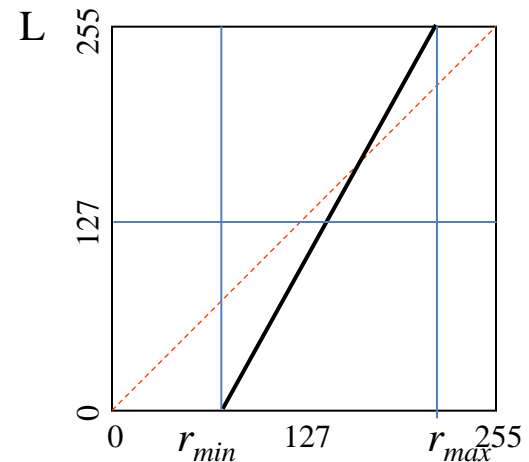
Improve the contrast in an image by 'stretching' the range of intensity values it contains to span a desired range of values, *e.g.* the full range of pixel values



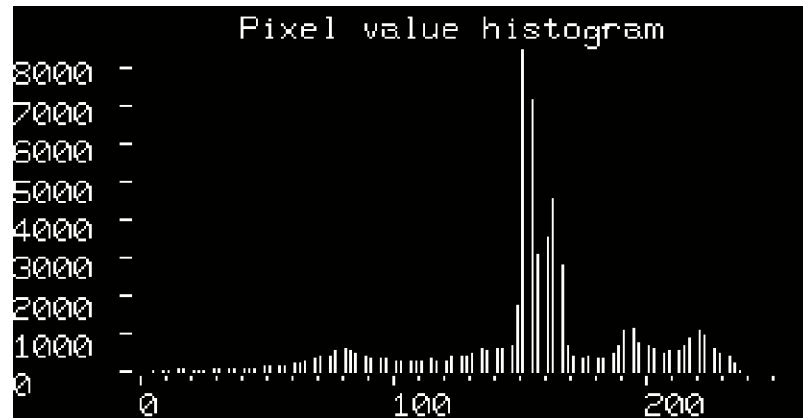
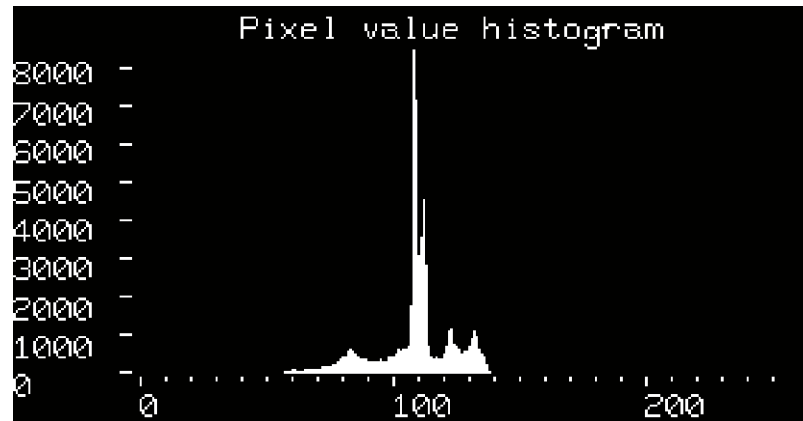
Contrast Stretching

If r_{max} and r_{min} are the maximum and minimum gray level of the input image and L is the total gray levels of output image, the transformation function for contrast stretch will be

$$s = T(r) = (r - r_{min}) \left[\frac{L-1}{r_{max} - r_{min}} \right]$$

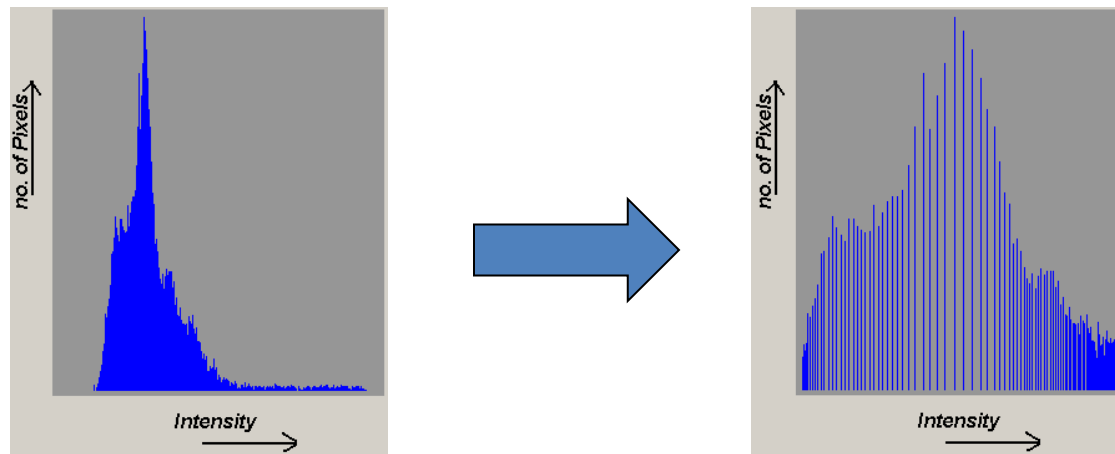


Contrast Stretching

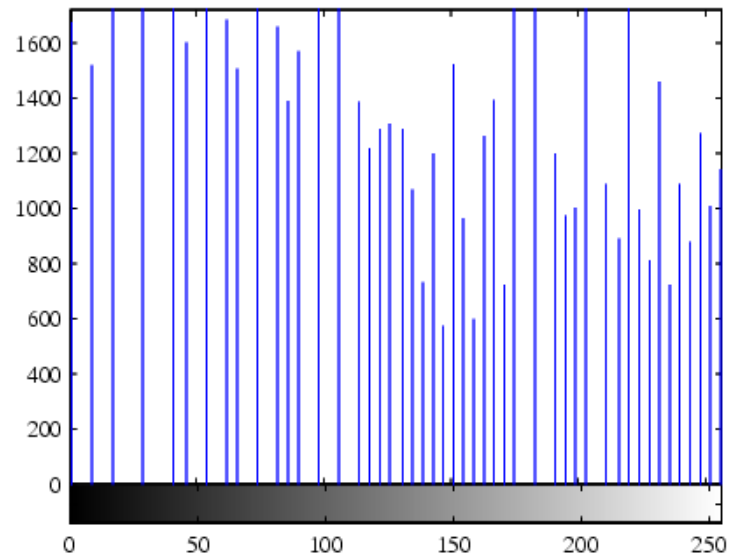
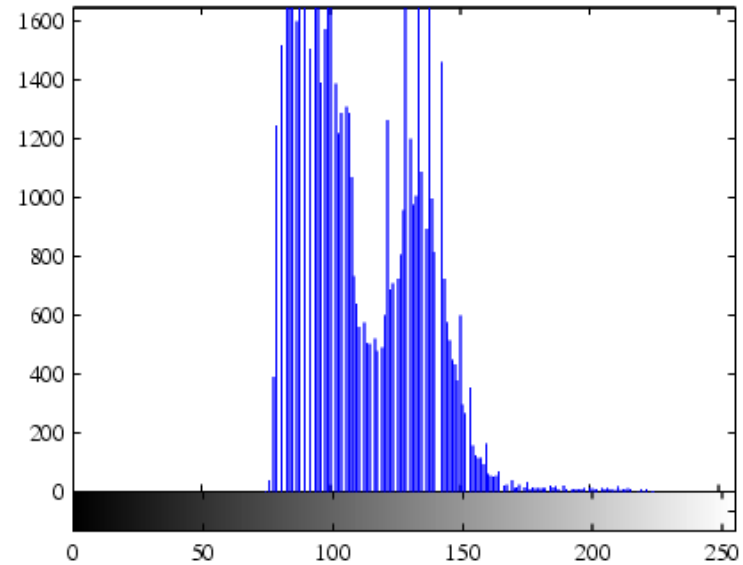


Histogram Equalization

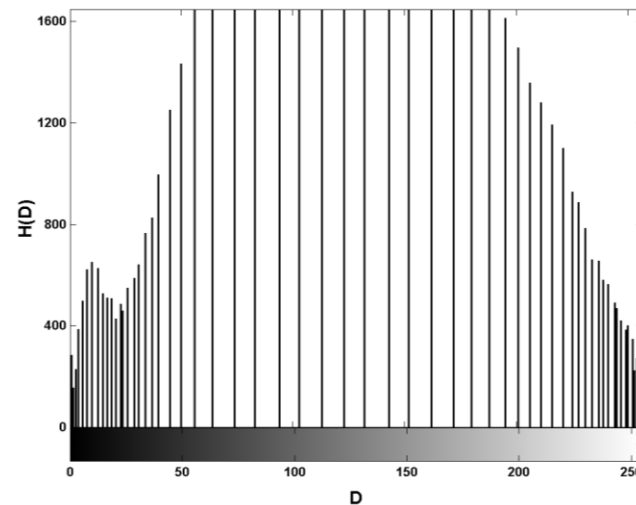
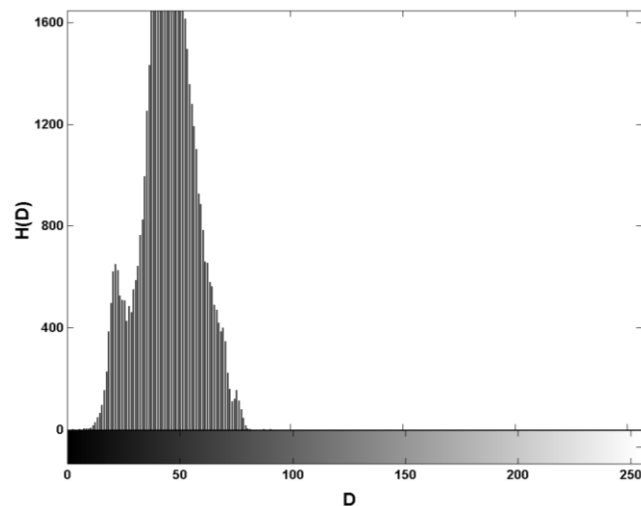
Histogram equalization re-assigns the intensity values of pixels in the input image such that the output image contains a uniform distribution of intensities



HISTOGRAM EQUALIZATION



AERIAL PHOTOGRAPH OF THE PENTAGON



**Resulting image uses more of dynamic range.
Resulting histogram almost, but not completely, flat.**

The Probability Distribution Function of an Image

$$\text{Let } A = \sum_{g=0}^{255} h_I(g)$$

Note that since $h_I(g)$ is the number of pixels in I with value g ,

A is the number of pixels in I . That is if I is R rows by C columns then $A = R \times C$.

Then,

$$p_I(g) = \frac{1}{A} h_I(g)$$

This is the probability that an arbitrary pixel from I has value g .

The Probability Distribution Function of an Image

- $p(g)$ is the fraction of pixels in an image that have intensity value g .
- $p(g)$ is the probability that a pixel randomly selected from the given image has intensity value g .
- Whereas the sum of the histogram $h(g)$ over all g from 0 to 255 is equal to the number of pixels in the image, the sum of $p(g)$ over all g is 1.
- p is the **normalized histogram** of the image

The Cumulative Distribution Function of an Image

Let $q = I(r,c)$ be the value of a randomly selected pixel from I . Let g be a specific gray level. The probability that $q \leq g$ is given by

$$P_I(g) = \sum_{\gamma=0}^g p_I(\gamma) = \frac{1}{A} \sum_{\gamma=0}^g h_I(\gamma) = \frac{\sum_{\gamma=0}^g h_I(\gamma)}{\sum_{\gamma=0}^{255} h_I(\gamma)},$$

where $h_I(\gamma)$ is the histogram of image I .

This is the probability that any given pixel from I has value less than or equal to g .

The Cumulative Distribution Function of an Image

Let $\mathbf{q} = I(r,c)$ be the value of a randomly selected pixel from I . Let g be a specific gray level. The probability that $q \leq g$ is given by

$$P_I(g) = \sum_{\gamma=0}^g p_I(\gamma) = \frac{1}{A} \sum_{\gamma=0}^g h_I(\gamma) = \frac{\sum_{\gamma=0}^g h_I(\gamma)}{\sum_{\gamma=0}^{255} h_I(\gamma)},$$

where $h_I(\gamma)$ is the histogram of image I .

Also called CDF for "Cumulative Distribution Function".

This is the probability that any given pixel from I has value less than or equal to g .

The Cumulative Distribution Function of an Image

- $P(g)$ is the fraction of pixels in an image that have intensity values less than or equal to g .
- $P(g)$ is the probability that a pixel randomly selected from the given band has an intensity value less than or equal to g .
- $P(g)$ is the cumulative (or running) sum of $p(g)$ from 0 through g inclusive.
- $P(0) = p(0)$ and $P(255) = 1$;

Histogram Equalization

Task: remap image I so that its histogram is as close to constant as possible

Let $P_I(\gamma)$

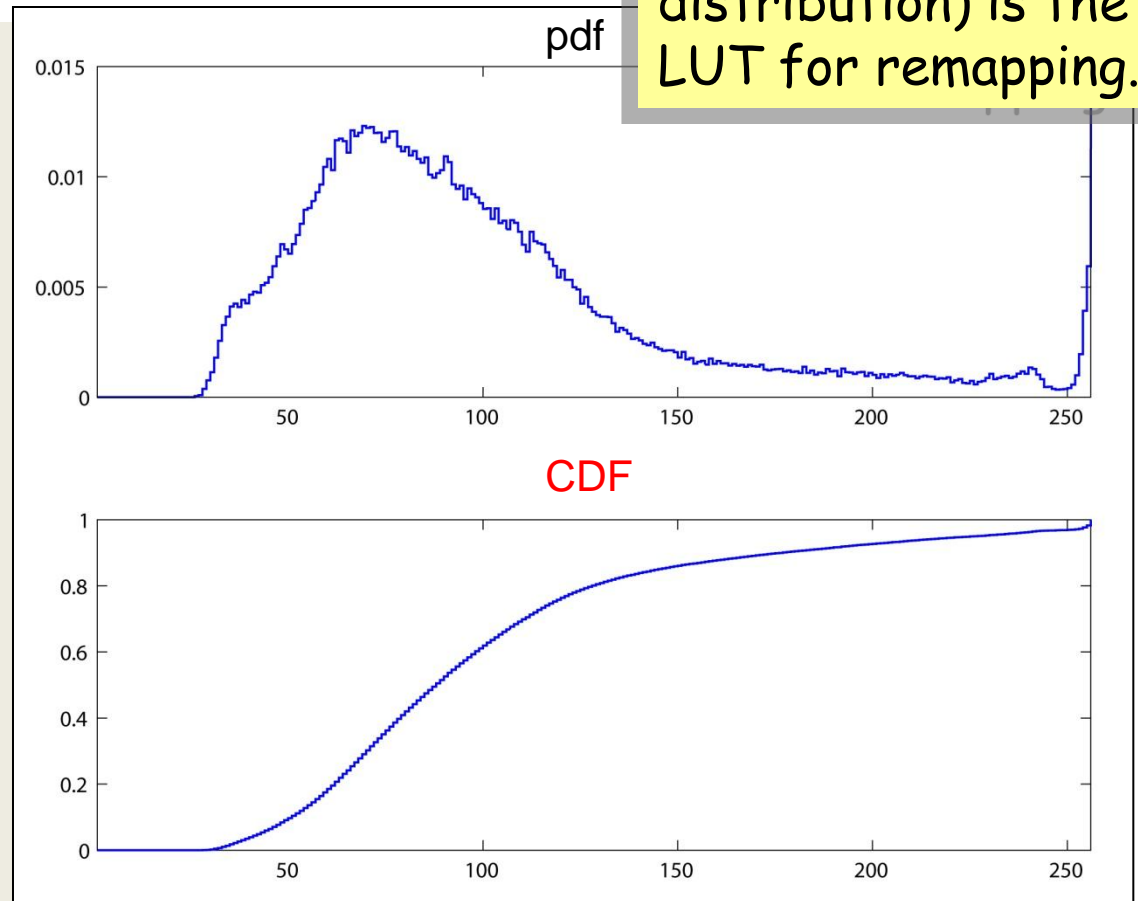
be the cumulative (probability) distribution function of I .

The CDF itself is used as the LUT.

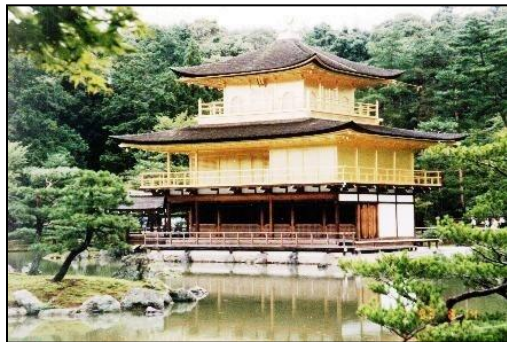
Histogram Equalization



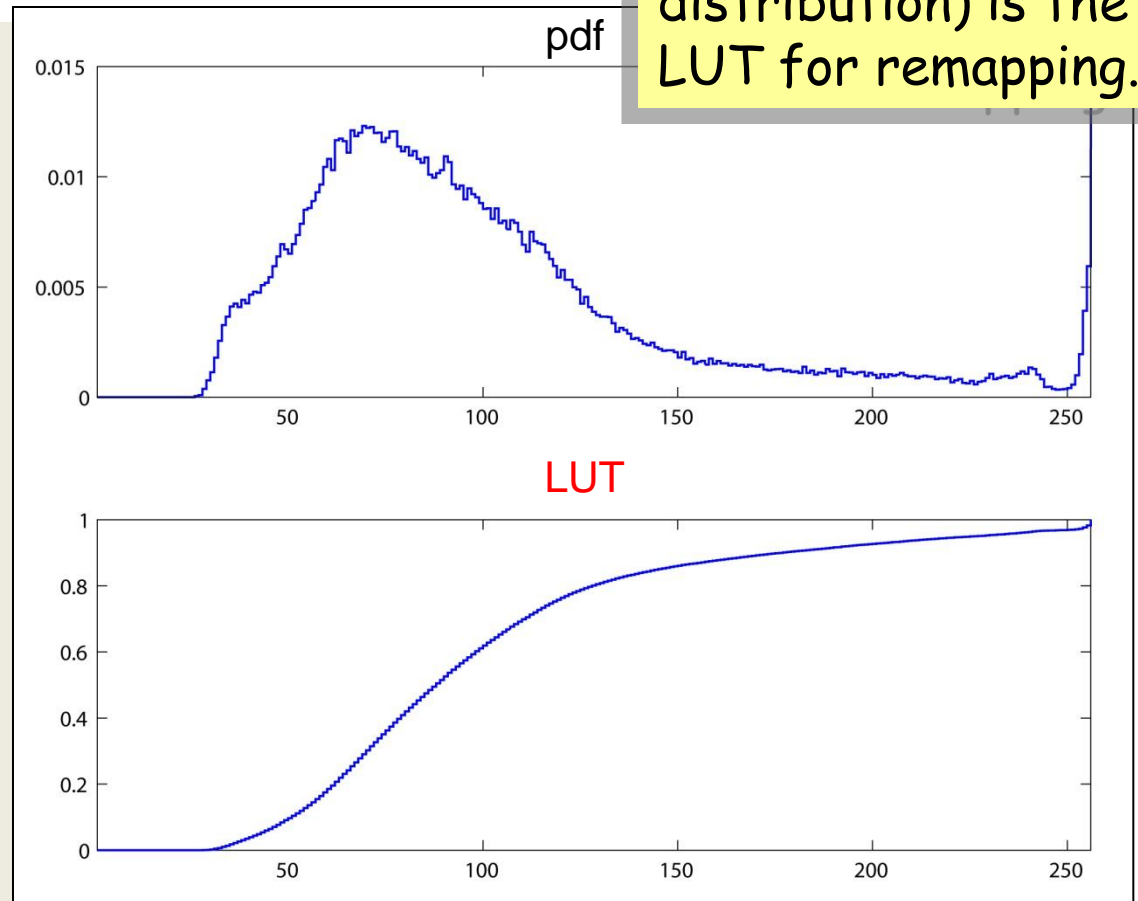
The CDF (cumulative distribution) is the LUT for remapping.



Histogram Equalization



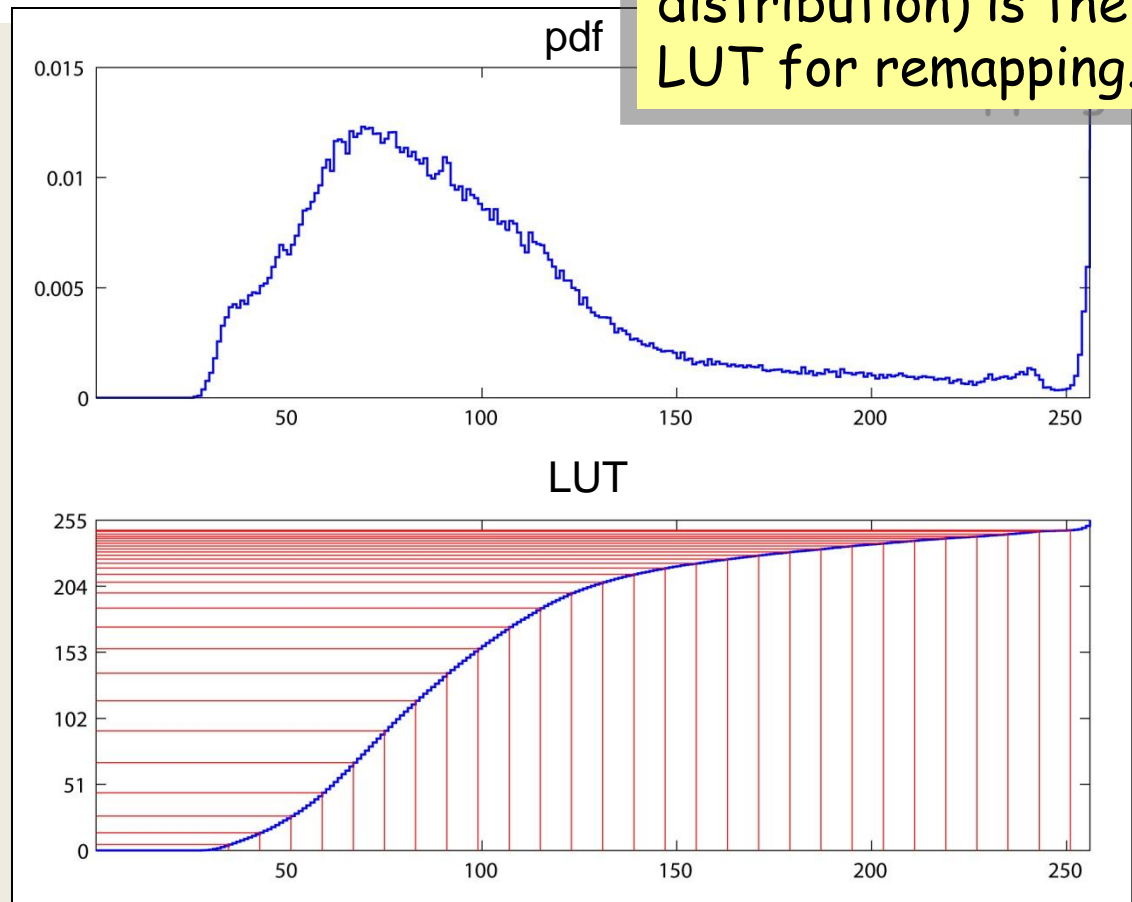
The CDF (cumulative distribution) is the LUT for remapping.



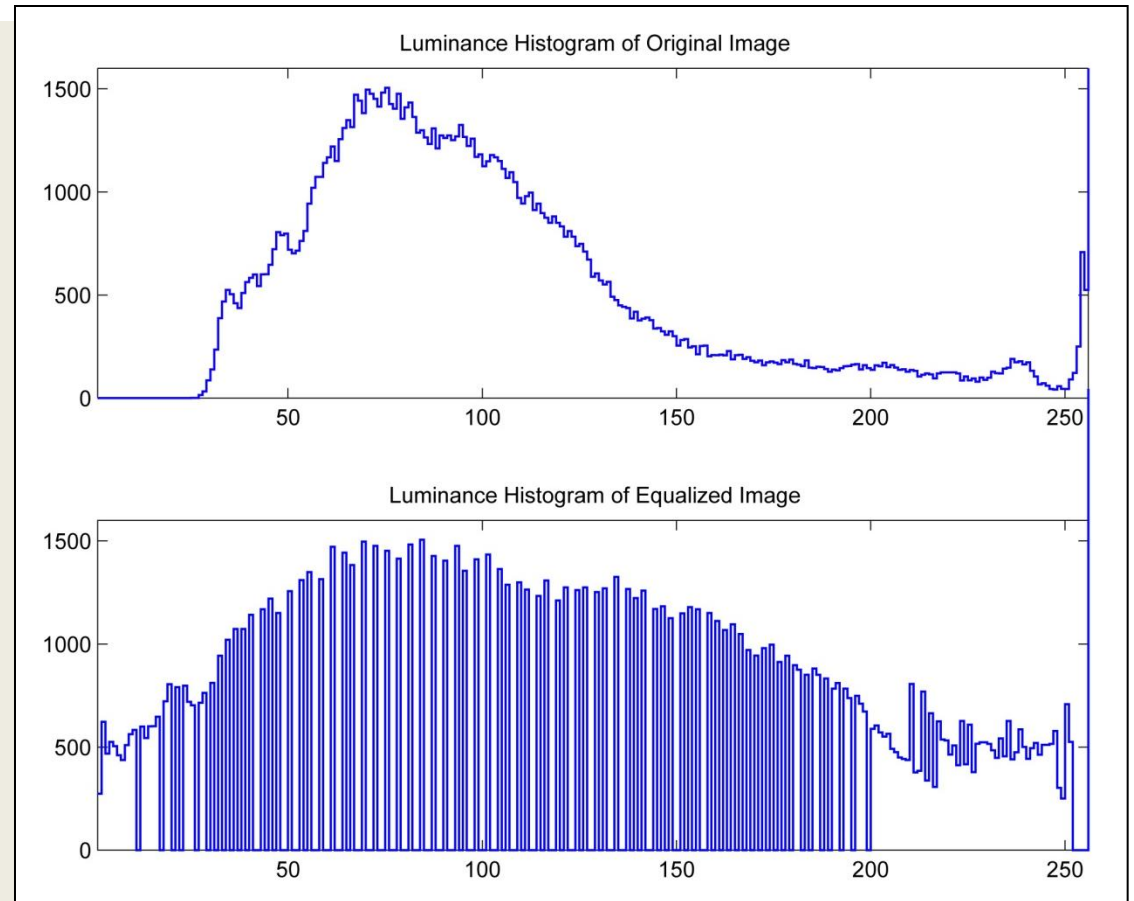
Histogram Equalization



The CDF (cumulative distribution) is the LUT for remapping.



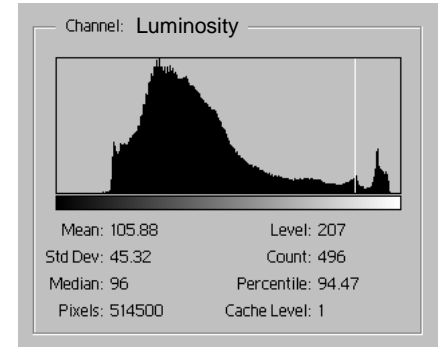
Histogram Equalization



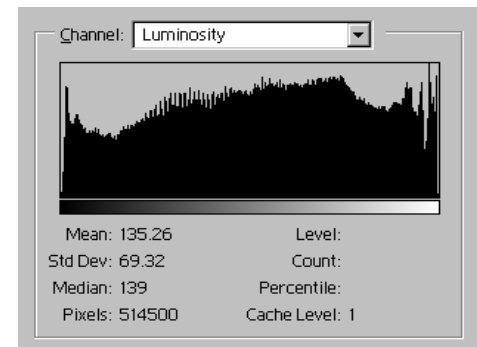
Histogram Equalization



$$J(r,c) = 255 \cdot P_I[I(r,c)].$$



before



after

HISTOGRAM EQUALIZATION IMPLEMENTATION

0	0	0	0	0
1	1	1	1	4
4	5	6	6	6
8	8	8	8	9

HISTOGRAM EQUALIZATION IMPLEMENTATION

0	0	0	0	0
1	1	1	1	4
4	5	6	6	6
8	8	8	8	9

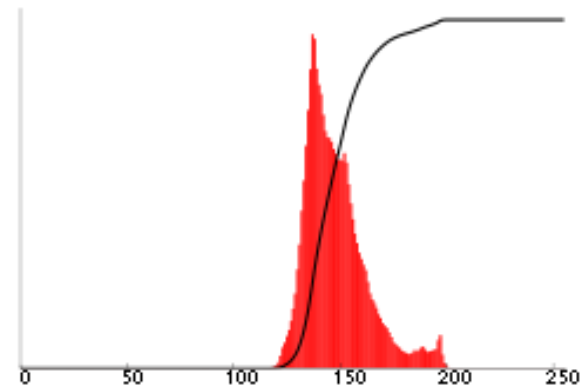
2	2	2	2	2
4	4	4	4	5
5	5	7	7	7
9	9	9	9	9

Gray levels	0	1	2	3	4	5	6	7	8	9
Counts ($h(r_k)$)	5	4	0	0	2	1	3	0	4	1
	r_0	r_1			r_2	r_3	r_4		r_5	r_6
Normalized h ($P(r_k)$)	5/20	4/20	0	0	2/20	1/20	3/20	0	4/20	1/20
cdf $F(r_k)$	5/20	9/20			11/20	12/20	15/20		19/20	20/20
$s_k = \text{round}(9 \cdot F(r_k))$	2	4			5	5	7		9	9
	s_0	s_1			s_2	s_3	s_4		s_5	s_6

Histogram Equalization: Example



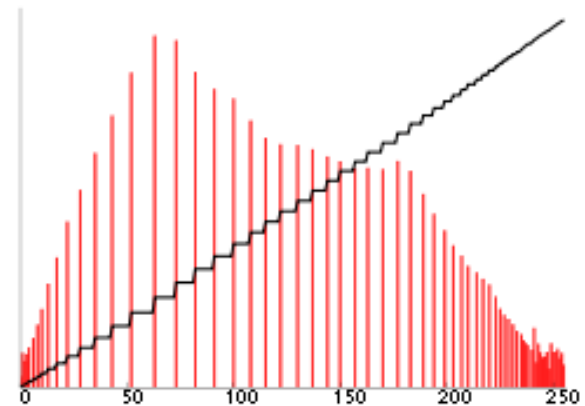
Original Image



Corresponding histogram (red) and cumulative histogram (black)



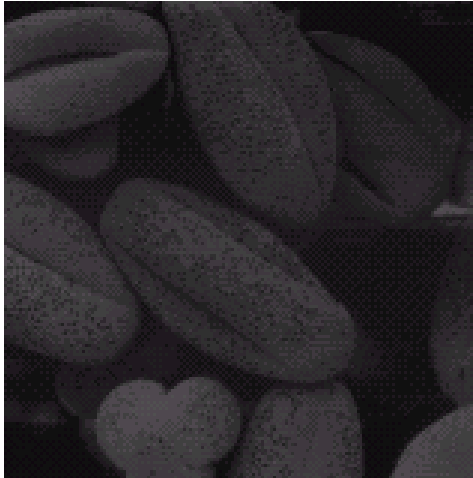
Image after histogram equalization



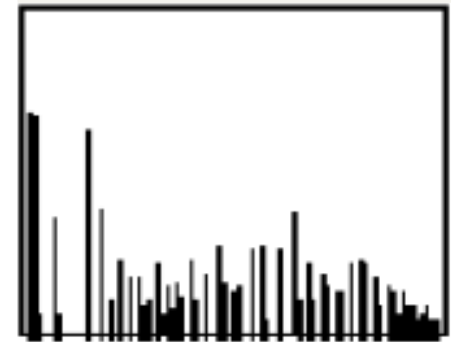
Corresponding histogram (red) and cumulative histogram (black)

Histogram Equalization: Example

Dark image



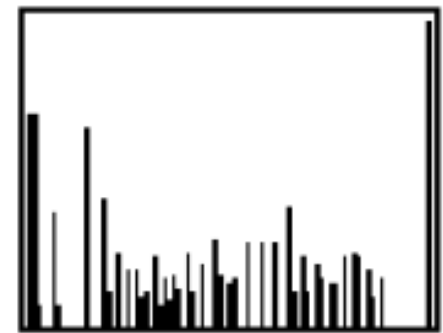
Equalized Histogram



Bright image

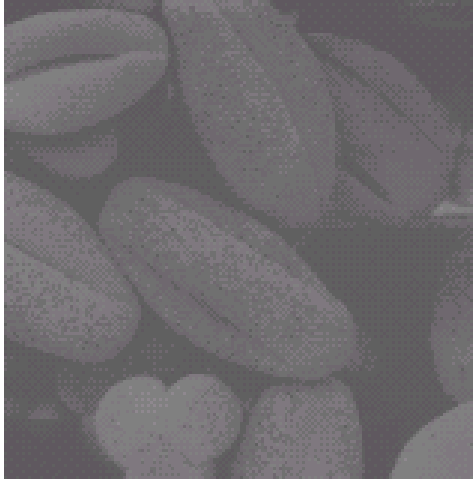


Equalized Histogram

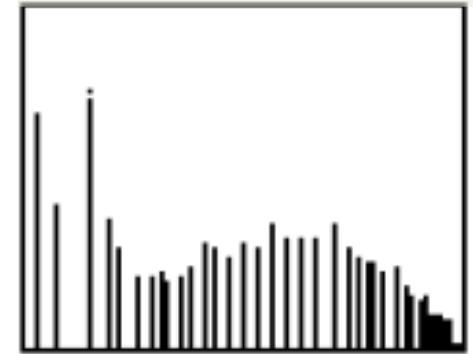


Histogram Equalization: Example

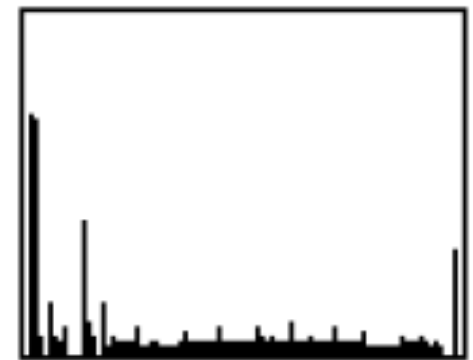
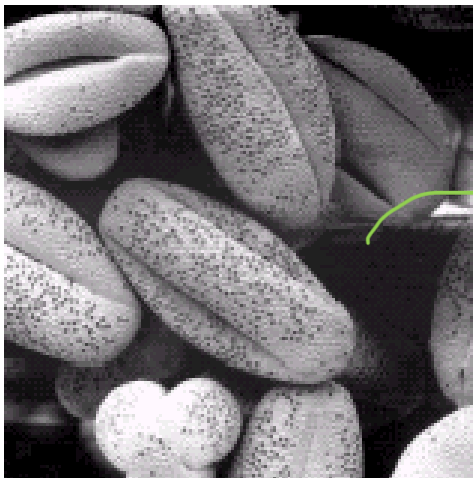
Low contrast



Equalized Histogram



High Contrast



Equalized Histogram

HISTOGRAM MATCHING (SPECIFICATION)

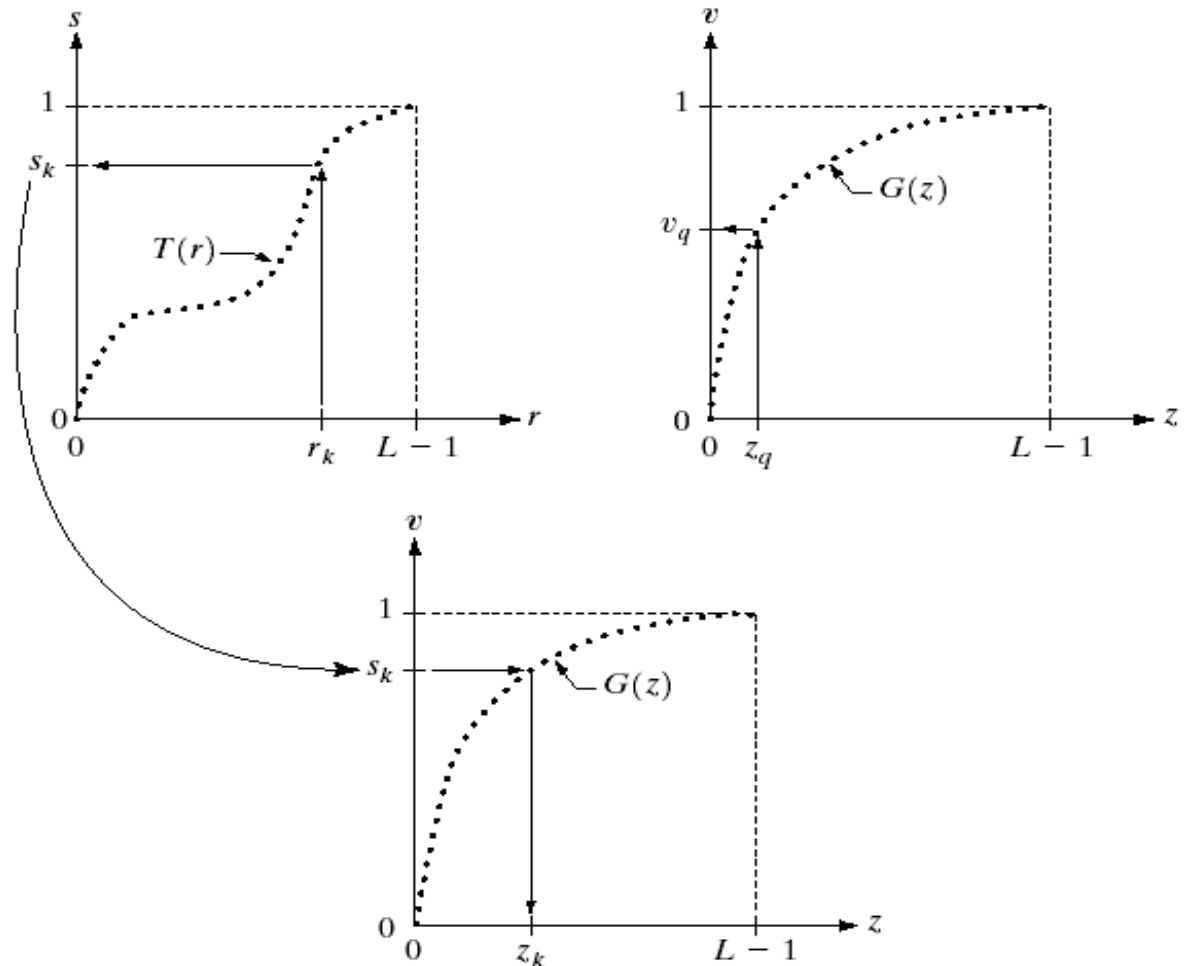
- HISTOGRAM EQUALIZATION DOES NOT ALLOW INTERACTIVE IMAGE ENHANCEMENT AND GENERATES ONLY ONE RESULT: AN APPROXIMATION TO A UNIFORM HISTOGRAM.
- SOMETIMES THOUGH, WE NEED TO BE ABLE TO SPECIFY PARTICULAR HISTOGRAM SHAPES CAPABLE OF HIGHLIGHTING CERTAIN GRAY-LEVEL RANGES.

MAPPINGS

a b
c

FIGURE 3.19

(a) Graphical interpretation of mapping from r_k to s_k via $T(r)$.
 (b) Mapping of z_q to its corresponding value v_q via $G(z)$.
 (c) Inverse mapping from s_k to its corresponding value of z_k .



HISTOGRAM SPECIFICATION

- OBTAIN THE HISTOGRAM OF THE GIVEN IMAGE
- MAP EACH LEVEL r_k TO A LEVEL S_k
- OBTAIN THE TRANSFORMATION FUNCTION G FROM THE GIVEN $P_z(z)$
- PRECOMPUTE z_k FOR EACH VALUE OF S_k
- FOR EACH PIXEL IN THE ORIGINAL IMAGE, IF THE VALUE OF THAT PIXEL IS r_k MAP THIS VALUE TO ITS CORRESPONDING LEVEL S_k , THEN MAP LEVEL S_k INTO THE FINAL VALUE z_k

HISTOGRAM SPECIFICATION

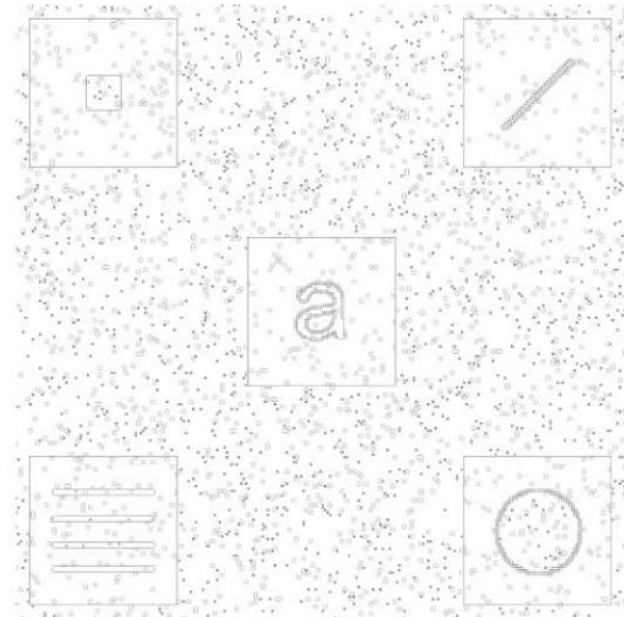
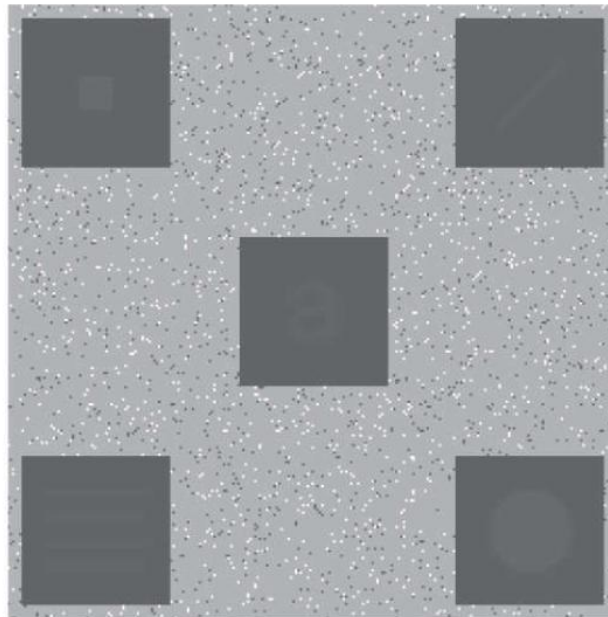
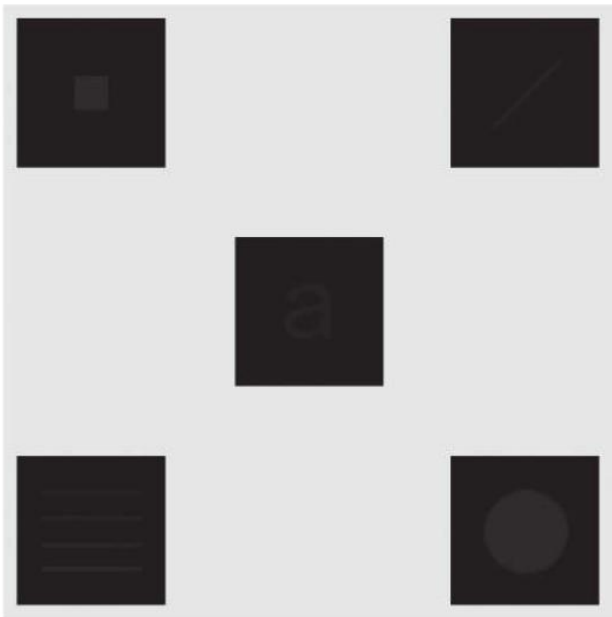
k	n_k	$p_r(r_k)$	s_k	$p_z(z_k)$	v_k	n_k
0	790	0.19	0.19	0	0	0
1	1023	0.25	0.44	0	0	0
2	850	0.21	0.65	0	0	0
3	656	0.16	0.81	0.15	0.15	790
4	329	0.08	0.89	0.2	0.35	1023
5	245	0.06	0.95	0.3	0.65	850
6	122	0.03	0.98	0.2	0.85	985
7	81	0.02	1.0	0.15	1.0	448

A 64X64 (4096 PIXELS) IMAGE WITH 8 GRAY LEVELS

GLOBAL/LOCAL HISTOGRAM EQUALIZATION

- IT MAY BE NECESSARY TO ENHANCE DETAILS OVER SMALL AREAS IN THE IMAGE
- THE NUMBER OF PIXELS IN THESE AREAS MAY HAVE NEGLIGIBLE INFLUENCE ON THE COMPUTATION OF A GLOBAL TRANSFORMATION WHOSE SHAPE DOES NOT NECESSARILY GUARANTEE THE DESIRED LOCAL ENHANCEMENT
- DEVISE TRANSFORMATION FUNCTIONS BASED ON THE GRAY LEVEL DISTRIBUTION IN THE NEIGHBORHOOD OF EVERY PIXEL IN THE IMAGE
- THE PROCEDURE IS:
 - DEFINE A SQUARE (OR RECTANGULAR) NEIGHBORHOOD AND MOVE THE CENTER OF THIS AREA FROM PIXEL TO PIXEL.
 - AT EACH LOCATION, THE HISTOGRAM OF THE POINTS IN THE NEIGHBORHOOD IS COMPUTED AND EITHER A HISTOGRAM EQUALIZATION OR HISTOGRAM SPECIFICATION TRANSFORMATION FUNCTION IS OBTAINED.
 - THIS FUNCTION IS FINALLY USED TO MAP THE GRAY LEVEL OF THE PIXEL CENTERED IN THE NEIGHBORHOOD.
 - THE CENTER IS THEN MOVED TO AN ADJACENT PIXEL LOCATION AND THE PROCEDURE IS REPEATED.

GLOBAL/LOCAL HISTOGRAM EQUALIZATION



USE OF HISTOGRAM STATISTICS FOR IMAGE ENHANCEMENT (Global)

- LET r REPRESENT A GRAY LEVEL IN THE IMAGE $[0, L-1]$, AND LET $p(r_i)$ DENOTE THE NORMALIZED HISTOGRAM COMPONENT CORRESPONDING TO THE i^{th} VALUE OF r .
- THE n^{th} MOMENT OF r ABOUT ITS MEAN IS DEFINED AS

$$\mu_n(r) = \sum_{i=0}^{L-1} (r_i - m)^n p(r_i)$$

- WHERE m IS THE MEAN VALUE OF r (AVERAGE GRAY LEVEL)

$$m = \sum_{i=0}^{L-1} r_i p(r_i)$$

USE OF HISTOGRAM STATISTICS FOR IMAGE ENHANCEMENT (Global)

- THE SECOND MOMENT IS GIVEN BY

$$\mu_2(r) = \sum_{i=0}^{L-1} (r_i - m)^2 p(r_i)$$

- WHICH IS THE VARIANCE OF r
- MEAN AS A MEASURE OF AVERAGE GRAY LEVEL IN THE IMAGE
- VARIANCE AS A MEASURE OF AVERAGE CONTRAST

USE OF HISTOGRAM STATISTICS FOR IMAGE ENHANCEMENT (Local)

- LET (x,y) BE THE COORDINATES OF A PIXEL IN AN IMAGE, AND LET $S_{x,y}$ DENOTE A NEIGHBORHOOD OF SPECIFIED SIZE, CENTERED AT (x,y)
- THE MEAN VALUE $m_{s_{xy}}$ OF THE PIXELS IN $S_{x,y}$ IS

$$m_{s_{xy}} = \sum_{(s,t) \in S_{xy}} r_{s,t} p(r_{s,t})$$

- THE GRAY LEVEL VARIANCE OF THE PIXELS IN REGION $S_{x,y}$ IS GIVEN BY

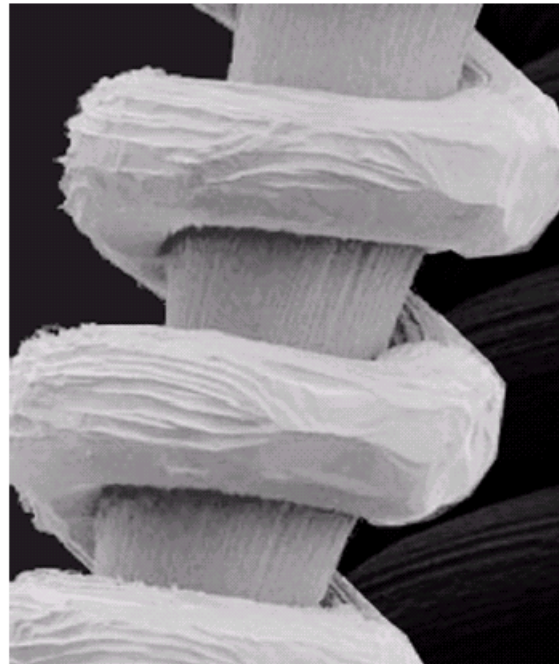
$$\sigma_{s_{xy}}^2 = \sum_{(s,t) \in S_{xy}} [r_{s,t} - m_{s_{xy}}]^2 p(r_{s,t})$$

USE OF HISTOGRAM STATISTICS FOR IMAGE ENHANCEMENT

- THE GLOBAL MEAN AND VARIANCE ARE MEASURED OVER AN ENTIRE IMAGE AND ARE USEFUL FOR GROSS ADJUSTMENTS OF OVERALL INTENSITY AND CONTRAST.
- A USE OF THESE MEASURES IN LOCAL ENHANCEMENT IS, WHERE THE LOCAL MEAN AND VARIANCE ARE USED AS THE BASIS FOR MAKING CHANGES THAT DEPEND ON IMAGE CHARACTERISTICS IN A PREDEFINED REGION ABOUT EACH PIXEL IN THE IMAGE.

TUNGSTEN FILAMENT IMAGE

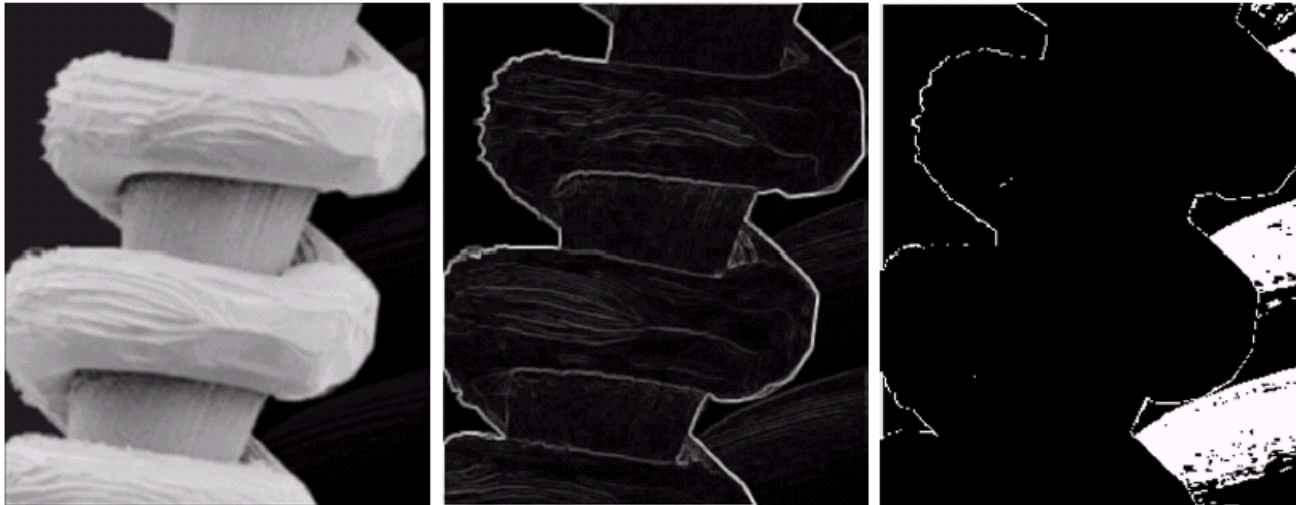
FIGURE 3.24 SEM image of a tungsten filament and support, magnified approximately 130 \times . (Original image courtesy of Mr. Michael Shaffer, Department of Geological Sciences, University of Oregon, Eugene).



USE OF HISTOGRAM STATISTICS FOR IMAGE ENHANCEMENT

- A PIXEL AT POINT (x,y) IS CONSIDERED IF:
 - $m_{sxy} \leq k_0 M_G$, where k_0 is a positive constant less than 1.0, and M_G is global mean
 - $\sigma_{sxy} \leq k_2 D_G$, where D_G is the global standard deviation and k_2 is a positive constant
 - $k_1 D_G \leq \sigma_{sxy}$, with $k_1 < k_2$
- A PIXEL THAT MEETS ALL ABOVE CONDITIONS IS PROCESSED SIMPLY BY MULTIPLYING IT BY A SPECIFIED CONSTANT, E, TO INCREASE OR DECREASE THE VALUE OF ITS GRAY LEVEL RELATIVE TO THE REST OF THE IMAGE.
- THE VALUES OF PIXELS THAT DO NOT MEET THE ENHANCEMENT CONDITIONS ARE LEFT UNCHANGED.

IMAGE ENHANCEMENT IN THE SPATIAL DOMAIN



a b c

FIGURE 3.25 (a) Image formed from all local means obtained from Fig. 3.24 using Eq. (3.3-21). (b) Image formed from all local standard deviations obtained from Fig. 3.24 using Eq. (3.3-22). (c) Image formed from all multiplication constants used to produce the enhanced image shown in Fig. 3.26.

IMAGE ENHANCEMENT IN THE SPATIAL DOMAIN



FIGURE 3.26
Enhanced SEM
image. Compare
with Fig. 3.24. Note
in particular the
enhanced area on
the right side of
the image.

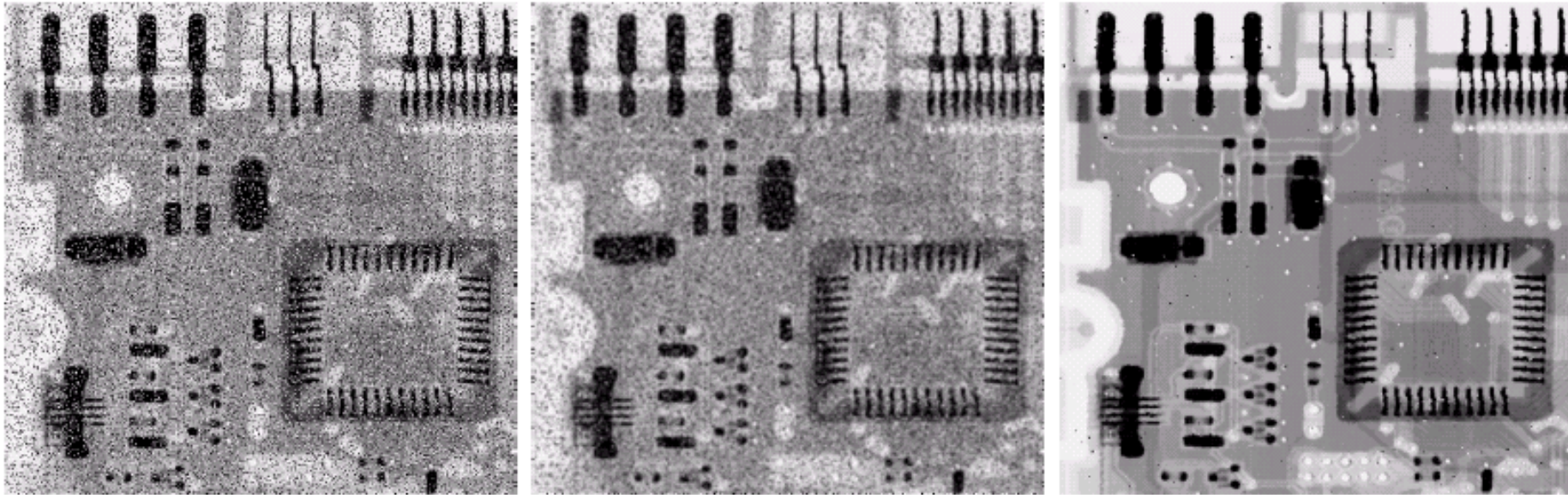
Spatial Filtering



a b c

FIGURE 4.50 (a) Original image (784×732 pixels). (b) Result of filtering using a GLPF with $D_0 = 100$. (c) Result of filtering using a GLPF with $D_0 = 80$. Note the reduction in fine skin lines in the magnified sections in (b) and (c).

Spatial Filtering



a b c

FIGURE 3.37 (a) X-ray image of circuit board corrupted by salt-and-pepper noise. (b) Noise reduction with a 3×3 averaging mask. (c) Noise reduction with a 3×3 median filter. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

Spatial Filtering



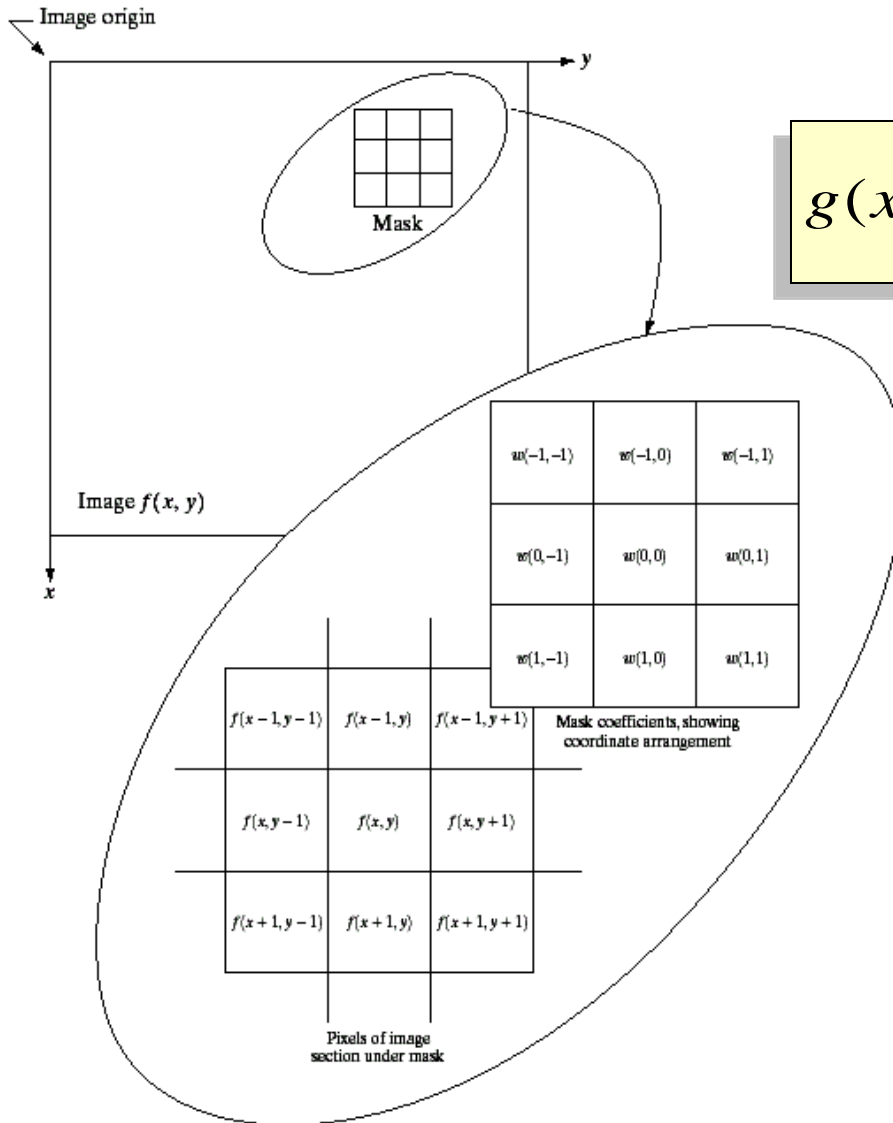
a b

FIGURE 4.58
(a) Original, blurry image.
(b) Image enhanced using the Laplacian in the frequency domain. Compare with Fig. 3.38(e).

Spatial Filtering: Basics

- ◆ The output intensity value at (x,y) depends not only on the input intensity value at (x,y) but also on the specified number of neighboring intensity values around (x,y)
- ◆ Spatial masks (also called window, filter, kernel, template) are used and **convolved** over the entire image for local enhancement (spatial filtering)
- ◆ The size of the masks determines the number of neighboring pixels which influence the output value at (x,y)
- ◆ The values (coefficients) of the mask determine the nature and properties of enhancing technique

Spatial Filtering: Basics



$$g(x, y) = \sum_{s=-a}^a \sum_{t=-b}^b w(s, t) f(x + s, y + t)$$

where $a = \frac{m-1}{2}$, $b = \frac{n-1}{2}$

$x = 0, 1, 2, \dots, M-1, y = 0, 1, 2, \dots, N-1$

Filtering can be given in equation form as shown above

Spatial Filtering: Basics

- ◆ Given the 3×3 mask with coefficients: w_1, w_2, \dots, w_9
- ◆ The mask cover the pixels with gray levels: z_1, z_2, \dots, z_9

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

z_1	z_2	z_3
z_4	z_5	z_6
z_7	z_8	z_9

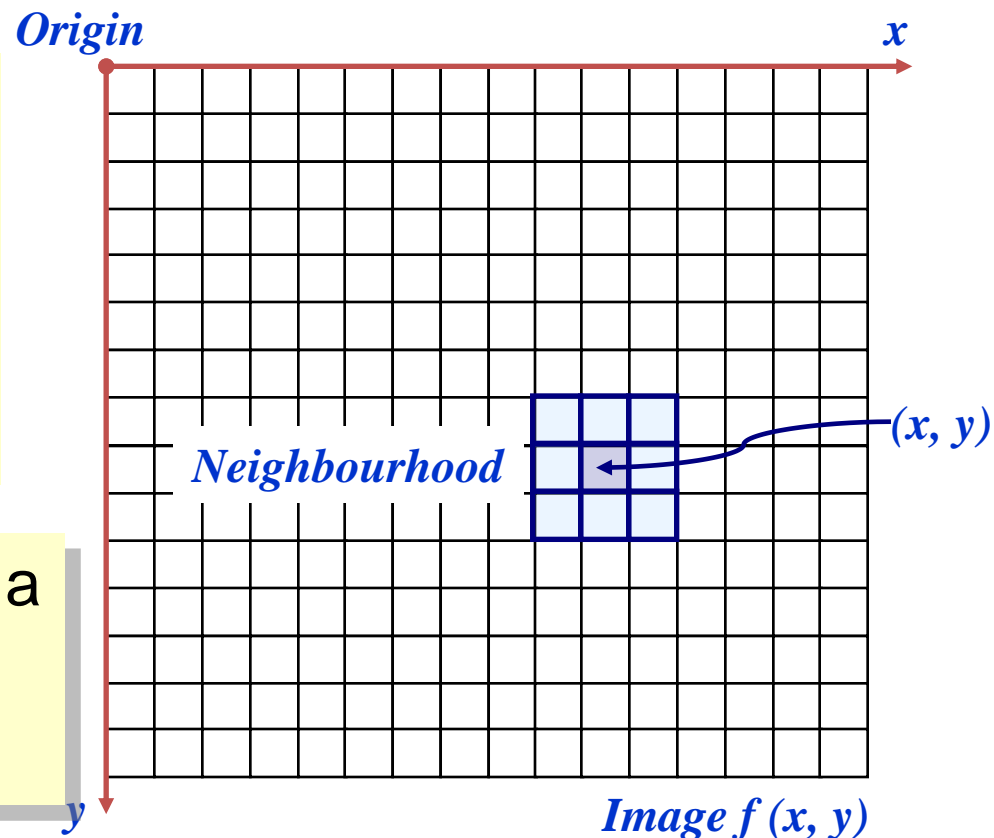
$$z \longleftarrow z_1 w_1 + z_2 w_2 + z_3 w_3 + \dots + z_9 w_9 = \sum_{i=1}^9 z_i w_i$$

- ◆ z gives the output intensity value for the processed image (to be stored in a new array) at the location of z_5 in the input image

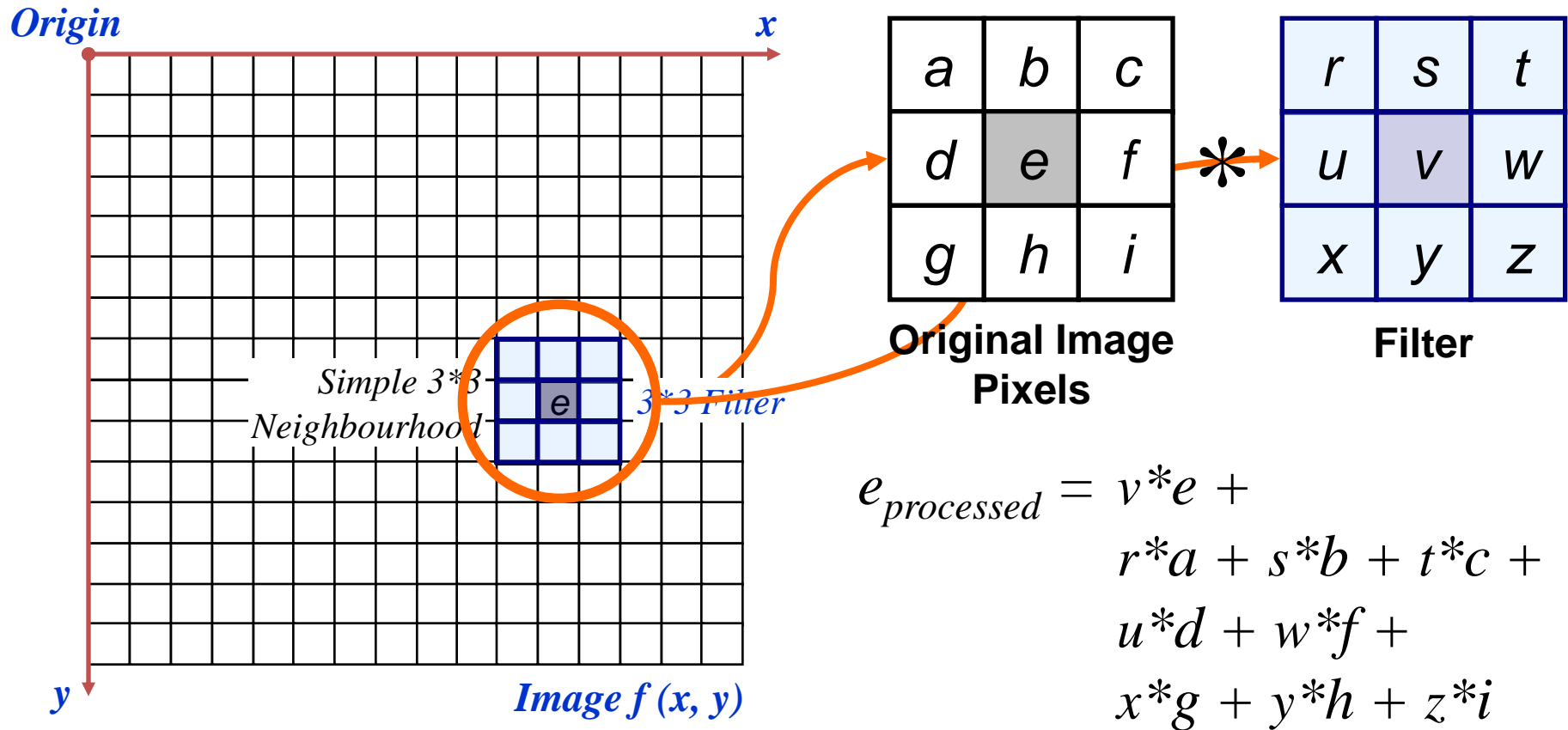
Spatial Filtering: Basics

Neighbourhood operations: Operate on a larger neighbourhood of pixels than point operations

Neighbourhoods are mostly a rectangle around a central pixel



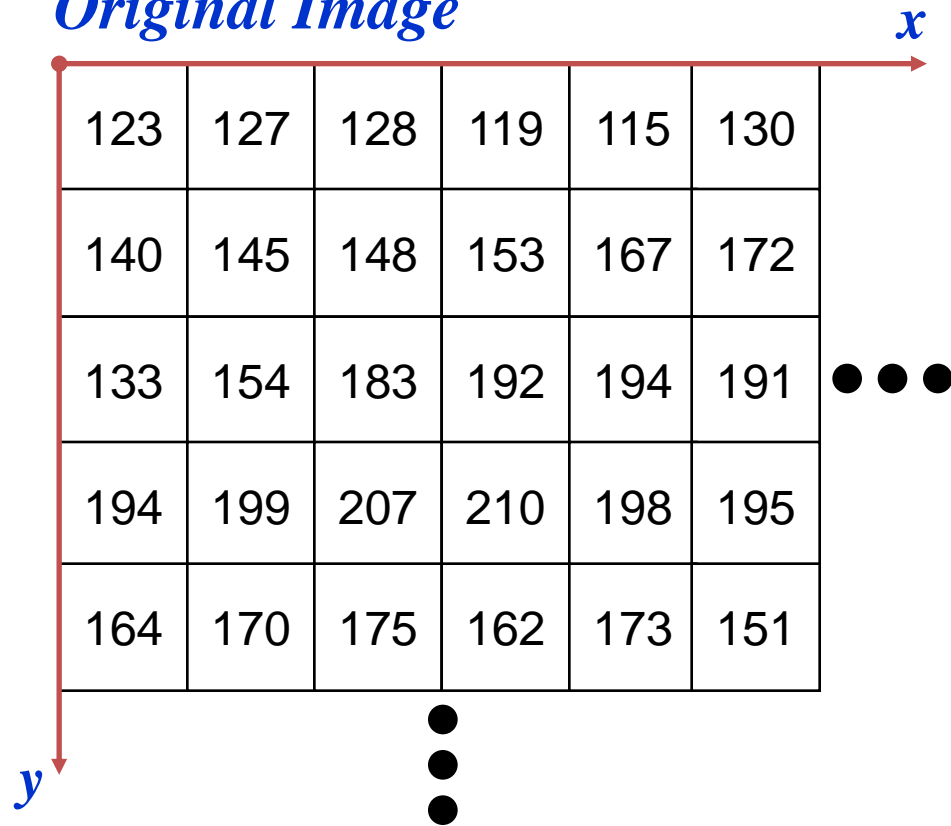
Spatial Filtering: Basics



The above is repeated for every pixel in the original image to generate the filtered image

Spatial Filtering: Basics

Original Image

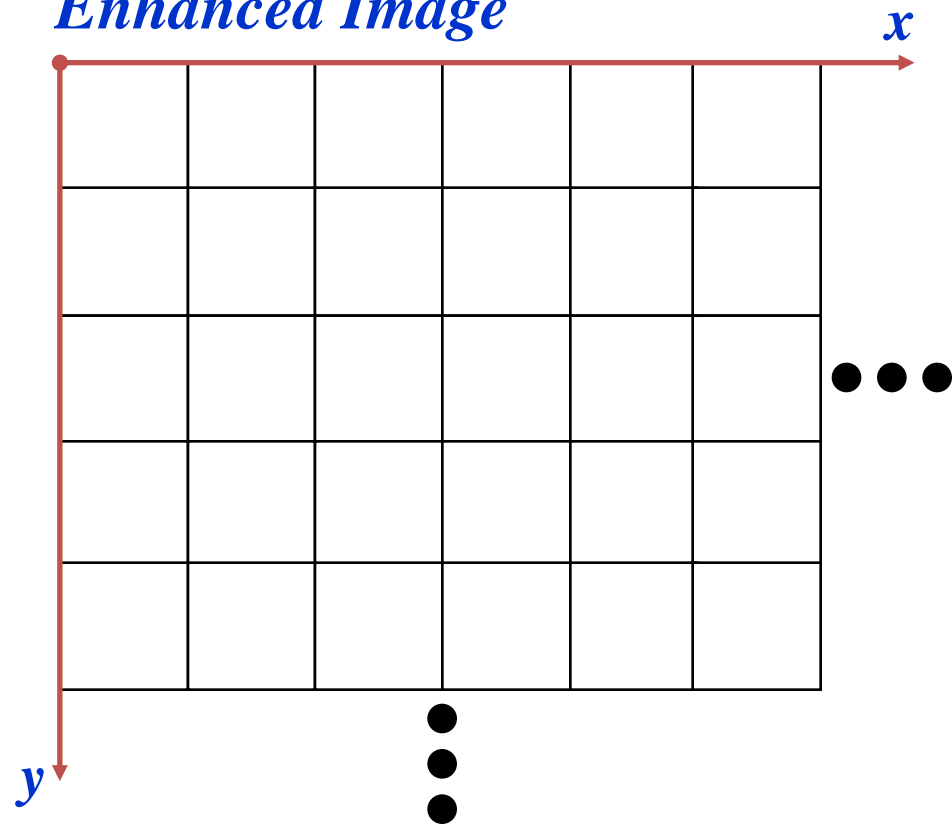


A 6x5 grid of numerical values representing an original image. The grid is labeled with a red 'x' axis at the top and a blue 'y' axis on the left. The values are as follows:

123	127	128	119	115	130
140	145	148	153	167	172
133	154	183	192	194	191
194	199	207	210	198	195
164	170	175	162	173	151
⋮					

Three black dots to the right of the grid indicate it continues horizontally.

Enhanced Image



A 6x5 grid of empty cells representing an enhanced image. The grid is labeled with a red 'x' axis at the top and a blue 'y' axis on the left. Three black dots to the right of the grid indicate it continues horizontally.

⋮					

Spatial Filtering: Basics

Mask operation near the image border: Problem arises when part of the mask is located outside the image plane

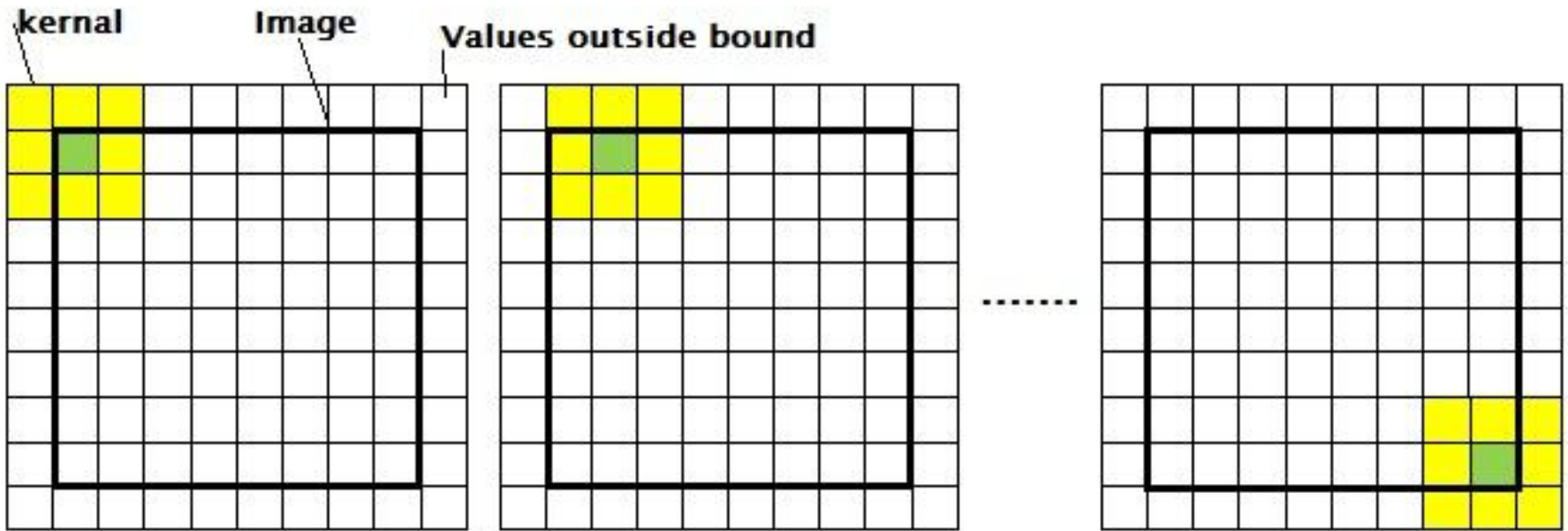
Discard the problem pixels (e.g. 512x512 input 510x510 output if mask size is 3x3)

Zero padding: Expand the input image by padding zeros (512x512 original image, 514x514 padded image, 512x512 output)

Zero padding is not recommended as it creates artificial lines or edges on the border

Pixel replication: We normally use the gray levels of border pixels to fill up the expanded region (for 3x3 mask). For larger masks a border region equal to half of the mask size is mirrored on the expanded region.

Spatial Filtering: Basics



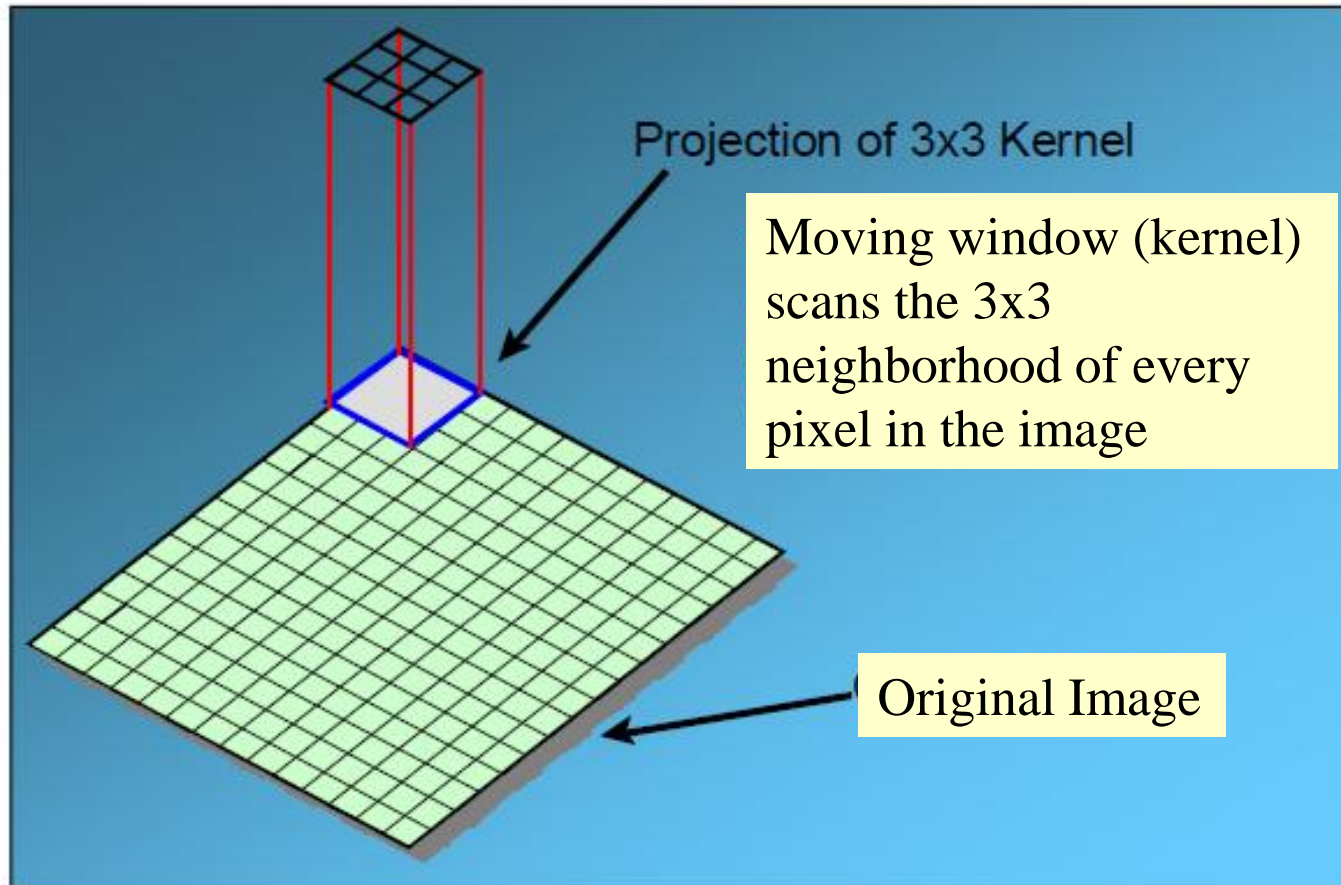
Mask operation near the border: Pixel replication

102	102	130	143	123	115
102	102	130	143	123	115
93	93			
98	98	...					
82	82	...					
65	65						
...	...						
...	...						

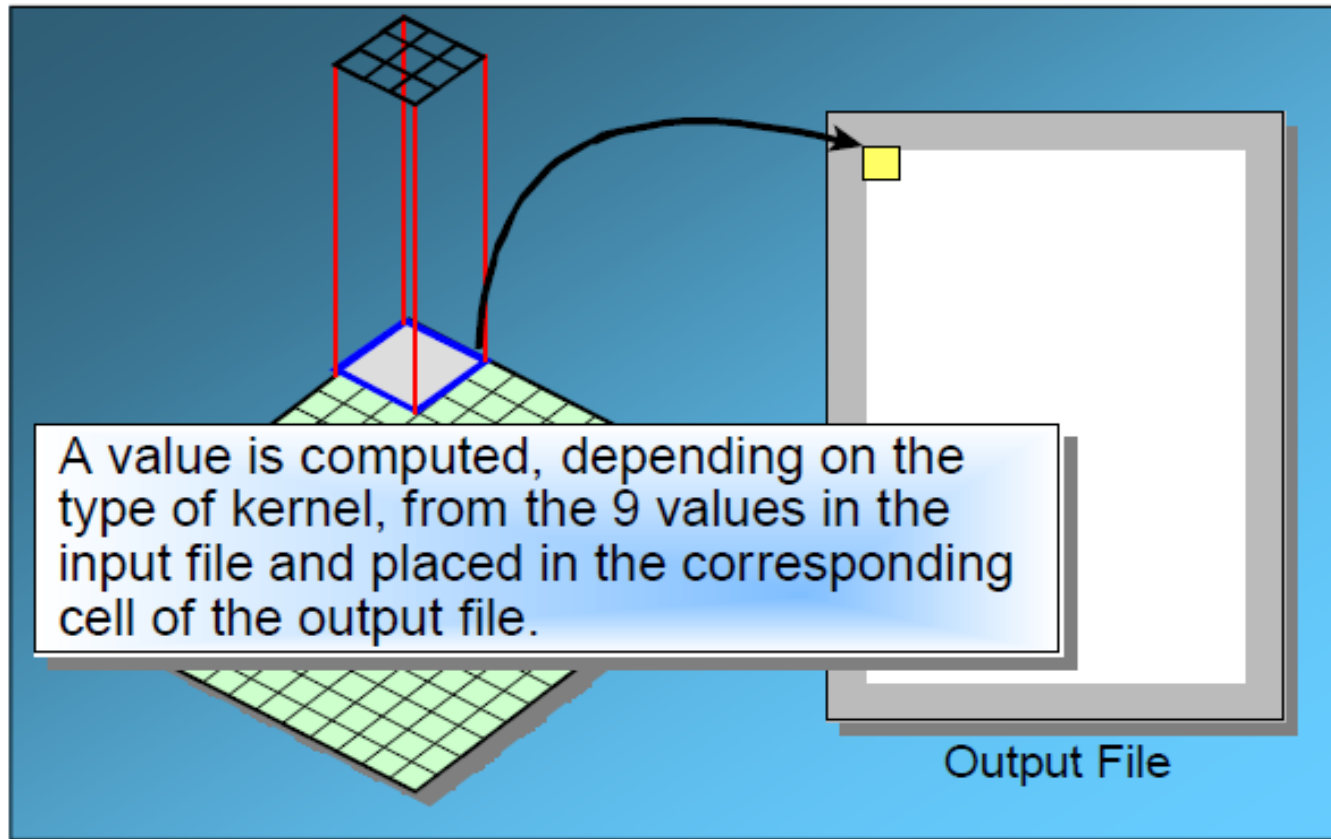
Expanded area

Original image size
(shaded area)

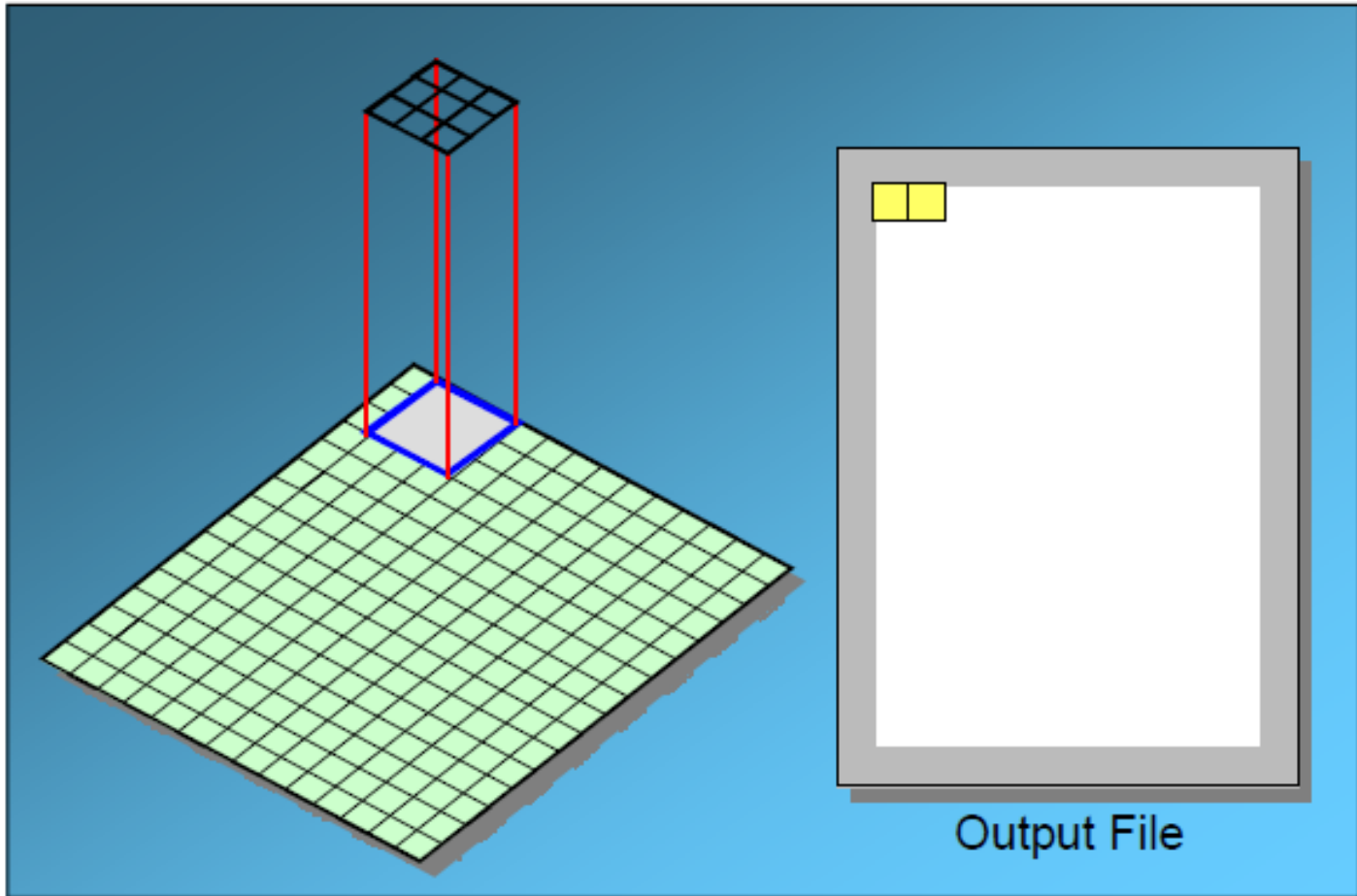
Spatial Filtering: Basics



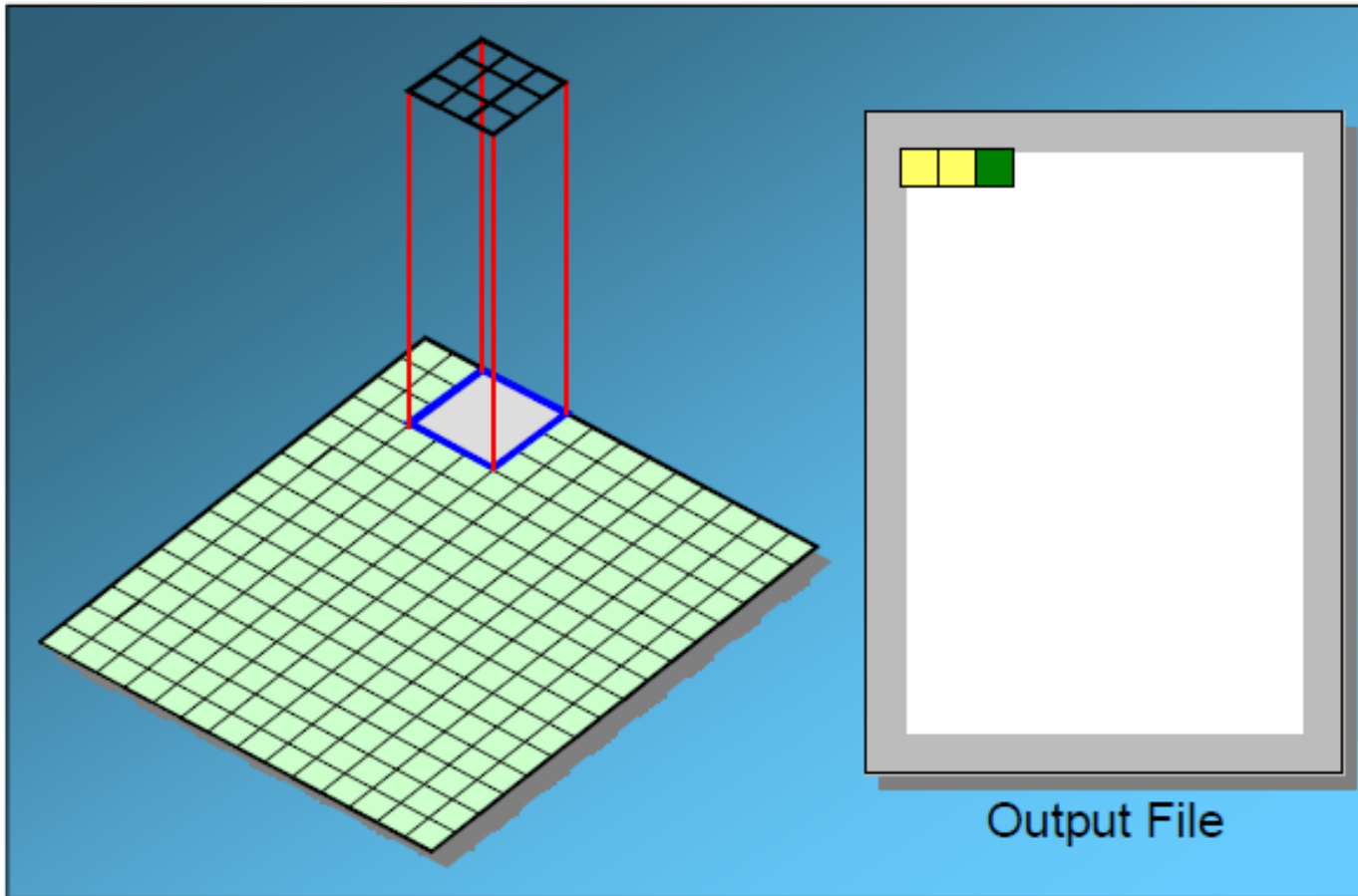
Spatial Filtering: Basics



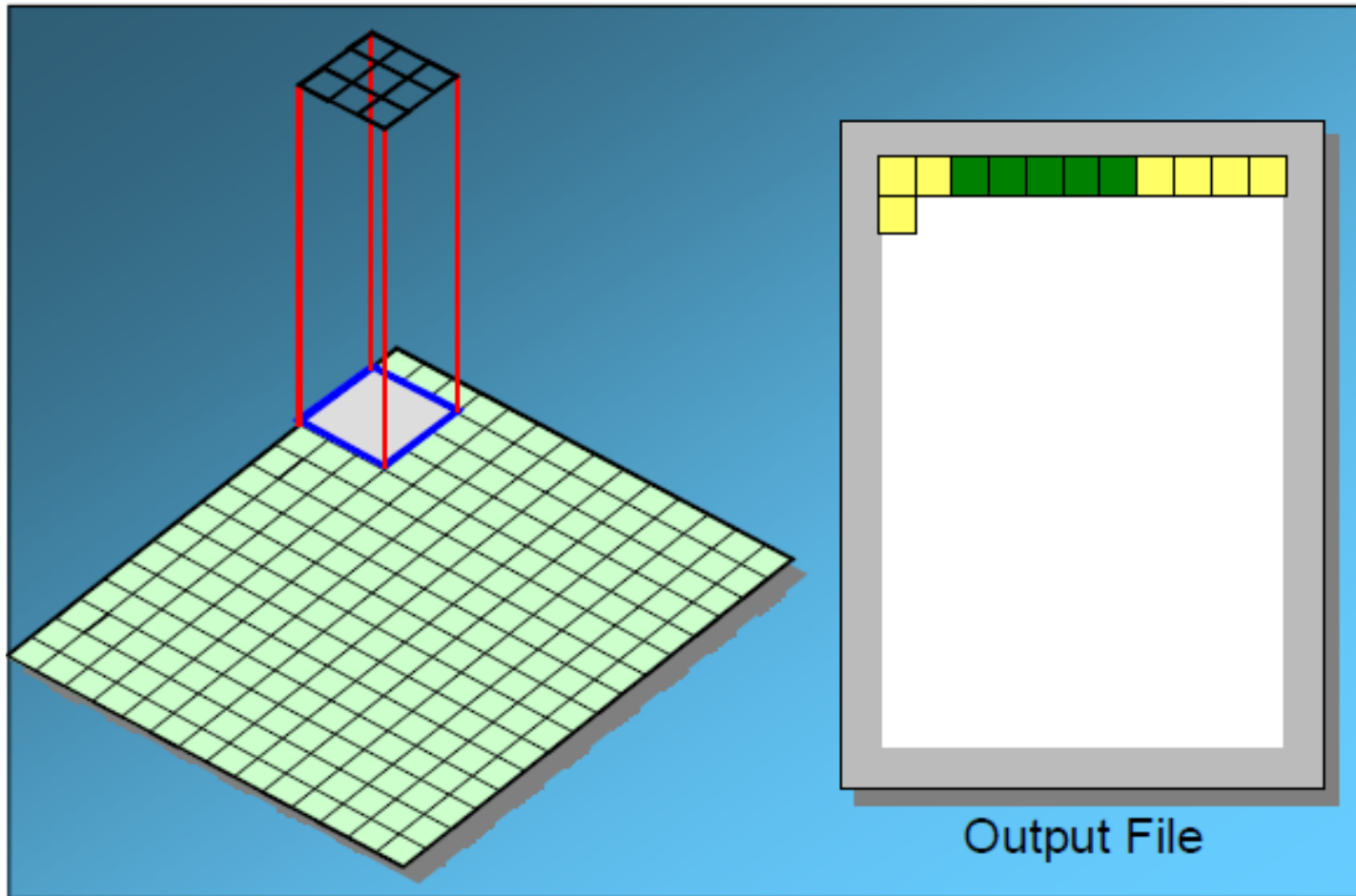
Spatial Filtering: Basics



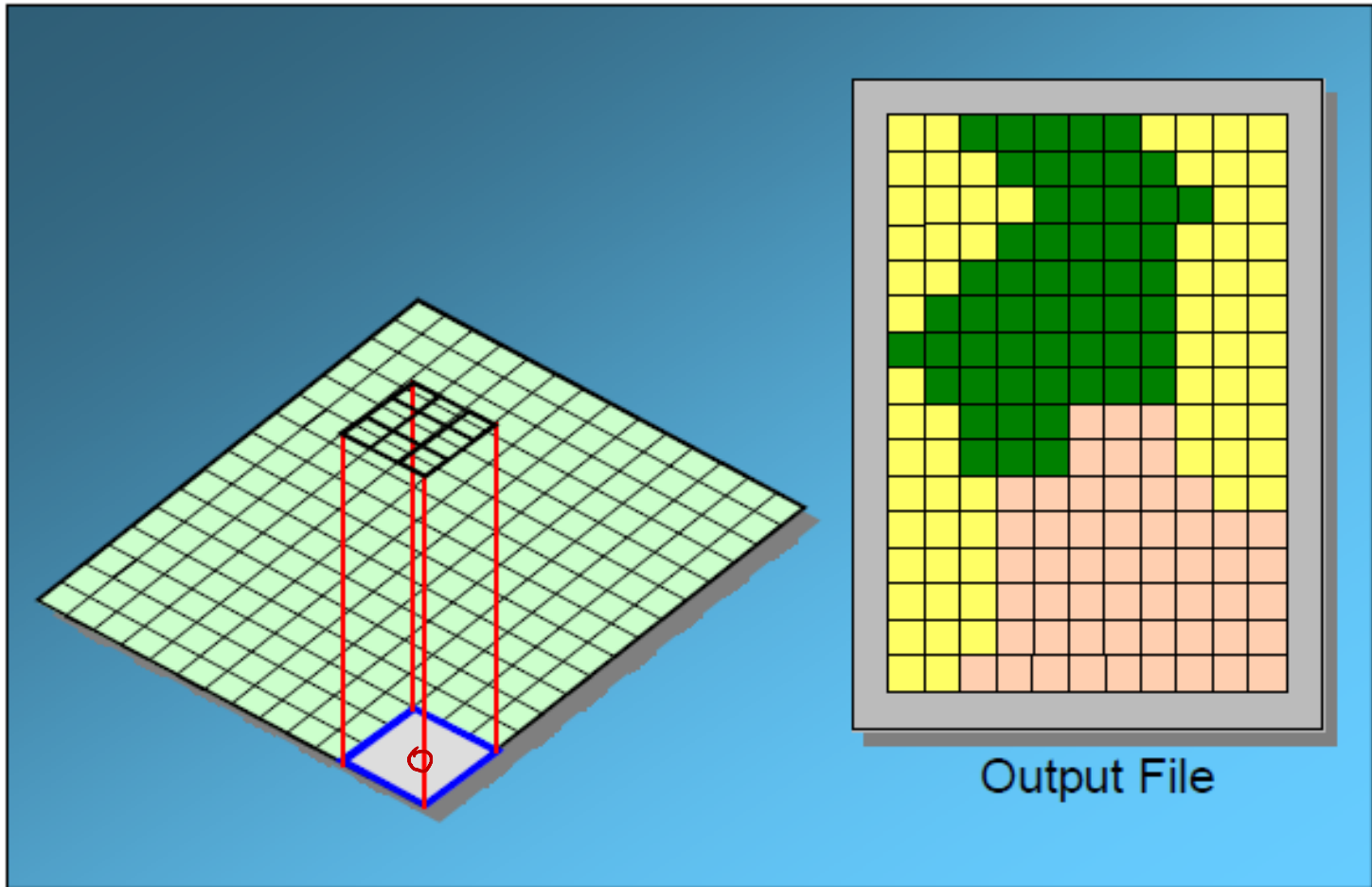
Spatial Filtering: Basics



Spatial Filtering: Basics



Spatial Filtering: Basics



Readings from Book (4th Edn.)

- Chapter – 3
 - 3.1
 - 3.2
 - 3.3 till histogram matching
 - 3.4



Acknowledgements

- ◆ Statistical Pattern Recognition: A Review – A.K Jain et al., PAMI (22) 2000
- ◆ Pattern Recognition and Analysis Course – A.K. Jain, MSU
- ◆ *Pattern Classification*” by Duda et al., John Wiley & Sons.
- ◆ Digital Image Processing”, Rafael C. Gonzalez & Richard E. Woods, Addison-Wesley, 2002
- ◆ Machine Vision: Automated Visual Inspection and Robot Vision”, David Vernon, Prentice Hall, 1991
- ◆ www.eu.aibo.com/
- ◆ Advances in Human Computer Interaction, Shane Pinder, InTech, Austria, October 2008