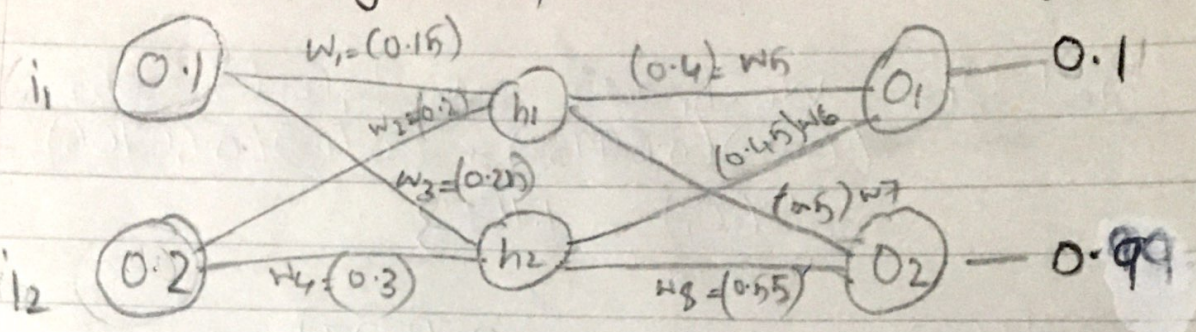
PROJECT ASSIGNMENT

Wadood Riaz Cheema

Back propagation / Backward Propagation of



$b_1 = (0.35)$

$b_2 = (0.6)$

Forward Pass:

$$h_1(net) = w_1 i_1 + w_2 i_2 + b_1$$
$$= (0.15)(0.1) + (0.2)(0.2) + 0.35$$
$$h_1(net) = 0.405$$

Sigmoid: $\sigma(h_1) = Out(h_1) = \dfrac{1}{1+e^{-h_1}}$

$$= \frac{1}{1+e^{-(0.405)}} = 0.59988$$

$$h_2(net) = w_3 i_1 + w_4 i_2 + b_1$$
$$= (0.25)(0.1) + (0.3)(0.2) + 0.35$$
$$h_2(net) = 0.435$$

$$Out(h_2) = \frac{1}{1+e^{-h_2}} = \frac{1}{1+e^{-0.435}} = 0.60706$$

$$net\ O_1 = W_5\, h_1(out) + W_6\, h_2(out) + b_2$$
$$= (0.4)(0.59988) + (0.45)(0.60706) + 0.6$$
$$net(O_1) = 1.11313$$

$$Out(O_1) = \frac{1}{1+e^{-O_1}} = 0.75271$$

$$net\ O_2 = W_7 (h_1)_{out} + W_8 (h_2)_{out} + b_2$$
$$= (0.5)(0.59988) + (0.55)(0.60706) + 0.6$$
$$net\ O_2 = 1.23382$$

$$Out(O_2) = \frac{1}{1+e^{-O_2}} = 0.77448$$

Error Calculation:-

$$E_{O_1} = \frac{1}{2}\left(\text{Target } O_1 - \text{Output } O_1\right)^2$$
$$= \frac{1}{2}\left(0.1 - 0.75271\right) = E_{O_1} = 0.21301$$
$$E_{O_2} = \frac{1}{2}\left(0.90 - 0.77448\right)^2 = E_{O_2} = 0.00787$$

Total Error $= 0.21301 + 0.00787$

T. Err. $= 0.22088$

Backward Pass.

$$\frac{\partial E_{total}}{\partial w_5} = \frac{\partial E_{tot}}{\partial out\,O_1} \times \frac{\partial out\,O_1}{\partial net\,O_1} \times \frac{\partial net\,O_1}{\partial w_5}$$

We know that

$$Tot\ Err = E_{O_1} + E_{O_2}$$

$$Err_{Tot} = \frac{1}{2}(Tar\,O_1 - Out\,O_1)^2 + \frac{1}{2}(Tar\,O_2 - Out\,O_2)^2$$

Take partial derivative on Both side wrt out O_1

$$\frac{\partial E_{total}}{\partial Out\,O_1} = \cancel{2} \times \frac{1}{\cancel{2}}(tar\,O_1 - Out\,O_1)^{2-1}(0-1) + [0]$$

$$= Out\,O_1 - Tar\,O_1$$

$$= 0.75271 - 0.1$$

$$\frac{\partial E_{tot}}{\partial out\,O_1} = 0.65271$$

Now, find $\frac{\partial out\,O_1}{\partial net\,O_1} = \partial\left(\frac{1}{1+e^{-net\,O_1}}\right)$

$$\frac{\partial out\,O_1}{\partial net\,O_1} = Out\,O_1(1 - Out\,O_1)$$

$$= 0.18613$$

$$\frac{\partial net\,O_1}{\partial W_5} = W_5\, Out\,h_1 + W_6\, Out\,h_2 + b_2$$

$$\frac{\partial net\,O_1}{\partial W_5} = Out\,h_1 = 0.59988$$

$$\boxed{\frac{\partial E_{tot}}{\partial W_5} = 0.07287}$$

Update the weight ($\alpha = 0.5$)

$$W_5^{+} = W_5 - \alpha \frac{\partial E_{tot}}{\partial W_5}$$

$$\boxed{W_5^{+} = 0.36356}$$

$$\frac{\partial E_{tot}}{\partial W_6} = \left.\frac{\partial E_{tot}}{\partial Out\,O_1} \times \frac{\partial Out\,O_1}{\partial net\,O_1} \times \frac{\partial net\,O_1}{\partial W_6}\right] \rightarrow III$$

for $\frac{\partial net\,O_1}{\partial W_6} = W_6\, Out\,h_2 + W_5\, Out\,h_1 + b_2$

$$= Out\,h_2 = 0.60706$$

Put the value in equ III

$$\frac{\partial E_{tot}}{\partial W_6} = 0.65271 \times 0.18613 \times 0.60706 = \cancel{0.7703}\ 0.073715$$

$$w_6^+ = w_6 - \alpha \frac{\partial tot}{\partial w_6} = \overset{0.45}{\cancel{0.50}} - 0.5(0.07375)$$

$$\boxed{w_6^+ = 0.46312}$$

$$\frac{\partial E_{tot}}{\partial w_7} = \frac{\partial E_{tot}}{\partial out\,O_2} \times \frac{\partial out\,O_2}{\partial net\,O_2} \times \frac{\partial net\,O_2}{\partial w_7} \rightarrow IV$$

find $\frac{\partial E_{tot}}{\partial out\,O_2} = Out\,O_2 - Tar\,O_2$

$$\frac{\partial E_{tot}}{\partial out\,O_2} = -0.12552$$

$$\frac{\partial out\,O_2}{\partial net\,O_2} = out\,O_2(1 - out\,O_2)$$

$$= 0.17466$$

$$\frac{\partial net\,O_2}{\partial w_7} = \cancel{net\,O_2}\ Out\,h_1 = 0.59988$$

Put in equ IV

$$\frac{\partial E_{tot}}{\partial w_7} = -0.12552 \times 0.17466 \times 0.59988$$

$$= \frac{\partial E_{tot}}{\partial w_7} = -0.01315$$

$$w_7^+ = w_7 - \alpha \frac{\partial E_{tot}}{\partial w_7} = 0.50 - 0.5(-0.01315$$

$$\boxed{w_7^+ = 0.45657} \quad w_7^* = 0.5065775$$

$$\frac{\partial E_{total}}{\partial w_8} = \frac{\partial E_{tot}}{\partial Out_{O_2}} \times \frac{\partial Out_{O_2}}{\partial net_{O_2}} \times \frac{\partial net_{O_2}}{\partial w_8} \rightarrow V$$

$$\frac{\partial net}{\partial w_8} = Out_{h_2} = 0.60706$$

Put in equ V

$$\frac{\partial E_{tot}}{\partial w_8} = -0.12552 \times 0.17466 \times 0.60706$$

$$\frac{\partial E_{tot}}{\partial w_8} = -0.013308$$

$$w_8^+ = 0.55 - 0.5(-0.013308)$$

$$\boxed{w_8^+ = 0.55665}$$

Now the Hidden layer: w_1

$$\frac{\partial E_{tot}}{\partial w_1} = \frac{\partial E_{tot}}{\partial out\,h_1} \times \frac{\partial out\,h_1}{\partial net\,h_1} \times \frac{\partial net\,h_1}{\partial w_1} \rightarrow C$$

for

$$\frac{\partial E_{tot}}{\partial out\,h_1} = \frac{\partial E_{O_1}}{\partial out\,h_1} + \frac{\partial E_{O_2}}{\partial out\,h_1}$$

$$\frac{\partial E_{O_1}}{\partial out\,h_1} = \frac{\partial E_{out_1}}{\partial net\,O_1} \times \frac{\partial net\,O_1}{\partial out\,h_1}$$

$$\frac{\partial E_{out_1}}{\partial net\,O_1} = \frac{\partial E_{O_1}}{\partial O_1} \times \frac{\partial O_1}{\partial net\,O_1}$$

$$= 0.65271 \times 0.18613$$
$$= 0.12149$$

$$\frac{\partial net\,O_1}{\partial out\,h_1} = w_5 = 0.4$$

$$\frac{\partial E_{O_1}}{\partial out\,h_1} = 0.12149 \times 0.4 = \boxed{0.048595}$$

$$\frac{\partial E_{O_2}}{\partial out\,h_1} = \frac{\partial E_{O_2}}{\partial net\,O_2} \times \frac{\partial net\,O_2}{\partial out\,h_1}$$

$$\frac{\partial E_{O_2}}{\partial net\,O_2} = \frac{\partial E_{O_2}}{\partial out\,O_2} \times \frac{\partial out\,O_2}{\partial net\,O_2}$$

$$\frac{\partial EO_2}{\partial net\,O_2} = -0.12552 \times 0.17466 = \boxed{-0.0219233}$$

$$\frac{\partial net\,O_2}{\partial Out\,h_1} = w_7 = 0.5$$

$$\frac{\partial EO_2}{\partial Out\,h_1} = -0.0219233 \times 0.5 = \boxed{-0.0109616}$$

$$\frac{\partial E\,tot}{\partial Out\,h_1} = \frac{\partial EO_1}{\partial Out\,h_1} + \frac{\partial EO_2}{\partial Out\,h_1}$$

$$= 0.048595 - (-0.0219233 \times 0.5)$$

$$\boxed{\frac{\partial E\,tot}{\partial Out\,h_1} = 0.059556}$$

Put the value in equ "c"

$$\frac{\partial tot\,E}{\partial w_1} = \frac{\partial tot}{\partial Out\,h_1} \times \frac{\partial Out\,h_1}{\partial net\,h_1} \times \frac{\partial net\,h_1}{\partial w_1}$$

$$= 0.059556 \times [out\,h_1(1 - out\,h_1)] \times i_1$$

$$\frac{\partial tot\,E_1}{\partial w_1} = 0.2400239 \times 0.059556 \times i_1$$

$$= \boxed{0.00142948}$$

$$w_1^* = w_1 - \alpha \frac{\partial E_{total}}{\partial w_1} = 0.15 - 0.5 \times 0.000142948$$

$$\boxed{w_1^* = 0.1499285}$$

Now, the hidden layer: w_2

$$\frac{\partial E_{tot}}{\partial w_2} = \frac{\partial E_{tot}}{\partial out_{h_1}} \times \frac{\partial out_{h_1}}{\partial net_{h_1}} \times \frac{\partial net_{h_1}}{\partial w_2}$$

$$= 0.059556 \times 0.2400239 \times 0.2$$

$$\frac{\partial E_{tot}}{\partial w_2} = 0.002858$$

$$w_2^* = w_2 - \alpha \times \frac{\partial E_{tot}}{\partial w_2} = 0.2 - 0.5 \times 0.002858$$

$$\boxed{w_2^* = 0.19857}$$

The hidden layer: w_3.

$$\frac{\partial E_{tot}}{\partial w_3} = \frac{\partial E_{tot}}{\partial out_{h_2}} \times \frac{\partial out_{h_2}}{\partial net_{h_2}} \times \frac{\partial net_{h_2}}{\partial w_3} \rightarrow 1)$$

$$\cancel{\frac{\partial E_{tot}}{\partial out_{h_2}} = \frac{\partial E_{O_1}}{\partial net_{O_2}} + \frac{\partial O_2}{\partial net}}$$

$$\frac{\partial E_{tot}}{\partial out_{h_2}} = \frac{\partial E_{O_1}}{\partial out_{h_2}} + \frac{\partial E_{O_2}}{\partial out_{h_2}}$$

$$\frac{\partial E_{O_1}}{\partial out_{h_2}} = \frac{\partial E_{out_1}}{\partial net_{O_1}} \times \frac{\partial net_{O_1}}{\partial out_{h_2}} \rightarrow w_6$$

$$\frac{\partial E_{O_1}}{\partial out_{h_2}} = 0.12149 \times 0.45 = \boxed{0.05467}$$

$$\frac{\partial E_{out_1}}{\partial net_{O_1}} = \frac{\partial E_{out_1}}{\partial out_{O_1}} \times \frac{\partial out_{O_1}}{\partial net_{O_1}}$$

$$= 0.12149$$

$$\frac{\partial E_{out_2}}{\partial out_{h_2}} = \frac{\partial E_{out_{O_2}}}{\partial net_{O_2}} \times \frac{\partial net_{O_2}}{\partial out_{h_2}} \rightarrow w_8$$

$$= -0.0219233 \times 0.55 = \boxed{-0.0120578}$$

$$\frac{\partial E_{out_{O_2}}}{\partial net_{O_2}} = \frac{\partial E_{out_2}}{\partial out_{O_2}} \times \frac{\partial out_{O_2}}{\partial net_{O_2}}$$

$$= -0.12552 \times 0.17466 = -0.0219233$$

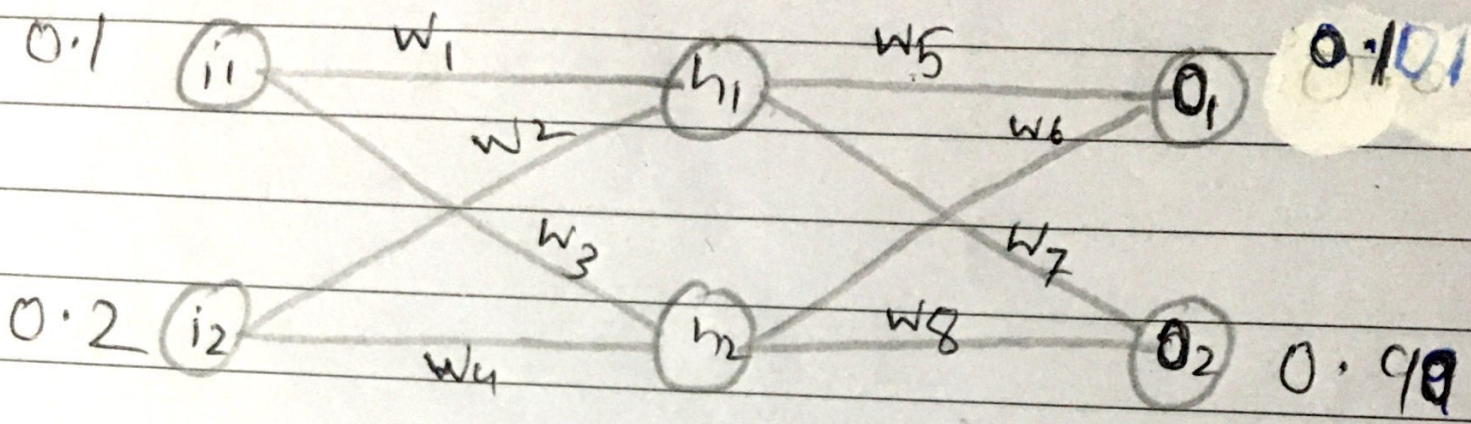
Put the value in eq n. 'D'

$$\frac{\partial E_{tot}}{\partial out_{h_2}} = 0.05467 + (-0.0120578) = \boxed{0.042612}$$

$$\frac{\partial E_{tot}}{\partial w_3} = 0.042612 \times [out_{h_2}(1 - out_{h_2})] \times i_1 \rightarrow 0.1$$

$$\boxed{\frac{\partial E_{tot}}{\partial w_3} = 0.0010164}$$

Updated "W"



$w_1 = 0.149928$

$w_2 = 0.19857$

$w_3 = 0.249491$

$w_4 = 0.29898$

$w_5 = 0.36356$

$w_6 = 0.413125$

$w_7 = 0.506575$

$w_8 = 0.55665$

$b_1 = 0.35$, $b = 0.6$

$$w_3^* = w_3 - \alpha \frac{\partial E_{tot}}{\partial w_3}$$

$$= 0.25 - 0.5 \times 0.0010164$$

$$\boxed{w_3^* = 0.249491}$$

The hidden layer: w_4

$$\frac{\partial E_{tot}}{\partial w_4} = \frac{\partial E_{tot}}{\partial out\,h_2} \times \frac{\partial out\,h_2}{\partial net\,h_2} \times \frac{\partial net\,h_2}{\partial w_4}$$

$$= 0.042612 \times [out\,h_2 (1 - out\,h_2) \times i_2$$

$$= 0.042612 \times 0.238538$$

$$\frac{\partial E_{tot}}{\partial w_4} = 0.0020329$$

$$w_4^* = w_4 - \alpha \frac{\partial E_{tot}}{\partial w_4}$$

$$\boxed{w_4^* = 0.29898}$$

Again, Applying Forward Pass:

$$net(h_1) = w_1 i_1 + w_2 i_2 + b_1$$

$$= 0.404706$$

$$out\,h_1 = \frac{1}{1 + e^{-net(h_1)}} = 0.5998179 =$$

DATE 2 0 1

$$net\,h_2 = W_3 i_1 + W_4 i_2 + b_1$$
$$= 0.434741$$

$$out\,h_2 = \frac{1}{1+e^{-net\,h_2}} = 0.6070061$$

$$net\,O_1 = W_5 \cdot out(h_1) + W_6 (out\,h_2) + b_2$$
$$= \cancel{6.46773849}\ 1.06883919$$
$$=$$

$$out(O_1) = \frac{1}{1+e^{-net\,O_1}} = 0.74437609$$

$$net\,O_2 = W_7 (out\,h_1) + W_8 (out\,h_2) + b_2$$
$$= 1.241742698$$
$$=$$

$$out(O_2) = \frac{1}{1+e^{-O_2}} = 0.7758672$$

Error Calculation

$$EO_1 = \frac{1}{2}\left[T_{O_1} - Out\,O_1\right]^2 = 0.2076102727$$

$$EO_2 = \frac{1}{2}\left[0.9 - out\,O_2\right]^2 = 0.00770447$$

$$Tot.\ Error = EO_1 + EO_2$$
$$Tot.\ Error = 0.2153147 5908$$