Digital Image Processing

Lecture # 5
Spatial Enhancement-III

Smoothing Spatial Filters

- For blurring/noise reduction
- Blurring is usually used in preprocessing steps, e.g., to remove small details from an image prior to object extraction, or to bridge small gaps in lines or curves
- Equivalent to Low-pass spatial filtering in frequency domain because smaller (high frequency) details are removed based on neighborhood averaging (averaging filters)

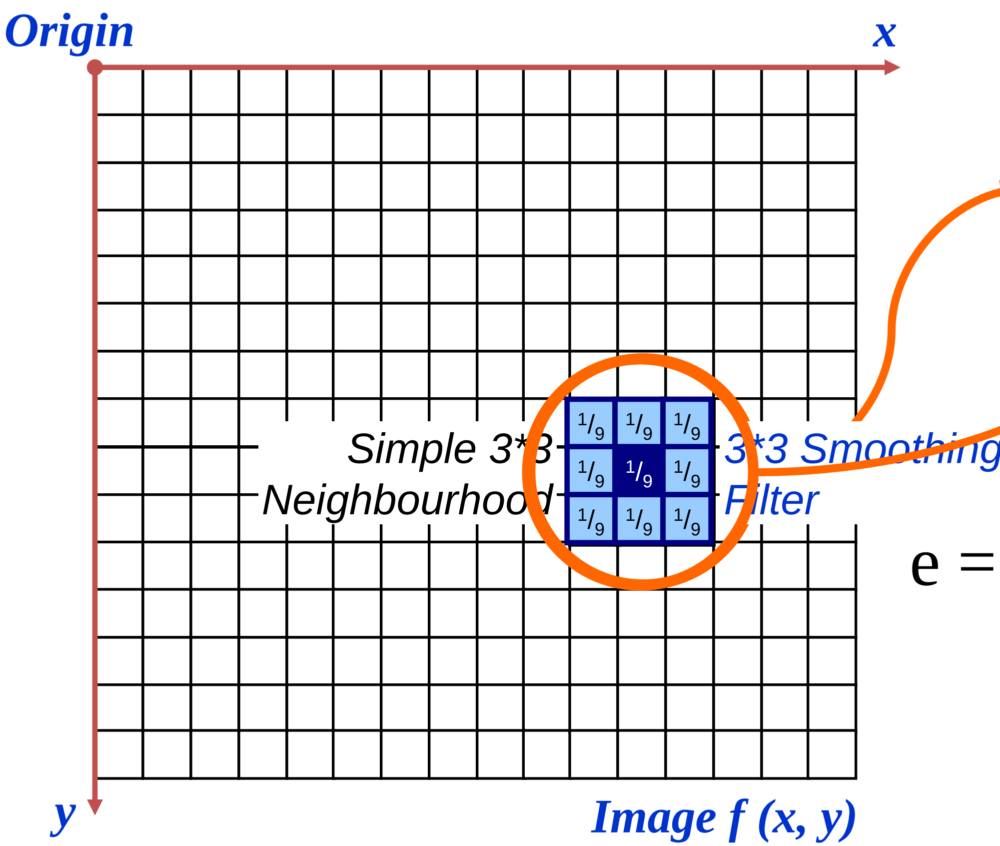
Smoothing Spatial Filters

Simply average all of the pixels in a neighbourhood around a central value

$^1/_9$	$^1/_9$	$^1/_9$
$^1/_9$	$^1/_9$	$^1/_9$
$^1/_9$	$^1/_9$	$^1/_9$

Simple averaging filter

Smoothing Spatial Filters



104	100	108
99	106	98
95	90	85

Original Image Pixels

*

$^1/_9$	$^1/_9$	$^1/_9$
$^1/_9$	$^1/_9$	$^1/_9$
$^1/_9$	$^1/_9$	$^1/_9$

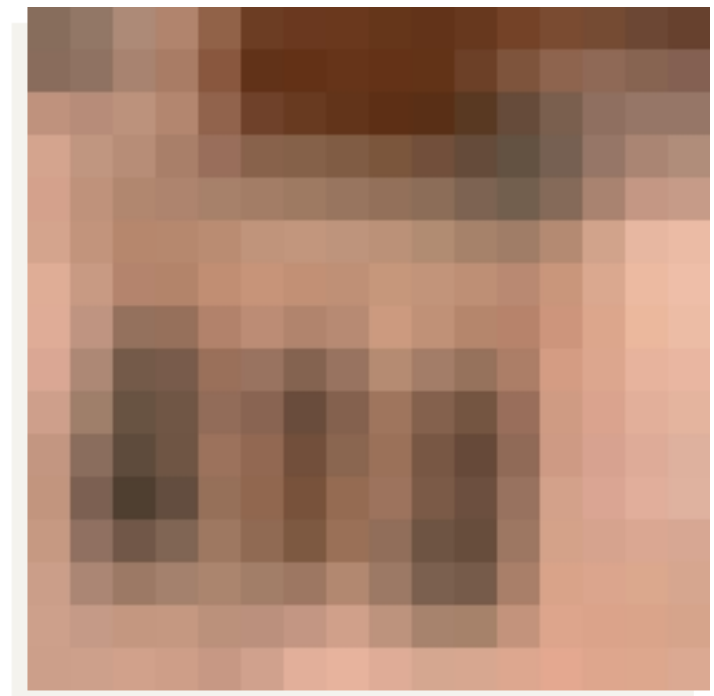
Filter

$$
\begin{aligned}
e = {} & {}^1/_9 * 106 + \\
& {}^1/_9 * 104 + {}^1/_9 * 100 + {}^1/_9 * 108 + \\
& {}^1/_9 * 99 + {}^1/_9 * 98 + \\
& {}^1/_9 * 95 + {}^1/_9 * 90 + {}^1/_9 * 85 \\
= {} & 98.3333
\end{aligned}
$$

The above is repeated for every pixel in the original image to generate the smoothed image

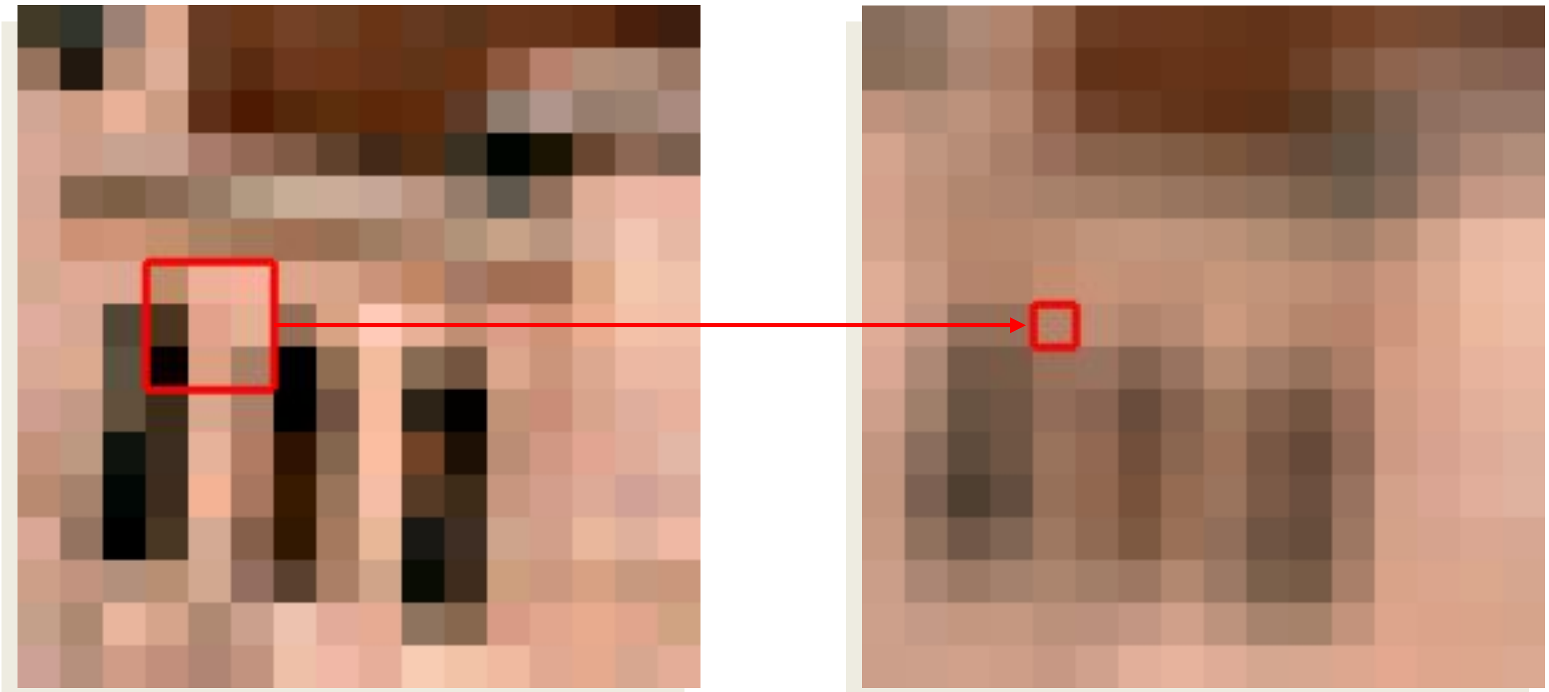
Smoothing Filter: Example

original



3x3 average

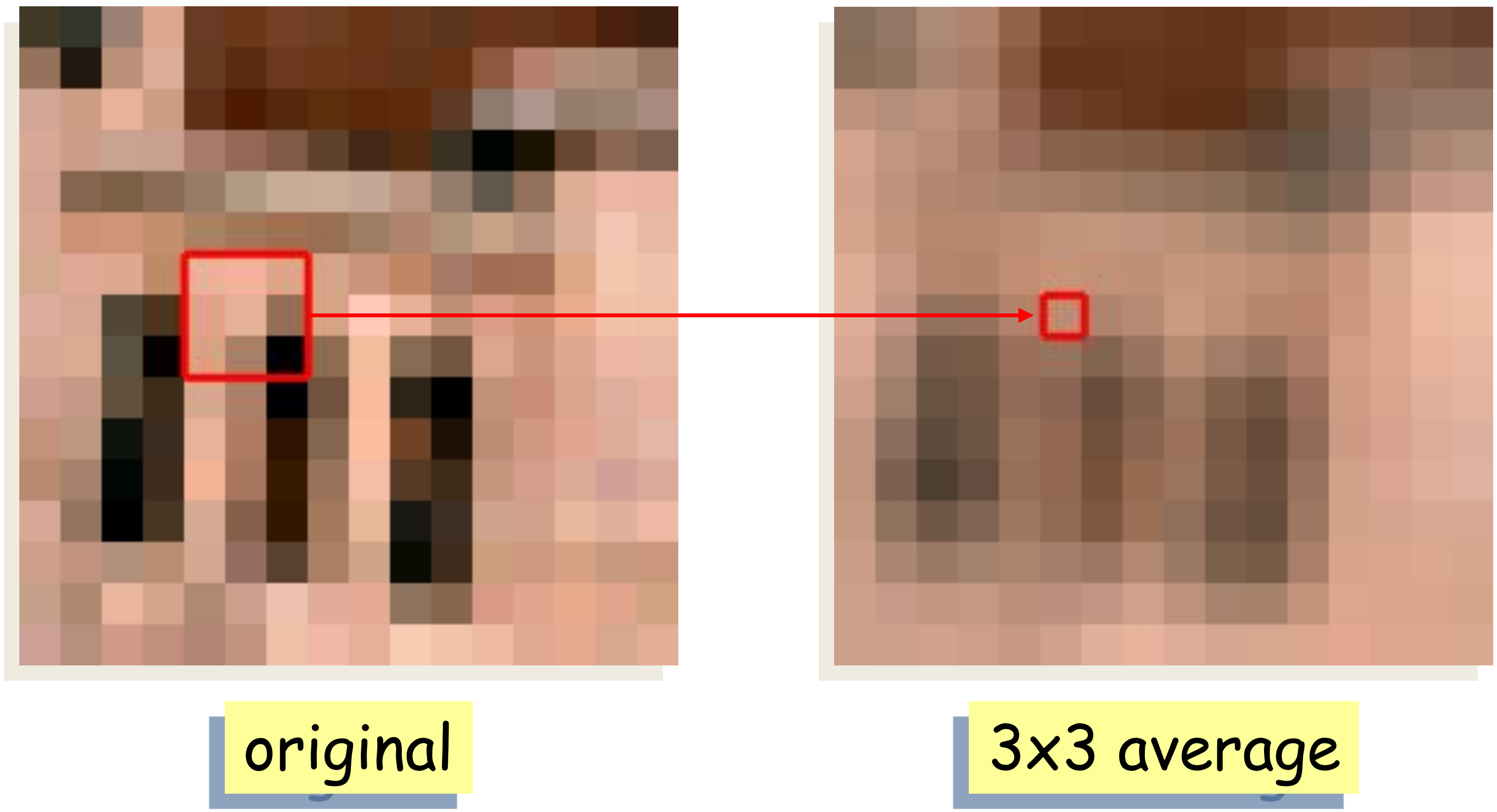
Smoothing Filter: Example



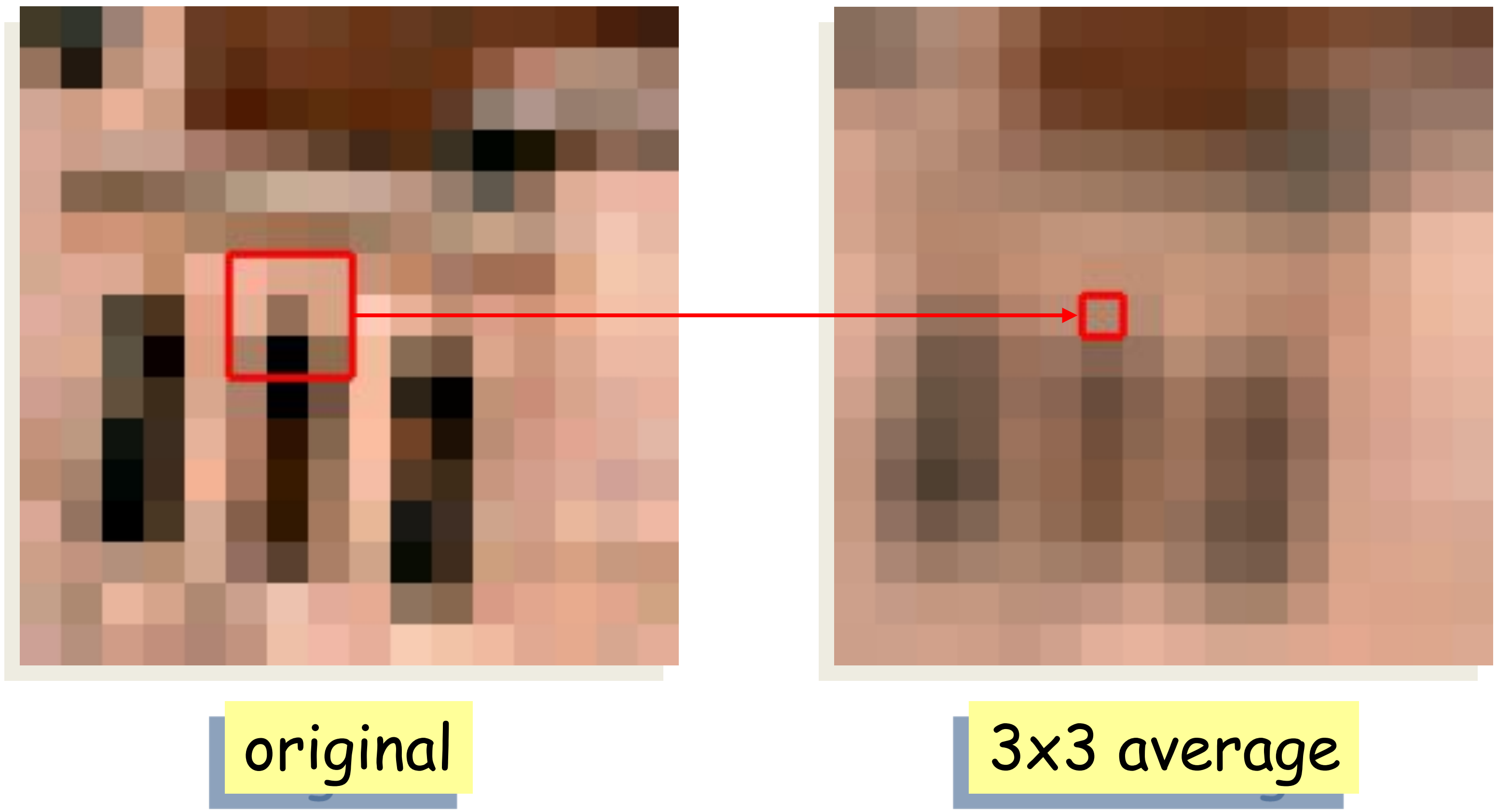
original

3x3 average

Smoothing Filter: Example



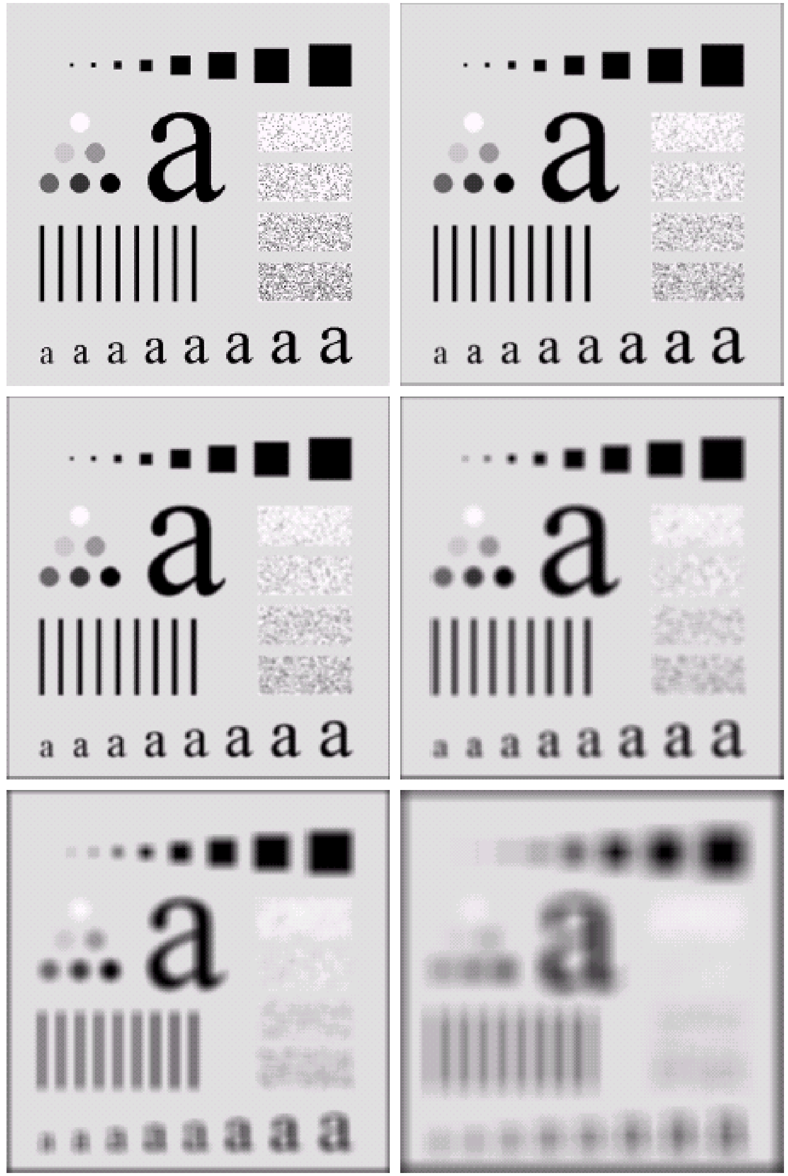
Smoothing Filter: Example



Original image

Size: 500x500

Smooth by 3x3 box filter



Smooth by 5x5 box filter

Smooth by 9x9 box filter

Smooth by 15x15 box filter

Smooth by 35x35 box filter

Notice how detail begins to disappear

Smoothing Spatial Filters

$$\frac{1}{9} \times \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \qquad \frac{1}{16} \times \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

Consider the output pixel is positioned at the center

Box Filter all coefficients are equal

Weighted Average give more (less) weight to near (away from) the output location

Sharpening Spatial Filters

Previously we have looked at smoothing filters which remove fine detail

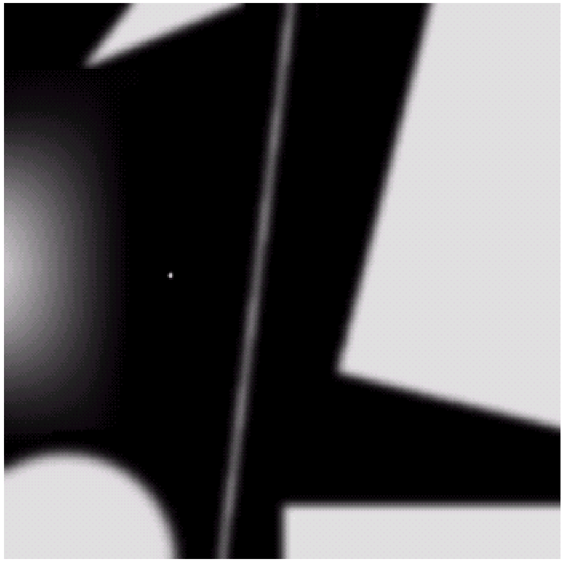
Sharpening spatial filters seek to highlight fine detail

- Remove blurring from images
- Highlight edges

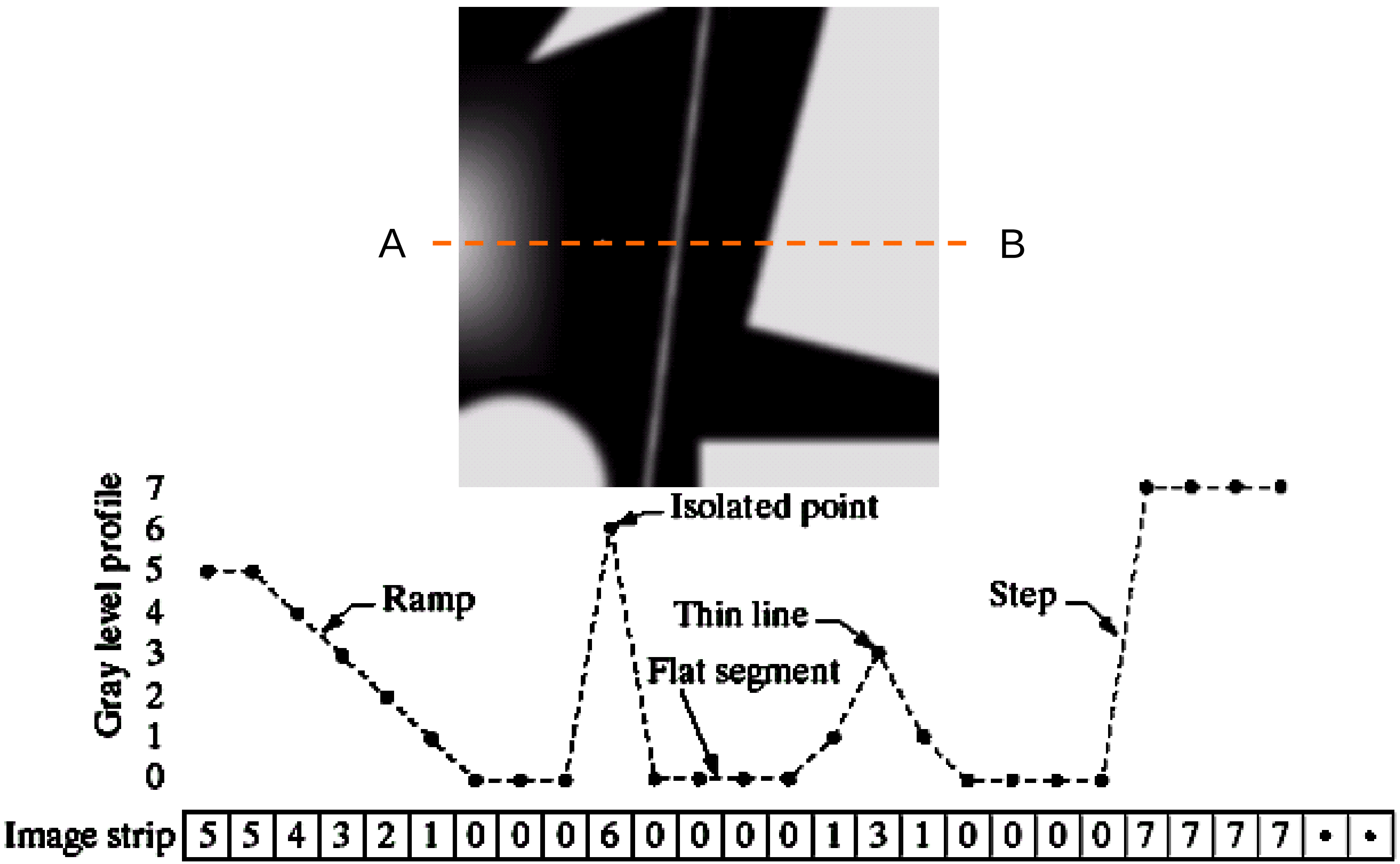
Sharpening filters are based on *spatial differentiation*

Spatial Differentiation

- Let's consider a simple 1 dimensional example



Spatial Differentiation



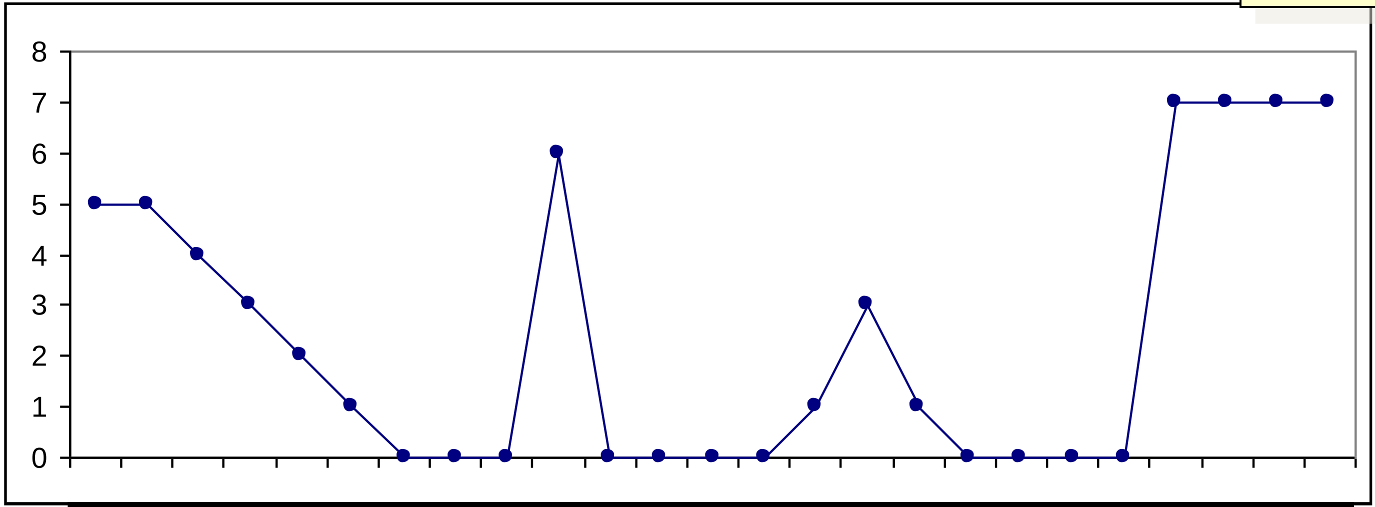
1^{st} Derivative

The 1^{st} derivative of a function is given by:

$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

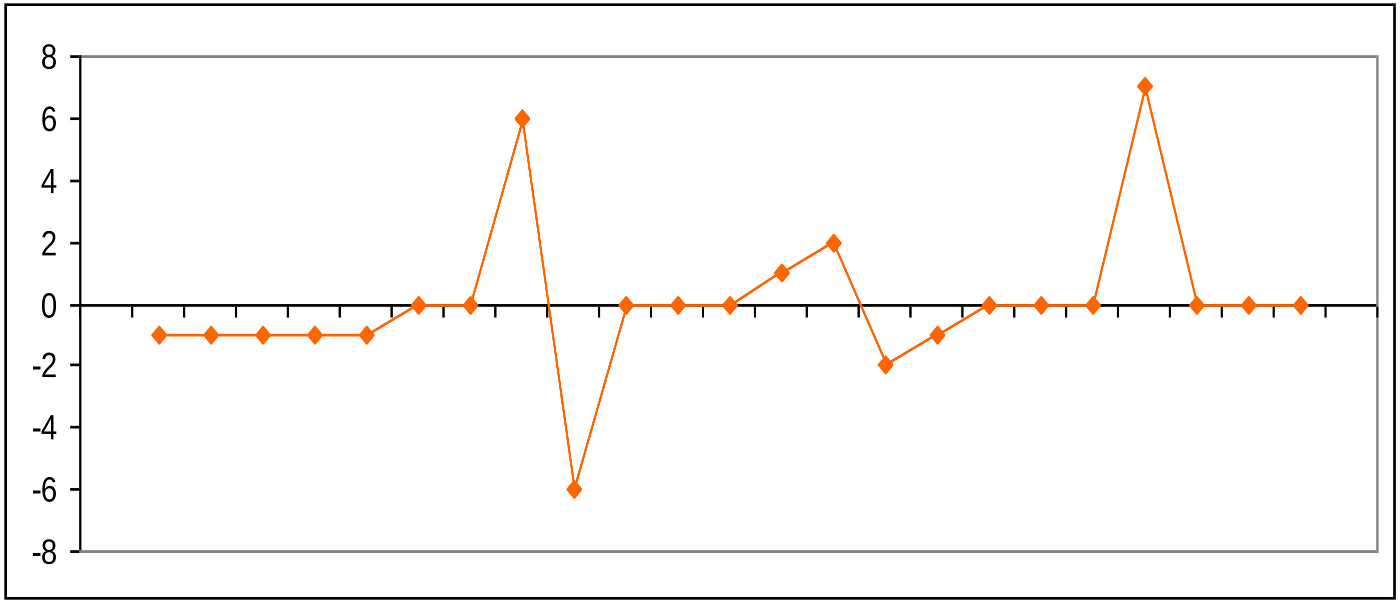
Its just the difference between subsequent values and measures the rate of change of the function

1st Derivative



5	5	4	3	2	1	0	0	0	6	0	0	0	0	1	3	1	0	0	0	0	7	7	7	7

	-1	-1	-1	-1	-1	0	0	6	-6	0	0	0	1	2	-2	-1	0	0	0	7	0	0	0	



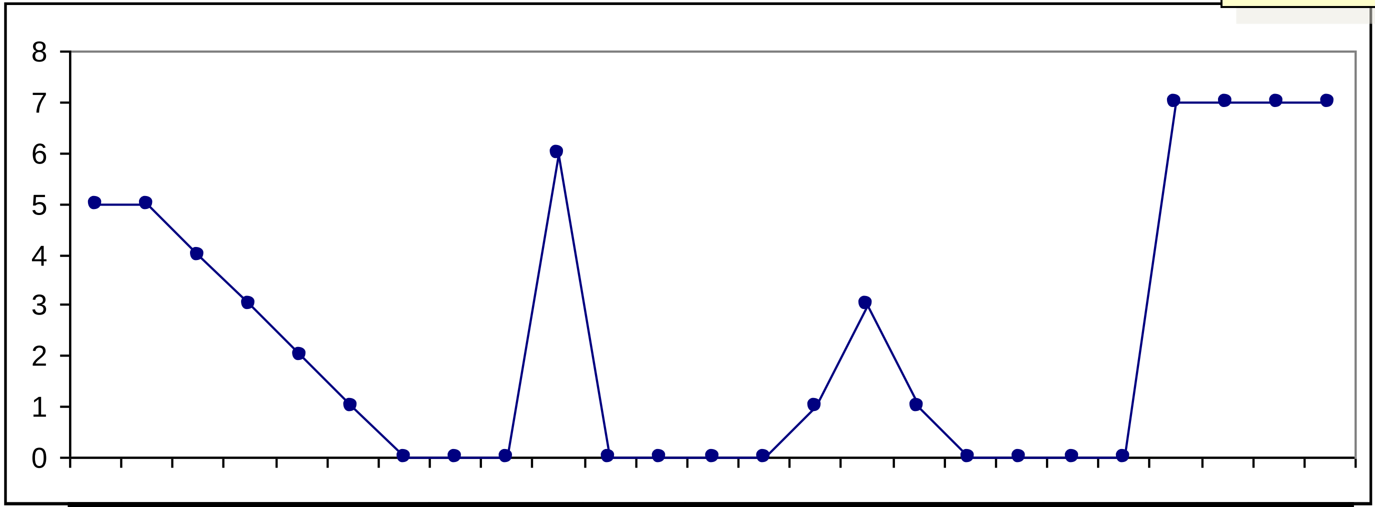
2^{nd} Derivative

The 2nd derivative of a function is given by:

Simply takes into account the values both before and after the current value

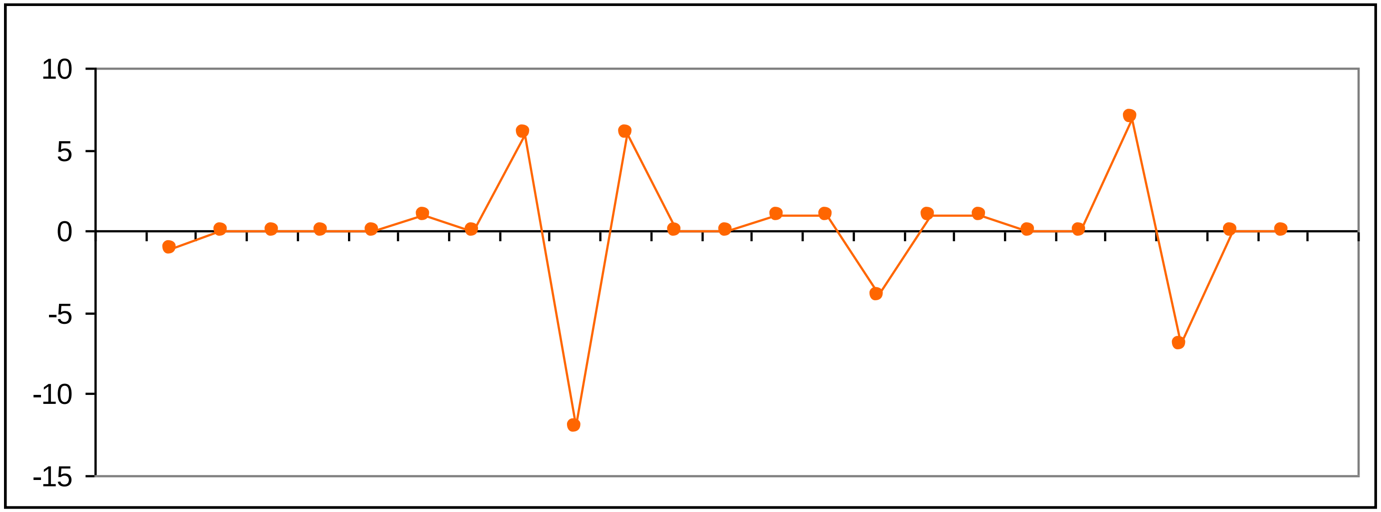
$$\frac{\partial^2 f}{\partial^2 x} = f(x+1) + f(x-1) - 2f(x)$$

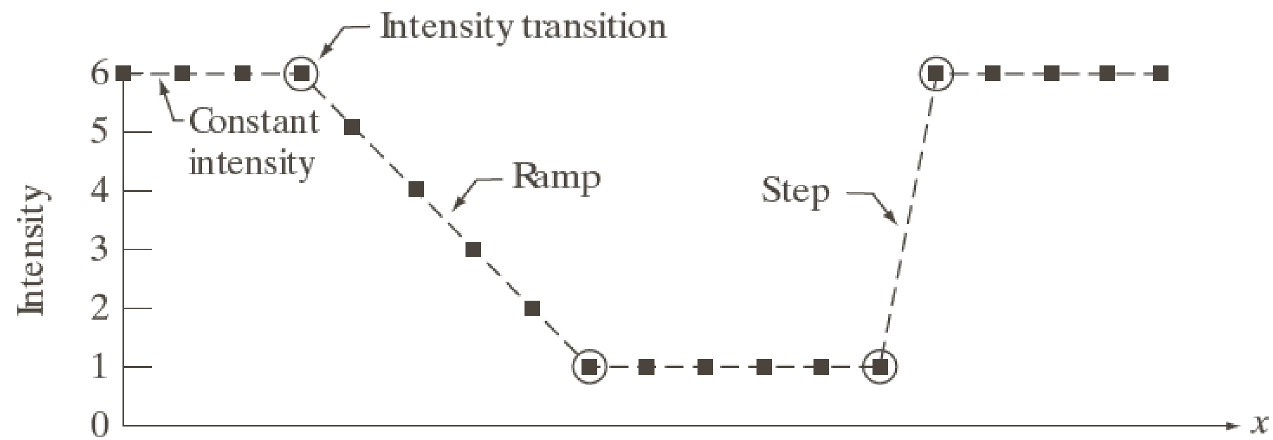
2^{nd} Derivative



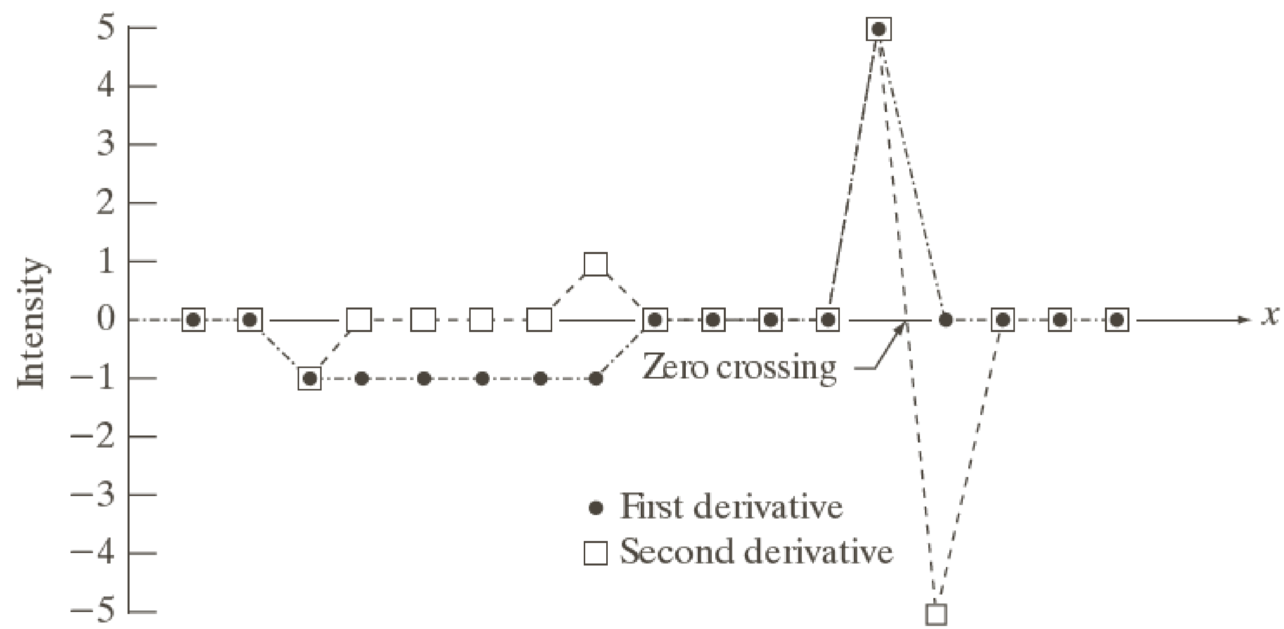
5	5	4	3	2	1	0	0	0	6	0	0	0	0	1	3	1	0	0	0	0	7	7	7	7

	-1	0	0	0	0	1	0	6	-12	6	0	0	1	1	-4	1	1	0	0	7	-7	0	0	





Scan line	6	6	6	6	5	4	3	2	1	1	1	1	1	1	6	6	6	6	6
1st derivative		0	0	−1	−1	−1	−1	−1	0	0	0	0	0	5	0	0	0	0	
2nd derivative		0	0	−1	0	0	0	0	1	0	0	0	0	5	−5	0	0	0	



2nd Derivative for Image Enhancement

The 2nd derivative is more useful for image enhancement than the 1st derivative - *Stronger response to fine detail*

We will come back to the 1st order derivative later on

The first sharpening filter we will look at is the *Laplacian*

Laplacian Filter

The Laplacian is defined as follows:

$$\boxed{\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}}$$

$$\frac{\partial^2 f}{\partial x^2} = f(x+1, y) + f(x-1, y) - 2f(x, y)$$

$$\frac{\partial^2 f}{\partial y^2} = f(x, y+1) + f(x, y-1) - 2f(x, y)$$

Laplacian Filter

So, the Laplacian can be given as follows:

$$\nabla^2 f = [f(x+1, y) + f(x-1, y) + f(x, y+1) + f(x, y-1)] - 4f(x, y)$$

Can we implement it using a filter/ mask?

0	1	0
1	-4	1
0	1	0

Laplacian Filter

0	1	0
1	−4	1
0	1	0

1	1	1
1	−8	1
1	1	1

0	−1	0
−1	4	−1
0	−1	0

−1	−1	−1
−1	8	−1
−1	−1	−1

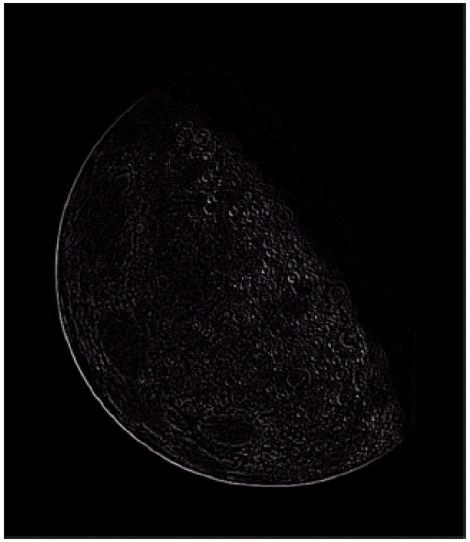
a	b
c	d

FIGURE 3.39 (a) Filter mask used to implement the digital Laplacian, as defined in Eq. (3.7-4). (b) Mask used to implement an extension of this equation that includes the diagonal neighbors. (c) and (d) Two other implementations of the Laplacian.

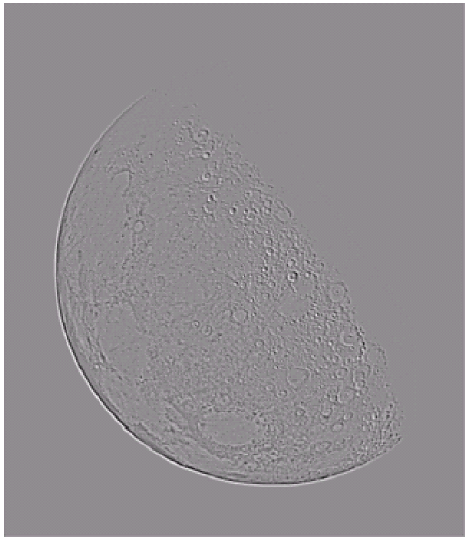
Laplacian Filter

Applying the Laplacian to an image we get a new image that highlights edges and other discontinuities

Original Image



Laplacian Filtered Image

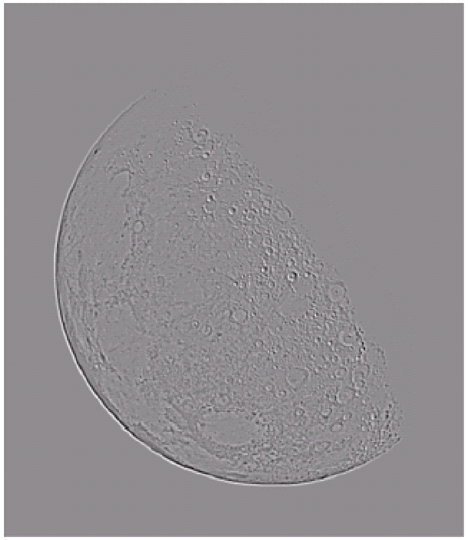


Laplacian Filtered Image Scaled for Display

Laplacian Image Enhancement

The result of a Laplacian filtering is not an enhanced image

To generate the final enhanced image



Laplacian
Filtered Image
Scaled for Display

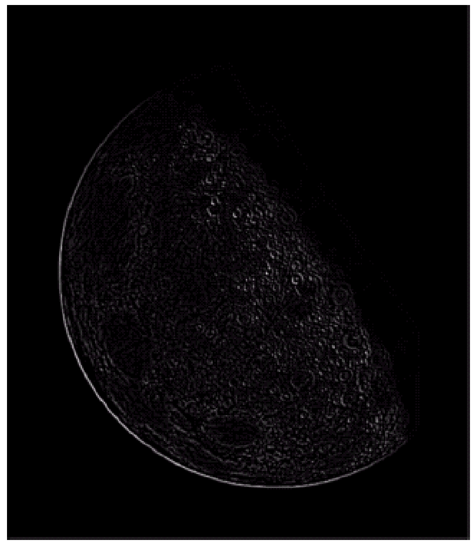
w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

$$g(x,y) = \begin{cases} f(x,y) - \nabla^2 f, & w_5 < 0 \\ f(x,y) + \nabla^2 f, & w_5 > 0 \end{cases}$$

Laplacian Image Enhancement

Original Image

-



Laplacian Filtered Image

=

Sharpened Image

In the final sharpened image edges and fine detail are much more obvious

Laplacian Image Enhancement

Simplified Image Enhancement

- The entire enhancement can be combined into a single filtering operation

$$
\begin{aligned}
g(x, y) &= f(x, y) - \nabla^2 f \\
&= f(x, y) - [f(x+1, y) + f(x-1, y) \\
&\qquad + f(x, y+1) + f(x, y-1) \\
&\qquad - 4f(x, y)]
\end{aligned}
$$

Simplified Image Enhancement

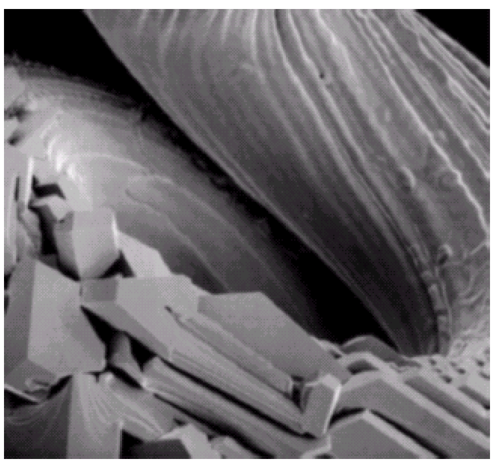
- The entire enhancement can be combined into a single filtering operation

$$
\begin{aligned}
g(x,y) &= f(x,y) - \nabla^2 f \\
&= 5f(x,y) - f(x+1,y) - f(x-1,y) \\
&\quad - f(x,y+1) - f(x,y-1)
\end{aligned}
$$

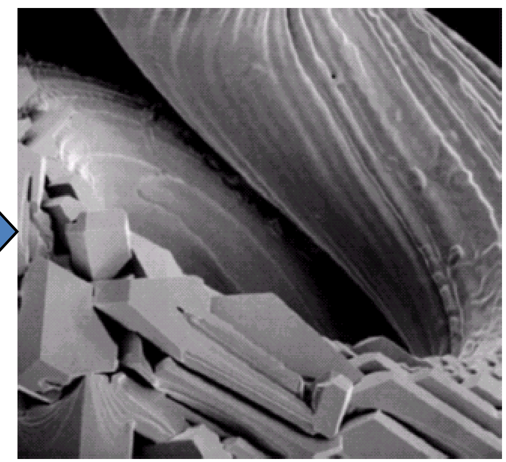
0	-1	0
-1	5	-1
0	-1	0

Simplified Image Enhancement

- This gives us a new filter which does the whole job for us in one step



0	-1	0
-1	5	-1
0	-1	0

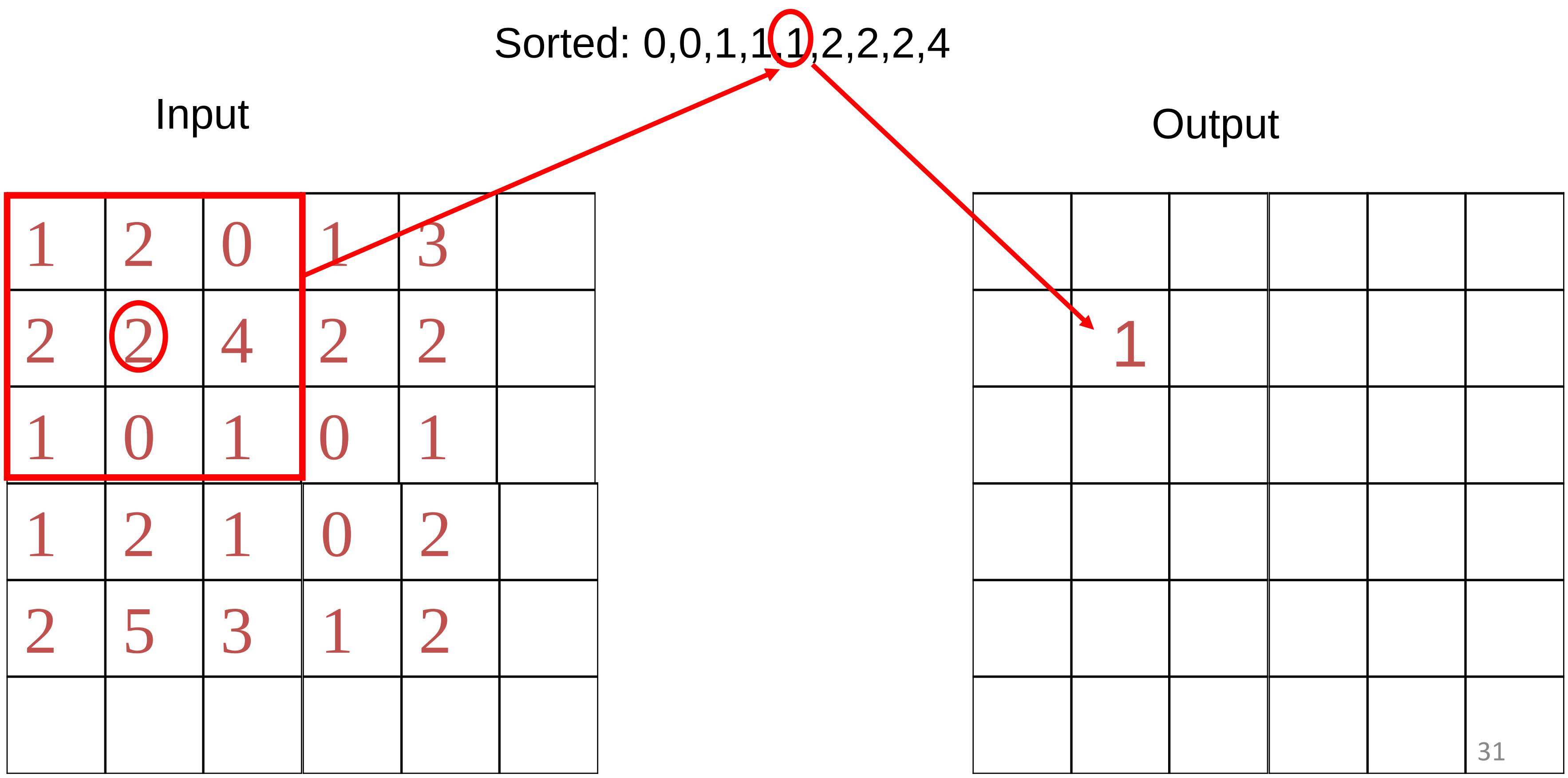


Order-Statistic Filtering

- Output is based on order of gray levels in the masked area
- Some simple neighbourhood operations include:
  - **Min:** Set the pixel value to the minimum in the neighbourhood
  - **Max:** Set the pixel value to the maximum in the neighbourhood
  - **Median:** The median value of a set of numbers is the midpoint value in that set

Median Filter

- For an image, mask symmetric: 3x3, 5x5, etc.



Median Filtering

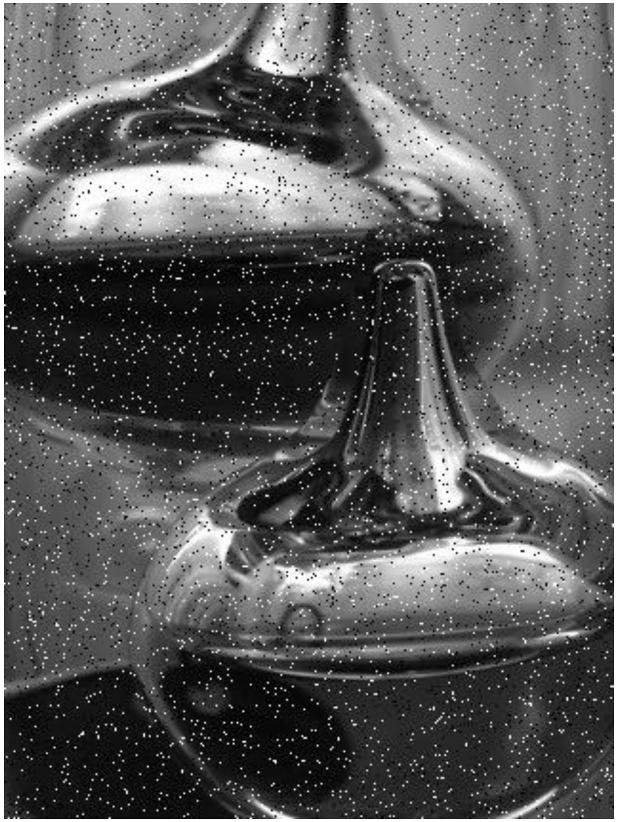
10	20	20
20	15	20
20	25	100

Sort the values
Determine the median

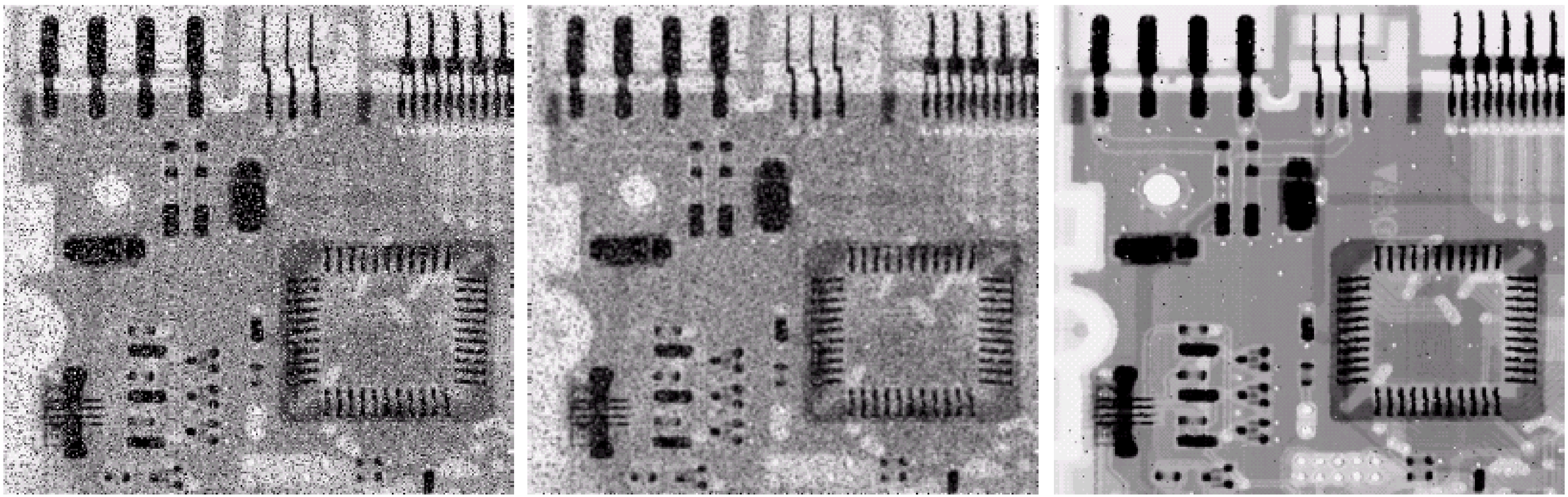
Median = ? 20

- Particularly effective when
  - The noise pattern consists of strong impulse noise (salt-and-pepper)

Salt and Pepper Noise



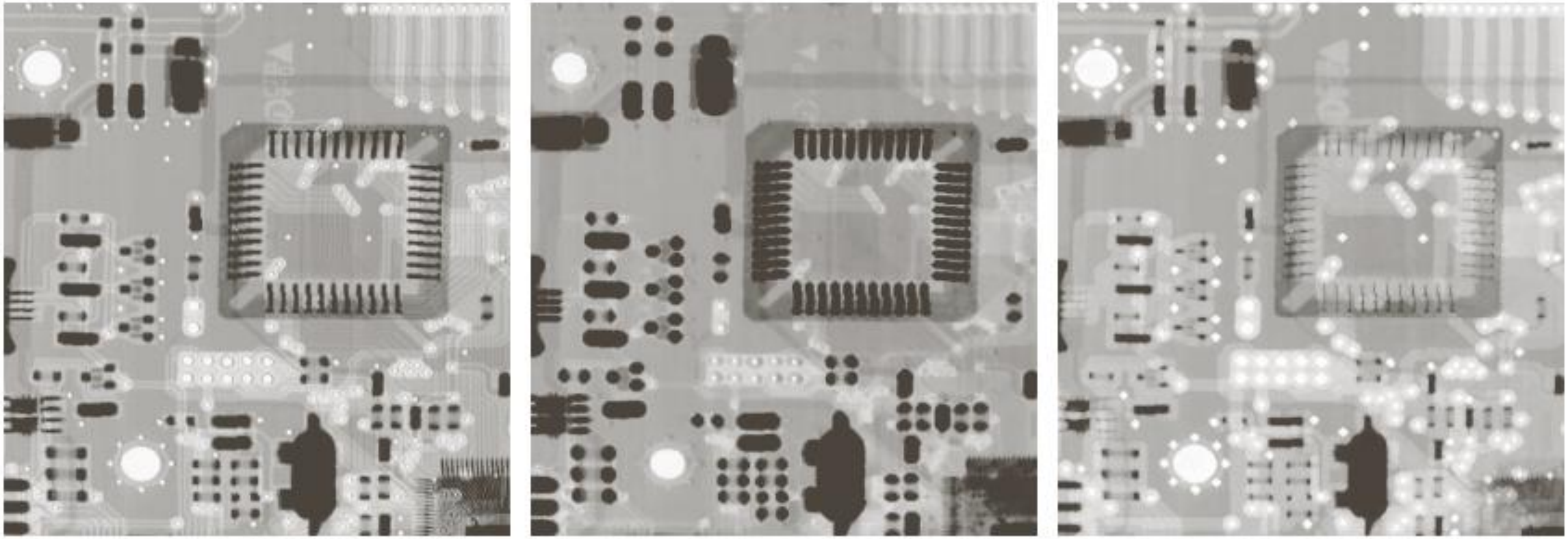
Median Filtering



a b c

FIGURE 3.37 (a) X-ray image of circuit board corrupted by salt-and-pepper noise. (b) Noise reduction with a 3 × 3 averaging mask. (c) Noise reduction with a 3 × 3 median filter. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

Min/Max Filtering

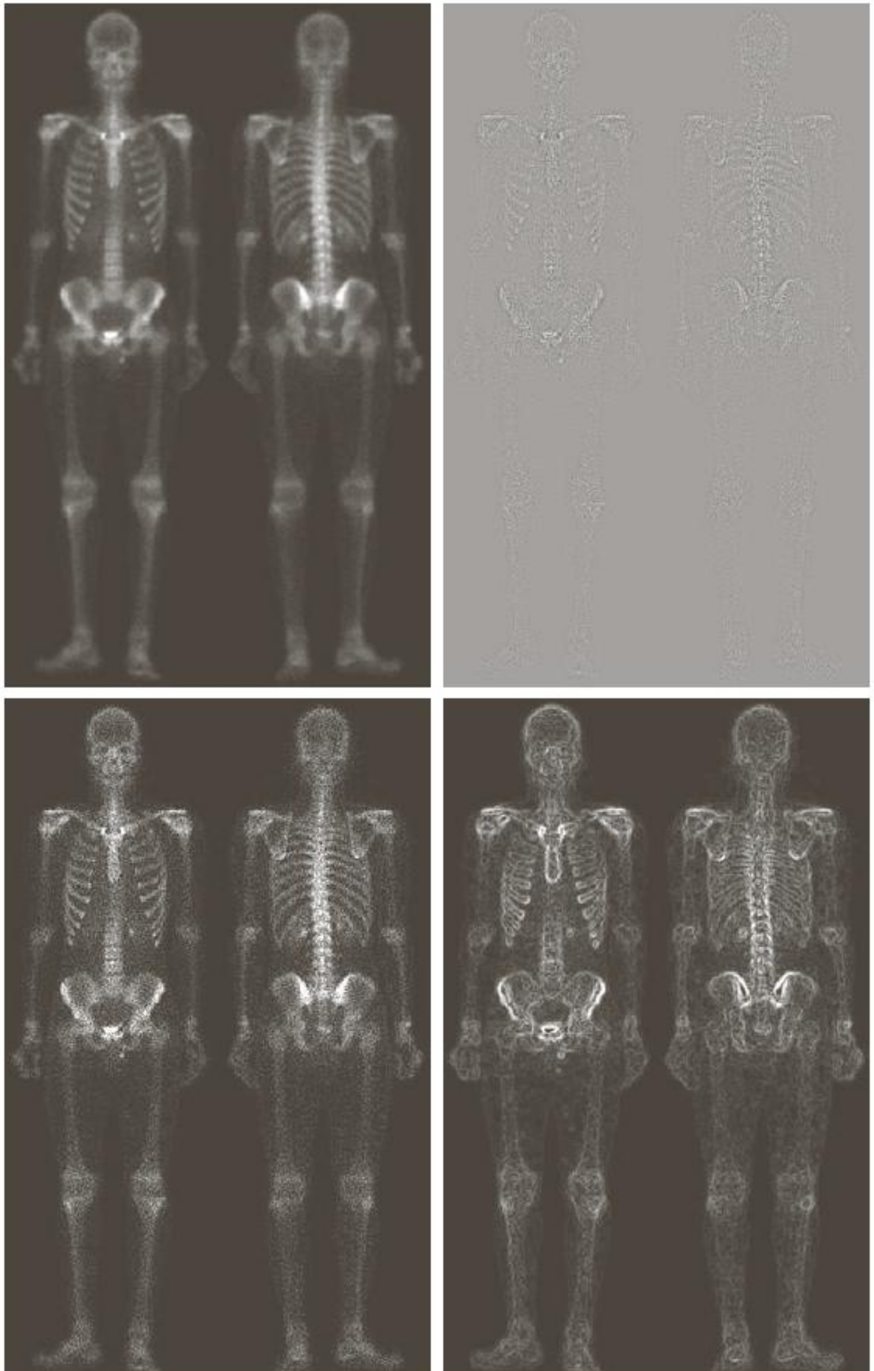


Combining Spatial Enhancement Methods

Successful image enhancement is typically not achieved using a single operation

Rather we combine a range of techniques in order to achieve a final result

This example will focus on enhancing the bone scan



a b
c d

FIGURE 3.43
(a) Image of whole body bone scan. (b) Laplacian of (a). (c) Sharpened image obtained by adding (a) and (b). (d) Sobel gradient of (a).

Readings from Book (4^{th} Edn.)

- 3.4 Filtering
- 3.5 Smoothing Filters
- 3.6 Sharpening Filtering
- 3.8 Combining Filters

Acknowledgements

- Statistical Pattern Recognition: A Review – A.K Jain et al., PAMI (22) 2000
- Pattern Recognition and Analysis Course – A.K. Jain, MSU
- *Pattern Classification"* by Duda et al., John Wiley & Sons.
- Digital Image Processing", Rafael C. Gonzalez & Richard E. Woods, Addison-Wesley, 2002
- Machine Vision: Automated Visual Inspection and Robot Vision", David Vernon, Prentice Hall, 1991
- www.eu.aibo.com/
- Advances in Human Computer Interaction, Shane Pinder, InTech, Austria, October 2008

Material in these slides has been taken from, the following resources