# Digital Image Processing

**Lecture # 7**

**Morphological Operations**

# Morphology: Example

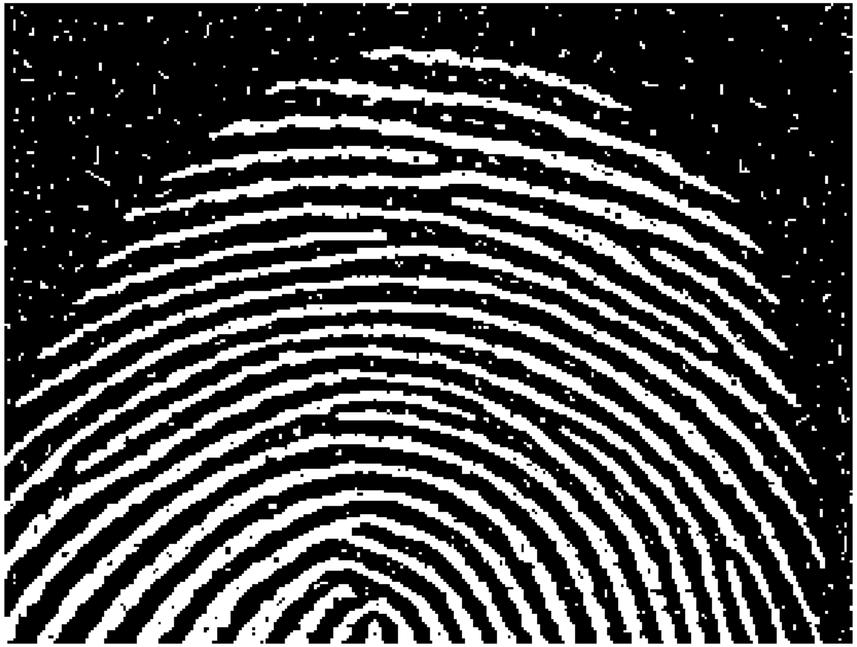


Image after segmentation

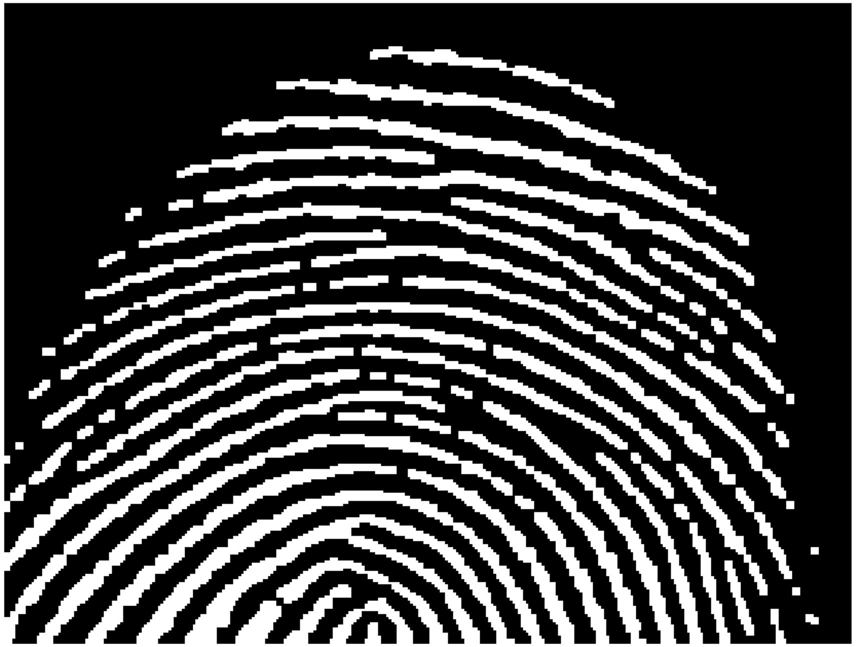
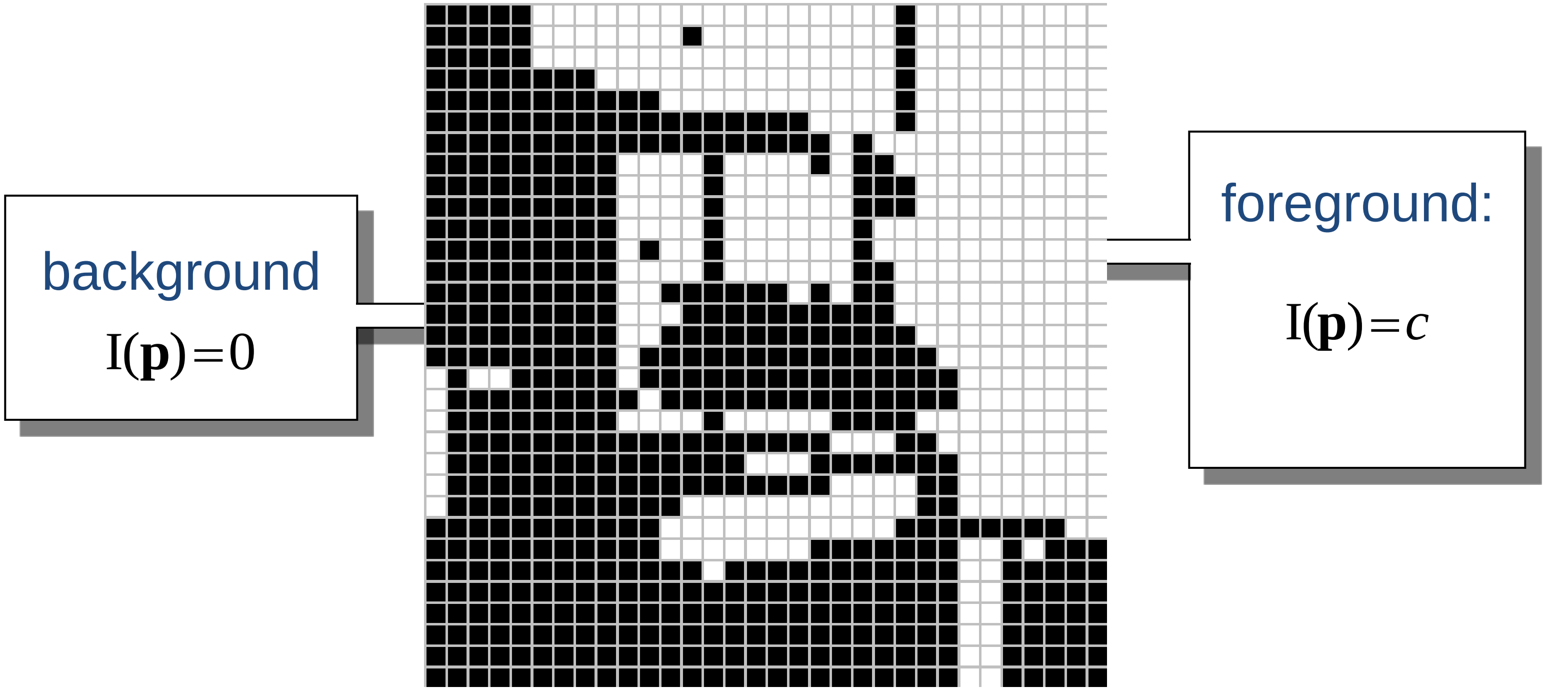


Image after segmentation and morphological processing

# Introduction



background

$I(\mathbf{p})=0$

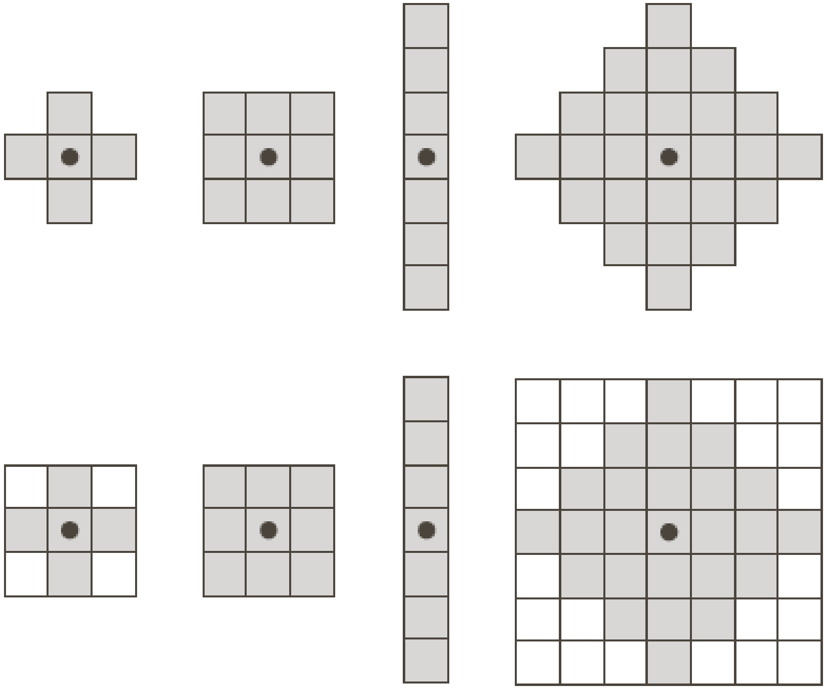
foreground:

$I(\mathbf{p})=c$

This represents a digital image. Each square is one pixel.

# Structuring Element

A structuring element is a small image – used as a moving window



Example Structuring Elements

Structuring Elements converted to rectangular arrays

# Structuring Element

For simplicity we will use rectangular structuring elements with their origin at the middle pixel

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |

| | | |
|---|---|---|
| 0 | 1 | 0 |
| 1 | 1 | 1 |
| 0 | 1 | 0 |

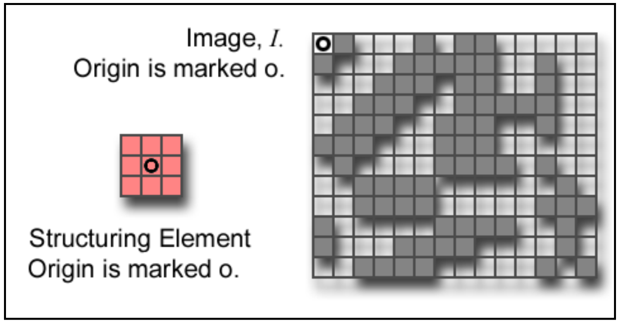
| | | | | |
|---|---|---|---|---|
| 0 | 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 0 | 0 |

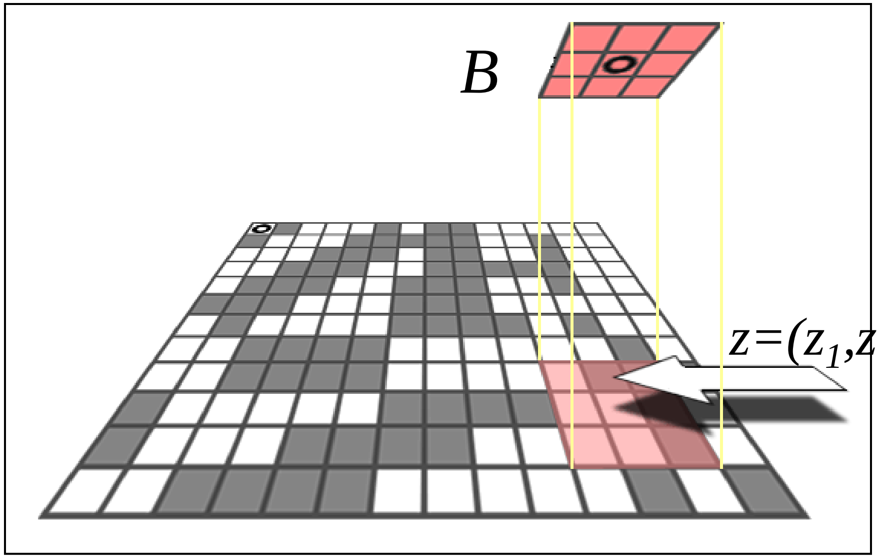
# Structuring Element: Translation

Let I be an image and *B* a SE.

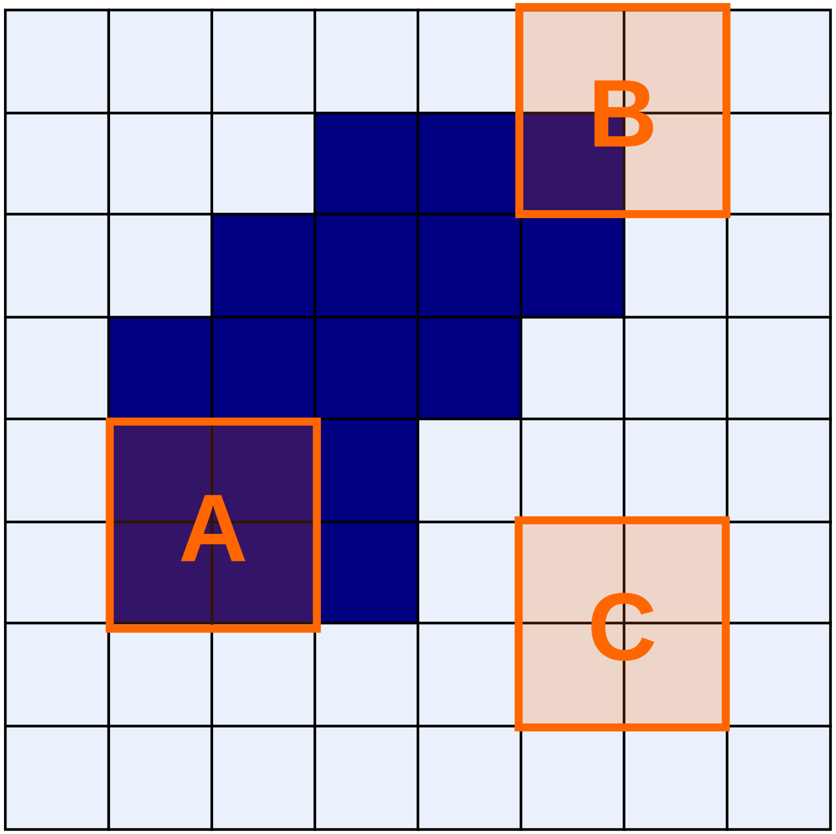
*$(B)_z$* means that *B* is moved so that its origin coincides with location z in image *I*.

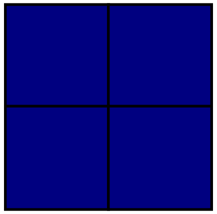
*$(B)_z$* is the *translate* of *B* to location *z* in *I*.





# Structuring Elements: Hits & Fits



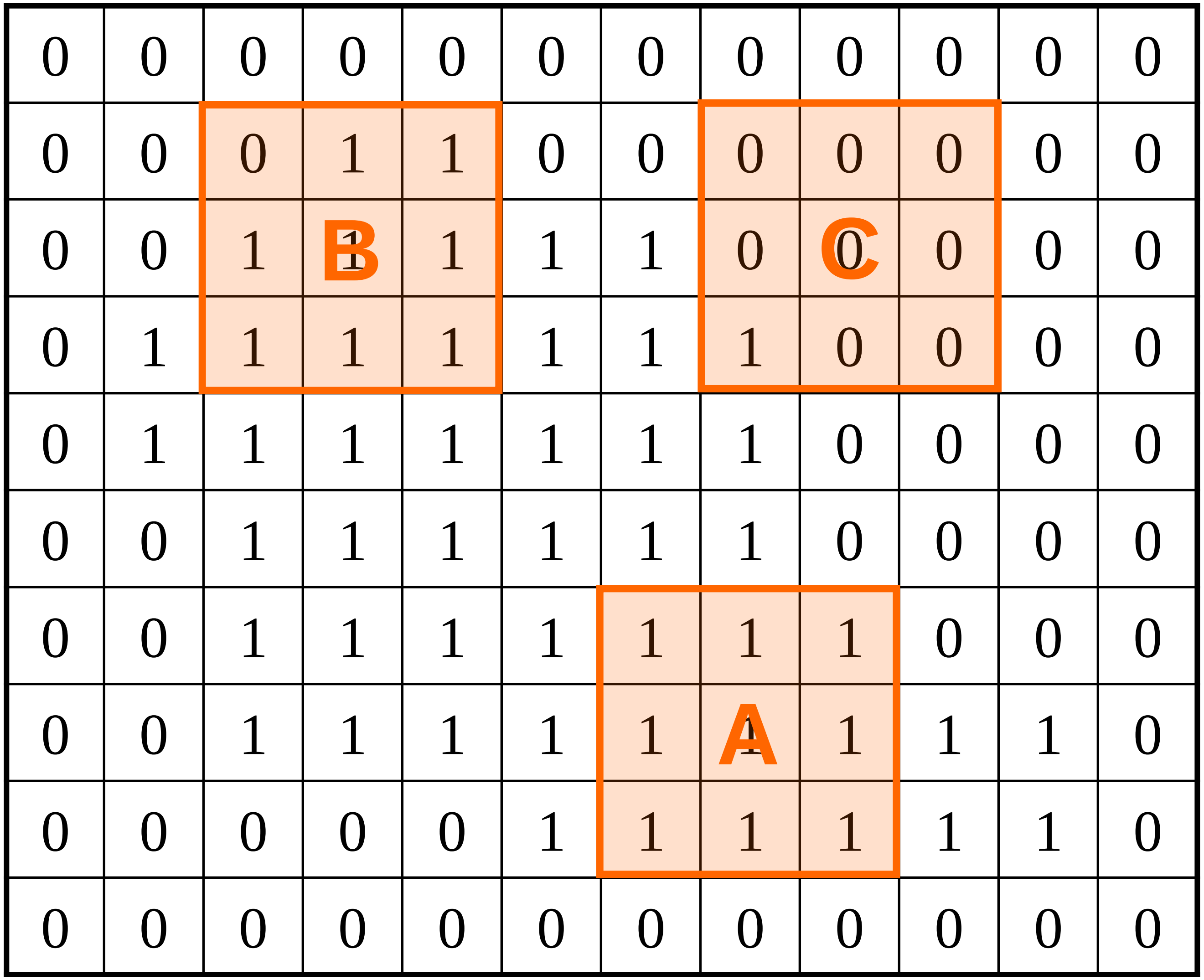


**Fit:** All *on pixels* in the structuring element cover *on pixels* in the image

**Hit:** Any *on pixel* in the structuring element covers an *on pixel* in the image

All morphological processing operations are based on these simple ideas

# Fitting & Hitting



| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |

Structuring Element 1

| | | |
|---|---|---|
| 0 | 1 | 0 |
| 1 | 1 | 1 |
| 0 | 1 | 0 |

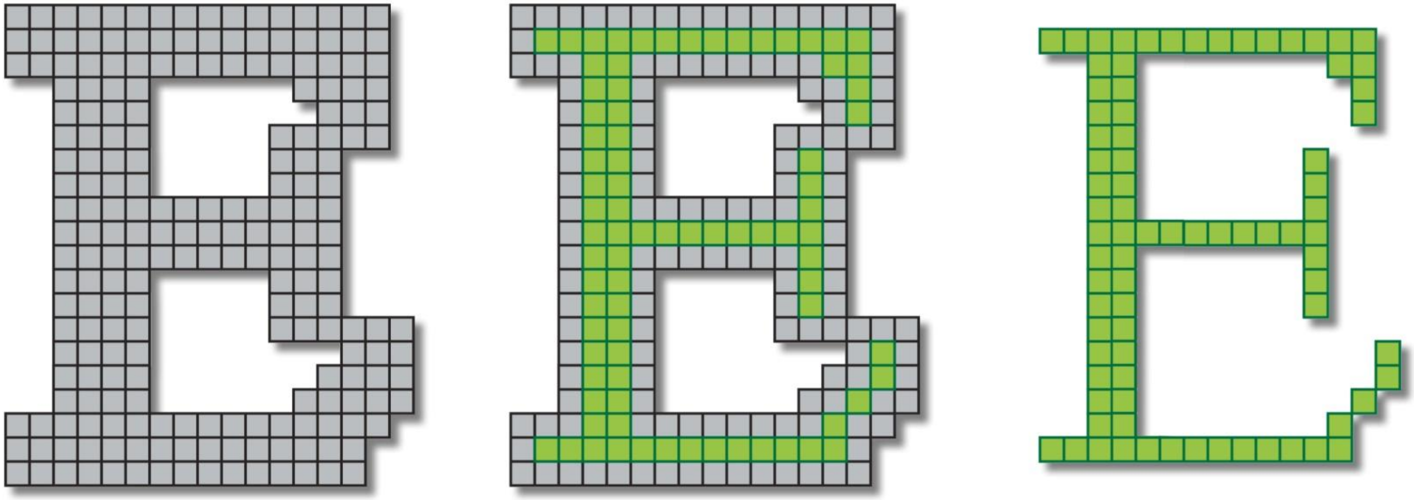
Structuring Element 2

# Fundamental Operations

- Fundamentally morphological image processing is very like spatial filtering
- The structuring element is moved across every pixel in the original image to give a pixel in a new processed image
- The value of this new pixel depends on the operation performed

There are two basic morphological operations: **erosion** and **dilation**

# Erosion



# Erosion

Definition

Erosion of image *f* by structuring element *s* is given by $f \ominus s$

The structuring element s is positioned with its origin at *(x, y)* and the new pixel value is determined using the rule:

$$g(x,y) = \begin{cases} 1 \text{ if } s \text{ fits } f \\ 0 \text{ otherwise} \end{cases}$$

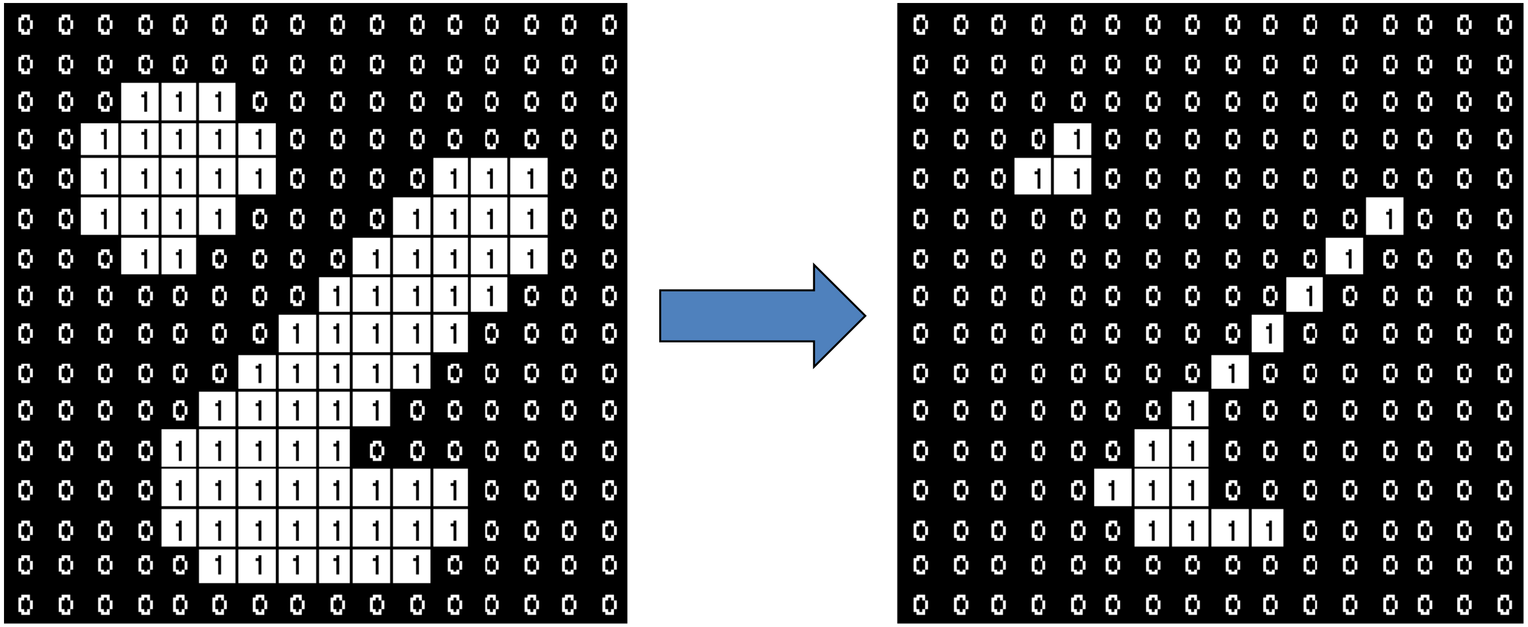
# Erosion – How to compute

- For each foreground pixel (which we will call the *input pixel*)
  - Superimpose the structuring element on top of the input image so that the origin of the structuring element coincides with the input pixel position.

  - If *for every* pixel in the structuring element, the corresponding pixel in the image underneath is a foreground pixel, then the input pixel is left as it is.

  - If any of the corresponding pixels in the image are background, however, the input pixel is also set to background value.

# Erosion – How to compute

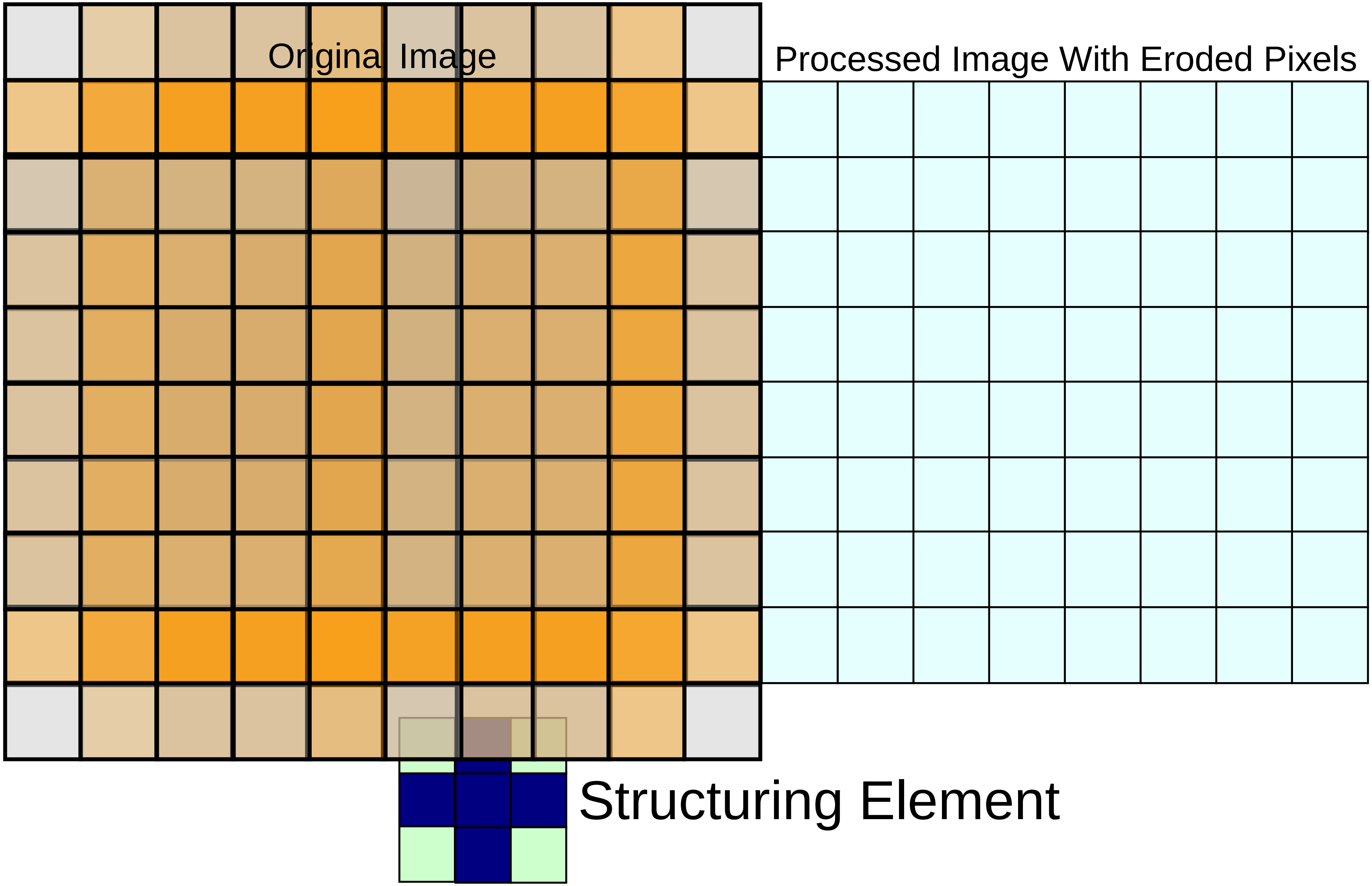


| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

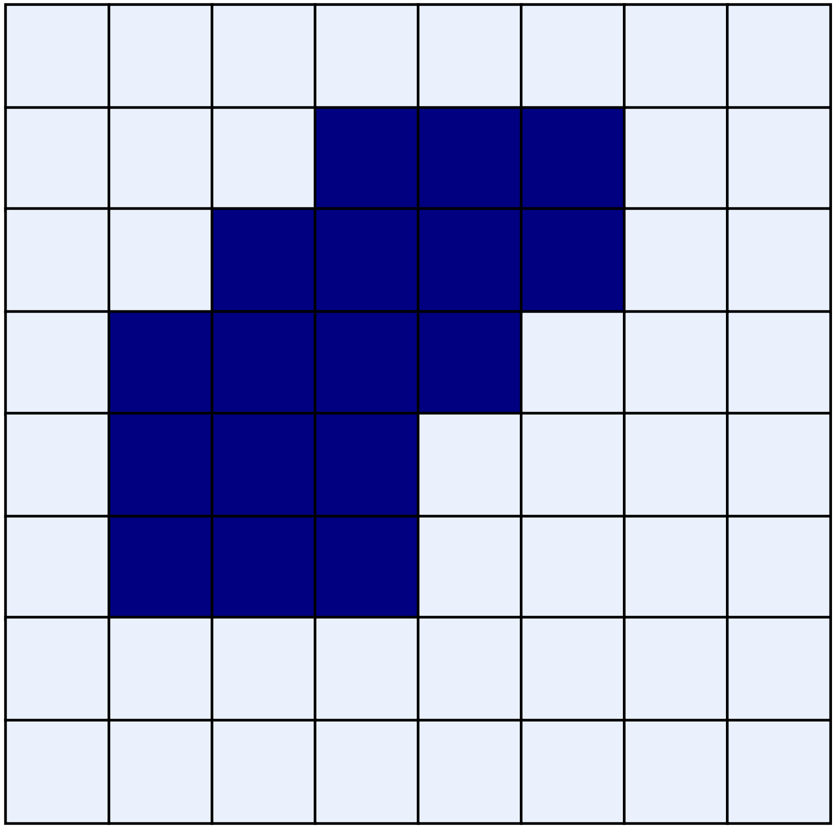
Erosion with a structuring element of size 3x3

# Erosion: Example

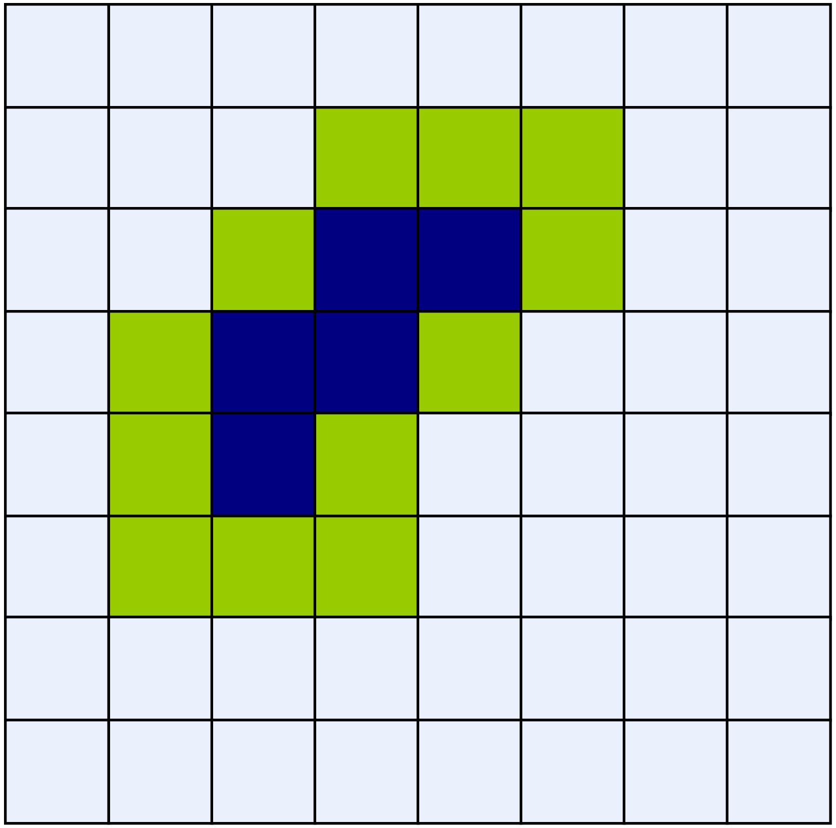


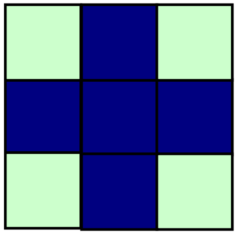
# Erosion: Example

Original Image



Processed Image





Structuring Element

# Erosion

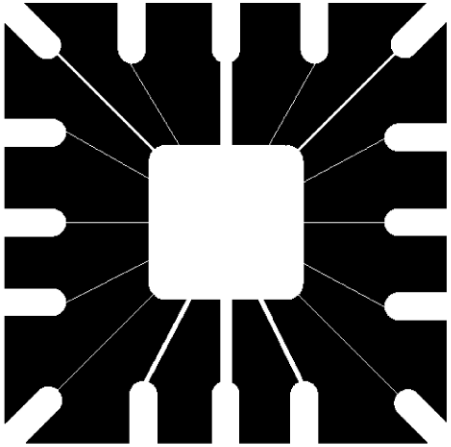
- Effects
  - Shrinks the size of foreground (1-valued) objects
  - Smoothes object boundaries
  - Removes small objects

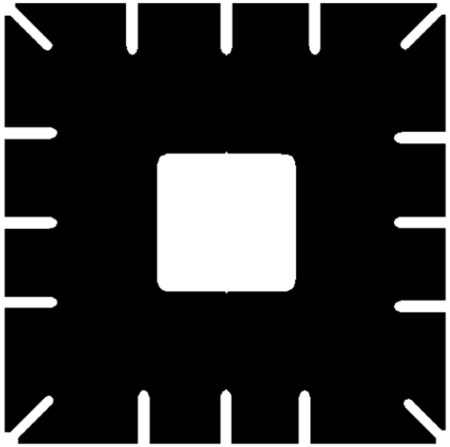
- Rule for Erosion

  In a binary image, if any of the pixel (in the neighborhood defined by structuring element) is 0, then output is 0

# Erosion: Example 1

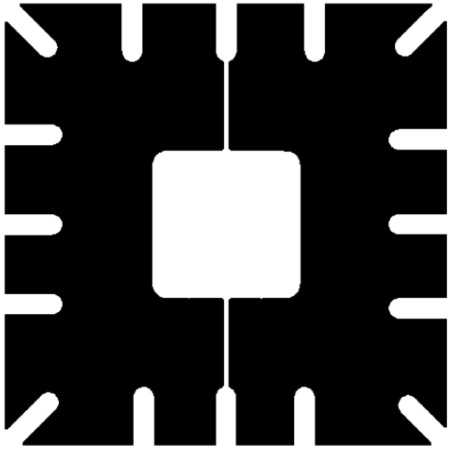
Original image

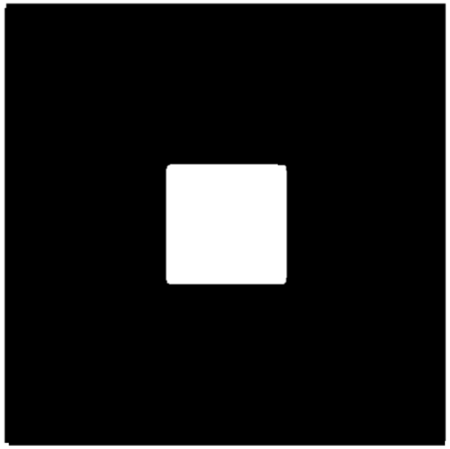




After erosion with a disc of radius 10

After erosion with a disc of radius 5

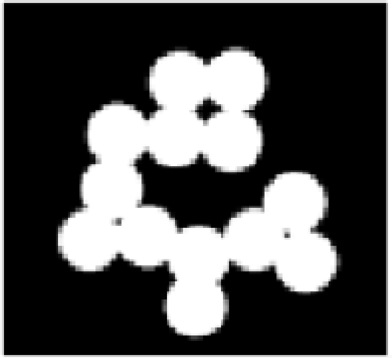


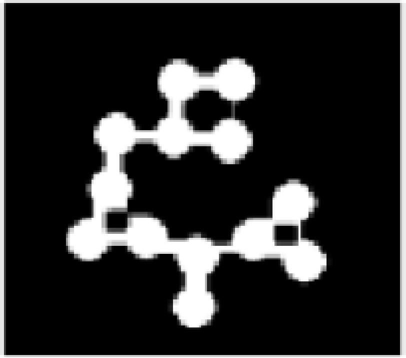


After erosion with a disc of radius 20

# Erosion: Example 2

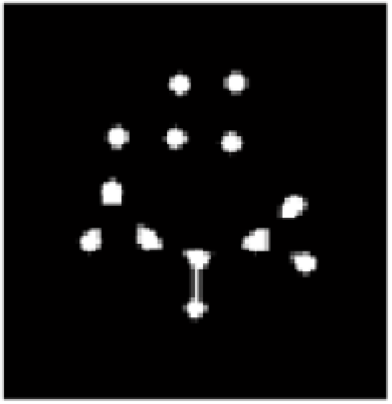
Original binary image *circles*

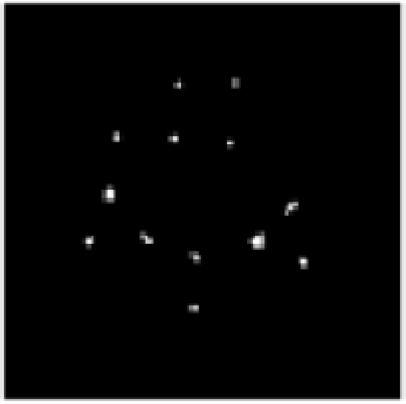




Erosion by 11x11 structuring element

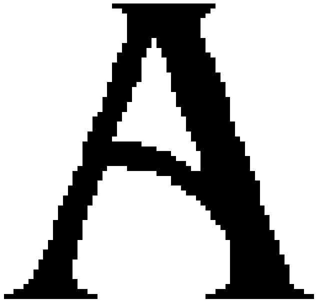
Erosion by 21x21 structuring element



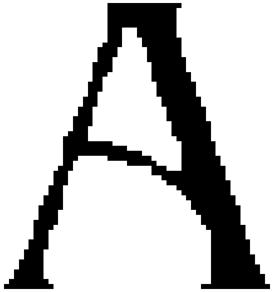


Erosion by 27x27 structuring element

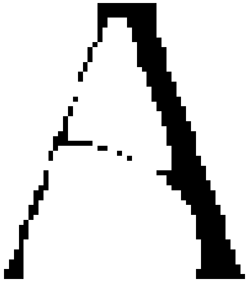
# Erosion: Example 3



Original image



Erosion by 3*3 square structuring element

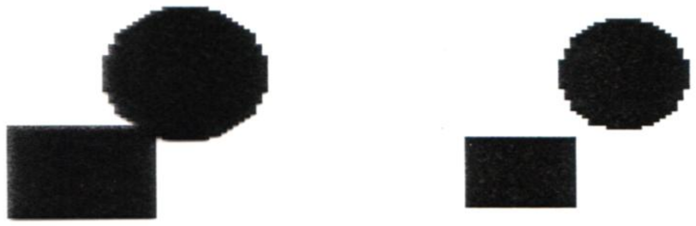


Erosion by 5*5 square structuring element

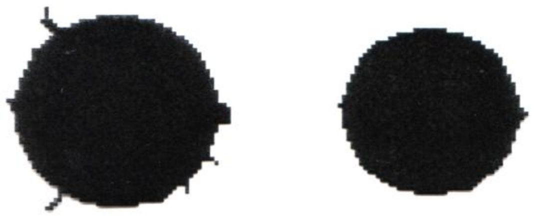
**Note:** In these examples a 1 refers to a black pixel!

# Erosion

Erosion can split apart joined objects



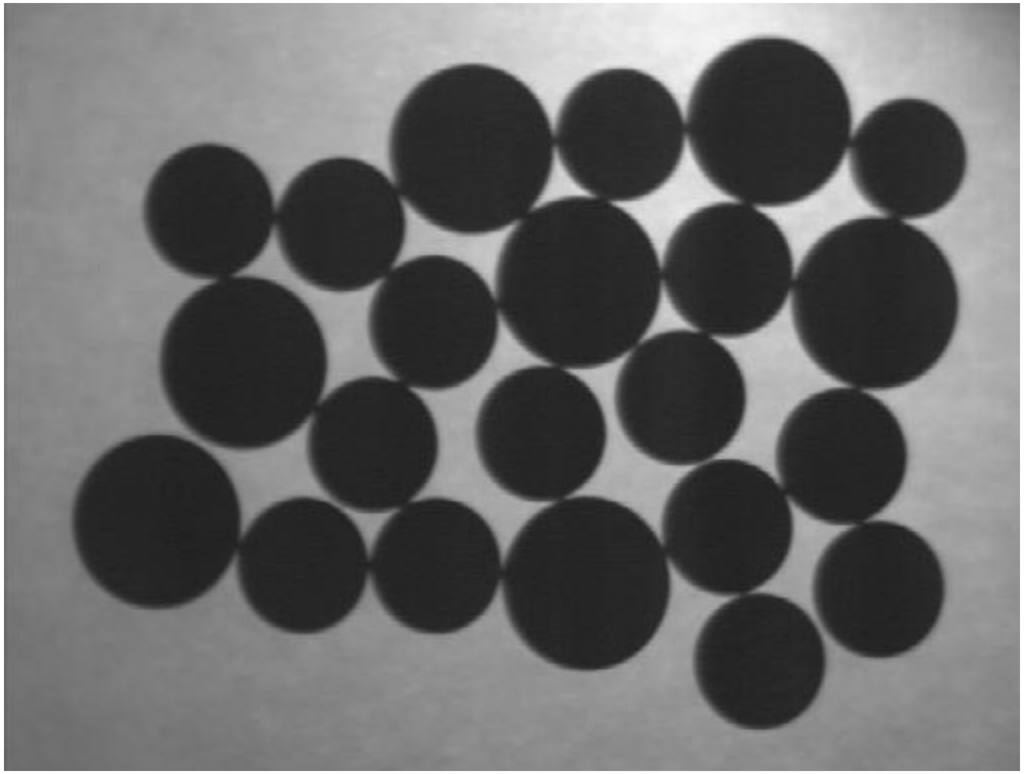
Erosion can strip away extrusions



**Watch out:** Erosion shrinks objects

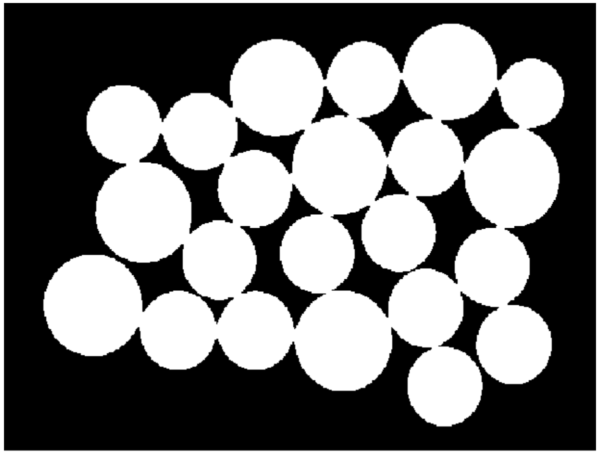
# Exercise

Count the number of coins in the given image

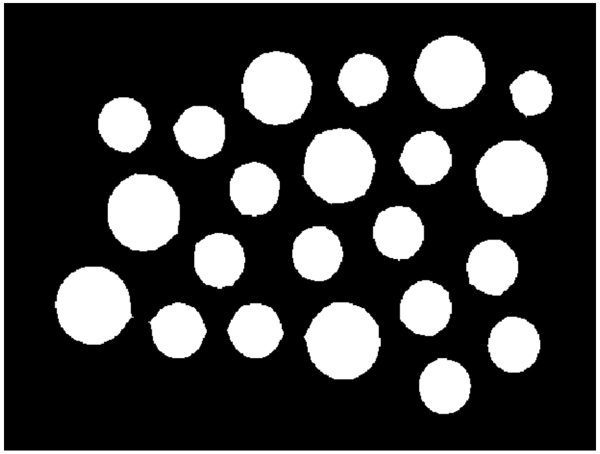


# Exercise: Solution

Binarize the image

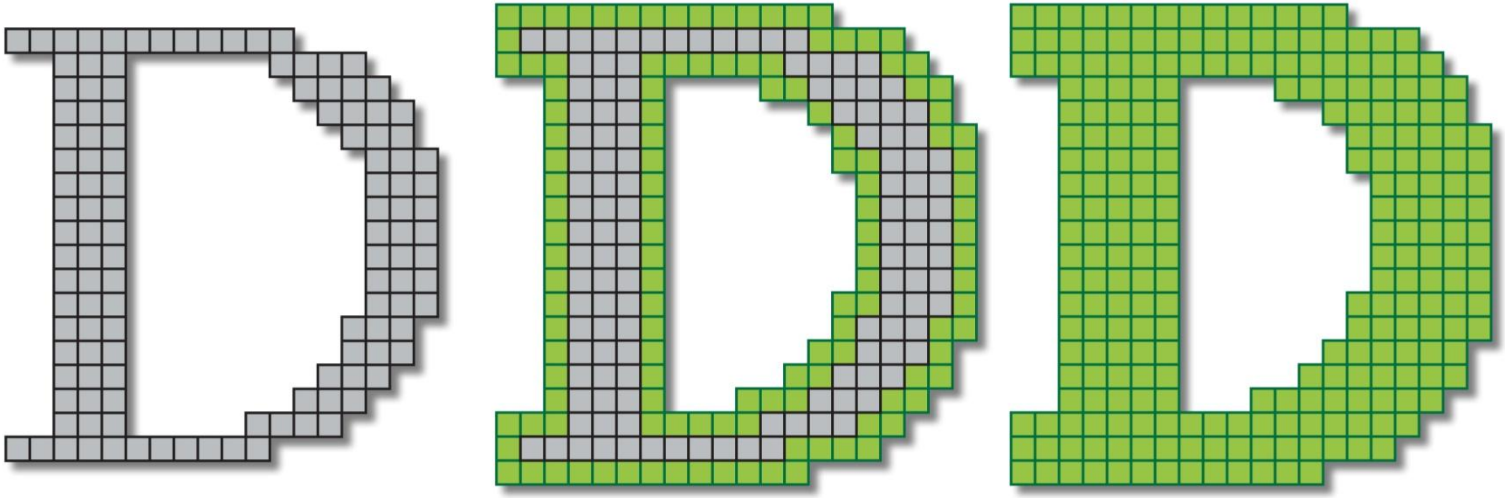


Perform Erosion



Use connected component labeling to count the number of coins

# Dilation



# Dilation

Definition:

Dilation of image *f* by structuring element *s* is given by

$f \oplus s$

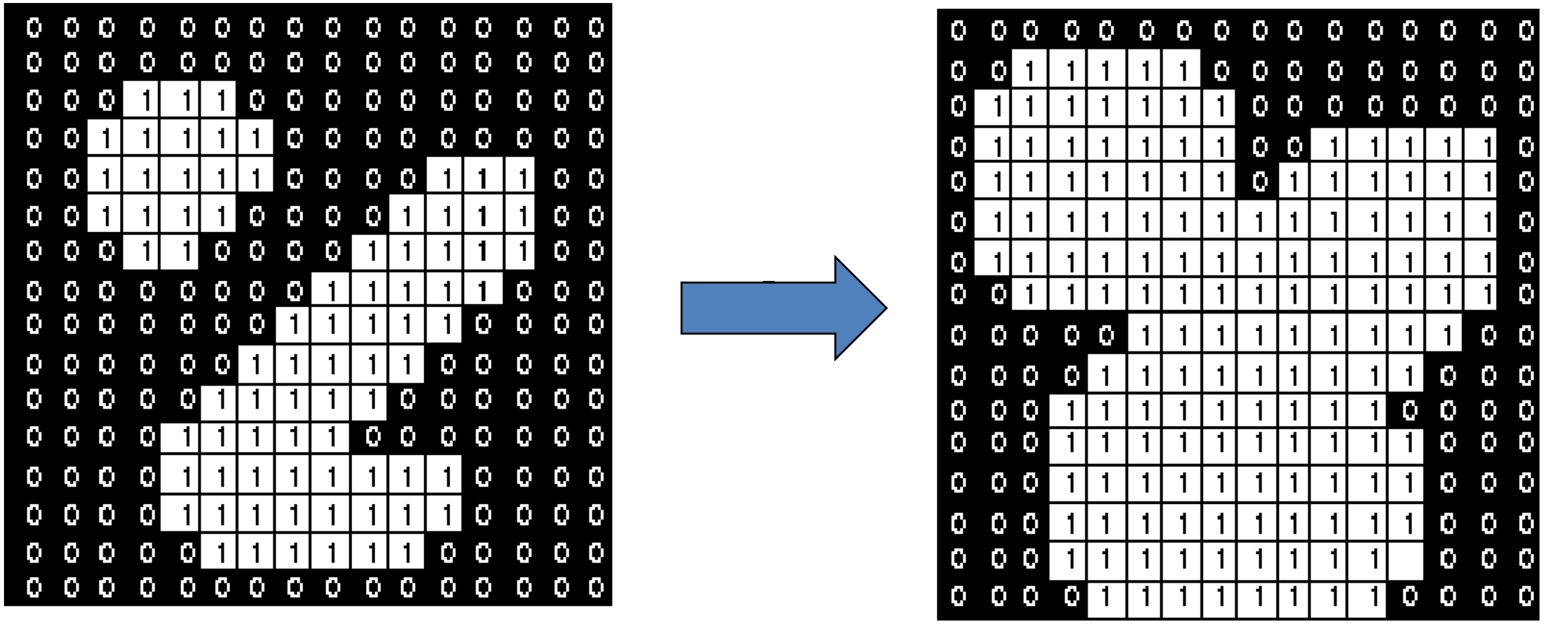
The structuring element s is positioned with its origin at *(x, y)* and the new pixel value is determined using the rule:

$$g(x, y) = \begin{cases} 1 \text{ if } s \text{ hits } f \\ 0 \text{ otherwise} \end{cases}$$

# Dilation – How to compute

- For each background pixel (which we will call the *input pixel*)
  - Superimpose the structuring element on top of the input image so that the origin of the structuring element coincides with the input pixel position
  - If *at least one* pixel in the structuring element coincides with a foreground pixel in the image underneath, then the input pixel is set to the foreground value
  - If all the corresponding pixels in the image are background, however, the input pixel is left at the background value

# Dilation: Example

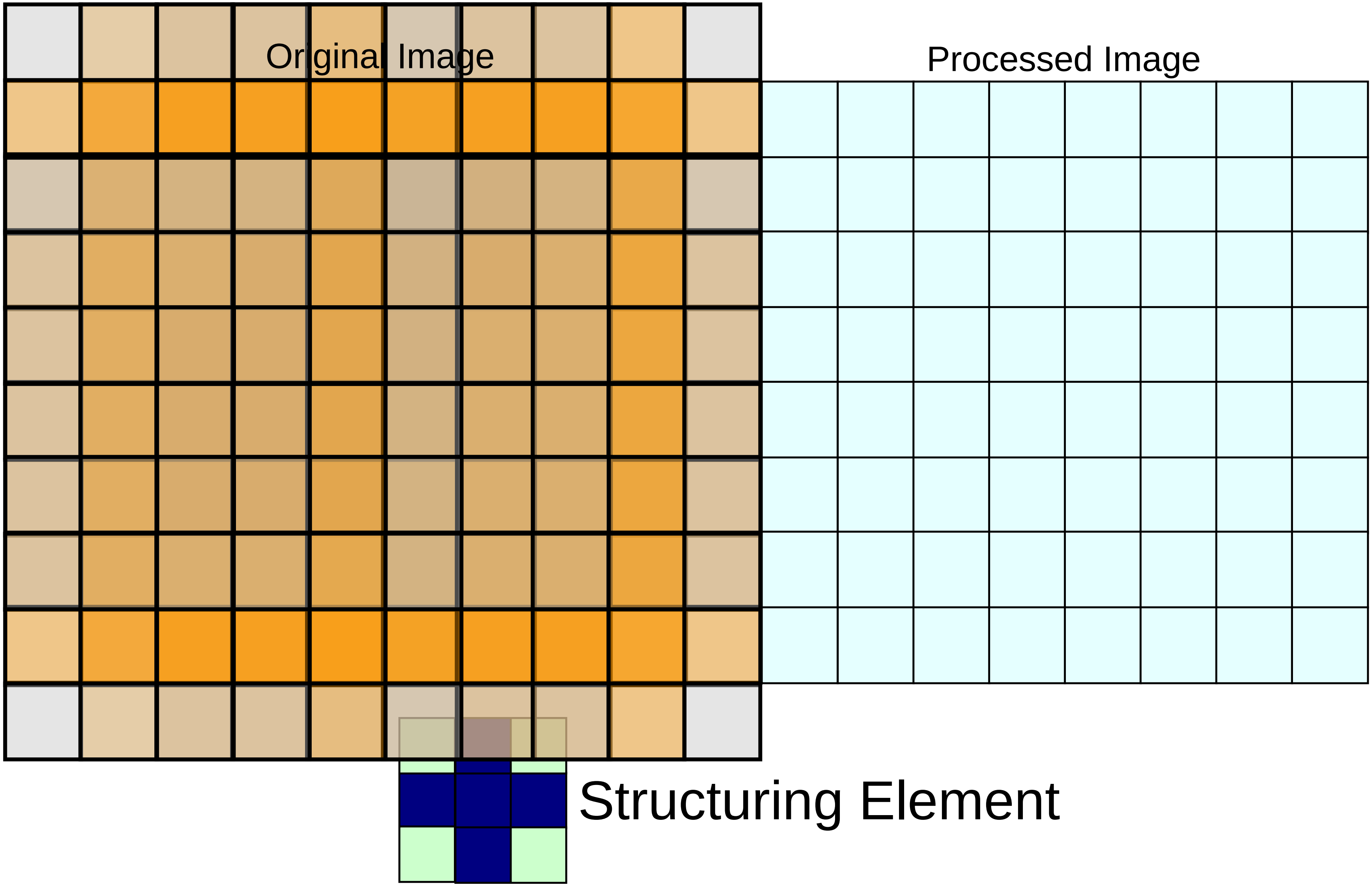


| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

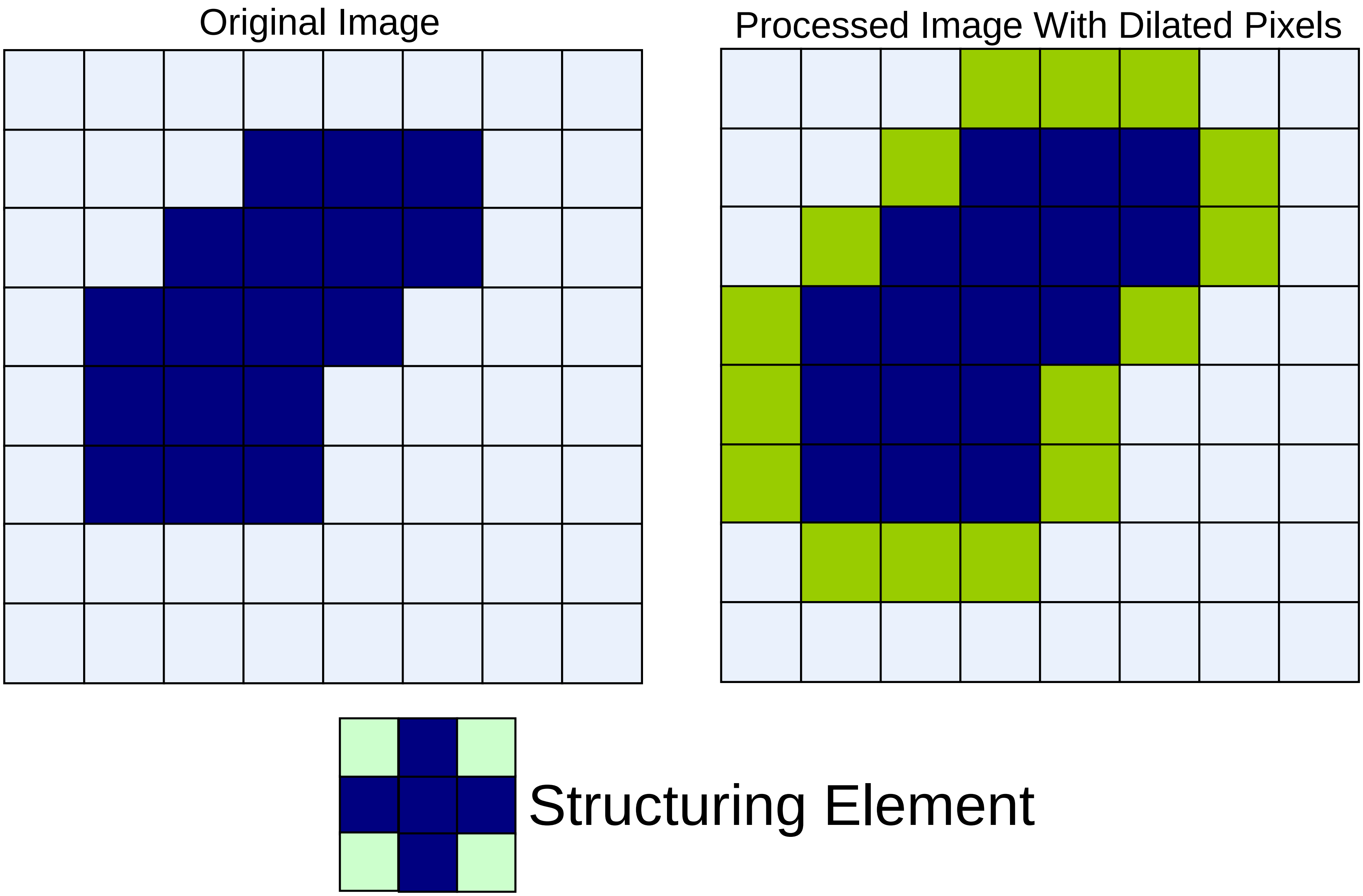
| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |  | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |

*Effect of dilation using a 3×3 square structuring element*

# Dilation: Example



# Dilation: Example



Original Image

Processed Image With Dilated Pixels

Structuring Element

# Dilation

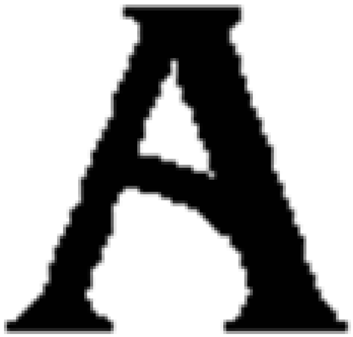
- Effects
  - Expands the size of foreground(1-valued) objects
  - Smoothes object boundaries
  - Closes holes and gaps

- Rule for Dilation

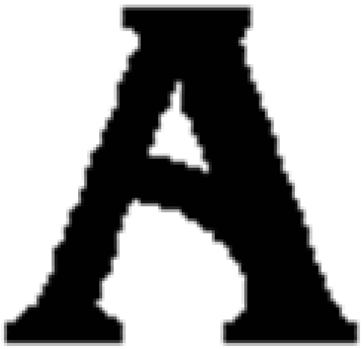
  In a binary image, if any of the pixel (in the neighborhood defined by structuring element) is 1, then output is 1

# Dilation: Example 1

Original image



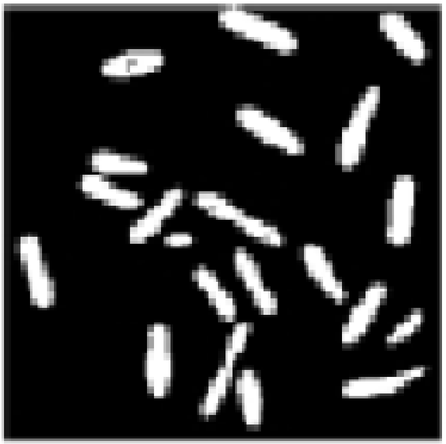
Dilation by 3*3 square structuring element



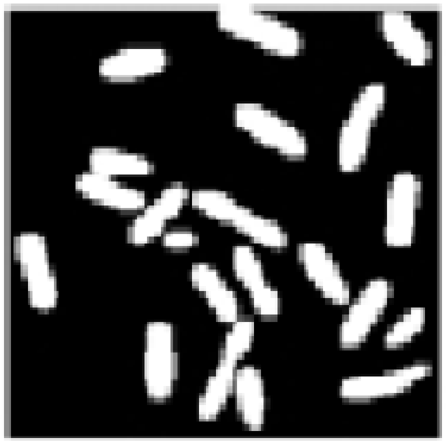
Dilation by 5*5 square structuring element

**Note:** In these examples a 1 refers to a black pixel!

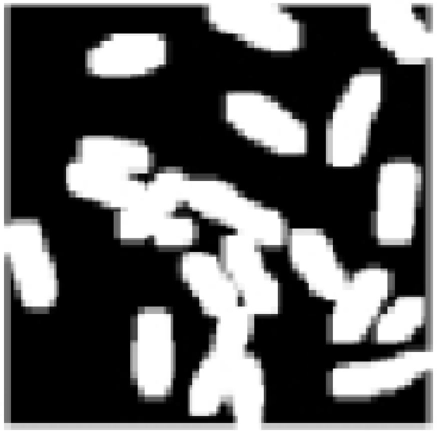
# Dilation: Example 2



Original (178x178)

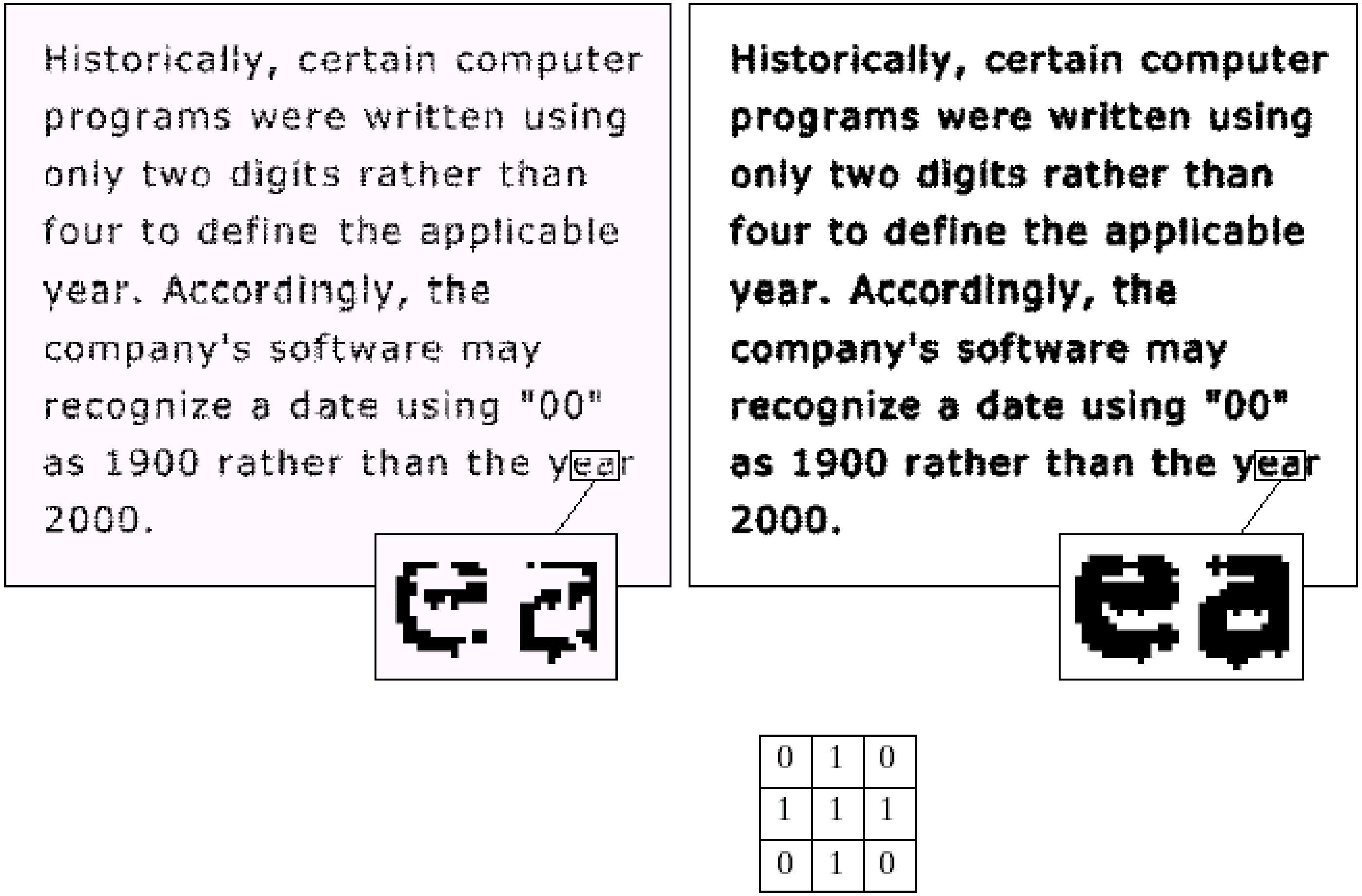


dilation with
3x3 structuring element



dilation with
7x7 structuring element

# Dilation: Example 3



Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.

Historically, certain computer programs were written using only two digits rather than four to define the applicable year. Accordingly, the company's software may recognize a date using "00" as 1900 rather than the year 2000.

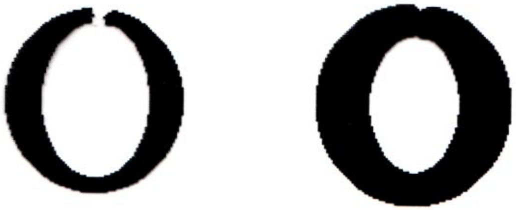
|   |   |   |
|---|---|---|
| 0 | 1 | 0 |
| 1 | 1 | 1 |
| 0 | 1 | 0 |

a c
b

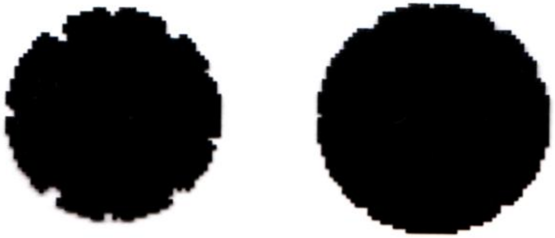
**FIGURE 9.5** (a) Sample text of poor resolution with broken characters (magnified view). (b) Structuring element. (c) Dilation of (a) by (b). Broken segments were joined.

# Dilation

Dilation can repair breaks



Dilation can repair intrusions



**Watch out:** Dilation enlarges objects

# Duality relationship between Dilation and Erosion

- Dilation and erosion are duals of each other:

$$(A!\ \ B)^c = A^c \oplus B$$

- For a symmetric structuring element:

$$(A!\ \ B)^c = A^c \oplus B$$

It means that we can obtain erosion of an image A by B simply by dilating its background (i.e. $A^c$) with the same structuring element and complementing the result.

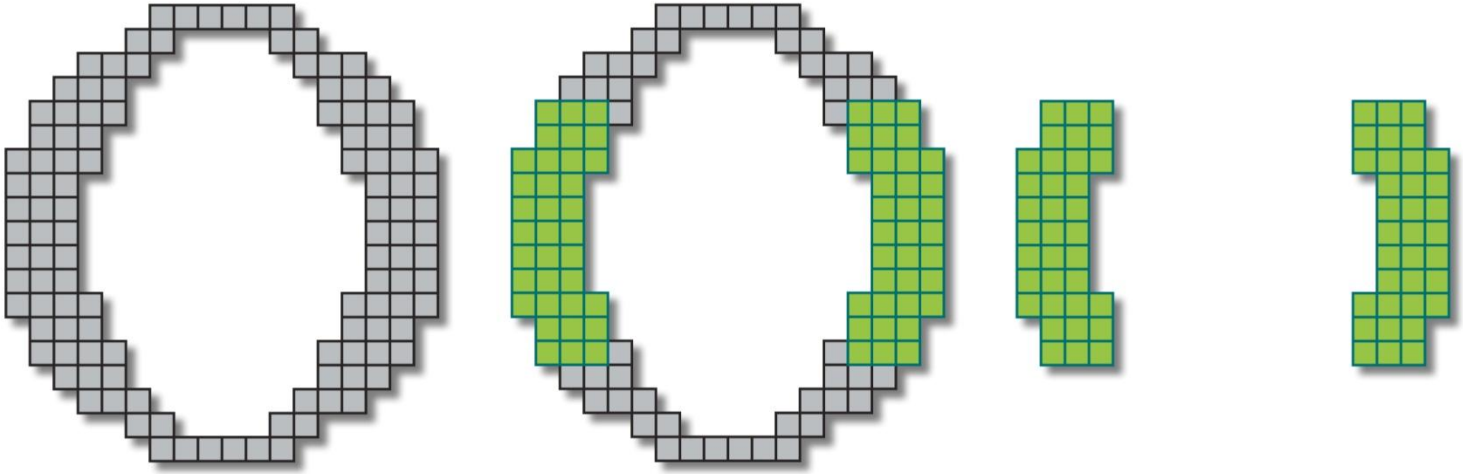
# Compound Operations

- More interesting morphological operations can be performed by performing combinations of erosions and dilations

The most widely used of these *compound operations* are:

- Opening
- Closing

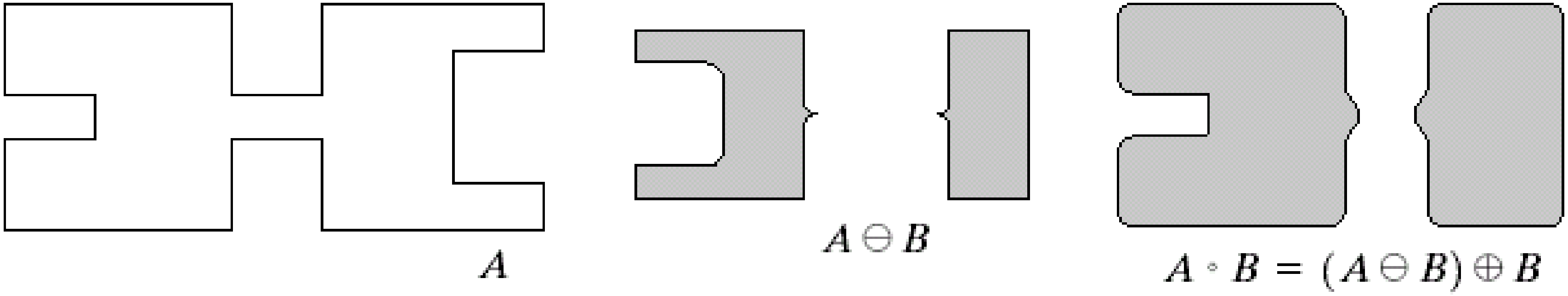
# Opening



# Opening

The opening of image *f* by structuring element *s*, denoted by *f* ○ *s* is simply an erosion followed by a dilation

$$f \circ s = (f \ominus s) \oplus s$$



Original shape

After erosion

After dilation (opening)

# Opening: Example

Original Image

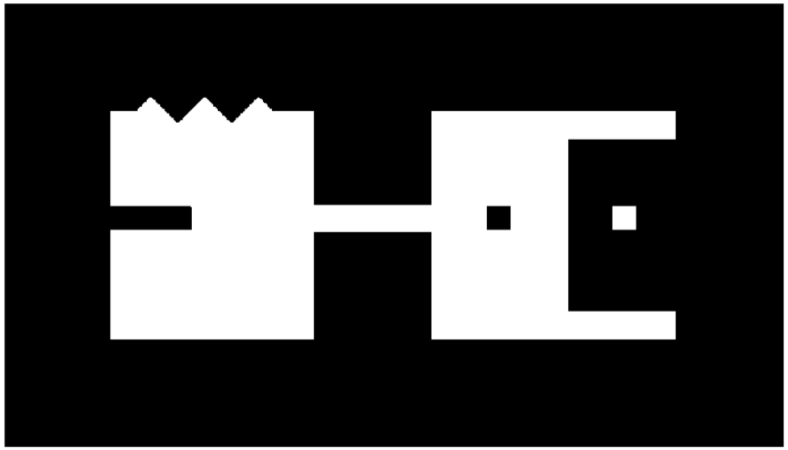
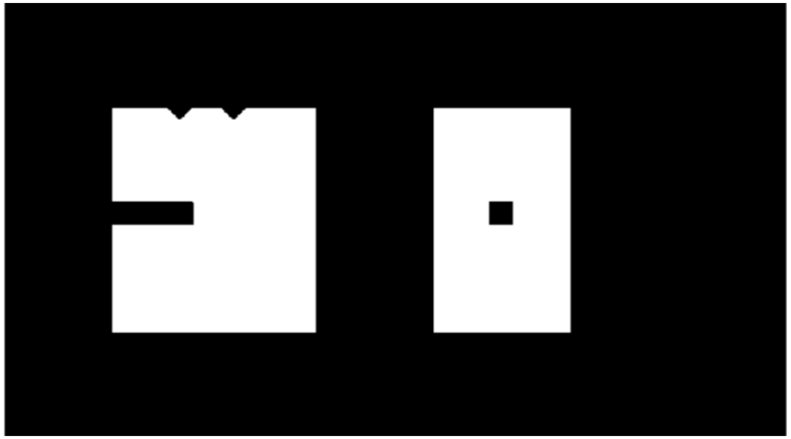


Image After Opening



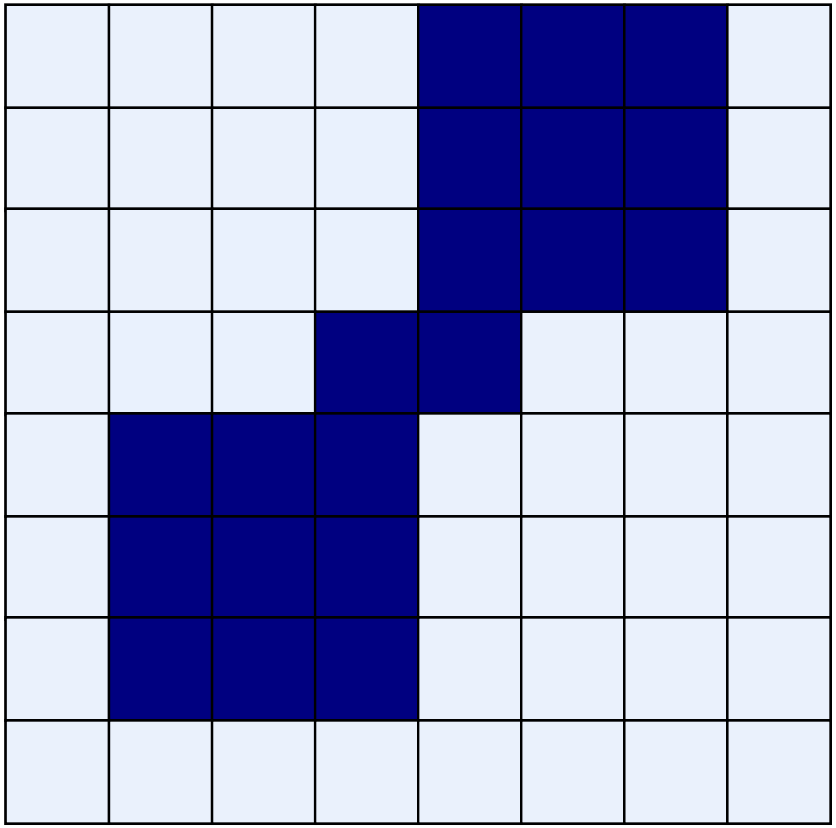
Opening

Breaks narrow joints

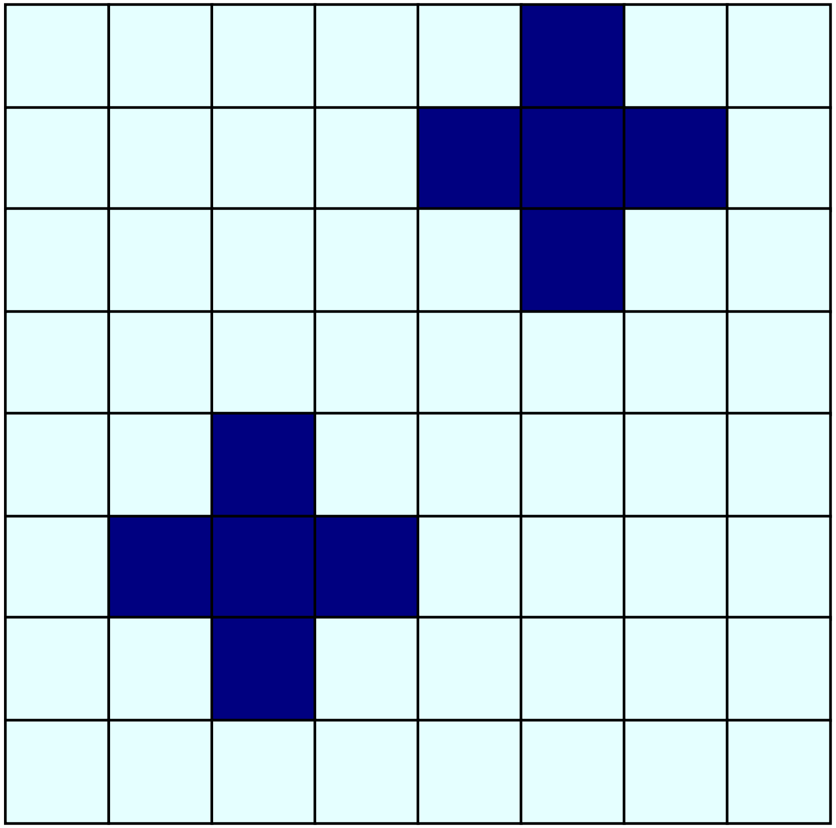
Removes ‘Salt’ noise

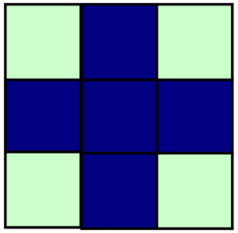
# Opening: Example

Original Image



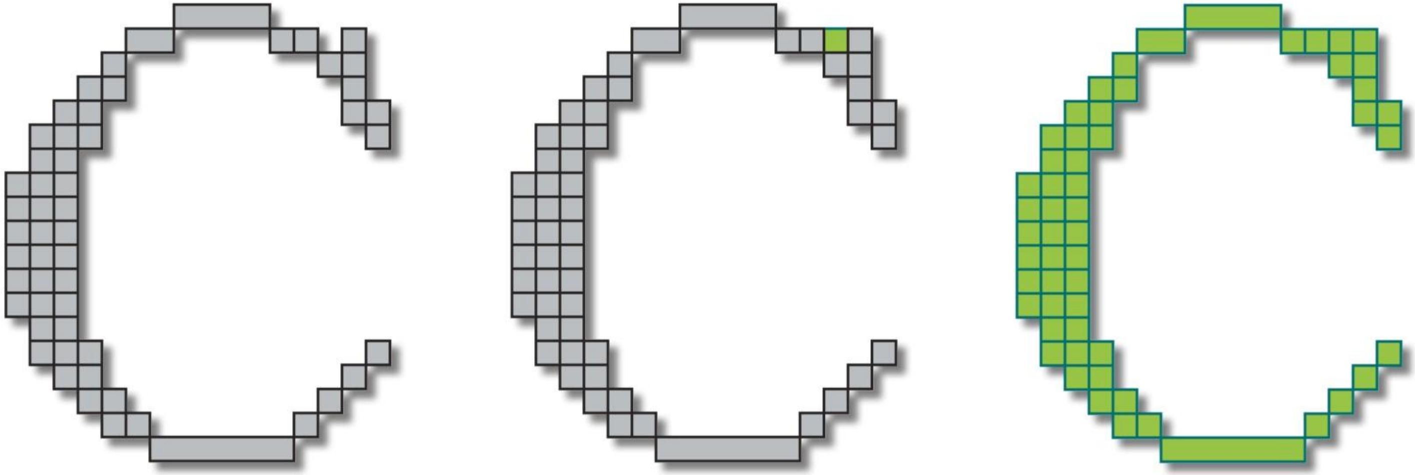
Processed Image





Structuring Element

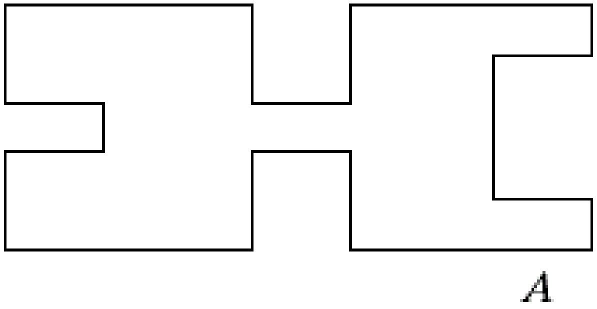
# Closing



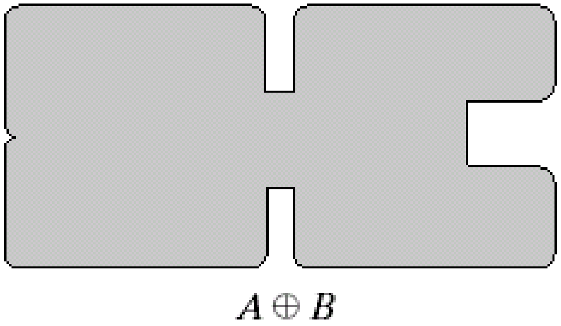
# Closing

The closing of image *f* by structuring element *s*, denoted by *f* • *s* is simply a dilation followed by an erosion

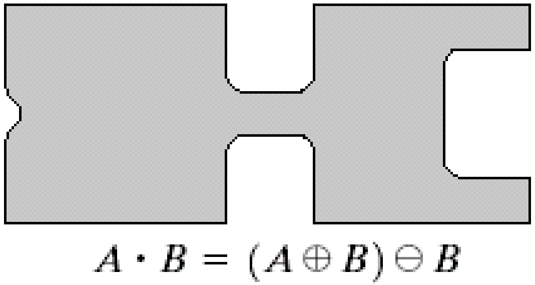
$$f \bullet s = (f \oplus s) \ominus s$$



Original shape



After dilation



After erosion (closing)

# Closing: Example

Original Image

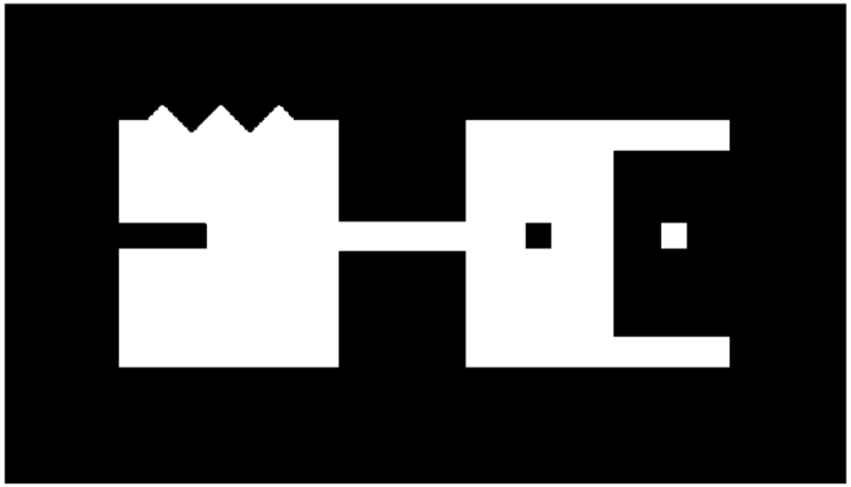
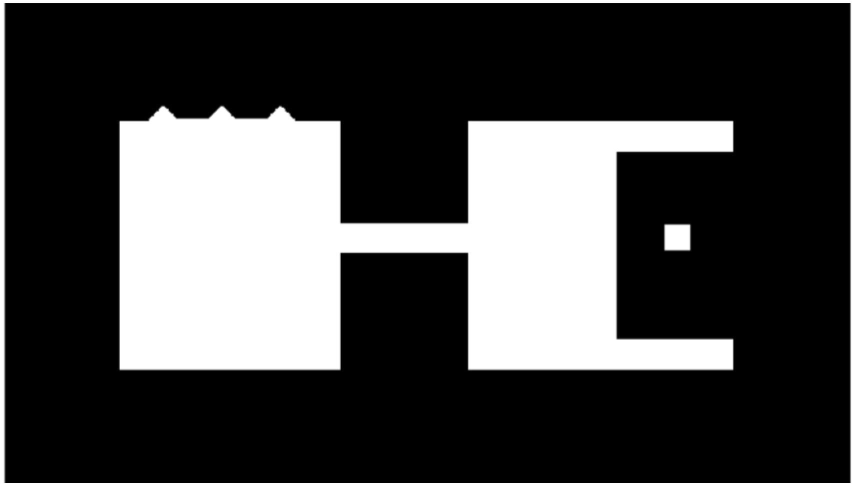


Image After Closing



Closing

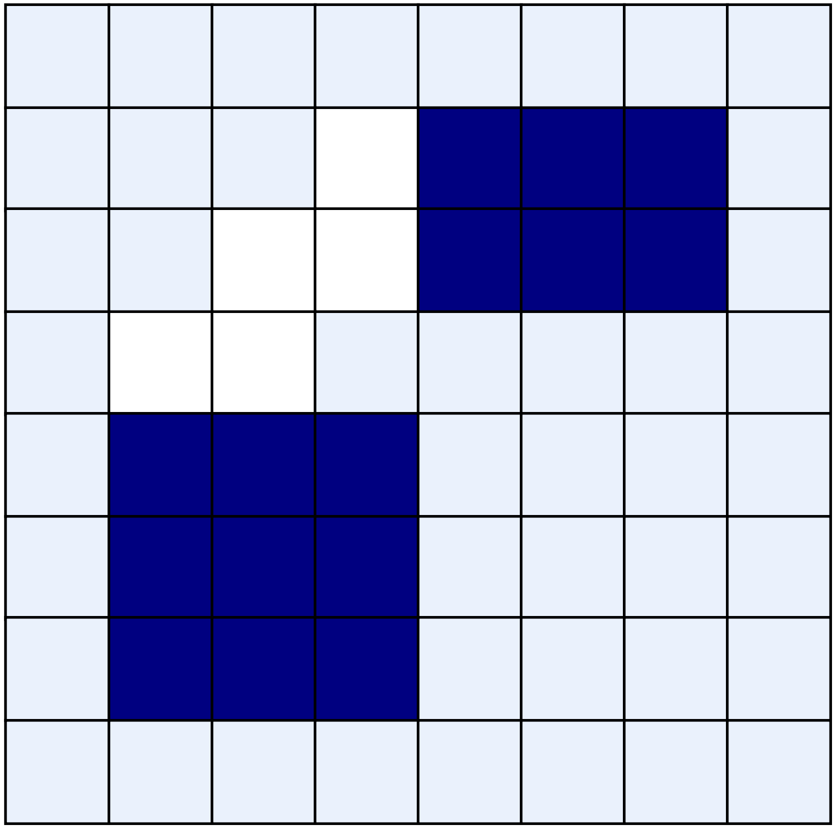
Eliminates small holes

Fills gaps

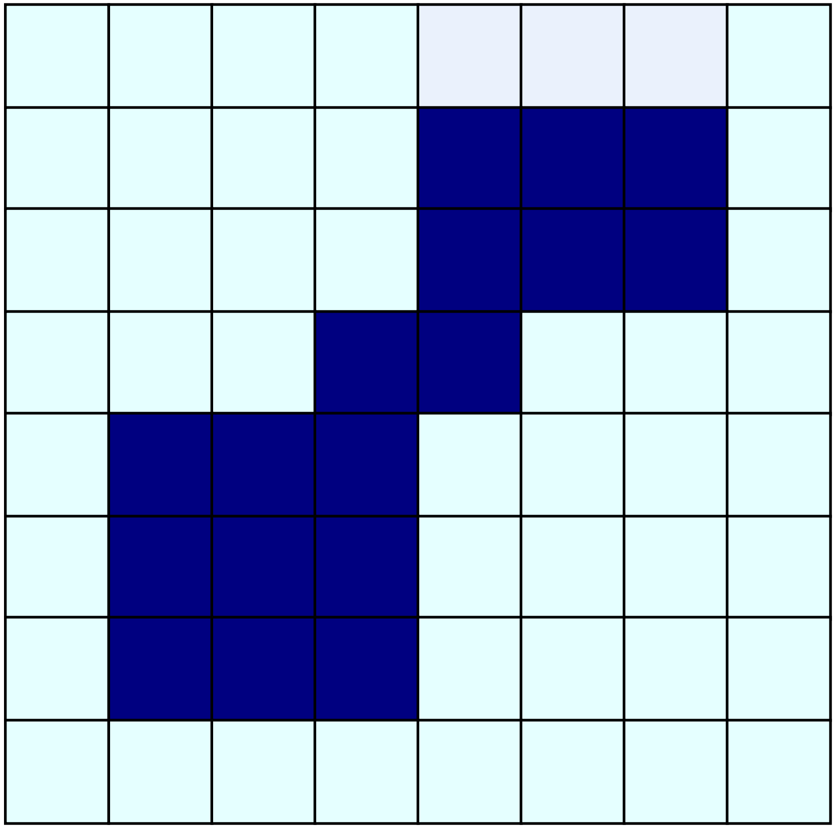
Removes ‘Pepper’ noise

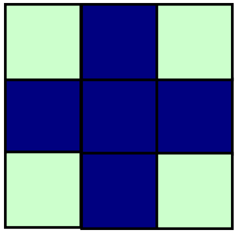
# Closing: Example

Original Image



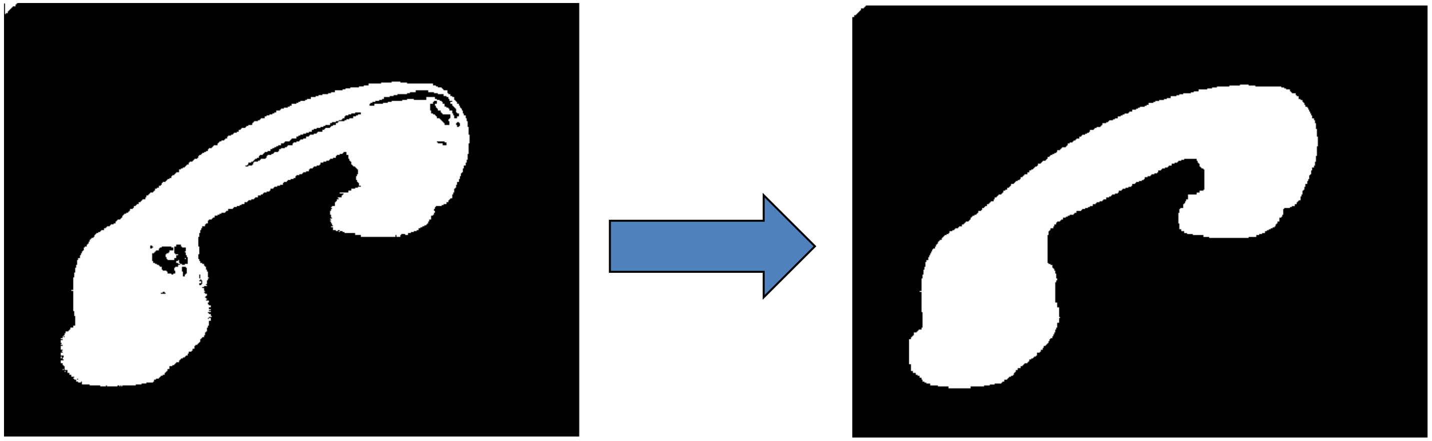
Processed Image





Structuring Element

# Closing



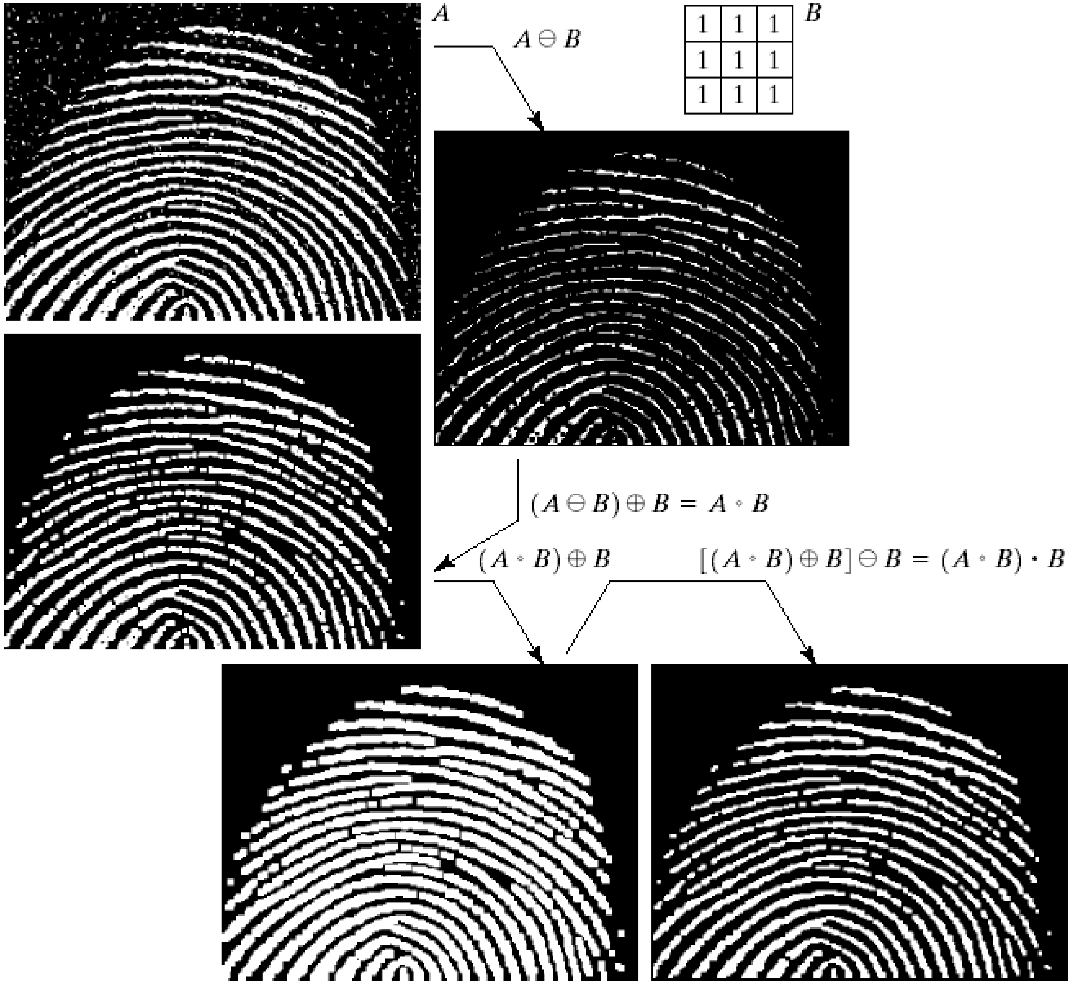
# Opening & Closing

- Opening and closing are duals of each others

$$(A \bullet B)^c = (A^c \circ B)$$

$$(A \circ B)^c = (A^c \bullet B)$$

# Morphological Processing Example



$A$
$A \ominus B$
$B$
1 1 1
1 1 1
1 1 1
$(A \ominus B) \oplus B = A \circ B$
$(A \circ B) \oplus B$
$[(A \circ B) \oplus B] \ominus B = (A \circ B) \bullet B$

a b
d c
e f

**FIGURE 9.11**
(a) Noisy image.
(b) Structuring element.
(c) Eroded image.
(d) Opening of $A$.
(e) Dilation of the opening.
(f) Closing of the opening.
(Original image courtesy of the National Institute of Standards and Technology.)

# Morphological Algorithms

Using the simple technique we have looked at so far we can begin to consider some more interesting morphological algorithms

We will look at:

- Boundary extraction
- Region filling
- Extraction of connected components

There are lots of others as well though:

- Thinning/thickening
- Skeletonization

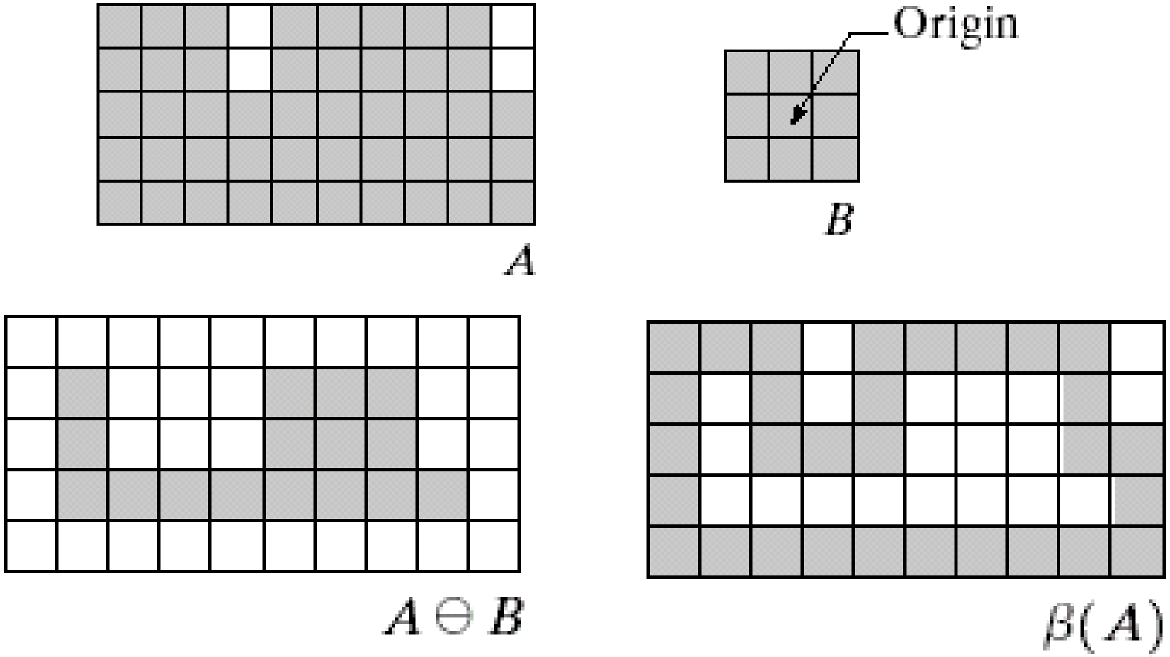
# Boundary Extraction

The boundary of set A denoted by *β(A)* is obtained by first eroding *A* by a suitable structuring element *B* and then taking the difference between *A* and its erosion.

$$\boxed{\beta(A) = A - (A!\ \ B)}$$

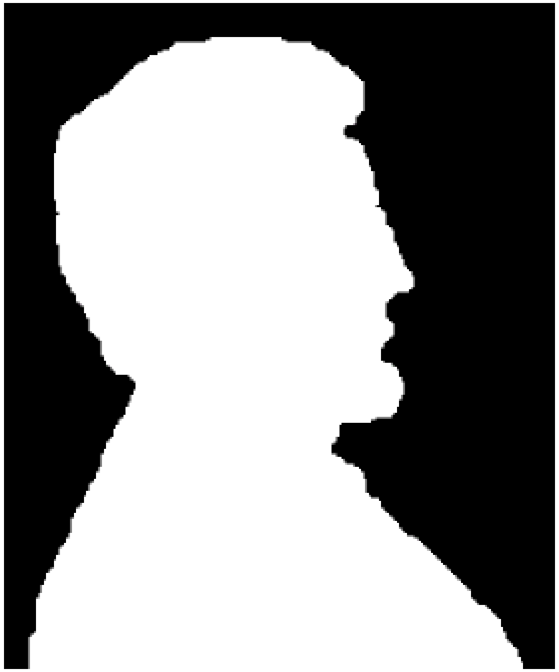
# Boundary Extraction

$$\boxed{\beta(A) = A - (A \ominus B)}$$

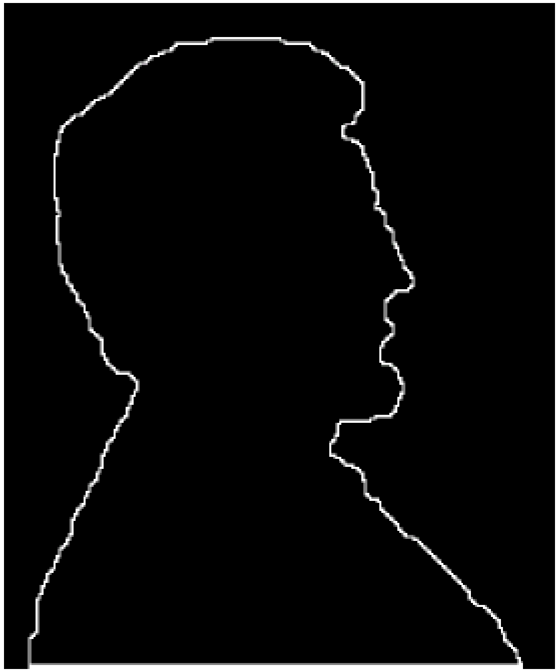


# Boundary Extraction

A simple image and the result of performing boundary extraction using a square 3*3 structuring element



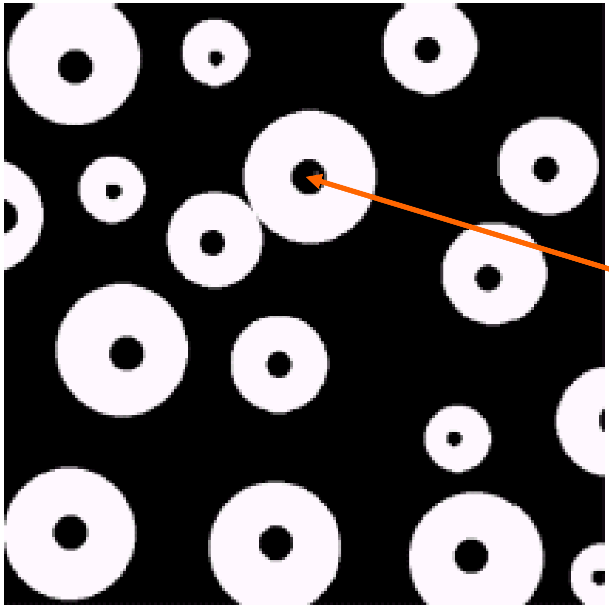
Original Image



Extracted Boundary

# Region (hole) Filling

Given a pixel inside a boundary, *region filling* attempts to fill that boundary with object pixels (1s)



Given a point inside here, can we fill the whole circle?

# Region Filling

Let A is a set containing a subset whose elements are 8-connected boundary points of a region, enclosing a background region i.e. hole

If all boundary points are labeled 1 and non boundary points are labeled 0, the following procedure fills the region:

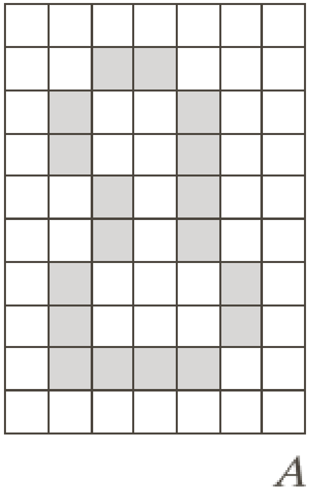
Inside the boundary

- Start from a known point $p$ and taking $X_0 = p$,
- Then taking the next values of $X_k$ as:

$$X_k = (X_k \oplus B) \bigcap A^c \qquad k = 1, 2, 3, \cdots$$

$B$ is suitable structuring element

- Terminate iterations if $X_{k+1} = X_k$
- The set union of $X_k$ and $A$ contains the filled set and its boundaries.



# Region Filling

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 1 | 0 | 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 | 0 | 1 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$A$

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 0 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |

$A^c$

| | | |
|---|---|---|
| 0 | 1 | 0 |
| 1 | 1 | 1 |
| 0 | 1 | 0 |

$B$

# Region Filling

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$X_0$

| | | |
|---|---|---|
| 0 | 1 | 0 |
| 1 | 1 | 1 |
| 0 | 1 | 0 |

$B$

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$(X_0 \oplus B)$

$$X_{k+1} = (X_k \oplus B) \bigcap A^c$$

$$k = 1, 2, 3, \cdots$$

# Region Filling

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$X_1 = (X_0 \oplus B)$

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 0 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |

$A^c$

| | | | | | | |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |

$X_1 = (X_0 \oplus B) \bigcap A^c$

$$X_{k+1} = (X_k \oplus B) \bigcap A^c$$

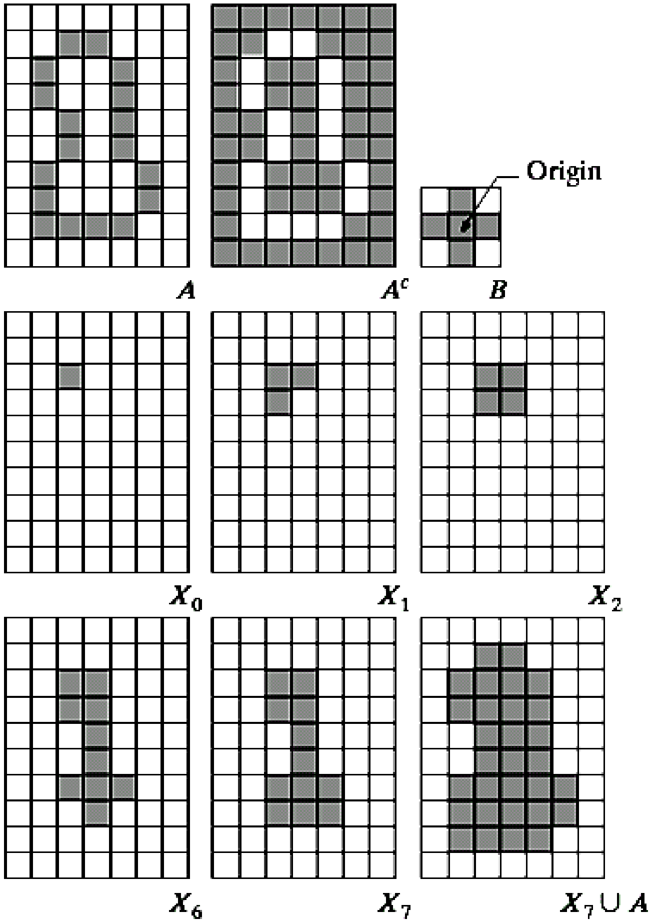
$$k = 1, 2, 3, \cdots$$

# Region Filling

$$X_{k+1} = (X_k \oplus B) \bigcap A^c$$

$$k = 1, 2, 3, \cdots$$

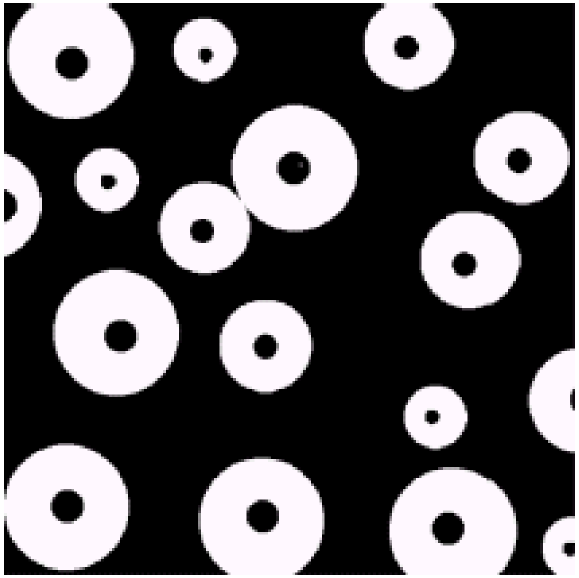
**NOTE:**
The intersection of dilation and the complement of A limits the result to inside the region of interest



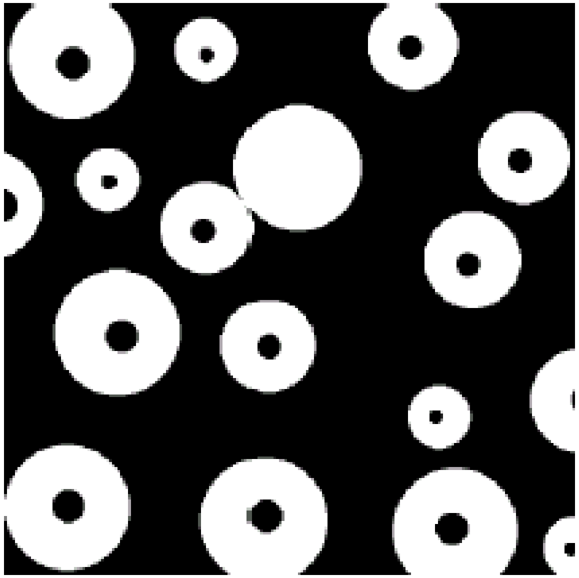
| | | |
|---|---|---|
| a | b | c |
| d | e | f |
| g | h | i |

**FIGURE 9.15** Hole filling. (a) Set *A* (shown shaded). (b) Complement of *A*. (c) Structuring element *B*. (d) Initial point inside the boundary. (e)–(h) Various steps of Eq. (9.5-2). (i) Final result [union of (a) and (h)].

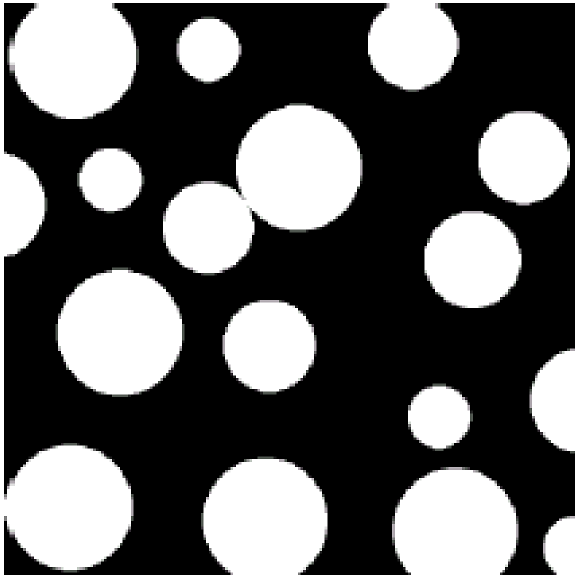
# Region Filling: Example



Original Image



One Region Filled



All Regions Filled

# Gray Level Image Morphological Operations

# Dilation & Erosion

$$(f \oplus b)(s,t) = max\{f(s-x,t-y)\}$$

$$(f \ominus b)(s,t) = min\{f(s-x,t-y)\}$$

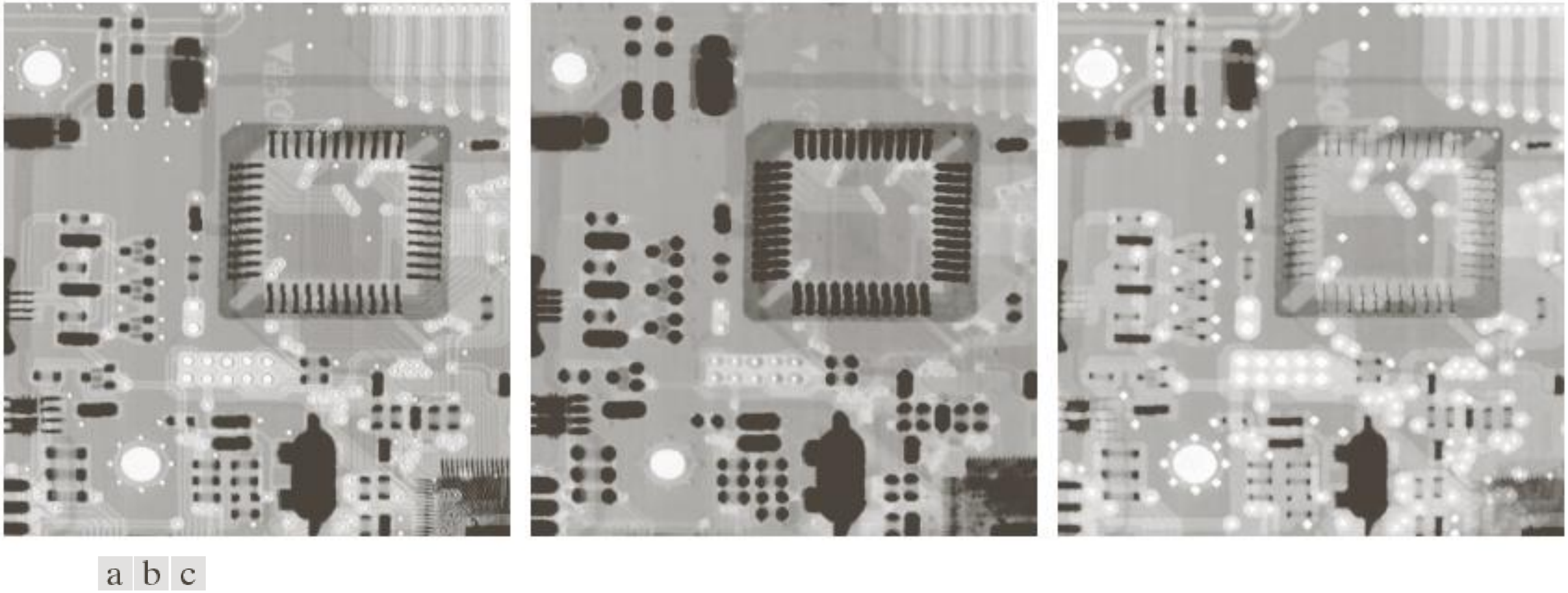
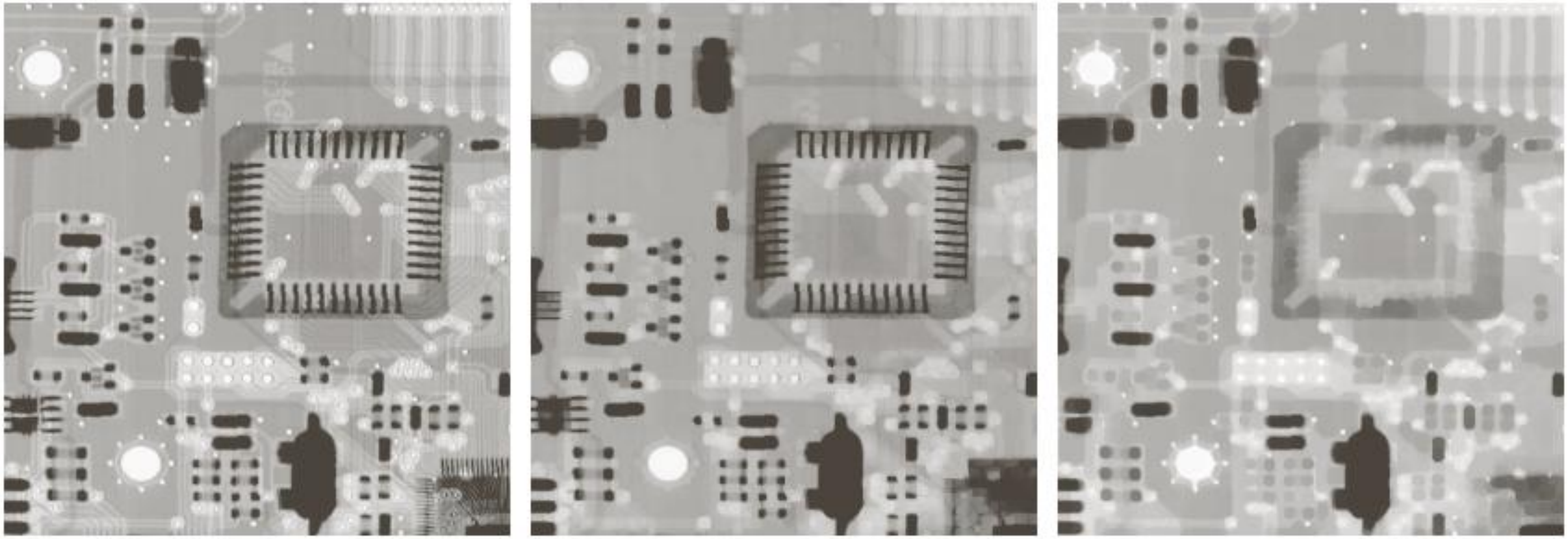


**FIGURE 9.35** (a) A gray-scale X-ray image of size 448 × 425 pixels. (b) Erosion using a flat disk SE with a radius of two pixels. (c) Dilation using the same SE. (Original image courtesy of Lixi, Inc.)

# Opening & Closing



a b c

**FIGURE 9.37** (a) A gray-scale X-ray image of size 448 × 425 pixels. (b) Opening using a disk SE with a radius of 3 pixels. (c) Closing using an SE of radius 5.

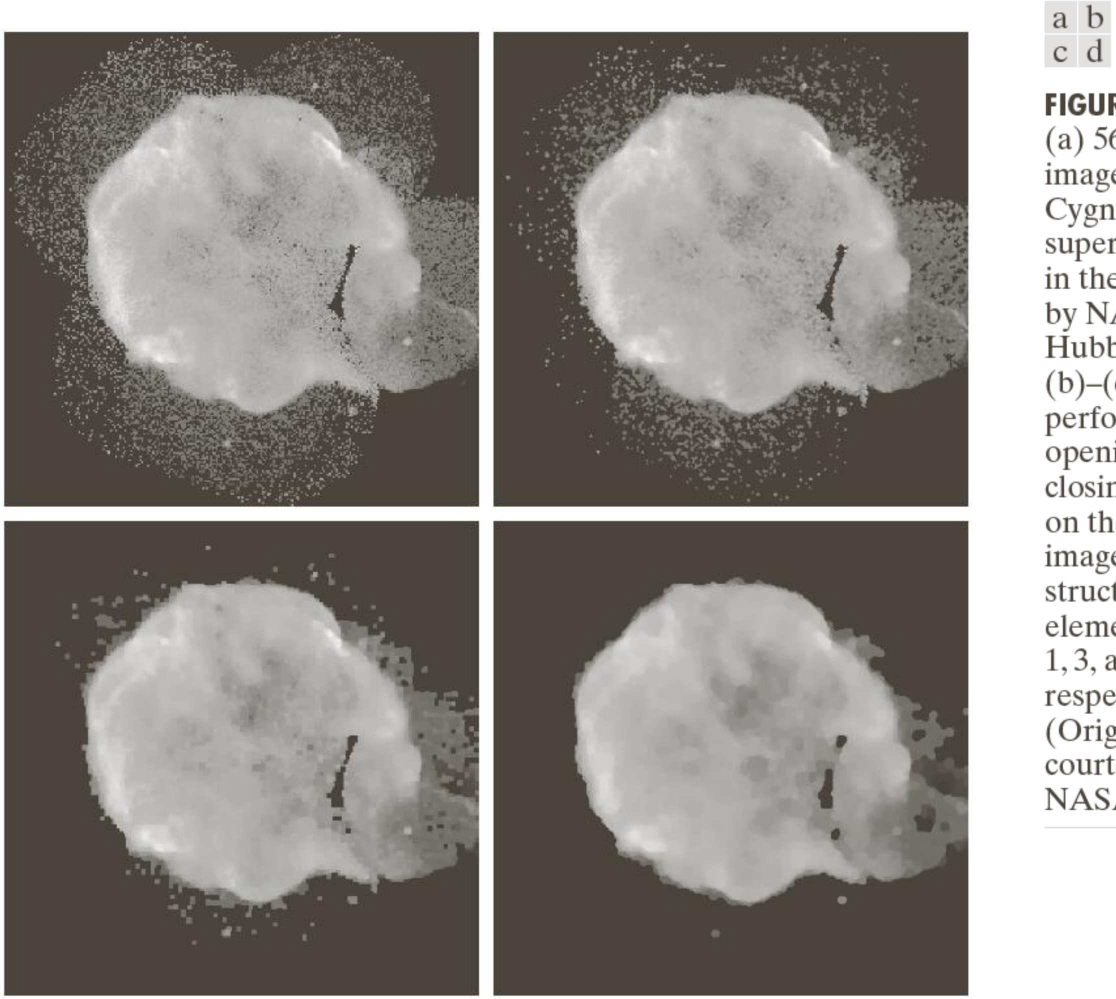
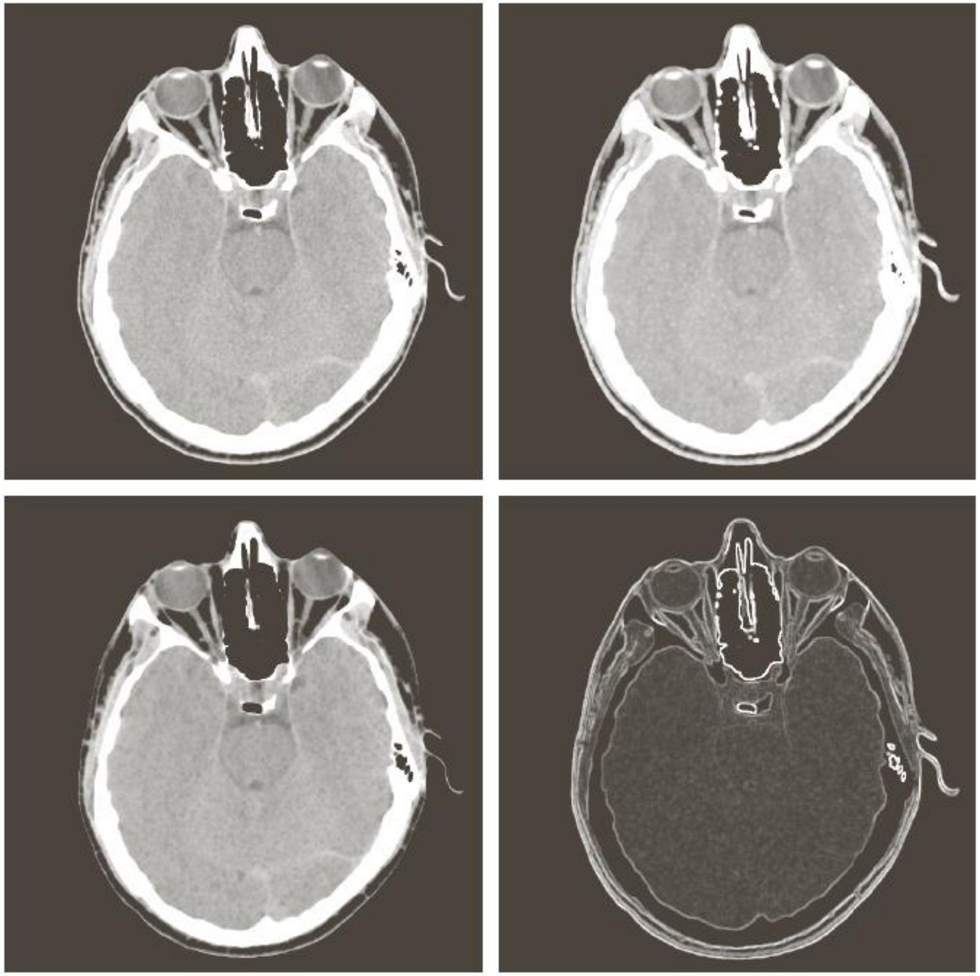


**FIGURE 9.38** (a) 566 × 566 image of the Cygnus Loop supernova, taken in the X-ray band by NASA's Hubble Telescope. (b)–(d) Results of performing opening and closing sequences on the original image with disk structuring elements of radii, 1, 3, and 5, respectively. (Original image courtesy of NASA.)

# Morphological Gradient

$$g = (f \oplus b) - (f \ominus b)$$

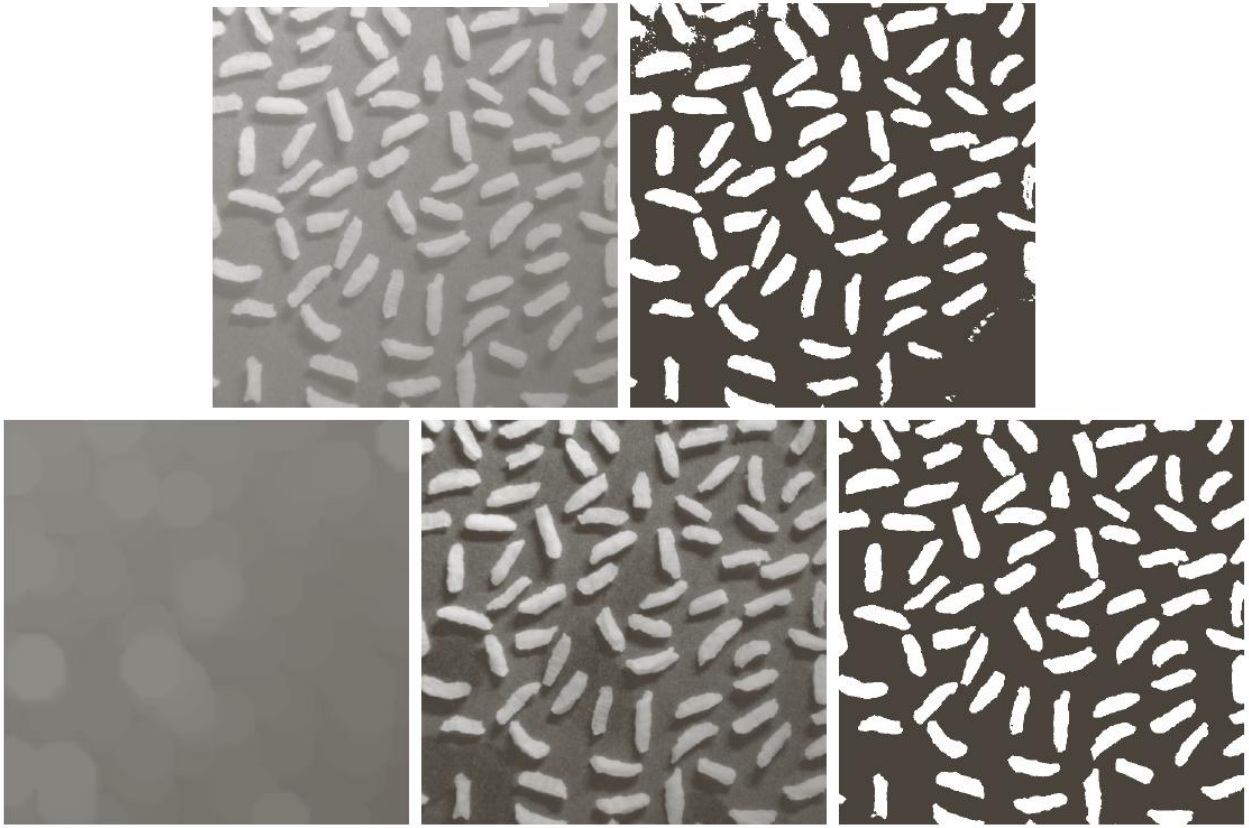


a b
c d

**FIGURE 9.39** (a) 512 × 512 image of a head CT scan. (b) Dilation. (c) Erosion. (d) Morphological gradient, computed as the difference between (b) and (c). (Original image courtesy of Dr. David R. Pickens, Vanderbilt University.)

# Top Hat & Bottom Hat Transformations

$$g_{top} = f - (f \circ b) \qquad g_{bot} = f - (f \bullet b)$$



a b
c d e

**FIGURE 9.40** Using the top-hat transformation for *shading correction*. (a) Original image of size 600 × 600 pixels. (b) Thresholded image. (c) Image opened using a disk SE of radius 40. (d) Top-hat transformation (the image minus its opening). (e) Thresholded top-hat image.

# Acknowledgements

Material in these slides has been taken from, the following resources

- Digital Image Processing”, Rafael C. Gonzalez & Richard E. Woods, Addison-Wesley, 2002
- Computer Vision for Computer Graphics, Mark Borg
- Computer Vision A modern Approach by Frosyth
- CSCI 1430: Introduction to Computer Vision by James Tompkin