Digital Signal Processing

Lecture-2

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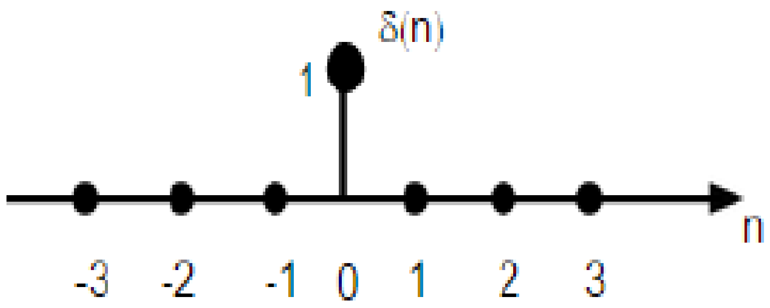
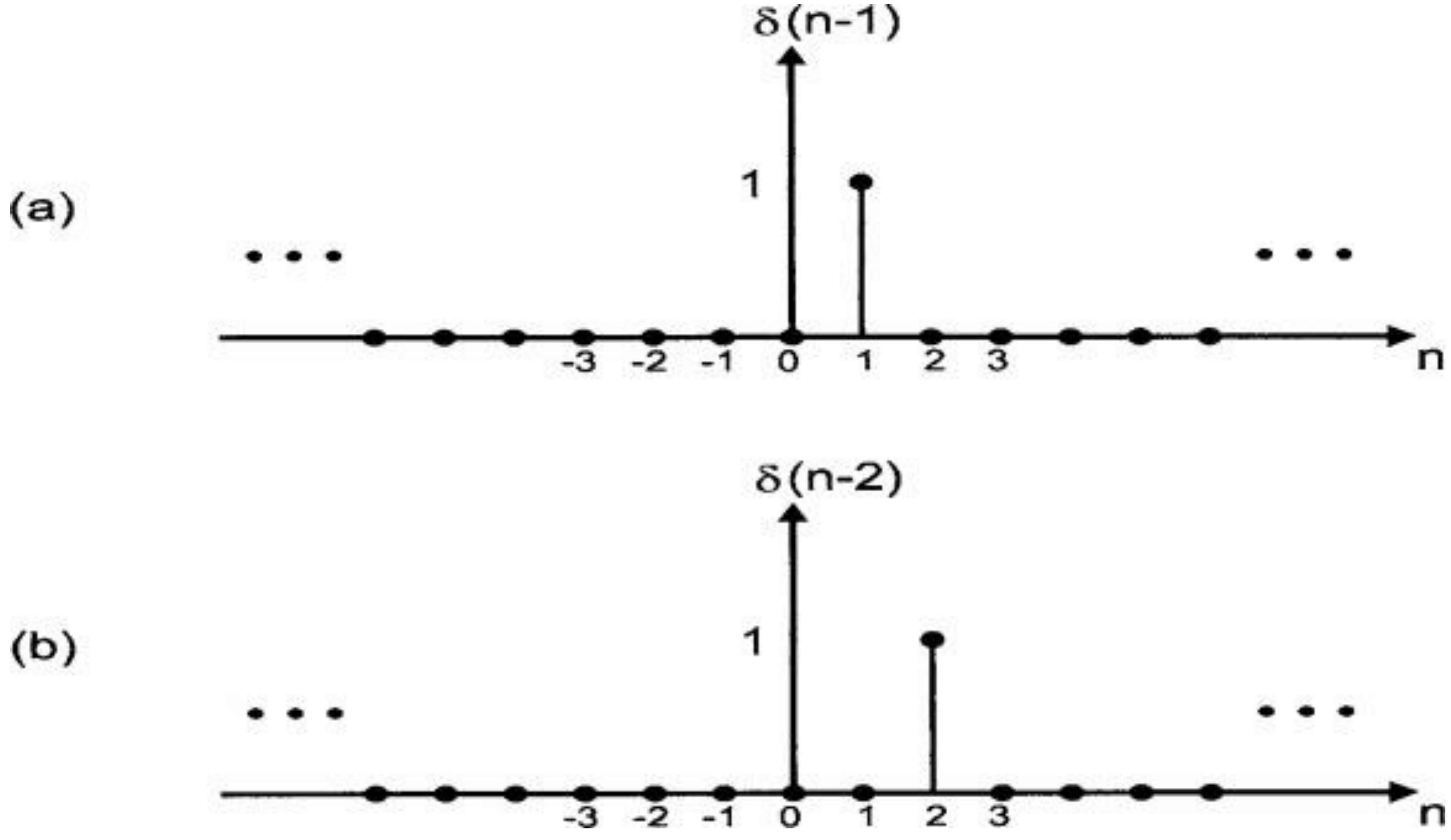
Standard Discrete Time Signals

Unit Impulse Sequence

- It is denoted as δ(n) in discrete time domain and can be defined as;
- δ(n)={1 for n= 0; Otherwise = 0}



- Shifted Impulse Signal?



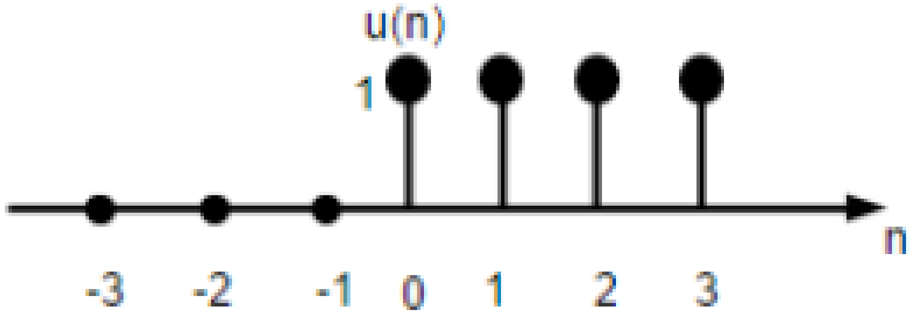
(a)
δ(n-1)
1
-3 -2 -1 0 1 2 3
n
(b)
δ(n-2)
1
-3 -2 -1 0 1 2 3
n

Standard Discrete Time Signals

Unit Step Sequence

Discrete time unit step signal is defined as;

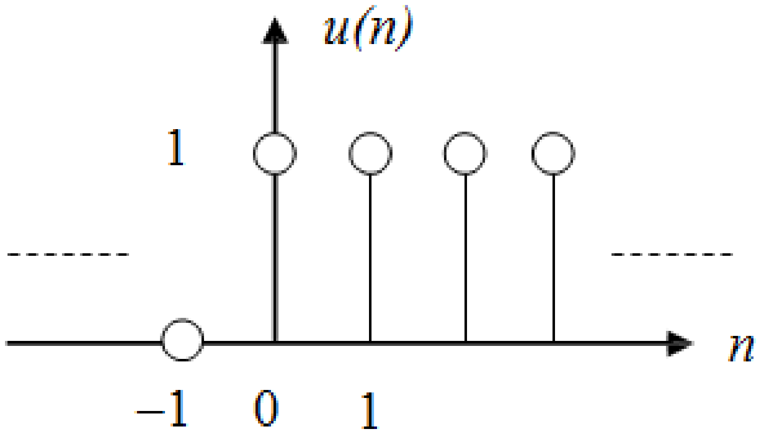
$$U(n) = \begin{cases} 1, & for \quad n \geq 0 \\ 0, & for \quad n < 0 \end{cases}$$

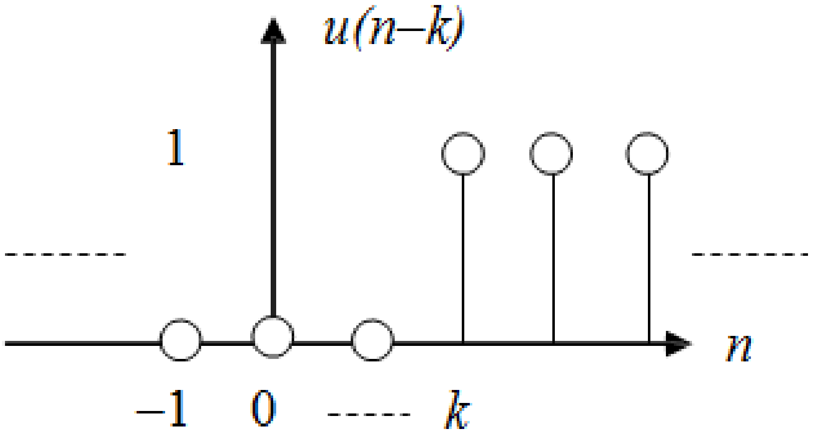


Shifted Unit Step Sequence

u(n-k)?

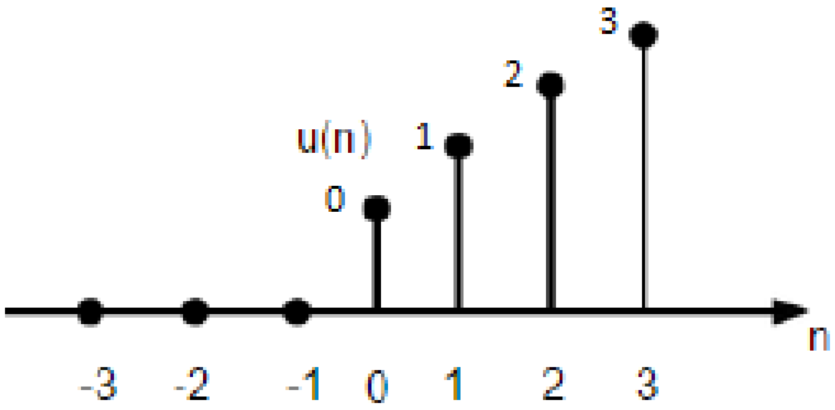
Standard Discrete Time Signals





Standard Discrete Time Signals

Unit Ramp Sequence

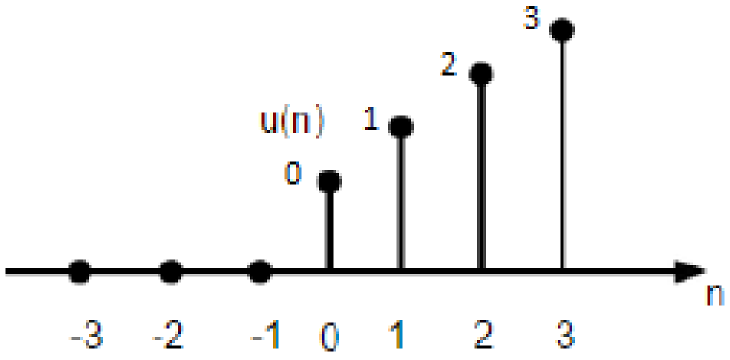


Standard Discrete Time Signals

Unit Ramp Function

A discrete unit ramp function can be defined as –

$$r(n) = \begin{cases} n, & for \quad n \geq 0 \\ 0, & for \quad n < 0 \end{cases}$$



The figure given above shows the graphical representation of a discrete ramp signal.

Standard Discrete Time Signals

Exponential Sequence

$G(n) = a^n$ for $n \geq 0$

Draw the sequence when a is between 0 and 1.

Standard Discrete Time Signals

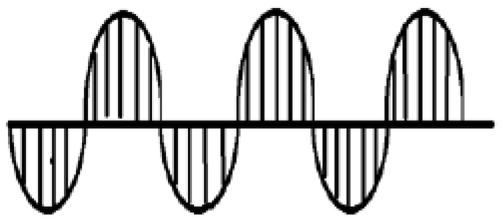
Discrete form of a sinusoidal signal can be represented in the format −

$$x(n) = A\sin(\omega n + \phi)$$

Here A,ω and φ have their usual meaning and n is the integer. Time period of the discrete sinusoidal signal is given by −

$$N = \frac{2\pi m}{\omega}$$

Where, N and m are integers.



A discrete time signal is periodic only if its frequency f_o is a rational number or w_o is a rational multiplier of 2Π

Complex Exponential Signals

A complex exponential signal which is defined at discrete instants of time is known as discrete-time complex exponential sequence. Mathematically, the discrete-time complex exponential sequence is defined as,

$$x(n) = a^n e^{j(\omega_0 n + \varphi)} = a^n \cos(\omega_0 n + \varphi) + j a^n \sin(\omega_0 n + \varphi)$$

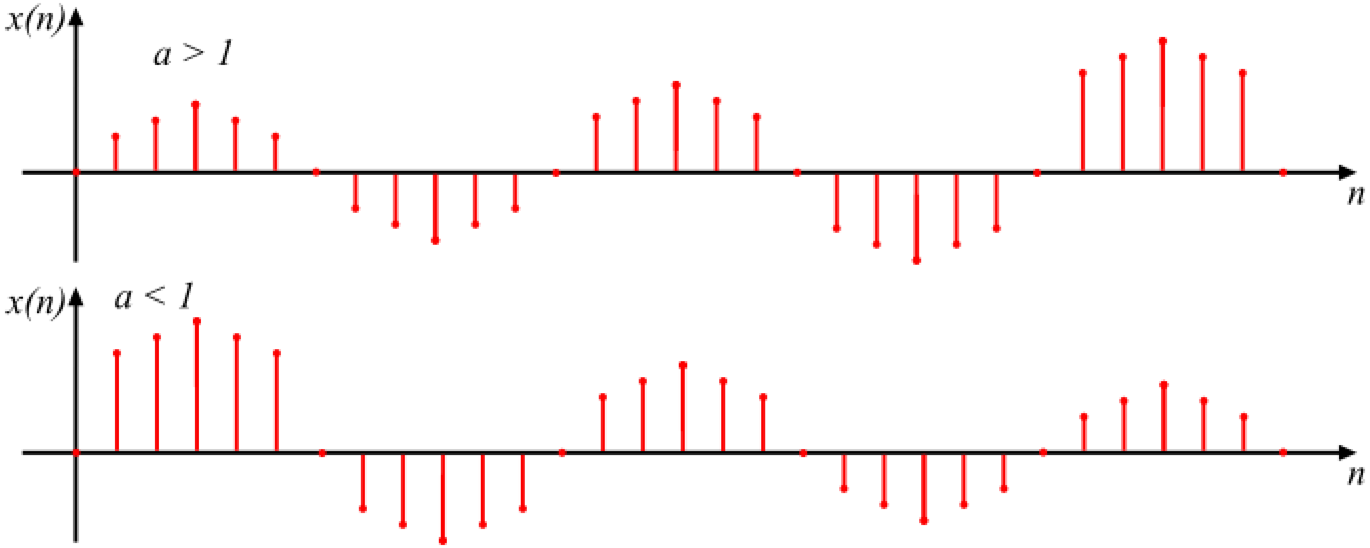
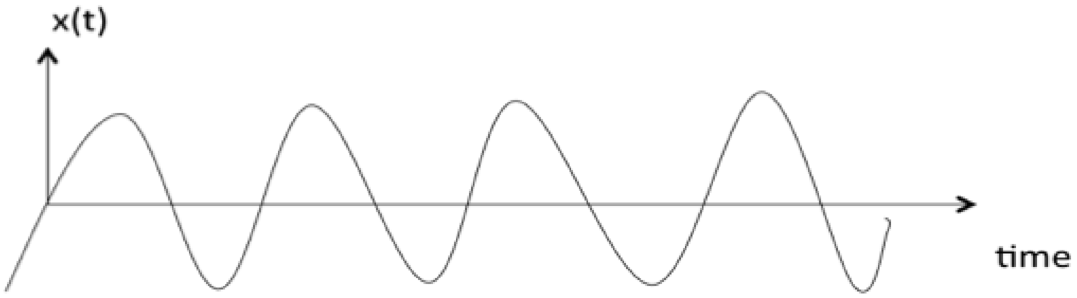


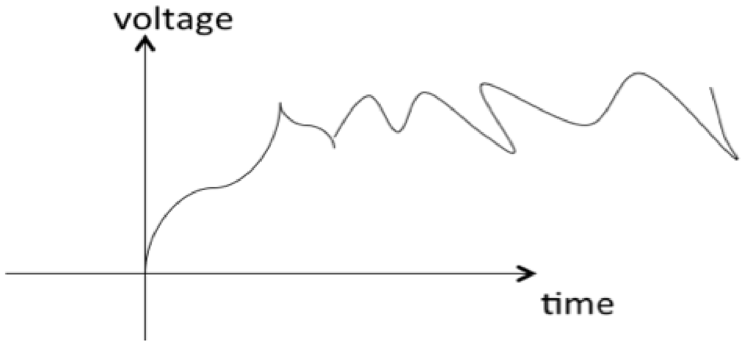
Figure-4

Classification of Discrete Time Signals

Deterministic and non-deterministic signal

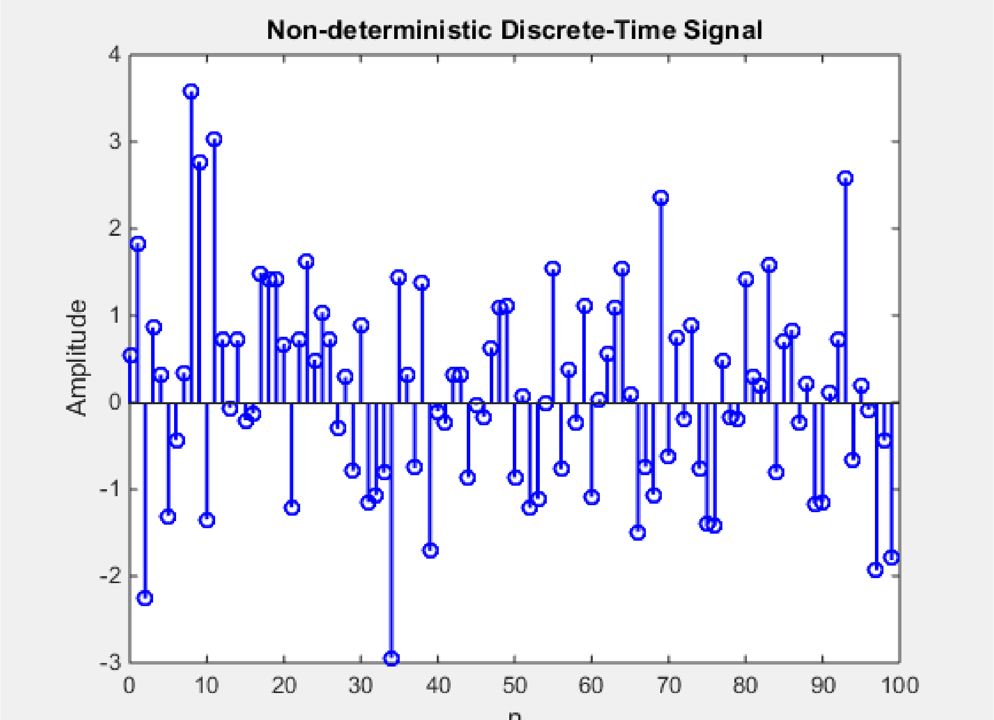
Signals which can be defined exactly by a mathematical formula are known as deterministic signals. There is no uncertainty with respect to its value at some instant of time. Non- deterministic signals are random in nature hence they are called random signals.





1. $x(n) = \sin(\pi n/4)$
2. Unit impulse signal

```
% Generate a non-deterministic signal with random values
n = 0:99;
x = randn(size(n))
% Plot the non-deterministic signal
stem(n, x, 'b', 'LineWidth', 1.5);
xlabel('n');
ylabel('Amplitude');
title('Non-deterministic Discrete-Time Signal’);
```



Classification of Discrete Time Signals

Periodic and aperiodic signals

A discrete-time signal x(n) is said to be periodic if it satisfies the following condition

$$x(n) = x(n + N); \quad \text{for all integers } n$$

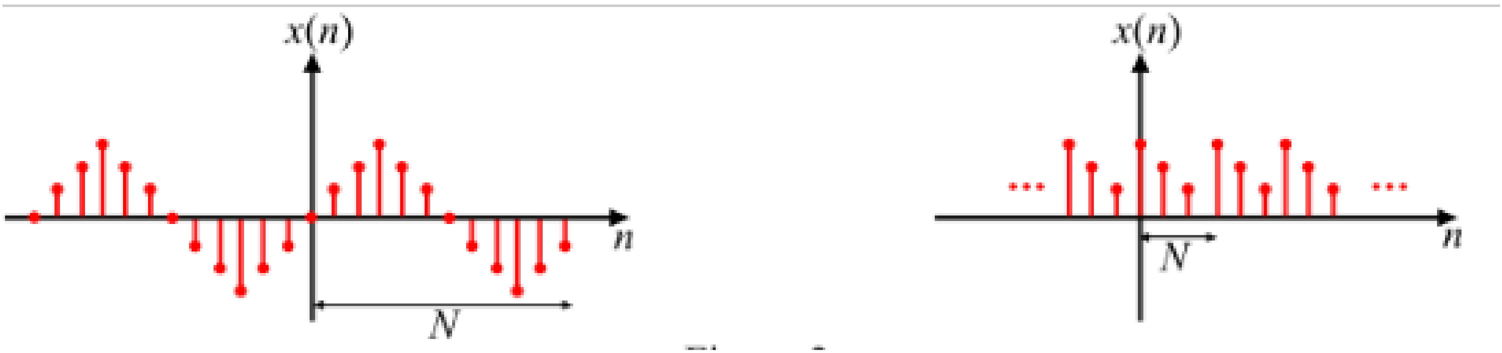
Here, N is the time period of the periodic signal and a positive integer. The smallest value of the time period (N) which satisfies the above condition is known as fundament time period of the signal. The fundamental time period (N) may be defined as the minimum number of samples taken by signal to repeat itself.

If there is no value of N for which the above condition is satisfied, then the signal is said to be aperiodic.

Sometimes aperiodic signals are said to have a period of infinity.

The angular frequency of the discrete time periodic sequences is given by, $\omega = 2\Pi/N$

Therefore, the time period of the sequence is,
$N = 2\Pi/\omega$ (The smallest value of N for which the condition $x(n) = x(n+N)$ is satisfied is known as fundamental period.



Sum of two discrete time periodic signals is also periodic. (Period is LCM of N1 and N2).

To check periodicity of discrete time sinusoidal complex signals, see if f_o is a rational number or prove that $e^{j\omega 0n}$ is periodic if $\omega 0/2\pi$ is rational

$$e^{j\omega_0(n+N)} = e^{j\omega_0 n} \implies e^{j\omega_0 N} = 1$$

$$e^{j\omega_0 N} = \cos(\omega_0 N) + j\sin(\omega_0 N)$$

For $e^{j\omega_0 N}$ to equal 1, the real part ($\cos(\omega_0 N)$) and the imaginary part ($\sin(\omega_0 N)$) should be equal to 1 and 0 respectively. Since the cosine function repeats with a period of 2π, $\omega_0 N$ must be an integer multiple of 2π.

Therefore, $\omega_0 N = 2\pi k$, where k is an integer.

Mention if the following signals are periodic or aperiodic. Also mention the time period of signals.

1. Sin (2Π /3)n + Cos (2Π /10)n
2. cos (4n)
3. $x(n)= e^{j7\Pi n/4}$

$$x[n] = (-1)^n \cos(\frac{2\pi n}{7})$$

Classification of Discrete Time Signals

Bounded Signals

A **bounded signal** is one whose absolute value always stays within a fixed limit. In other words, the signal never becomes infinitely large at any point, including at its extremes or as time progresses.

The amplitude of the sinusoidal signal $\sin(\omega n)$ is always between -1 and 1 for any value of ω and n. Therefore, it remains bounded within this range regardless of the values of ω and n.

$X[n] = 1/(1+n^2)$??

Classification of Discrete Time Signals

Energy and Power Signals

An energy signal is a signal for which the total energy over all time is finite.
A power signal is a signal for which the average power over all time is finite.
Power signals have infinite duration and do not decay to zero as *n* goes to infinity.
Examples of energy signals include finite-length sequences or decaying exponential signals.
Examples of power signals include sinusoidal signals and other periodic signals.

Hence, the total energy of the discrete time signal x(n) is defined as

$$E = \sum_{n=-\infty}^{\infty} |x(t)|^2$$

The average power of a discrete time signal x(t) is defined as

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(t)|^2$$

Q1. Is the given signal an energy signal?

$$x[n] = \begin{cases} 1 & \text{if } 0 \leq n \leq N \\ 0 & \text{otherwise} \end{cases}$$

Q2. Determine if the given signal is energy signal or power signal.

$X(n)= (1/2)^n u(n)$

Classification of Discrete Time Signals

Causal and Non-Causal Signals

Mathematically, a signal $x[n]$ is causal if $x[n]=0$ for $n<0$.

Mathematically, an anticausal signal $x[n]$ typically has non-zero values only for $n<0$, with $x[n]=0$ for $n \geq 0$.

X(n)= {2,3,4,1,0,-9}

Basic operations on Discrete Time Signals

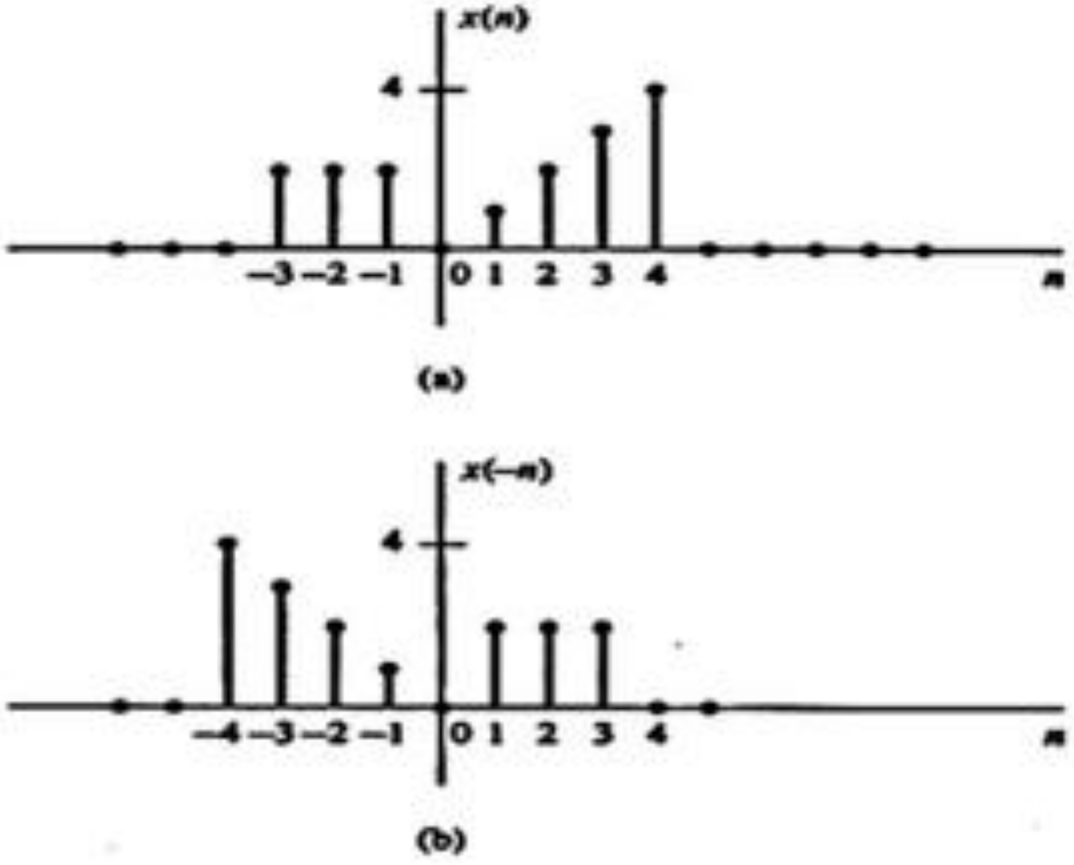
1. **Scaling of discrete time signals (Amplitude Scaling, Time scaling)**
2. **Folding of discrete time signals (Time reversal operation, Transpose, Reflection)**
3. **Shifting of discrete time signals (Time shifting)**
4. **Addition, Multiplication**

Basic operations on Discrete Time Signals

1. Folding of discrete time signals (Time reversal operation, Transpose, Reflection)

Folding of x(n) can be achieved by changing n to –n

x(n)= 0.5n for n=-2:2



Basic operations on Discrete Time Signals

1. Addition of two discrete time signals

Addition is performed on sample-by-sample basis. Addition is commutative.

Y(n)= x1(n) + x2(n)= x2(n)+x1(n)

X1(n)={1,2,3,4}
X2(n)={4,1,3,-1}
Y(n)=??

2. Multiplication of two discrete time signals

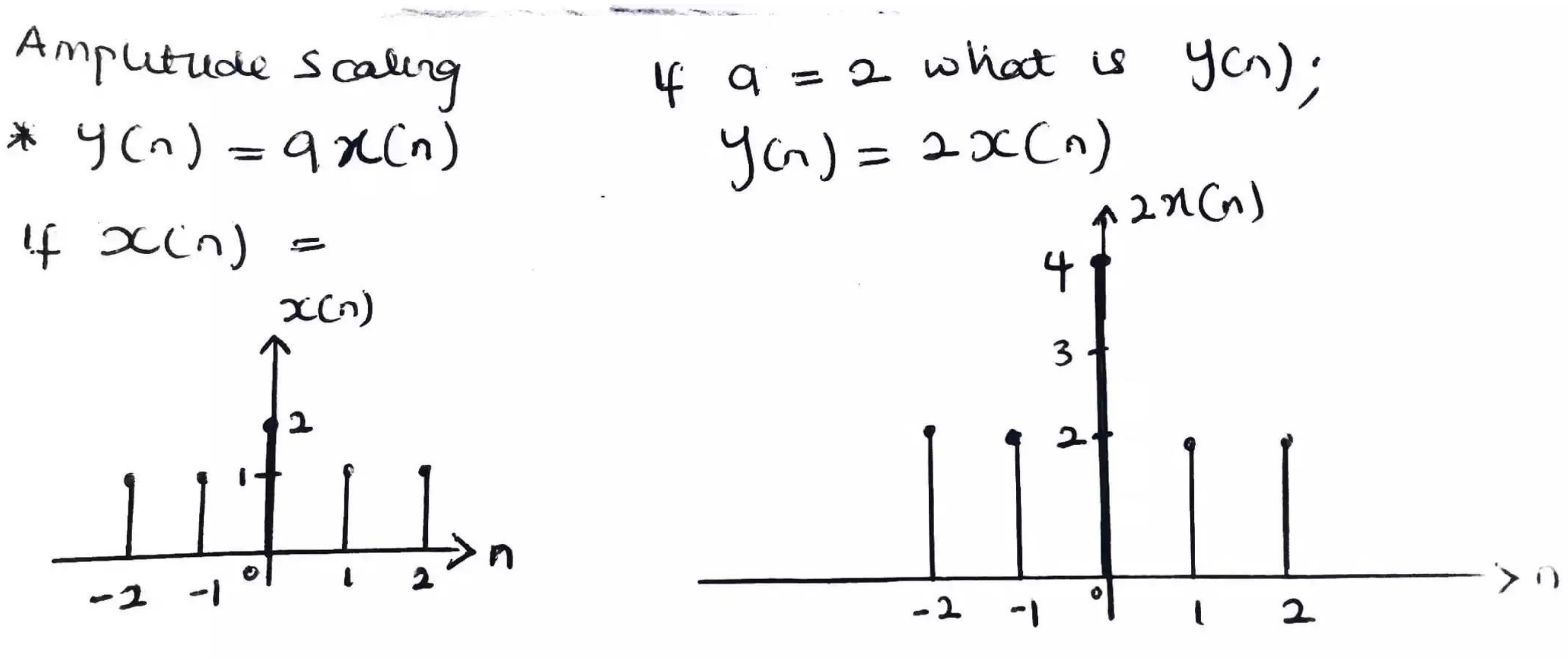
Multiplication is performed on sample-by-sample basis. Multiplication is commutative.

Y(n)= x1(n) * x2(n)= x2(n) * x1(n)

Basic operations on Discrete Time Signals

Scaling of discrete time signals

The **amplitude scaling** of a discrete time signal is performed by multiplying every sample of the signal by constant ‘A’.
y(n)= Ax(n)
If A>1 then its amplification. If A <1 then its attenuation.



Basic operations on Discrete Time Signals

Scaling of discrete time signals

The **time scaling** of a discrete time signal is performed by replacing n by an in x(n) i.e., x(an). If a>1 then its time compression. If a <1 then its time expansion.

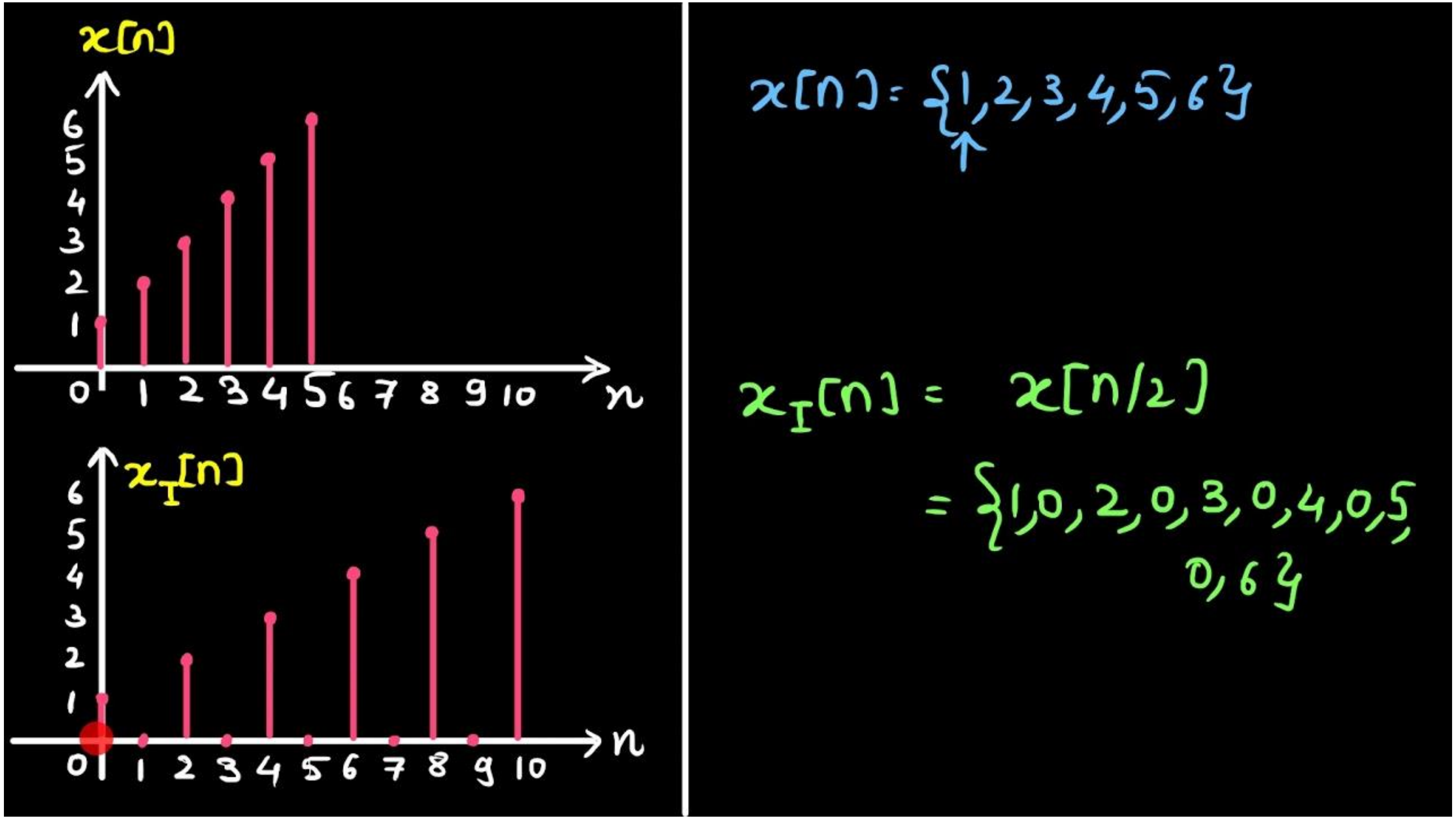
Example: y(n)=x(2n)

x(n)= {4,3,2,1,2,3,4}

$\uparrow$

Example: Take x(n)={1,2,3,4,5,6}. Find y(n)=x(n/2)

Basic operations on Discrete Time Signals

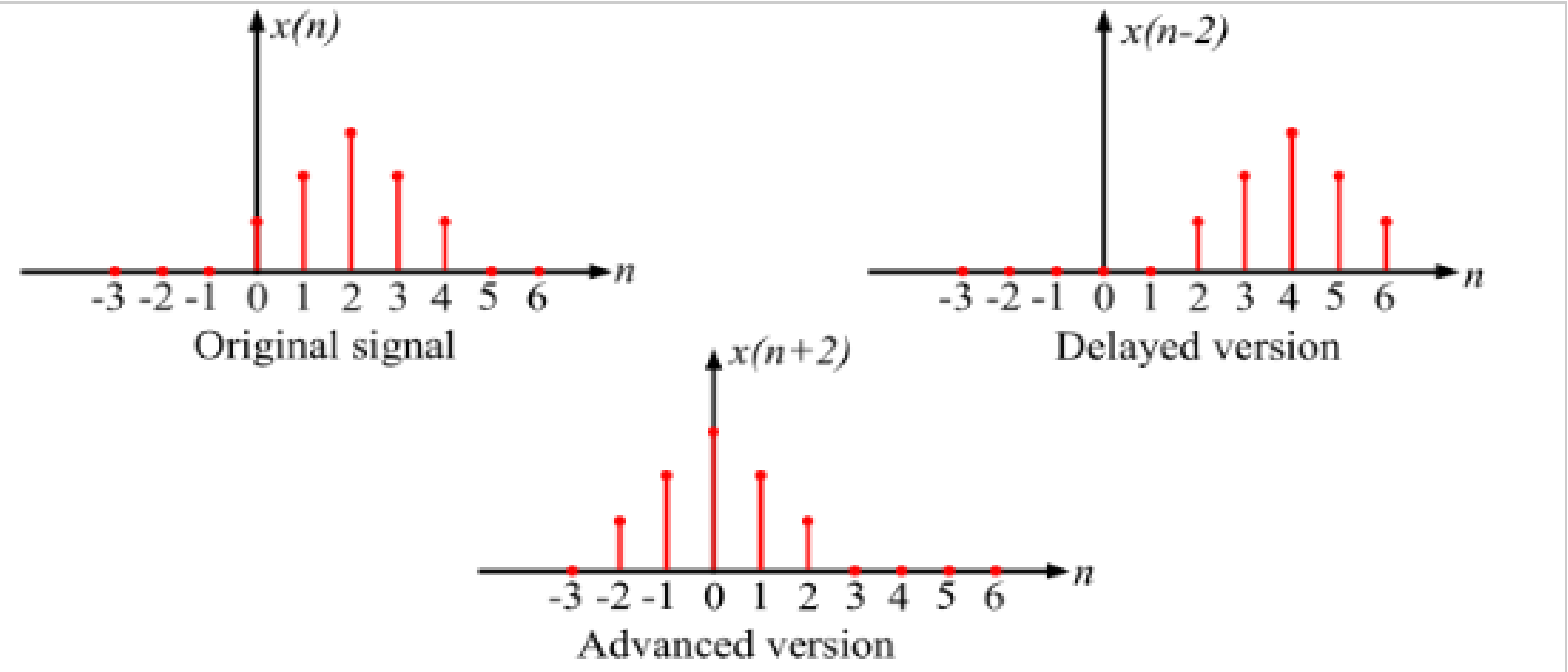


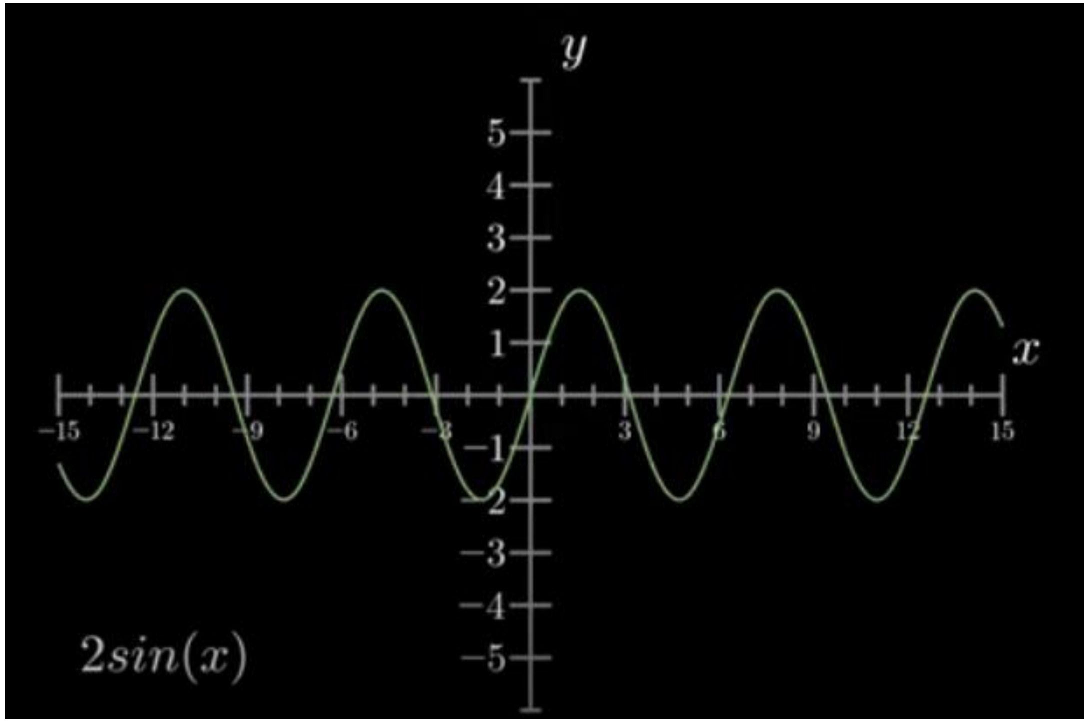
Basic operations on Discrete Time Signals

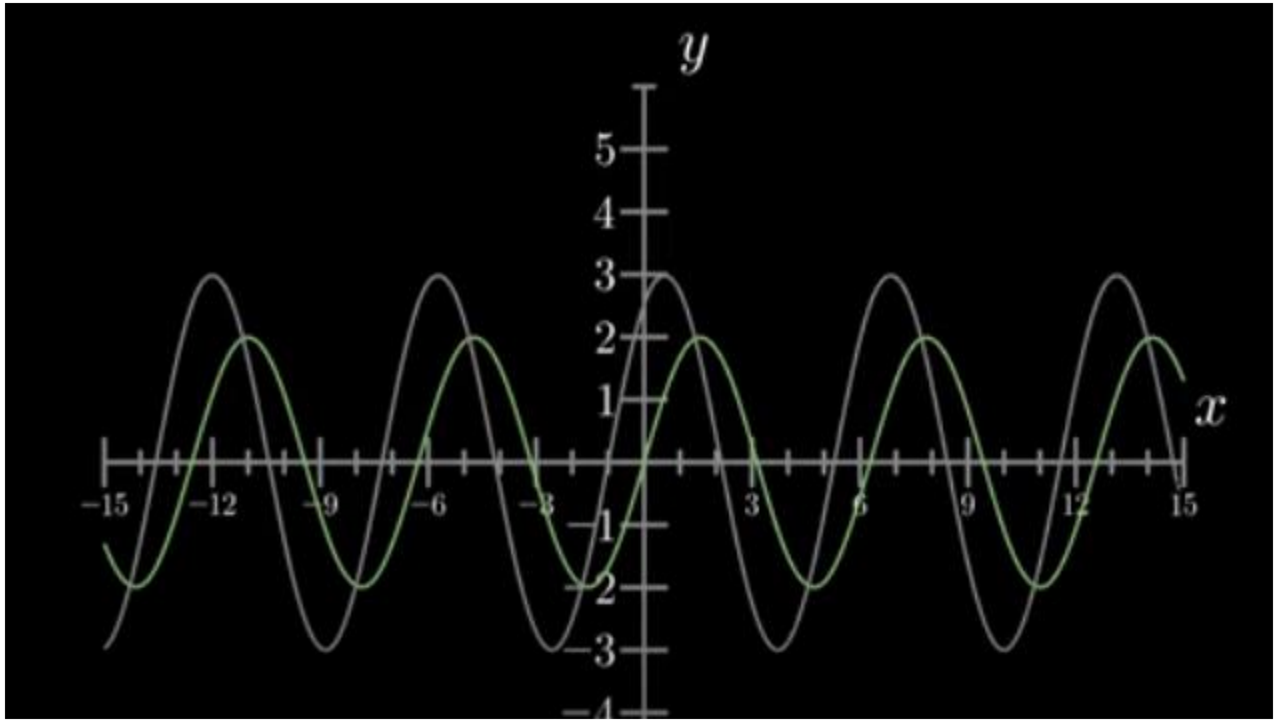
The **time shifting** operation of a discrete time signal x(n) is represented as

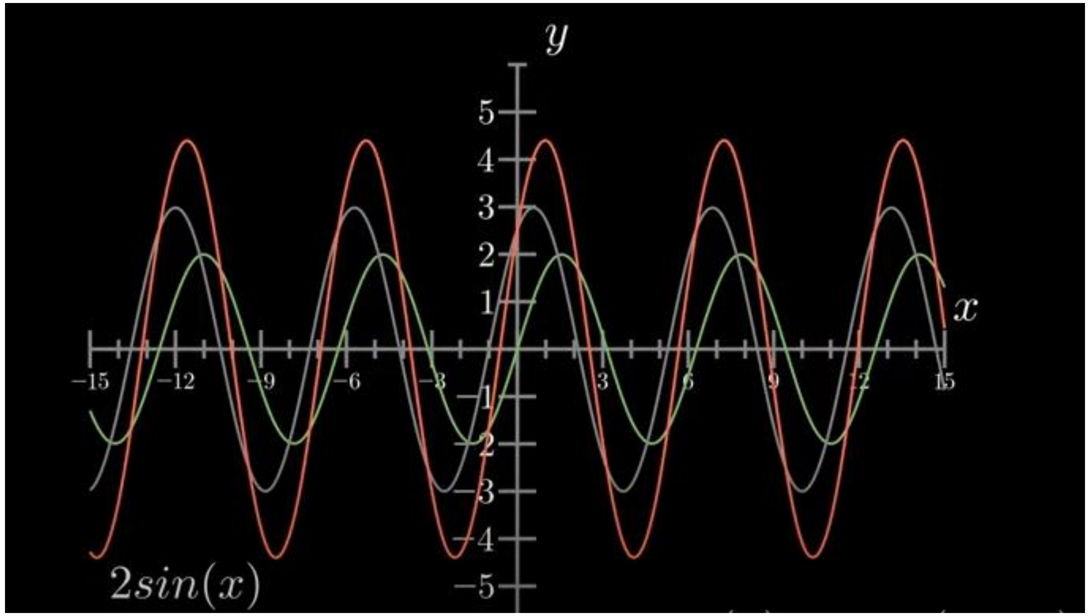
$$y(n) = x(n - n_0)$$

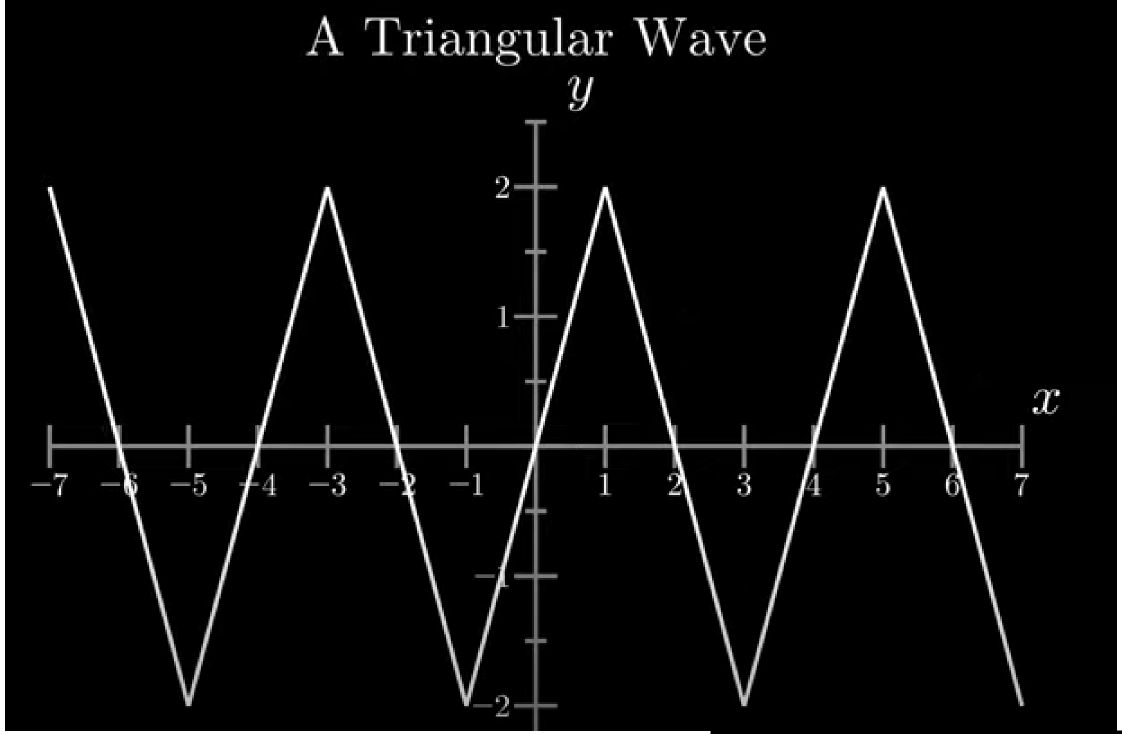
This equation shows that the signal y(n) can be obtained by time-shifting the signal x(n) by n_0 units.

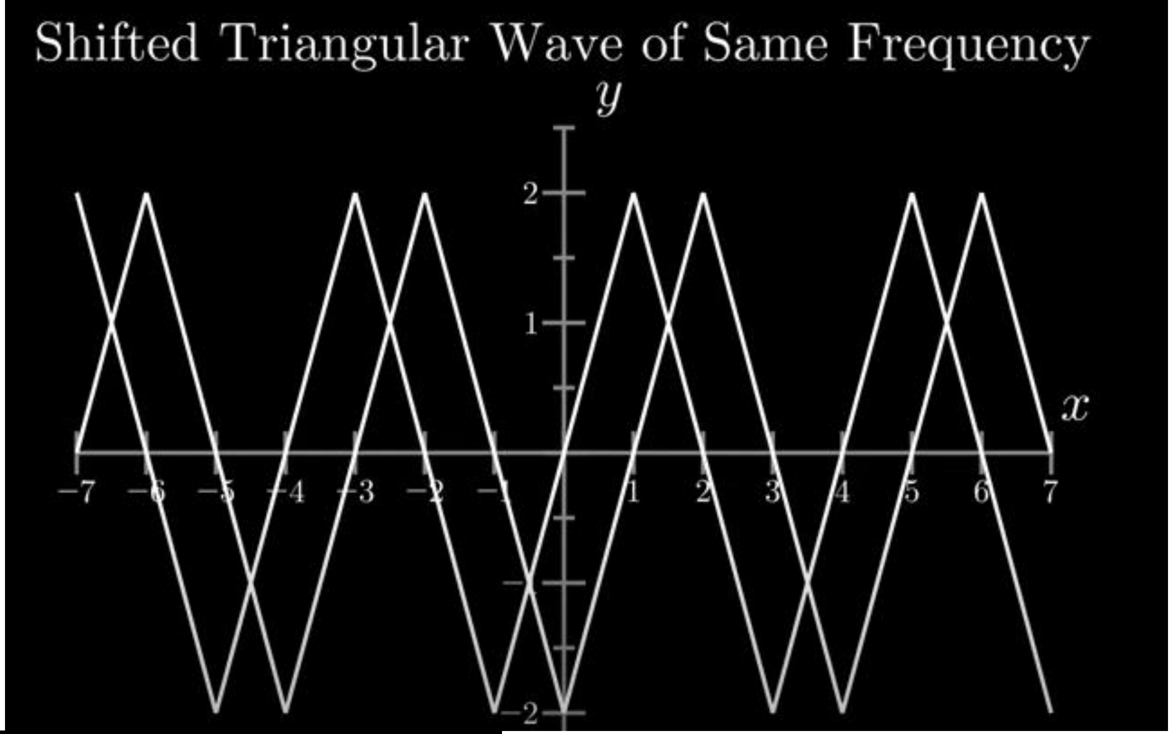


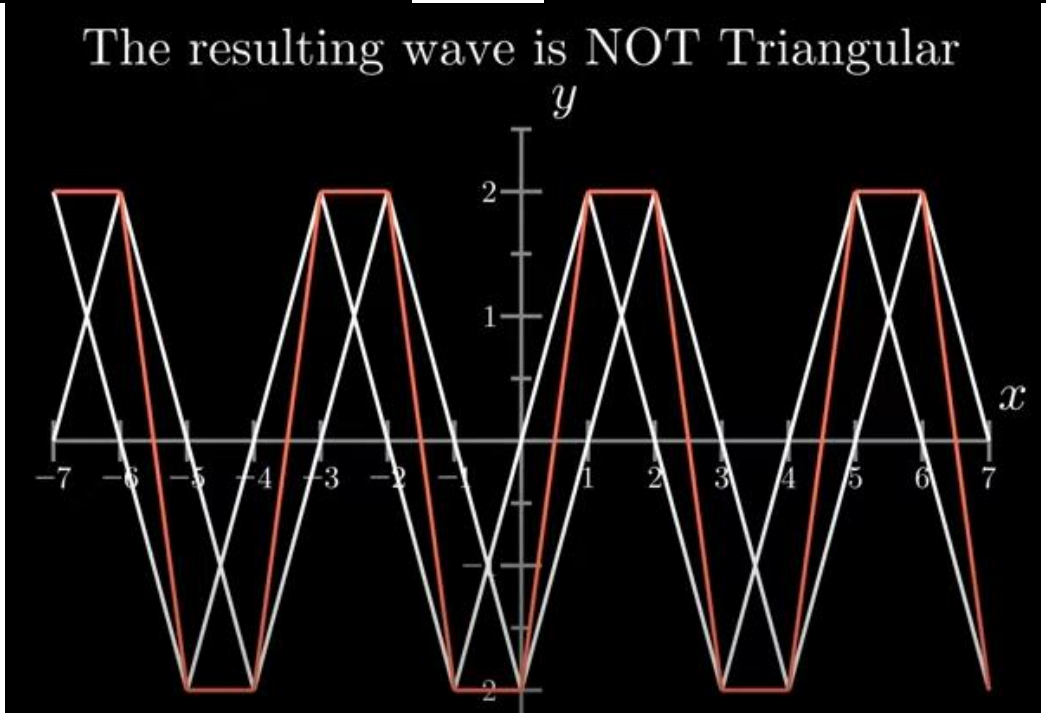

y
x
5
4
3
2
1
−1
−2
−3
−4
−5
−15
−12
−9
−6
−3
3
6
9
12
15
2sin(x)


y
x
5
4
3
2
1
−1
−2
−3
−15
−12
−9
−6
−3
3
6
9
12
15

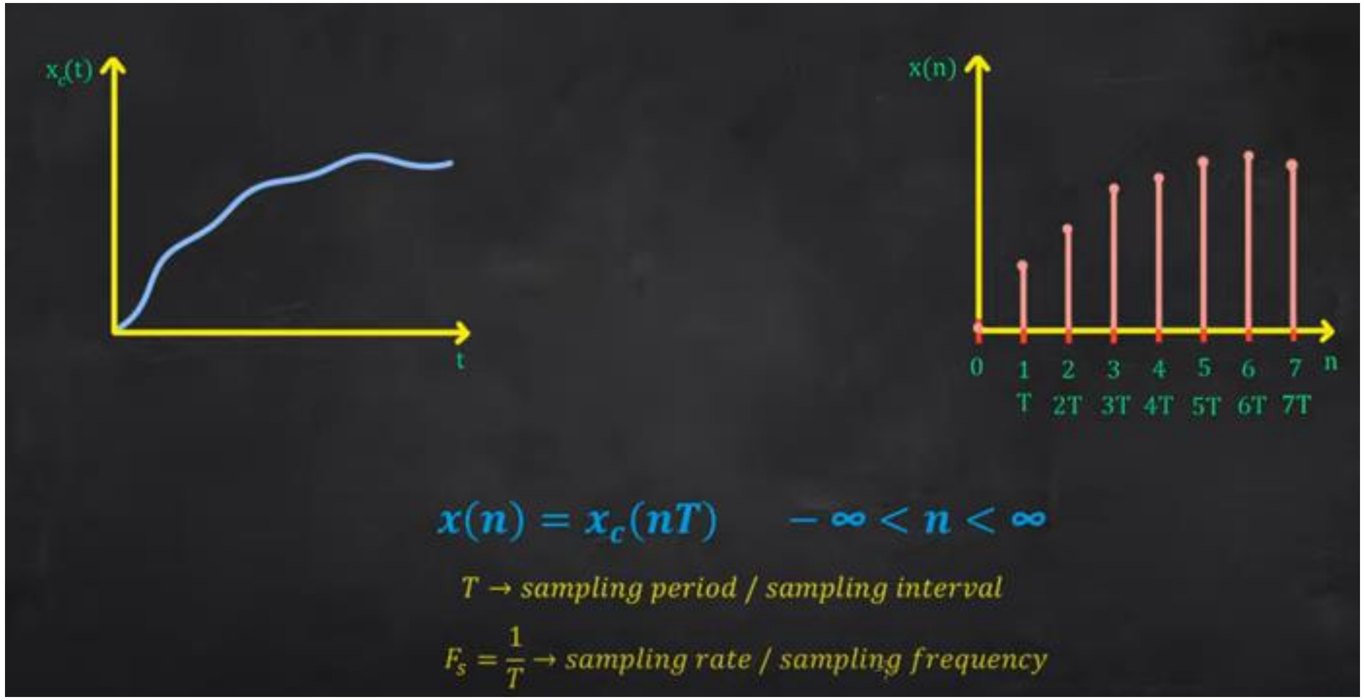

y
x
5
4
3
2
1
−1
−2
−3
−4
−5
−15
−12
−9
−6
−3
3
6
9
12
15
2sin(x)


A Triangular Wave
y
x
2
1
−1
−2
−7 −6 −5 −4 −3 −2 −1 1 2 3 4 5 6 7


Shifted Triangular Wave of Same Frequency
y
x
2
1
−2
−7 −6 −5 −4 −3 −2 −1 1 2 3 4 5 6 7


The resulting wave is NOT Triangular
y
x
2
1
−7 −6 −5 −4 −3 −2 −1 1 2 3 4 5 6 7

Normalized frequency



- t=nT=n/Fs

$\Omega = 2\pi F \quad \Omega \rightarrow$ *angular frequency of CTS,* $\quad F \rightarrow$ *frequency of CTS*

$\omega = 2\pi f \quad \omega \rightarrow$ *angular frequency of DTS,* $\quad f \rightarrow$ *frequency of DTS*

$F_s = \frac{1}{T} \rightarrow$ *sampling rate / sampling frequency*

$$x_C(t) = A\cos(\Omega t + \theta)$$

$$\boxed{f = \frac{F}{F_s}}$$ → normalized frequency / relative frequency