Digital Signal Processing

Lecture-4

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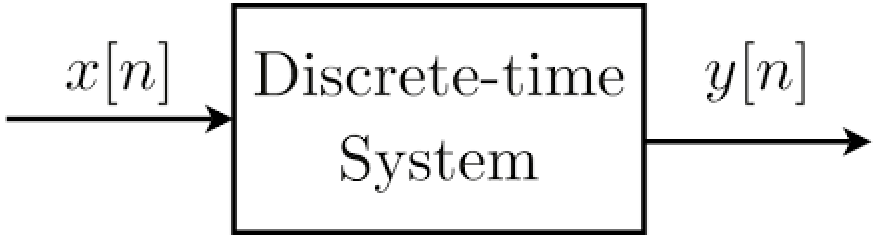
A discrete-time system is anything that takes a discrete-time signal as input and generates a discrete-time signal as output. The concept of a system is very general.

Response depends upon the transfer function of a system.

Classification of discrete time systems:

1. Static and dynamic
2. Causal and non-causal
3. Time invariant and time variant
4. Linear and non-linear
5. Stable and unstable
6. FIR and IIR
7. Recursive and non-recursive



Linear Time Invariant Systems (LTI)

If the discrete time system satisfies the properties of linearity and time invariance, then the system is called as linear time-invariant system.

What is Linearity:
Principle of superposition

The impulse response refers to the output of a system when an impulse function is applied as the input. In practical terms, the impulse response describes how the system reacts over time to an impulse input, allowing engineers and analysts to predict the system's behaviour to arbitrary inputs.

Representation of Systems

Block representation
Signal Flow Graph Representation (SFGR)
Constant Multiplier
Unit delay element
Unit advance element

Draw the block diagram and signal flow diagram of the discrete time system having the equation given below:

$$y(n) = 1.5\,x(n) + 1.5\,x(n-1)$$

Static or dynamic systems

The output at any given time depends only on the input at that same time and not on past or future inputs and not on the past values of output.

$Y(n) = nx(n) + 2x^2(n)$???

Time shifting, time scaling?

Recursive and non-recursive systems

A system whose output y(n) depends on any number of past output values as well as present and past input values.

A system whose output does not depend on the past outputs but only depends on the present and past inputs is called non-recursive system.

Linear and non-linear systems

A **system** is said to be **linear** if it obeys the principle of homogeneity and principle of superposition.

Principle of Homogeneity

The principle of homogeneity says that a system which generates an output y(t) for an input x(t) must produce an output ay(t) for an input ax(t).

Superposition Principle

According to the principle of superposition, a system which gives an output $y_1(t)$ for an input $x_1(t)$ and an output $y_2(t)$ for an input $x_2(t)$ must produce an output $[y_1(t) + y_2(t)]$ for an input $[x_1(t) + x_2(t)]$.

A system is said to be a **non-linear system** if it does not obey the principle of homogeneity and principle of superposition.

For linearity :

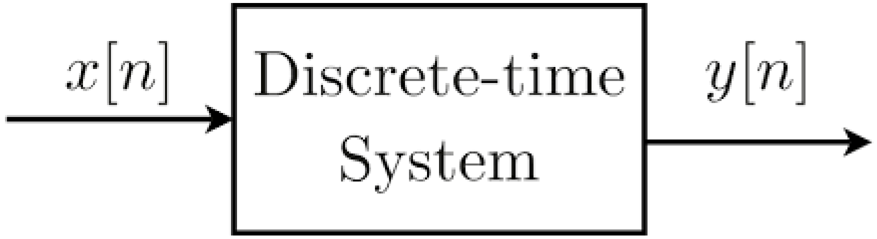
$$H\{a_1 x_1(n) + a_2 x_2(n)] = H\{a_1 x_1(n)\} + H\{a_2 x_2(n)\}$$

Procedure to test linearity:

$$H\{a_1 x_1(n) + a_2 x_2(n)\} = H\{a_1 x_1(n)\} + H\{a_2 x_2(n)\}$$

Let y(n)=nx(n)

Generally:



Here y(n)=H{x(n)}=nx(n)

$x_1(n)$

$y_1(n) = H\{x_1(n)\} = n\, x_1(n)$

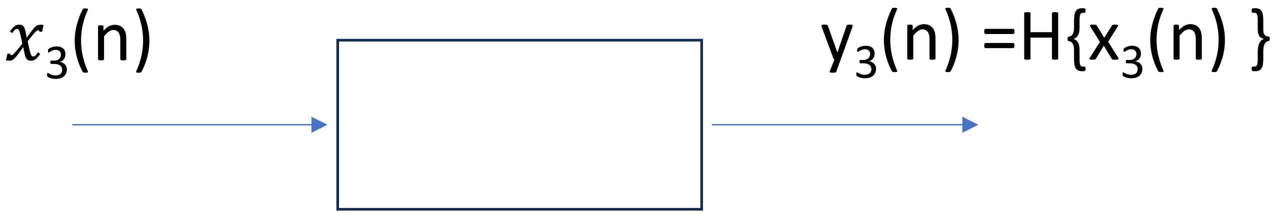
$x_2(n)$

$y_2(n) = H\{x_2(n)\} = n\, x_2(n)$

$$\{a_1 y_1(n)\} + \{a_2 y_2(n)\} = a_1 n\, x_1(n) + a_2\, n\, x_2(n)$$

Let's consider $x_3(n) = a_1 x_1(n) + a_2 x_2(n)$

Let $y_3(n)$ be the response of $x_3(n)$



$y_3(n) = H\{a_1 x_1(n) + a_2 x_2(n)\} = n\{a_1 x_1(n) + a_2 x_2(n)\}$

See $y_3(n) = a_1 y_1(n) + a_2 y_2(n)$

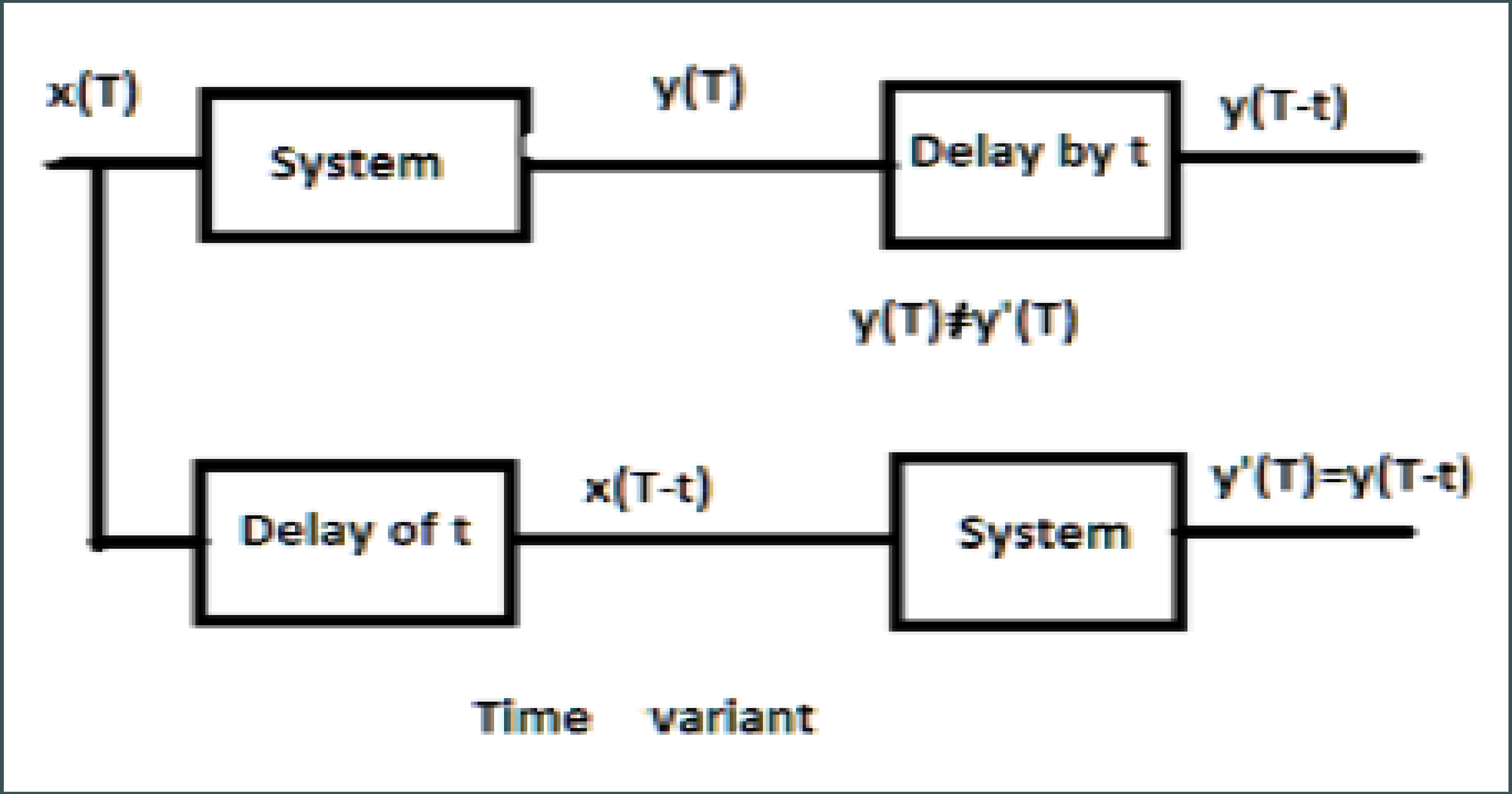
Check the following for linearity

a) $y(n) = 2x(n) + 5$
b) $y(n) = x(n) - 2x(n-1)$
c) $y(n) = 1/N \sum_{k=0}^{N-1} x(n-k)$
d) $y(n) = \mathrm{Re}\{x(n)\}$

2. Shift-Invariance
3. Stability

A system is said to be time invariant if the response of the system to an input is not a function of time. Also, any delay provided in the input must be reflected in the output for a time invariant system.

$Y(n) = nx(n)$??


x(T)
System
y(T)
Delay by t
y(T-t)
y(T)≠y'(T)
Delay of t
x(T-t)
System
y'(T)=y(T-t)
Time variant

Stable or unstable system

A system is said to be stable if every bounded input produces a bounded output.

A bounded signal is one whose magnitude is always a finite value.

Example: sine wave

Stability Condition of an LTI Discrete-Time System

- **BIBO Stability Condition** - A discrete-time is BIBO stable if and only if the output sequence $\{y[n]\}$ remains bounded for all bounded input sequence $\{x[n]\}$
- An LTI discrete-time system is BIBO stable if and only if its impulse response sequence $\{h[n]\}$ is absolutely summable, i.e.

$$S = \sum_{n=-\infty}^{\infty} |h[n]| < \infty$$

Stable or unstable system

$h(n)= 0.2^{n} u(n)$

$h(n)= 4^{n} u(-n)$

$Y(n)= e^{-x(n)}$

$y(n)= x(n) + \frac{1}{2} x(n-1) + \frac{1}{4} x(n-2)$

Causal or non-causal system

$Y(n)= x(n) - x(-n-1) + x(n-1)$?