National University
of Computer & Emerging Sciences FAST Peshawar Campus

Name: _____   Roll No: _____

Program: BS (CS, SE)
Semester: Spring – 2020
Time Allowed: 3 Hours + 1 hour for uploading
Course: Discrete Structures

Examination: Final
Weight 60%   Total Marks: 118
Date: June, 2020
Instructor: Dr. Nouman Azam

NOTE: Attempt all questions on a sheet of paper.
Try attempting all questions in the order they are given in the paper.
Attempt on a paper using pen / pencil, scan and then send me in soft preferably through slate
And if slate is not working then through slack using direct message.
If both slate and slack are out of order then email me directly (consider this as a last option)
Make sure you paper is not visible to anyone else or it will be cheating
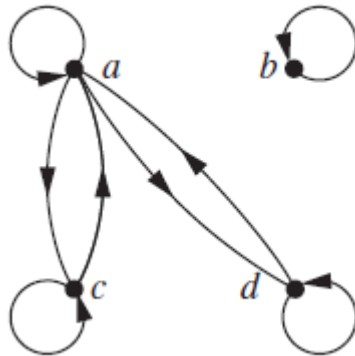

Relations

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1. Consider the relation R given by $\{(a,b) \mid 2a + 3b$ is divisible by $5\}$, (**Points 3*3 = 9**)
   a) Computer the relation R on defined on the set A = {1,2,3,4}
   b) Determine if R is reflexive when it is define on the set A = Z^+
   c) Determine if R is reflexive when it is define on the set A = Z-

2. Explain why the relation R = $\emptyset$ defined on a non empty set A, cannot be reflexive but is symmetric and transitive. (**Points 3**)

3. Determine if the relation R = $\{(a,b) \mid a$ is taller than $b \}$ defined on the set of all people is (**Points 3*4 = 12**)
   a) Reflexive
   b) Symmetric,
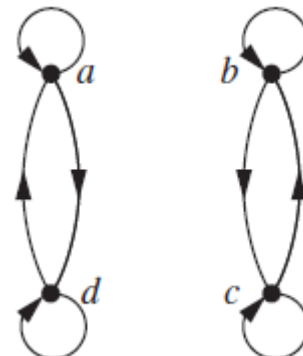   c) Ant symmetric,
   d) Transitive,
   Note: Explain each part with proper definition and explanation

4. Consider a relation R which is given based on the matrix given below. The relation is defined based on the set A = {1, 2, 3, 4} and the rows and columns of the matrix corresponds to the integers in set A listed in increasing order. (**Points 3*3 = 9**)
   M_R = [1   1   0   1
        1   0   1   0
        1   1   0   0
        1   1   0   0]
   a) Compute the tuples in R
   b) Compute the tuples in R^{-1}
   c) Compute the tuples in R U R^{-1} and R $\cap$ R^{-1}

5. Explain with reasons if the relations given below in the form of graphs are equivalence relations? If they are equivalence relations then provide the equivalence classes and partitions (**Points 3*2 = 6**)

a)   b)

6. Consider the following partial order ({2, 3, 6, 12, 24, 36, 48, 60, 72}, |) (**Points 5*3 = 15**)
   a. Define the partial order relation (Hint: I need a set containing tuples satisfying the relation)
   b. Construct the corresponding Hasse diagram of the relation defined in part (a)
   c. Is this a lattice. Explain
   d. What is the greatest and least element if any exits.
   e. What are maximal and minimal elements if any exist.

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Graphs

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6. The degree sequence of a graph is the sequence of the degrees of the vertices of the graph in a decreasing order. What is the degree sequence of the following graphs (**Points 3*4 = 12**)
   a. W_5
   b. $K_{3,3}$
   c. C_6
   d. K_6

7. Draw a circular graph for the graph represented by the following sets (**Points 3**)

   a. Set of vertices = V = {a_1,a_2,a_3,a_4}
   b. Set of edges = E = {(a_1,a_2), (a_3,a_4), (a_1,a_3), (a_2,a_4)}

8. Draw a circular graph representing K_4. (**Points 3**)

9. Draw the adjacency matrix for the graph $K_{2,3}$. Assume that the first group/set of vertices include {a,b} and the second group/set of vertices include {c,d,e} (**Points 3**)
   Hint: One adjacency matrix is needed

10. Determine whether the graphs corresponding to the following two matrices are isomorphic or not (**Points 3**)

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \quad \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$
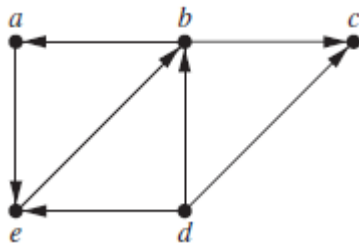
Hint: I need the **9 step procedure** outlined in the class. You will first need to draw the graphs.

11. Answer the following questions with justification (**Points 4 * 3 = 12**)
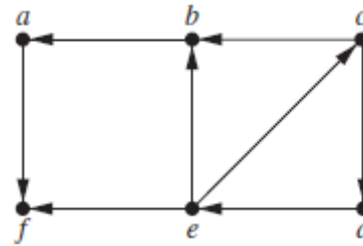   a. Does the graphs W_N and C_N has a euler path when N is odd.

    b. Does the graphs W_N and C_N has a euler circuit when N is even.

    c. Does the graphs W_N and C_N has a hamilton path when N is even.

    d. Does the graph W_N and C_N has a hamilton circuit when N is odd.
       Note: Each part must have two answers with justifications

12. Find the strongly connected components of each of the graphs given below (**Points 3*2=6**)

    a.   b.

13. What is the chromatic number of a graph with (**Points 3*2 = 6**)
    a. K_N where N is odd.
    b. C_N where N is even.

    Hint: Provide justifications for your answer

    _____   _____

                                Trees

    _____

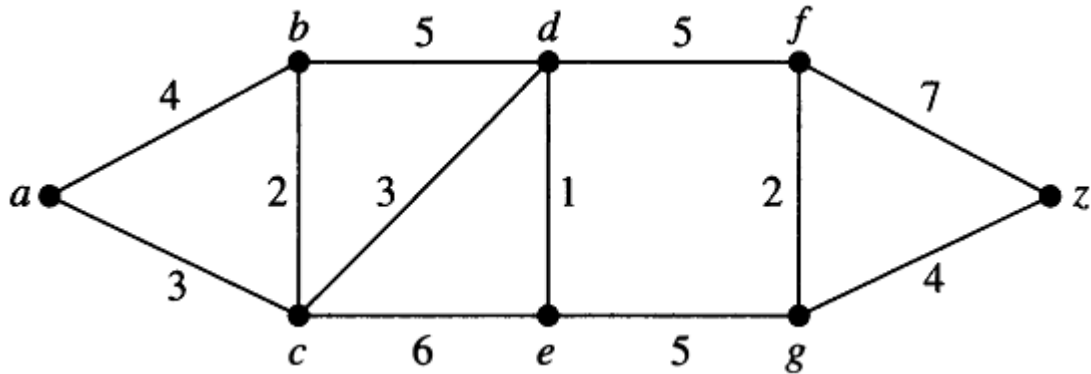14. Consider the following set theoretic expression ((A + B)*C) - (A * (B - A)) (**Points 4*3 = 12**)
    c. Represent the above expression using ordered rooted tree.
    d. Write this expression in prefix notation.
    e. Write this expression postfix notation.
    f. Write this expression infix notation.
       Hint: It's a friendly question. In part (a) I expect a tree and in the (b – c) I expect expressions

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Shortest Paths and Spanning Trees

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15. Consider the weighted graph shown below. Use the Dijistra algorithm to find the minim path between f and a. (**Points 3**)

Hint: I need

1. A table which shows the application of the dijistra algorithm.
2. The minimum path
3. The minimum distance

16. Consider the graph of W_5 and K_6, construct the spanning tree of these graphs using (**Points 3*2 = 6**)
   g. Construct spanning trees of these graphs using depth first search.
   h. Construct spanning trees of these graphs using breadth first search.

Good Luck