Empirical Finance – ETF Performance Analysis

We use R as a tool for empirical analysis and Microsoft Excel to organize and analyze the collected data, as well as to create graphical representations of the results. All Excel files and figures used in the analysis are provided along with this document in the email. In addition, the complete R script and corresponding R outputs are included in the PDF file attached by email, which will make the verification of the results and replication of the analysis easier if necessary.

We select three ETF and present them briefly (what do they represent, composition, currency, localization, …).

MSCI World UCITS ETF (Market ETF):

The Market ETF we chose is the MSCI World UCITS ETF. The latter tracks the MSCI World Index, which represents large and mid-capitalization stocks in developed nations worldwide. It is highly diversified both geographically and sectorally, comprising companies from multiple countries and sectors. We use this ETF in our study as a proxy for the global equity market, and it is denominated in euros and traded on European stock exchanges.

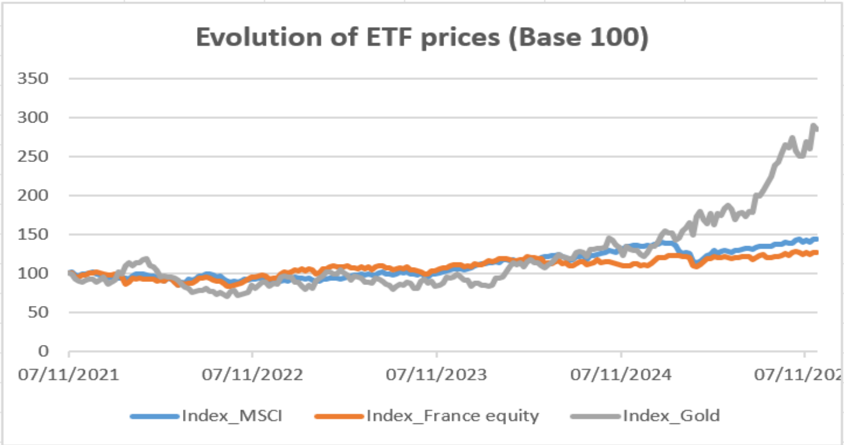
France Equity UCITS ETF (Narrow equity ETF):

The first narrow ETF we use in our assignment, which provides country-specific economic and financial information, is the France Equity UCITS ETF. The latter tracks a French equities index that concentrates on the performance of firms in France and provides exposure to a single national stock market. It invests primarily in large and medium-sized firms such as the CAC 40. It is denominated in euros and traded on European exchanges.

Gold UCITS ETF (Alternative asset ETF):

The second narrow ETF we chose in our assignment is the Gold UCITS ETF which gives information about gold price and provides an alternative asset class rather than equities. Its viability is determined by changes in gold prices, and its composition is connected to actual gold or gold-related contracts. ETF is traded on European markets and has a euro value. Particularly in times of financial market stress, gold is frequently seen as a safe-haven or diversification asset.

We graph the evolution of your three ETF within the same Graph.



In this question, we use a base 100 index to present the evolution of the three ETFs where all the prices are normalized to 100 at the beginning. We then compare their relative performance over the years. The market ETF shows a steady growth tendency, demonstrating its high level of diversification across countries and sectors. Given its exposure to a single national market, the France Equity ETF's growth has been quite moderate. Gold ETF highlights its potential as a source of diversification and an alternate asset with a different and more erratic trajectory.

We display the descriptive statistics of the returns of our ETFs. We choose between arithmetic and geometric returns.

We use arithmetic returns because they are suited for descriptive statistics and regression-based performance analysis and appropriate for weekly data. Geometric returns are more pertinent for long-term wealth building, but they are not necessary in this empirical framework.

The arithmetic returns formula is: $R_t = \frac{P_t - P_{t-1}}{P_{t-1}}$ with $P_t: price\ of\ the\ asset\ at\ t$ and $P_{t-1}: price\ of\ the\ asset\ at\ t-1$.

We can directly observe these returns, and they correspond to the one-period earnings of an investor holding the asset over a single week.

ETF	Mean	Median	Min	Max	Std. Dev.	Skewness	Excess Kurtosis	P5	P95
MSCI World	0.001912	0.002947	-0.08932	0.049227	0.019459	-0.635521	2.101434	-0.03100	0.032449
France Equity	0.001362	0.002176	-0.10122	0.058158	0.022539	-0.706354	2.026537	-0.03581	0.036369
Gold	0.006136	0.001500	-0.09765	0.160322	0.048904	0.333747	-0.05778	-0.06969	0.099373

From the table above, we see clear differences across ETFs. The Gold ETF has the highest mean (0,61%) but the most volatility (4,89%), demonstrating a high level of risk. Since the France Equity ETF employs a single-country approach, which is less diversified than a global market portfolio, it is more volatile (2.22%) than the MSCI World ETF (1.94%). This is confirmed by the extreme values. Gold shows the largest weekly gain (16%) and large losses (9,76%), demonstrating the significant tail risk. The France equity ETF also shows the presence of downside risk in equity markets (minimum weekly losses 10%).

In addition to the typical descriptive statistics, we also report skewness and excess kurtosis to know the asymmetry of the returns. The two equity ETFs show negative skewness, demonstrating that large negative returns happen more frequently than large positive returns. Their positive excess kurtosis indicates leptokurtic distributions, meaning that we should expect more extreme events than a normal distribution. On the other hand, gold returns show a distribution close to normality with a slightly positive skewness and excess kurtosis close to 0. These results are consistent with the stylized facts of financial returns.

<u>We test the normality hypothesis.</u>

Null hypothesis (H_0): Returns are normally distributed, implying zero skewness and zero excess kurtosis.

Alternative hypothesis (H_1): Returns are not normally distributed (asymmetry and excess kurtosis).

ETF	**JB Statistic**	**Degrees of Freedom**	**p-value**	**Normality (5%)**
MSCI World	53.279	2	2.70×10^{-12}	Rejected
France Equity	53.906	2	1.97×10^{-12}	Rejected
Gold	3.965	2	0.1377	Not rejected

To test the normality of returns we use the Jarque–Berra test. For the MSCI World and France Equity ETFs, the p-values are very low, so we reject the null hypothesis, and we conclude that they have asymmetric and leptokurtic return distributions. For Gold, the p-value is 0,1377 so we do not reject the null hypothesis (distribution is very close to normality). This confirms what we found in the previous question.

<u>We calculate and display the correlation matrix of your three ETFs.</u>

	MSCI World	France Equity
MSCI World	1.000	0.653
France Equity	0.653	1.000

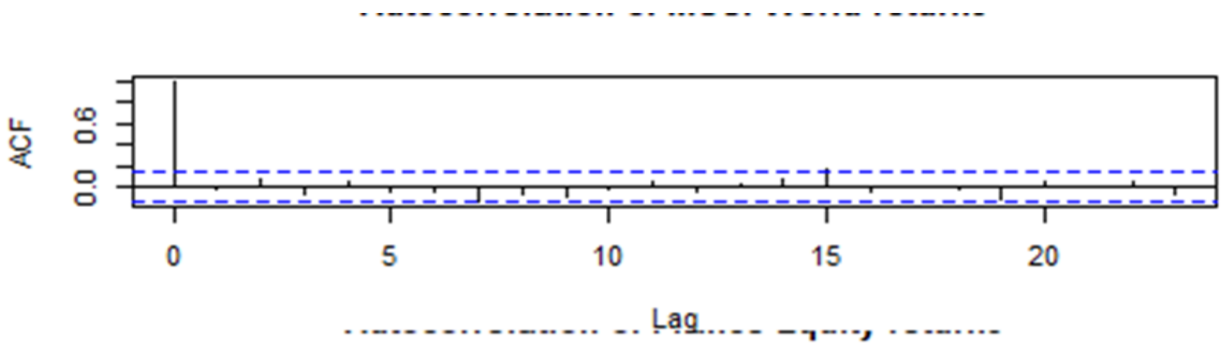
The correlation matrix above demonstrates a strong positive correlation between the MSCI World and France Equity ETFs. On the other hand, gold has a low correlation with the other two ETFs. These findings indicate that we have a high degree of integration between national and global equity markets and that gold can be used as an alternative asset when there's financial market uncertainty.

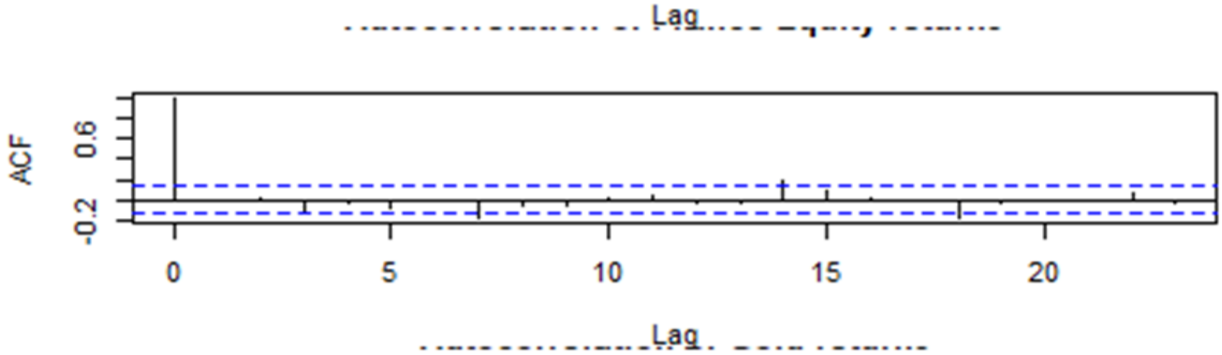
<u>We calculate and graph the autocorrelation of the ETFs.</u>

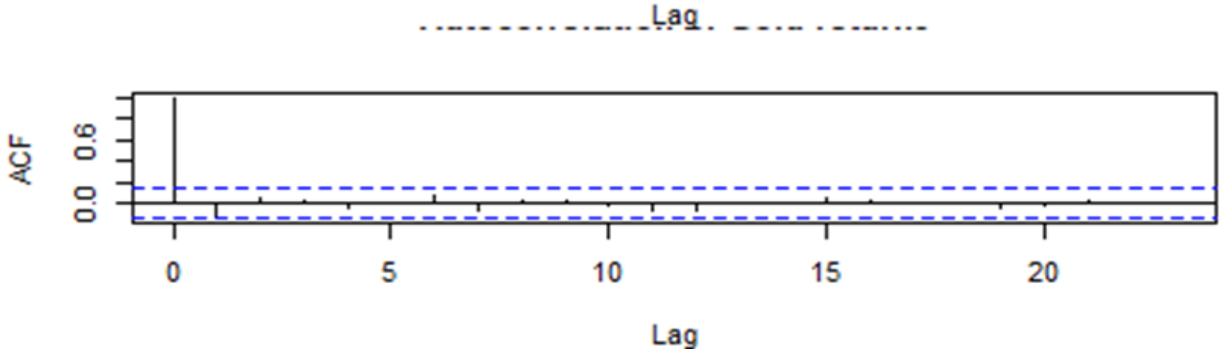
To capture short and medium term serial dependence, we compute autocorrelation up to lag 20 for each ETF. The findings show that we have small coefficients, and we don't have a persistent pattern across lags. This indicates that returns are behaving approximately as white noise. Past returns do not give information about future returns.

Lag	MSCI World	France Equity	Gold
1	-0.011	-0.012	-0.115
2	0.079	0.018	0.045
3	-0.081	-0.126	0.023
4	0.055	-0.035	-0.038
5	-0.039	-0.078	0.005
6	-0.042	0.003	0.064
7	-0.138	-0.167	-0.078
8	-0.070	-0.043	0.038
9	-0.090	-0.040	0.031
10	-0.031	0.030	-0.019
11	0.043	0.056	-0.064
12	-0.043	-0.031	-0.076
13	0.029	-0.017	0.015
14	0.073	0.197	-0.001

15	0.166	0.099	0.046
16	-0.040	0.022	0.028
17	0.000	-0.010	-0.005
18	-0.034	-0.170	-0.002
19	-0.117	-0.027	-0.040
20	0.046	0.004	-0.017







<u>We estimate the Jensen's equations of our "narrow" ETF and compare them to the market (large ETF).</u>

We estimated the Jensen's equations by : $R_{i,t} = \alpha_i + \beta_i \times R_{M,t} + \varepsilon_t$
with

- R_M = MSCI World (market ETF)
- R_i = narrow ETF (France or Gold narrow equity)

If we want to compare the equations to the market, we should compare β to 1 (market exposure), compare α to 0 (abnormal performance) and compare R^2 (degree of co-movement with the market).

<u>For France equity ETF vs Market (MSCI World)</u>

The estimated Jensen equation is : $\mathbf{R_{France,t} = -0.000083 + 0.756\ R_{M,t}}$

Min	1Q	Median	3Q	Max
-0.099598	-0.008384	-0.000988	0.010376	0.048414

	Estimate	Std. Error	t value	Pr(>|t|)	Significance
Intercept	-0.0000832	0.0011813	-0.07	0.944	n.s
Market ex	0.7559236	0.0605574	12.48	<2e-16 ***	***

Significance codes:

*** p < 0.001 ** p < 0.01 * p < 0.05 n.s. : non significant

```
Residual standard error: 0.01712 on 210 degrees of freedom
Multiple R-squared:  0.4259,    Adjusted R-squared:  0.4232
F-statistic: 155.8 on 1 and 210 DF,  p-value: < 2.2e-16
```

Market exposure (β) : We have β = 0.756, which means that France equity has strong exposure to global equity movements and is less volatile than the global market since β < 1.

Abnormal performance (α) : France equity's performance is entirely explained by its market exposure because α = 0 and not significant (p = 0.944) indicates no indication of abnormal performance in comparison to the MSCI World.

We have R^2 = 0.43, which means that about 43% of France Equity return variation is explained by global market movements.

This demonstrates strong integration with global equity markets: France equity behaves like a scaled-down version of the global equity market, with significant but less-than-one market exposure and no abnormal performance.

<u>For Gold ETF vs Market (MSCI World) :</u>

The estimated Jensen equation is : $\mathbf{R_{Gold,t} = 0.0051+0.559\ R_{M,t}}$

Min	1Q	Median	3Q	Max
-0.116702	-0.032807	-0.005731	0.031993	0.163212

	Estimate	Std. Error	t value	Pr(>|t|)	Significance
Intercept	0.005066	0.003298	1.536	0.1260	n.s
Market ex	0.559420	0.169071	3.309	0.0011	**

Significance codes:

*** p < 0.001 ** p < 0.01 * p < 0.05 n.s. : non-significant

```
Residual standard error: 0.04779 on 210 degrees of freedom
Multiple R-squared:  0.04955,   Adjusted R-squared:  0.04502
F-statistic: 10.95 on 1 and 210 DF,  p-value: 0.001103
```

Market exposure (β) : We have β =0.559, which means that Gold has positive but limited exposure to equity markets. This confirms gold is not an equity-like asset.

Abnormal performance (α) : We have 0.005, not significant (p = 0.126), which shows no statistically significant abnormal performance.

We have $R^2 = 0.05$, meaning only 5% of gold return variation is explained by the equity market. This explains gold returns, which are driven by other factors (inflation, uncertainty, monetary policy).

All things considered, gold offers diversification and has very little reliance on changes in the global equity market.

<u>We test the reliability of the coefficients, explain the methods, and the results.</u>

```
> # Robust (HC) t-tests
> coeftest(m_fr, vcov. = vcovHC(m_fr, type = "HC1"))

t test of coefficients:

               Estimate  Std. Error t value Pr(>|t|)
(Intercept) -8.3197e-05  1.1897e-03 -0.0699   0.9443
M_ex         7.5592e-01  6.6003e-02 11.4529   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

> coeftest(m_au, vcov. = vcovHC(m_au, type = "HC1"))

t test of coefficients:

             Estimate Std. Error t value Pr(>|t|)
(Intercept) 0.0050661  0.0032844  1.5425 0.124467
M_ex        0.5594203  0.1840329  3.0398 0.002668 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The Jensen regressions show that both narrow ETFs have statistically significant exposure to the global equities market, although the fact that the quantity of this exposure differs greatly. The France equity ETF depends heavily on the market, but gold has a smaller and more limited correlation with market returns. Moreover, there is no indication of aberrant performance in comparison to the MSCI World equity in any scenario, as Jensen's alpha is not statistically significant. The use of heteroskedasticity-robust standard errors confirms the reliability of these results.

<u>We test the stability of the coefficient, explain the method used, and the results.</u>

In order to check for stability, we must check whether the relationship between the narrow ETF and the market is constant over time. For that, we use the Chow test, as it is the standard econometric test for parameter stability when a potential break date is specified.

The alternative hypothesis assumes a structural change, whereas the null hypothesis states that the regression parameters remain constant throughout time. If the null hypothesis is rejected, this means a structural break and instability of the coefficient. Therefore, the sample is split into two subperiods at the midpoint of the observation window (break point). We set the hypotheses:

- **Null hypothesis (H_0):** The regression coefficients are stable over time (no structural break).
- **Alternative hypothesis (H_1):** At least one coefficient (α or β) changes across subperiods.

```
> # For France narrow ETF
> chow_fr <- sctest(F_ex ~ M_ex, type = "Chow", point = break_point, data = df)
> chow_fr

        Chow test

data:  F_ex ~ M_ex
F = 1.9637, p-value = 0.1429

>
> # For Gold narrow ETF
> chow_G <- sctest(G_ex ~ M_ex, type = "Chow", point = break_point, data = df)
> chow_G

        Chow test

data:  G_ex ~ M_ex
F = 1.6249, p-value = 0.1994
```

1. $\boldsymbol{ETF_{ex_t}}$ **~** $\boldsymbol{M_{ex_t}}$ **:** This specifies the regression model being tested:

$$\boldsymbol{ETF_{ex_t} = \alpha_t + \beta \times M_{ex_t} + \varepsilon_t}$$

- ETF_ex is the dependent variable, representing the excess return of the narrow ETF.
- M_ex is the independent variable, representing the market excess return.
- The test evaluates whether the intercept α and slope β remain constant across sub-periods.

2. type = "Chow"

This option shows that we use a Chow test, which is a convenient test when the break date is already known.

- The Chow test compares:
  - a regression estimated on the full sample, and
  - two regressions estimated before and after the break point.
- It tests whether the coefficients are equal across the two sub-samples.

3. point = break_point

This argument indicates where structural break is located in the dataset.

- break_point is the observation index at which the sample is split.
- The model is therefore estimated:
  - once for observations t ≤ break_point,
  - once for observations t > break_point.

Using the Chow test for Jensen’s regression coefficients, we do not reject the null hypothesis for the France Equity ETF (p-value = 0.1429), showing constant coefficients over time. The same thing goes for the Gold ETF; it’s not rejected (p-value = 0.1994). These results show that the market exposure and performance characteristics of both ETFs remain stable across the sample period, supporting the robustness of the Jensen regression results.

Using alternative model-based performance measures, we estimate the performance of our two “narrow” ETFs.

To evaluate the performance of the two narrow ETFs, we use the Sharpe and Treynor ratios. The former ratio uses total return volatility to measure risk-adjusted performance, while the latter ratio evaluates performance with respect to systematic market risk. We use weekly returns and the risk-free rate is set to zero given the data availability.

Sharpe Ratio

$$Sharpe_i = \frac{E\ (R_i - R_f)}{\sigma(R_i)}$$

Treynor Ratio

$$Treynor_i = \frac{E\ (R_i - R_f)}{\beta_i}$$

We use the weekly arithmetic returns, and since the weekly risk-free rate is not available, we set the risk-free return to zero. First, we calculate average weekly returns and return volatilities for each ETF. The formula for the Sharpe ratio is obtained by dividing the mean return by the standard deviation, which helps us measure performance adjusted for total risk. Next, we extract betas from the Jensen regressions previously estimated and the Treynor ratio is computed by scaling the mean return by the estimated beta, which measures performance relative to systematic market risk. Finally, we report on the Sharpe and Treynor ratios in a summary table to make the comparison of risk-adjusted performance across the two ETFs easier.

ETF	**Sharpe**	**Treynor**
France equity	0.0604	0.0018
Gold	0.1255	0.0110

Since the results show that the Gold ETF indicates a higher Sharpe ratio (0.1255) than the France Equity ETF (0.0604), this means that the performance per unit of total risk is superior. Furthermore, the Treynor ratio of gold (0.0018) is significantly higher, which shows a strong performance compared to its low exposure to the equity market. In contrast, the France Equity ETF (0.0110) indicates more modest risk-adjusted performance, consistent with its equity-like behavior and higher market dependence. Overall, these findings show how gold can help spread out risk and match what was found in past results using Jensen’s regressions.

<u>We use measures of performance adjusted to the risk to assess the performance of oour “narrow” ETFs. We compare them to the market.</u>

The same risk-adjusted performance measures are used as in the previous question; to evaluate the performance adjusted to the risk, we again use the Sharpe and Treynor ratio. The Sharpe ratio measures excess returns per unit of total risk, while the Treynor ratio assesses performance compared to systematic market risk. We use weekly returns and the risk-free rate is set to zero due to data availability. In this question, we use the market ETF (MSCI World) as a benchmark to test relative performance.

ETF	**Sharpe**	**Treynor**
Market (MSCI World)	0.0982	0.0019
France equity	0.0604	0.0018
Gold	0.1255	0.0110

From the results, we see that the France Equity ETF exhibits a moderate Sharpe ratio (0.0604), comparable but not greater than that of the market (0.0982). This performance is largely explained by exposure to global equity risk, as confirmed by a significant beta in the previous questions. The Treynor ratio (0.0018) is nearly unchanging compared with equity-like behavior (0.0019), indicating no outperformance relative to the market once market risk is accounted for.

As for the Gold ETF, it shows a higher Sharpe ratio (0.1255), indicating better performance per unit of total risk. Its Treynor ratio (0.0110) is substantially higher, reflecting strong performance relative to very low market exposure. Compared to the market, gold provides returns that are less dependent on equity market fluctuations. We say that Gold outperforms the market on a risk-adjusted basis and offers meaningful diversification benefits relative to the market.