Lab # 09: Sampling of Audio Signals and Aliasing

Objectives:

The objective of this lab is to perform sampling on audio signals while taking care of aliasing.

Description:

In signal processing, **sampling** is the reduction of a continuous signal to a discrete signal. A common example is the conversion of a sound wave (a continuous signal) to a sequence of samples (a discrete-time signal).

A **sample** is a value or set of values at a point in time and/or space. While a **sampler** is a subsystem or operation that extracts samples from a continuous signal.

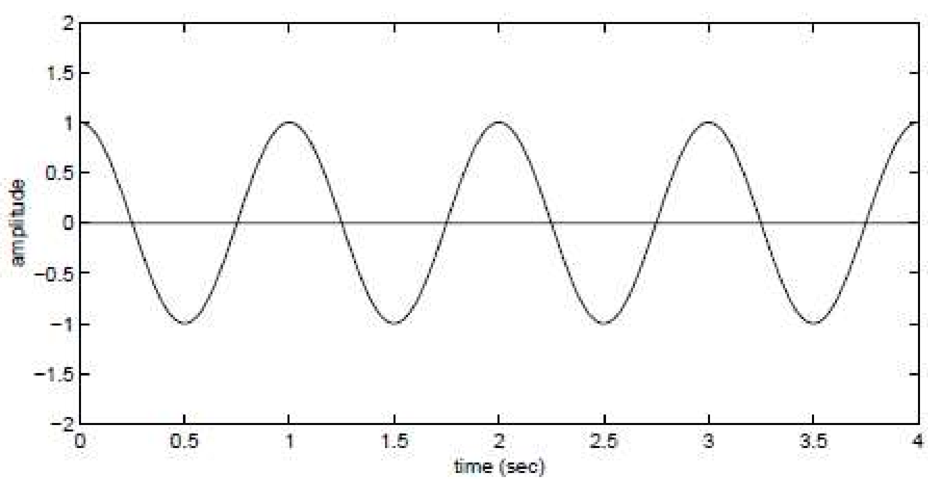
A theoretical **ideal sampler** produces samples equivalent to the instantaneous value of the continuous signal at the desired points.

The Nyquist sampling theorem provides a prescription for the nominal sampling interval required to avoid aliasing. It may be stated simply as follows:

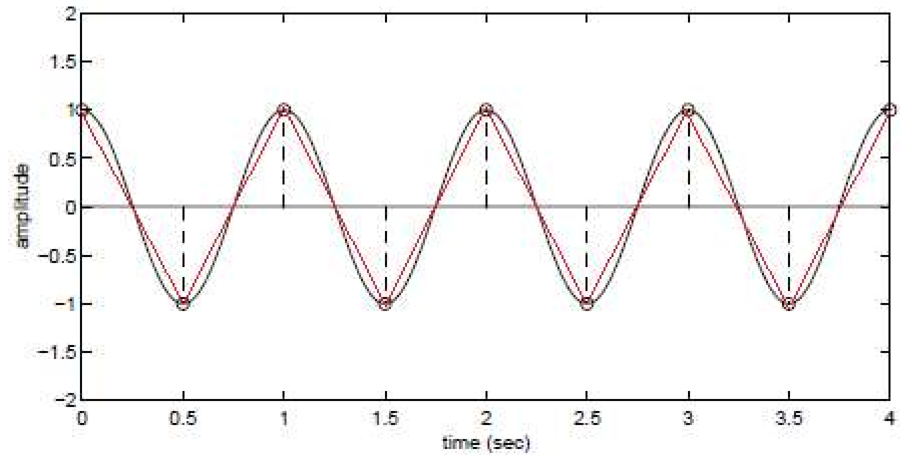
The sampling frequency should be at least twice the highest frequency contained in the signal.

$$F_s >= 2F_c$$

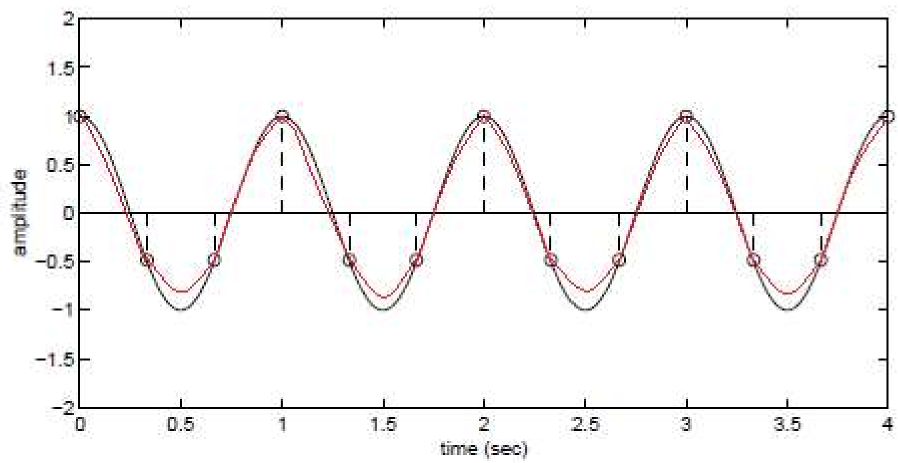
where fs is the sampling frequency (how often samples are taken per unit of time or space), and fc is the highest frequency contained in the signal. That this is so is really quite intuitive. Consider for example a signal composed of a single sinewave at a frequency of 1 Hz:



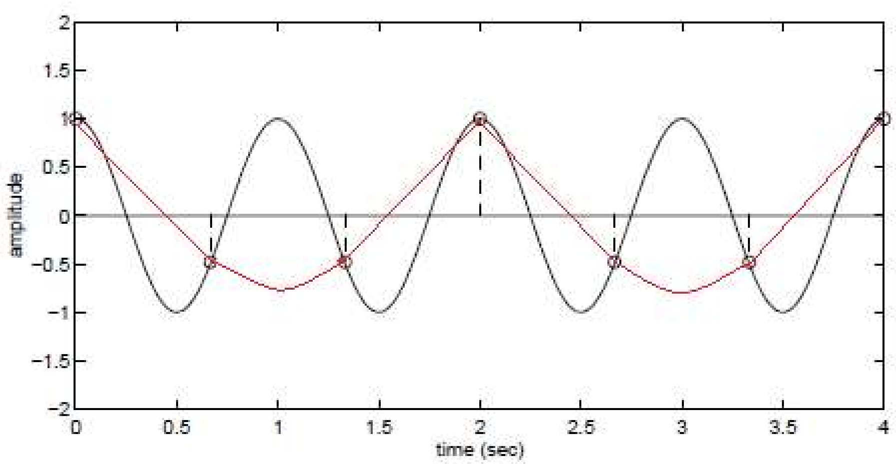
If we sample this waveform at 2 Hz (as dictated by the Nyquist theorem), that is sufficient to capture each peak and trough of the signal:



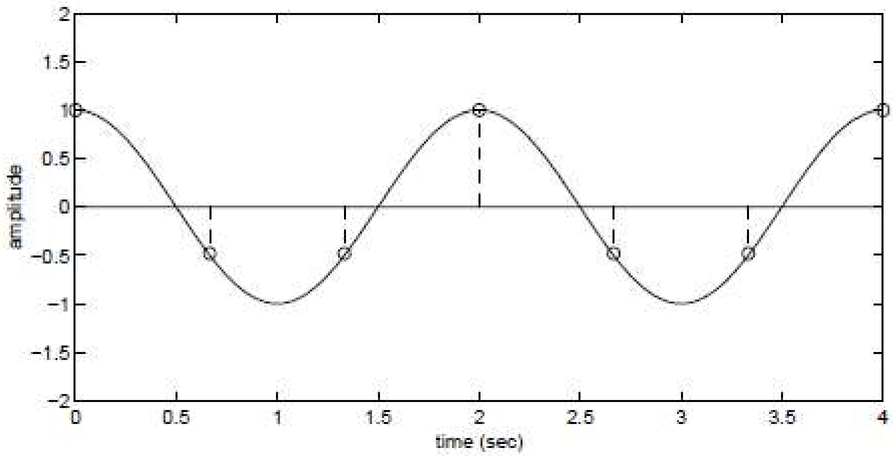
If we sample at a frequency higher than this, for example 3 Hz, then there are more than enough samples to capture the variations in the signal:



If, however we sample at a frequency lower than 2 Hz, for example at 1.5 Hz, then there are now not enough samples to capture all the peaks and troughs in the signal:

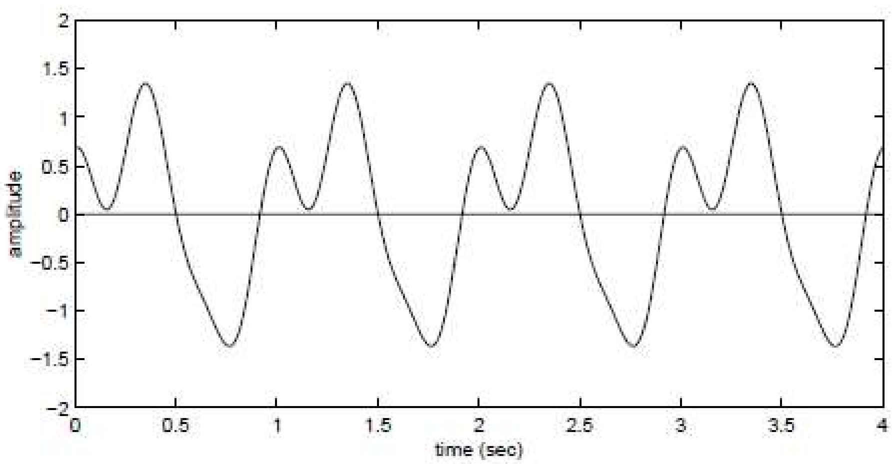


Note here that we are not only losing information, but we are getting the wrong information about the signal. The person receiving these samples, without any previous knowledge of the original signal, may well be misled into thinking that the signal has quite a different form:

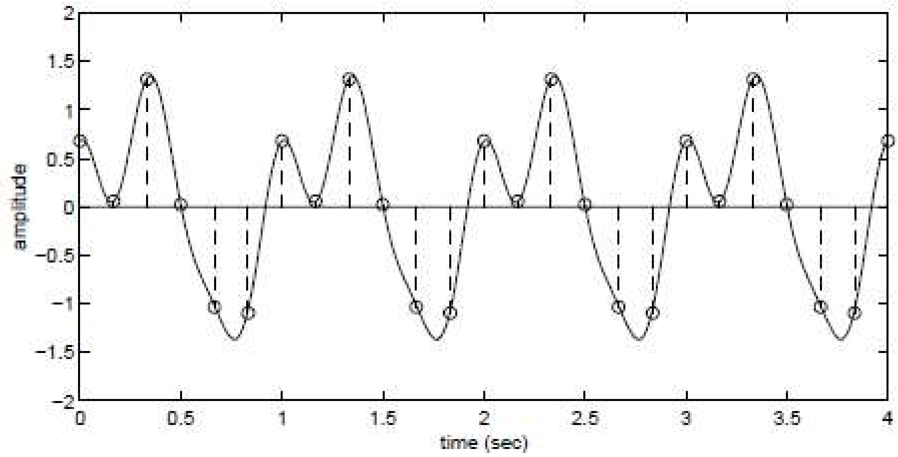


From this example, we can see the reason for the term aliasing. That is, the signal now takes on a different \persona," or a false presentation, due to being sampled at an insufficiently high frequency. Now we are ready to think about the sampling of a complex signal composed of many frequency components. By Fourier's theorem, we know that any continuous signal may be decomposed in terms of a sum of sines and cosines at different frequencies.

For example, the following waveform was composed by adding together sine waves at frequencies of 1 Hz, 2 Hz, and 3 Hz:



According to the Nyquist sampling theorem, the signal must be sampled at twice the highest frequency contained in the signal. In this case, we have fc=3 Hz, and so the Nyquist theorem tells us that the sampling frequency, fs, must be at least 6 Hz. And sure enough, this appears to be sufficient:



LAB TASK:

1. Prove Nyquist's Sampling Theorem, by sampling the following waves
   a) Y = Cos(2*pi*f*t)
      Where f=10Hz
   b) Y = Sin(2*pi*f1*t) + Cos(2*pi*f2*t)
      Where f1 = 50Hz and f2 = 200Hz
      at three possible sampling rates i.e.

   (1) $F_s = 2F_c$ (2) $F_s < 2F_c$ (3) $F_s > 2F_c$
2. Sample an audio with Fs=2Fc, Fs < 2Fc and Fs >2Fc. Observe the effect of all cases and plot signal.