# Methodic and Practical Foundations of Computer Science 1

## 13-Linear and Binary Search

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# Outline

a problem to be solved

**Divide**: Break into non-overlapping subproblems of the same type

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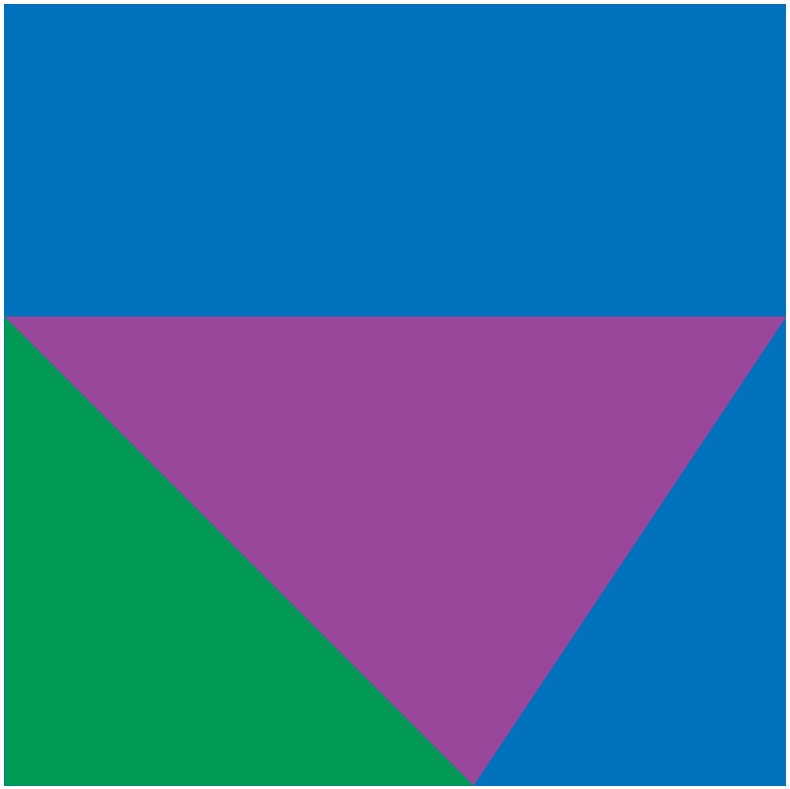
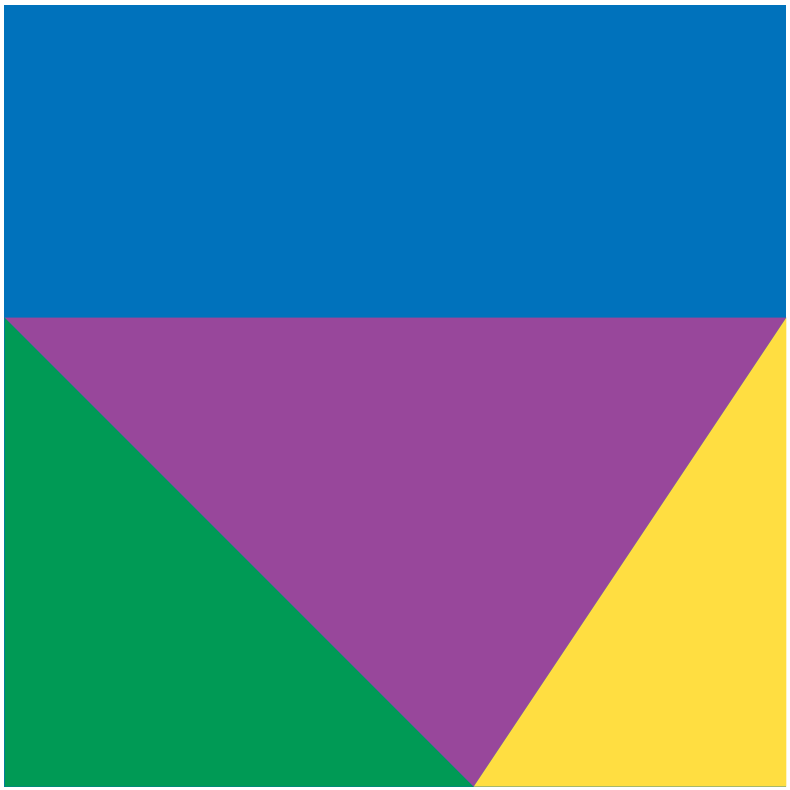
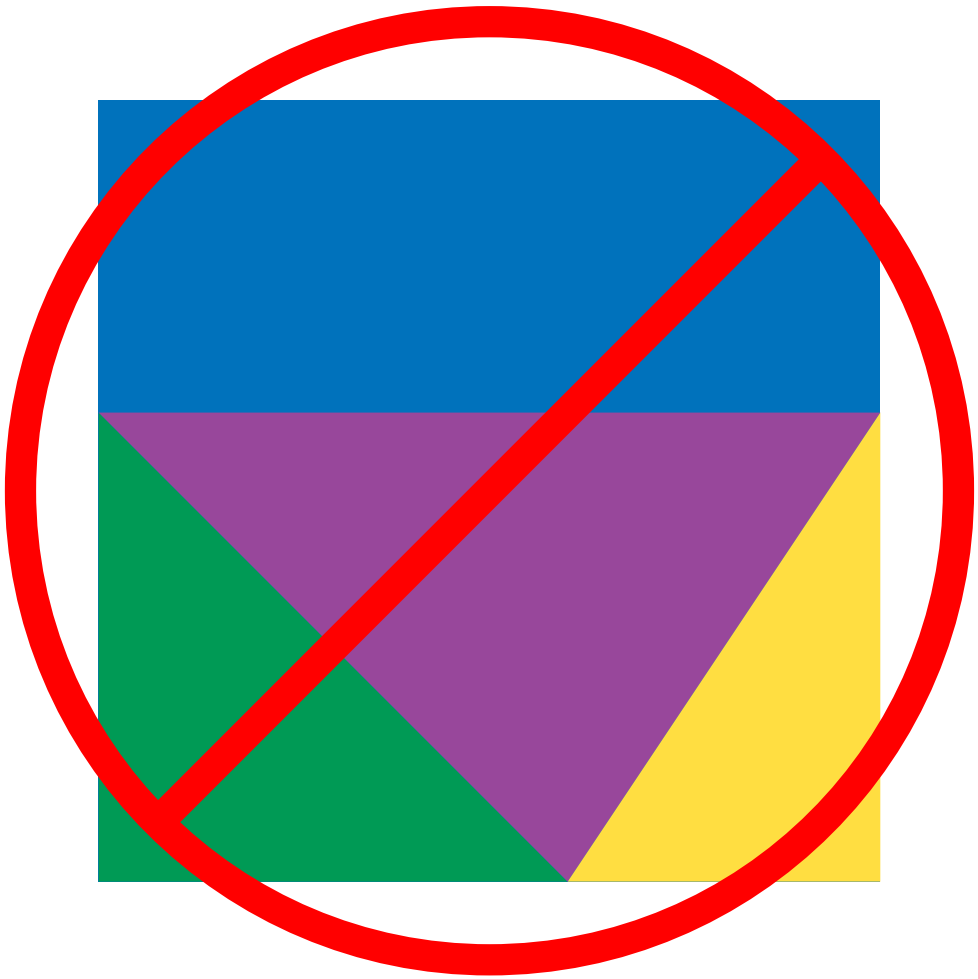
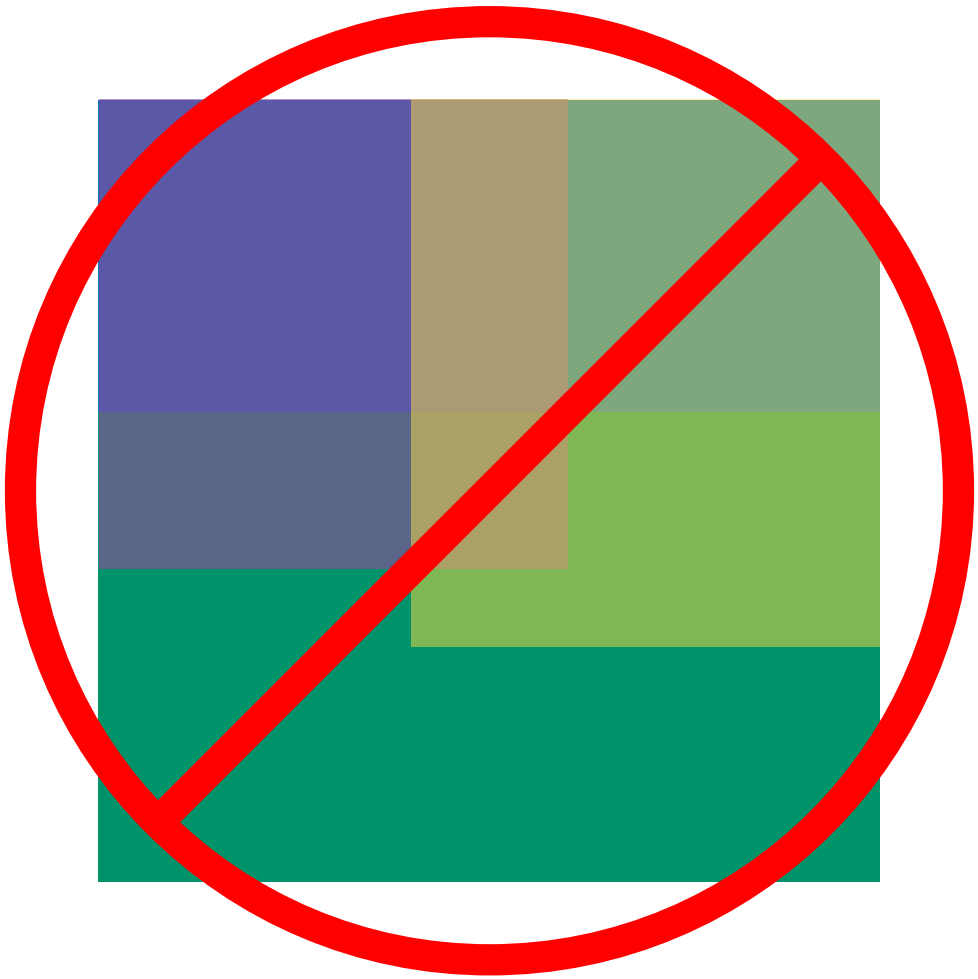






not the
same type



overlapping

# **Divide**: break apart

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# **Conquer**: solve subproblems

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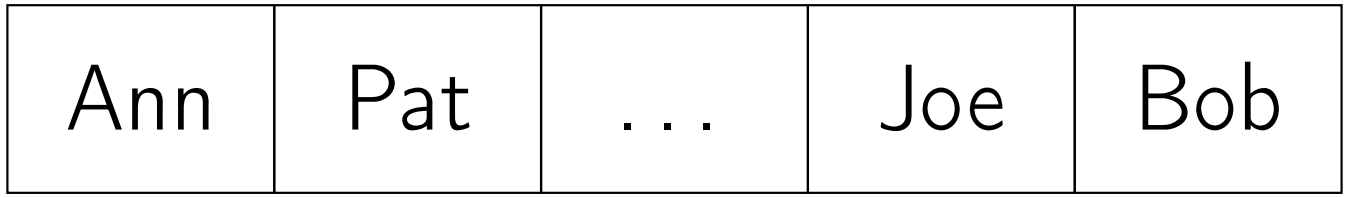
# Conquer: combine

1. Break into non-overlapping subproblems of the same type
2. Solve subproblems
3. Combine results

# Outline

# Linear Search in Array



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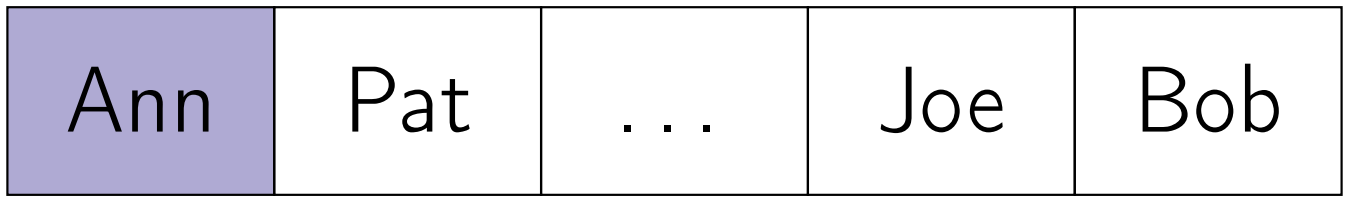

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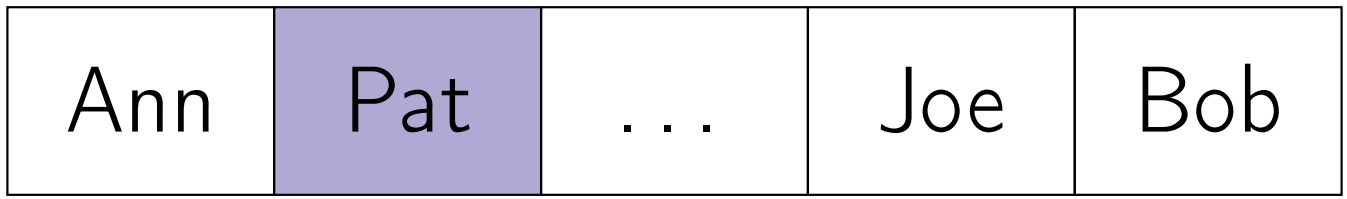

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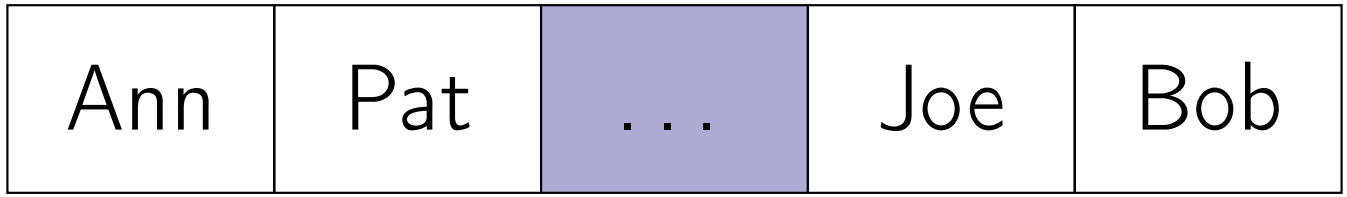

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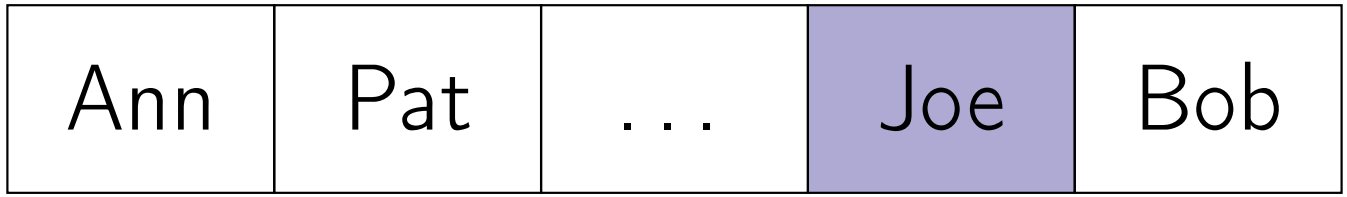

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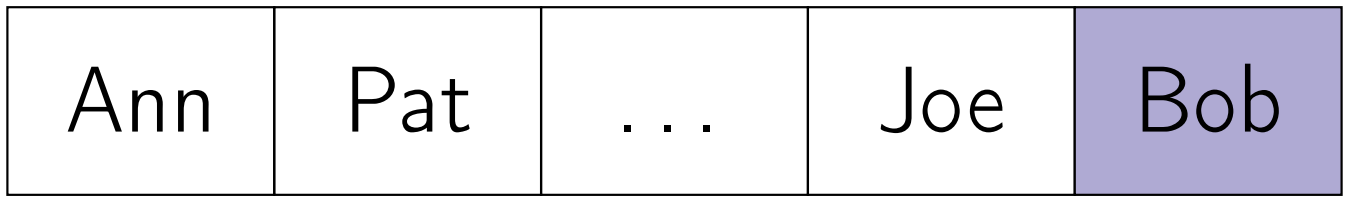

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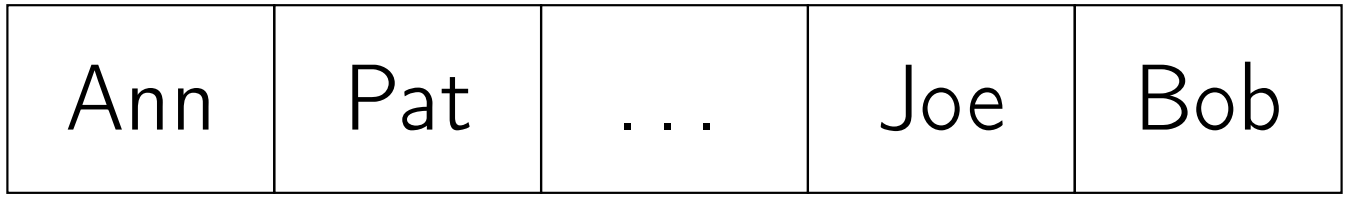

# Real-life Example

| **english** | **french** | **italian** | **german** | **spanish** |
|---|---|---|---|---|
| house | maison | casa | Haus | casa |
| car | voiture | auto | Auto | auto |
| table | table | tavola | Tabelle | mesa |

# Searching in an array

**Input:** An array $A$ with $n$ elements.
A key $k$.

**Output:** An index, $i$, where $A[i] = k$.
If there is no such $i$, then
NOT_FOUND.

# Recursive Solution

`LinearSearch`($A$, *low*, *high*, *key*)

# Recursive Solution

## LinearSearch($A$, $low$, $high$, $key$)

```
if high < low:
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if A[low] = key:
   return low
```

# Recursive Solution

## `LinearSearch`$(A, low, high, key)$

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## Definition

A recurrence relation is an equation recursively defining a sequence of values.

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## Fibonacci recurrence relation

$$F(n) = \begin{cases} 0 & \text{if } n = 0 \\ 1 & \text{if } n = 1 \\ F(n-1) + F(n-2) & \text{if } n > 1 \end{cases}$$

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$$0, 1, 1, 2, 3, 5, 8, \ldots$$

## LinearSearch($A$, $low$, $high$, $key$)

```
if high < low:
  return NOT_FOUND
if A[low] = key:
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return LinearSearch(A, low + 1, high, key)
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## LinearSearch($A, low, high, key$)

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if high < low:
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Recurrence defining worst-case time:

$$T(n) = T(n-1) + c$$

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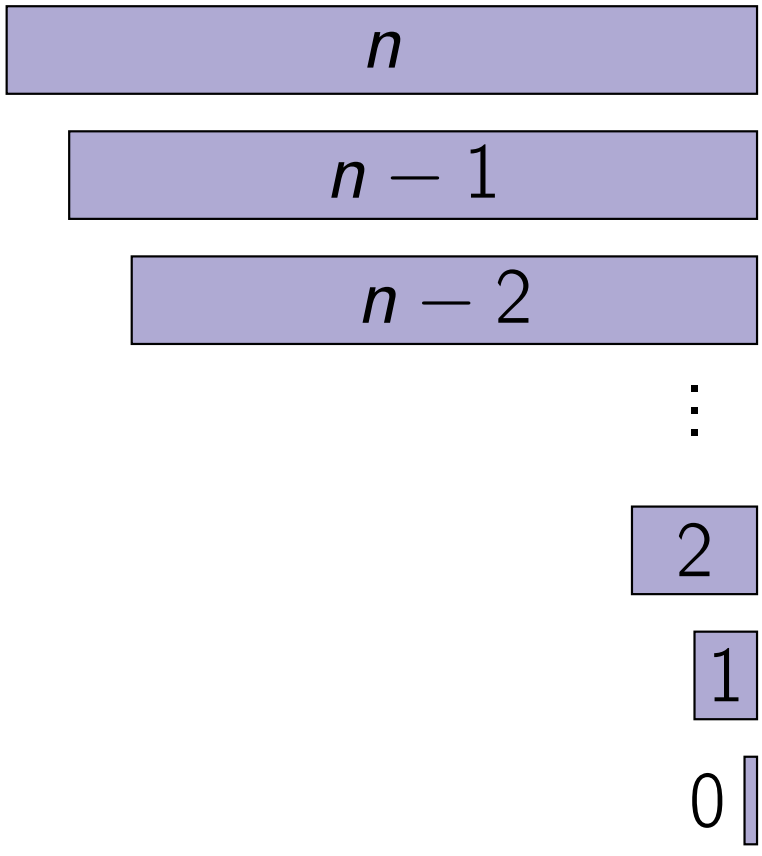
Recurrence defining worst-case time:

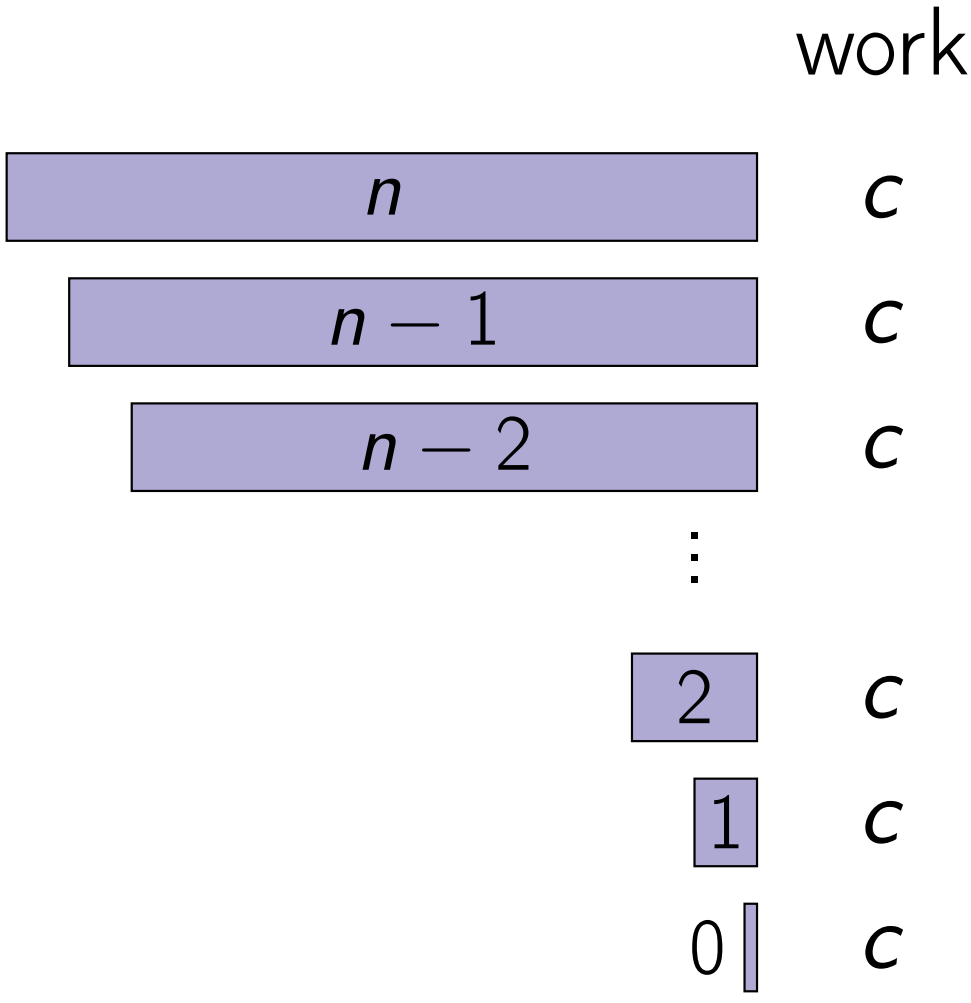
$$T(n) = T(n-1) + c$$

$$T(0) = c$$

# Runtime of Linear Search



# Runtime of Linear Search



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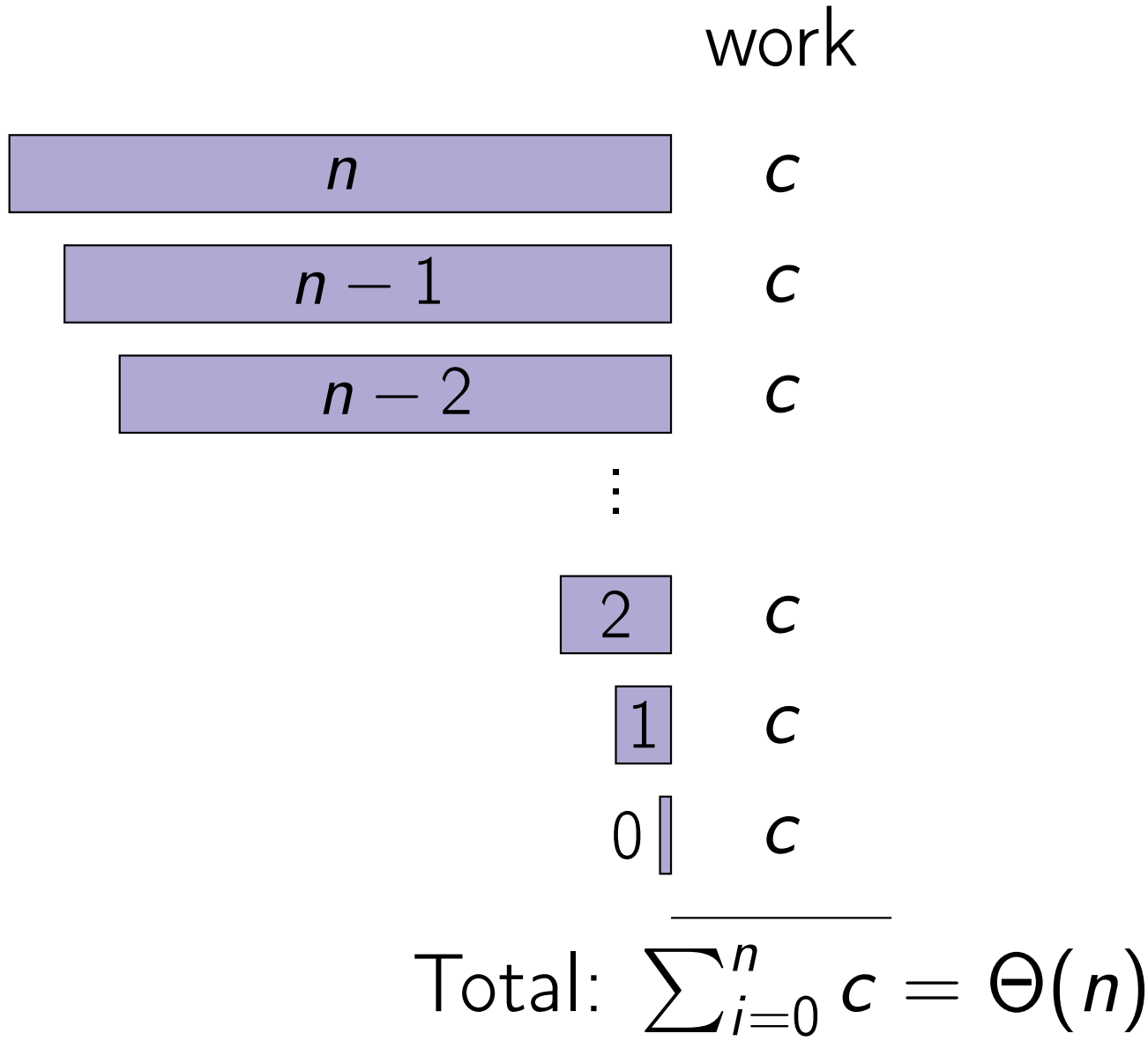

# Iterative Version

```
LinearSearchIt(A, low, high, key)
for i from low to high:
    if A[i] = key:
        return i
return NOT_FOUND
```

# Summary

- Create a recursive solution

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- Define a corresponding recurrence relation, $T$

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- Determine $T(n)$: worst-case runtime

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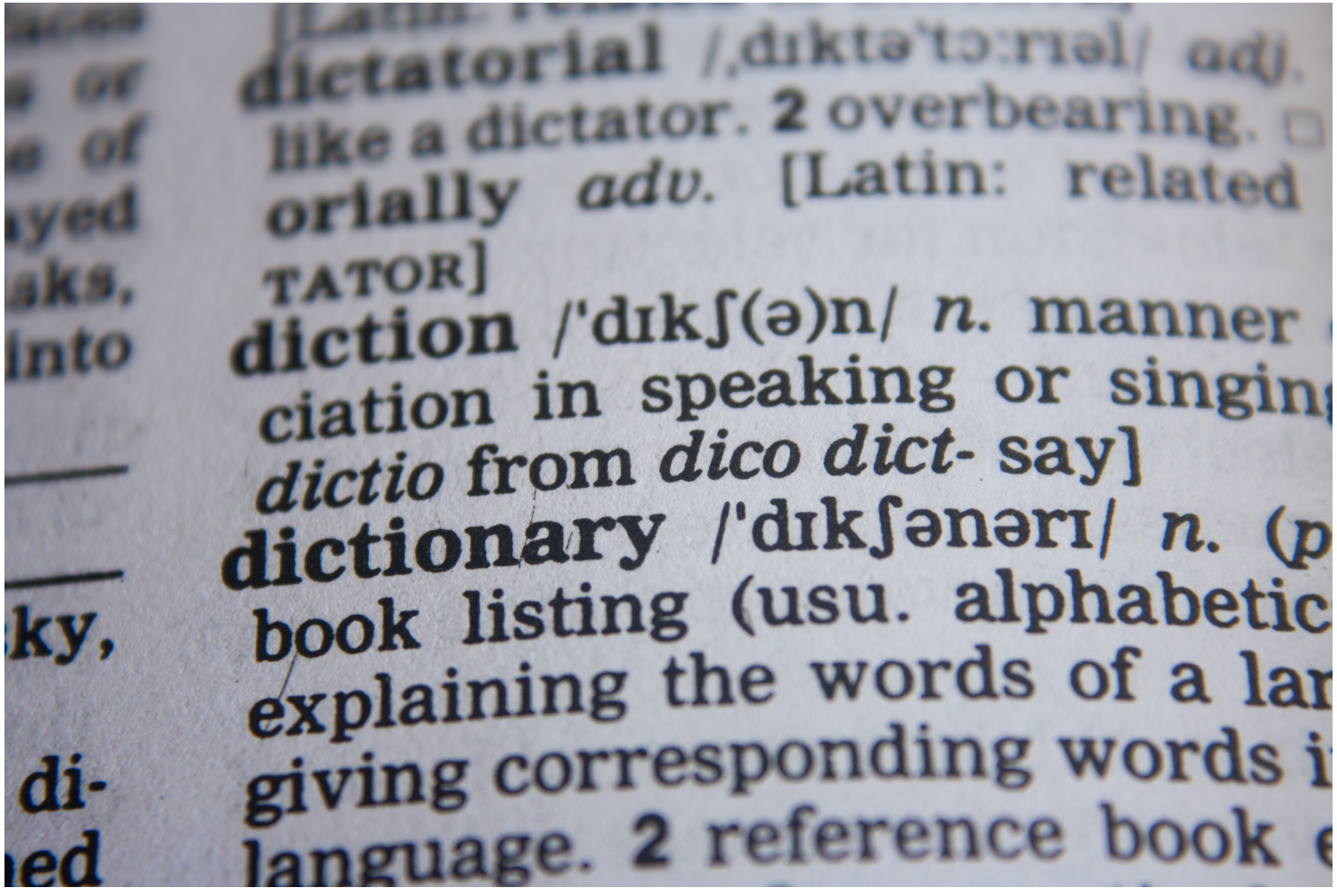
- Create a recursive solution
- Define a corresponding recurrence relation, $T$
- Determine $T(n)$: worst-case runtime
- Optionally, create iterative solution

# Outline

# Searching Sorted Data



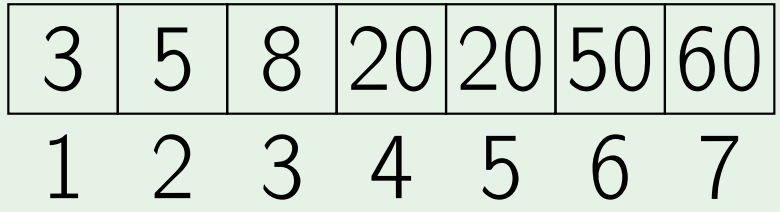
# Searching in a sorted array

**Input:** A sorted array $A[low \ldots high]$
$(\forall low \leq i < high : A[i] \leq A[i+1])$.
A key $k$.

**Output:** An index, $i$, $(low \leq i \leq high)$ where $A[i] = k$.
Otherwise, the greatest index $i$, where $A[i] < k$.
Otherwise $(k < A[low])$, the result is $low - 1$.

# Searching in a Sorted Array

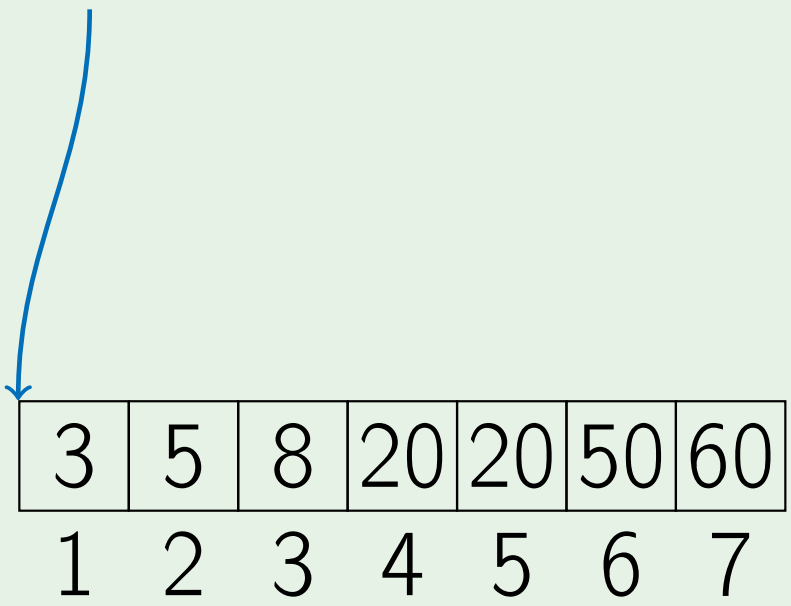
## Example



# Searching in a Sorted Array

## Example

$search(2) \rightarrow 0$

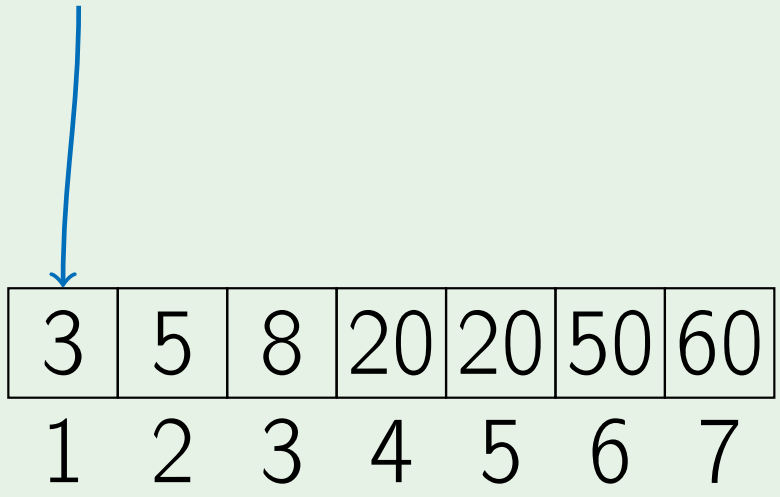


# Searching in a Sorted Array

## Example

$search(2) \rightarrow 0$

$search(3) \rightarrow 1$



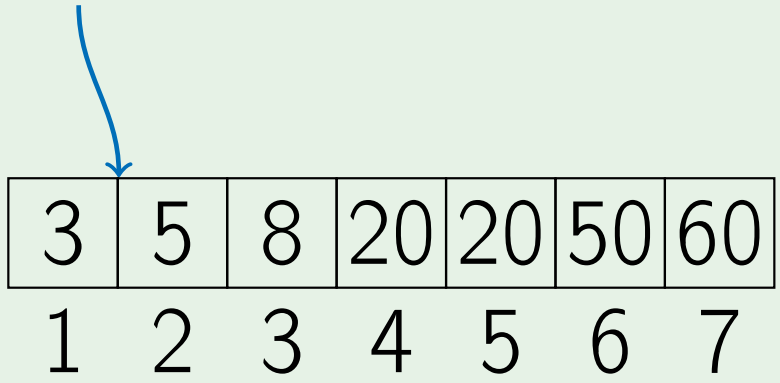
# Searching in a Sorted Array

## Example

$search(2) \rightarrow 0$

$search(3) \rightarrow 1$

$search(4) \rightarrow 1$



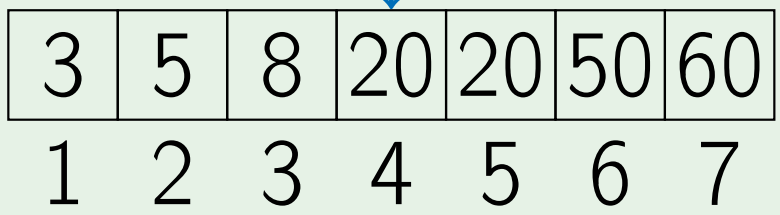
# Searching in a Sorted Array

## Example

$search(2) \rightarrow 0$ $search(20) \rightarrow 4$

$search(3) \rightarrow 1$

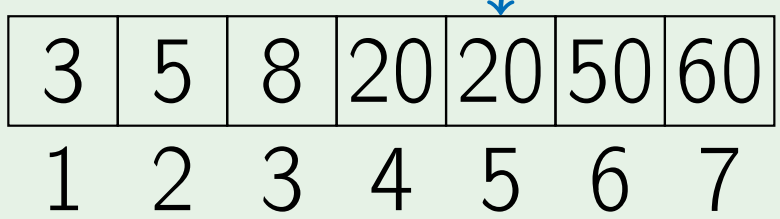
$search(4) \rightarrow 1$



# Searching in a Sorted Array

## Example

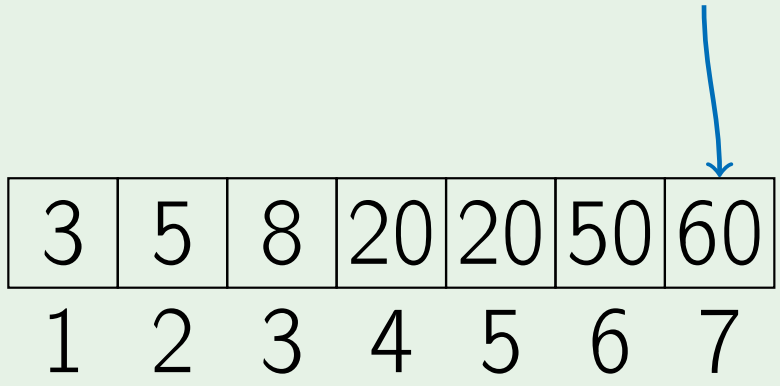
$search(2) \rightarrow 0$ $search(20) \rightarrow 4$
$search(3) \rightarrow 1$ $search(20) \rightarrow 5$
$search(4) \rightarrow 1$



# Searching in a Sorted Array

## Example

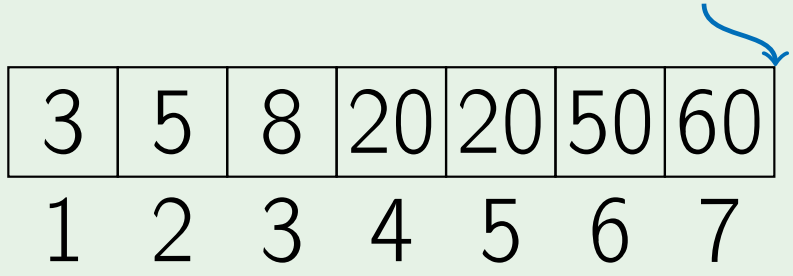
$$\textit{search}(2) \to 0 \quad \textit{search}(20) \to 4$$
$$\textit{search}(3) \to 1 \quad \textit{search}(20) \to 5$$
$$\textit{search}(4) \to 1 \quad \textit{search}(60) \to 7$$



# Searching in a Sorted Array

## Example

$search(2) \rightarrow 0$ $search(20) \rightarrow 4$
$search(3) \rightarrow 1$ $search(20) \rightarrow 5$
$search(4) \rightarrow 1$ $search(60) \rightarrow 7$
$search(90) \rightarrow 7$



# `BinarySearch`($A$, $low$, $high$, $key$)

# BinarySearch($A$, *low*, *high*, *key*)

```
if high < low:
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# BinarySearch($A$, $low$, $high$, $key$)

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$mid \leftarrow \left\lfloor low + \frac{high - low}{2} \right\rfloor$

# BinarySearch($A$, $low$, $high$, $key$)

if $high < low$:
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$mid \leftarrow \left\lfloor low + \frac{high - low}{2} \right\rfloor$
if $key = A[mid]$:
  return $mid$

# BinarySearch($A$, $low$, $high$, $key$)

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if high < low:
    return low − 1
mid ← ⌊low + (high − low)/2⌋
if key = A[mid]:
    return mid
else if key < A[mid]:
    return BinarySearch(A, low, mid − 1, key)
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if high < low:
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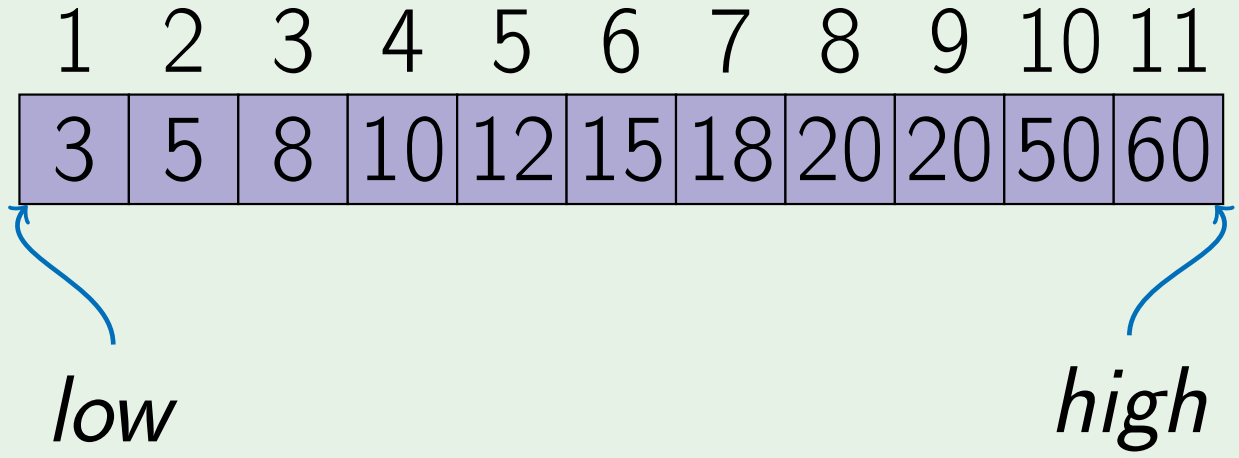
else:
    return BinarySearch(A, mid + 1, high, key)
```

# Example: Searching for the key 50

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|
| 3 | 5 | 8 | 10 | 12 | 15 | 18 | 20 | 20 | 50 | 60 |

# Example: Searching for the key 50

$\texttt{BinarySearch}(A, 1, 11, 50)$



# Example: Searching for the key 50

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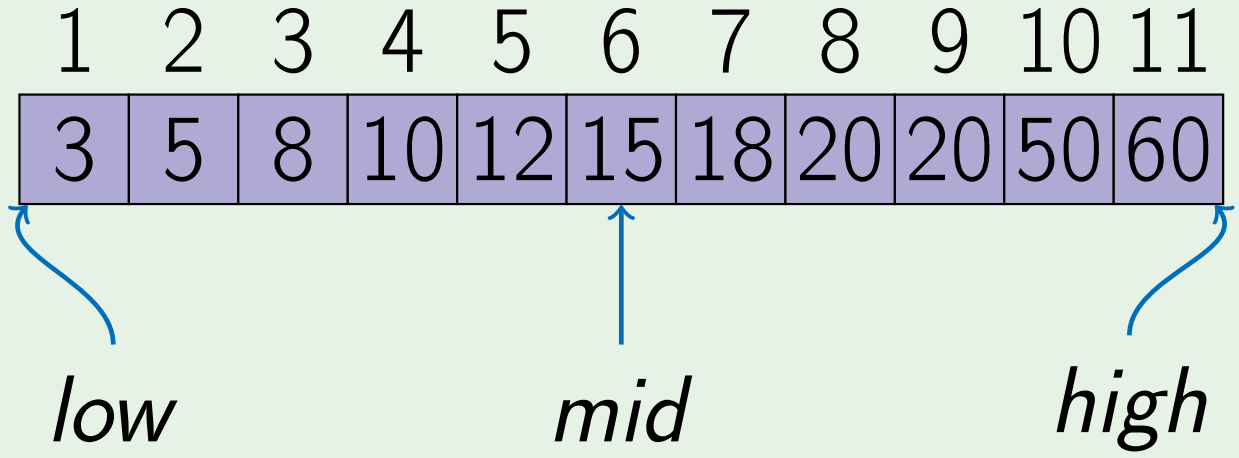
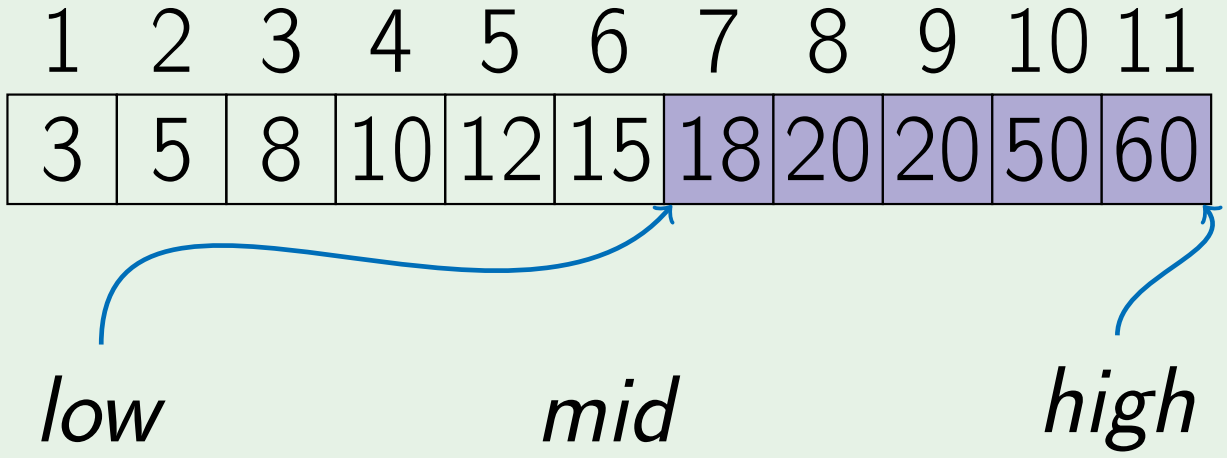


# Example: Searching for the key 50

$\texttt{BinarySearch}(A, 1, 11, 50)$
$\texttt{BinarySearch}(A, 7, 11, 50)$



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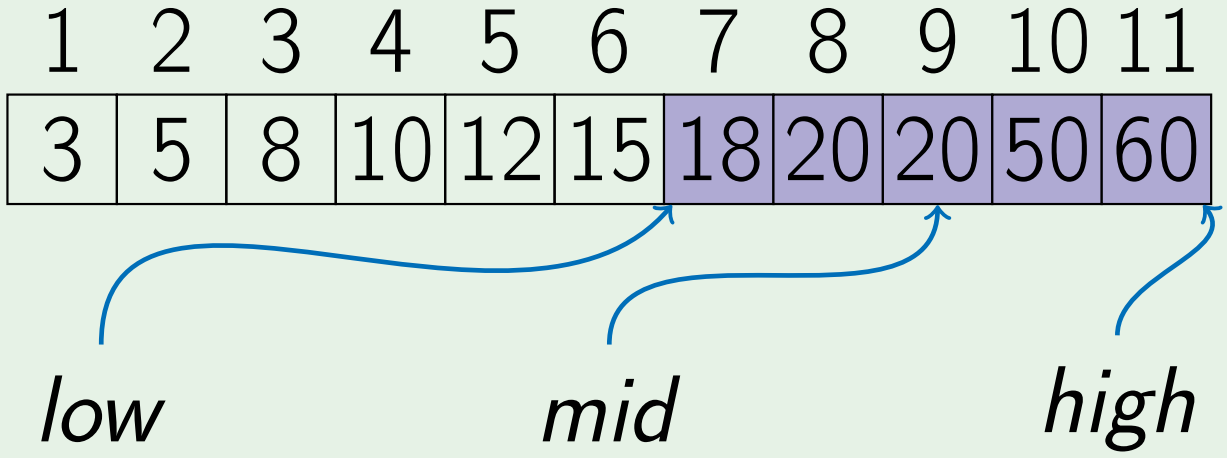

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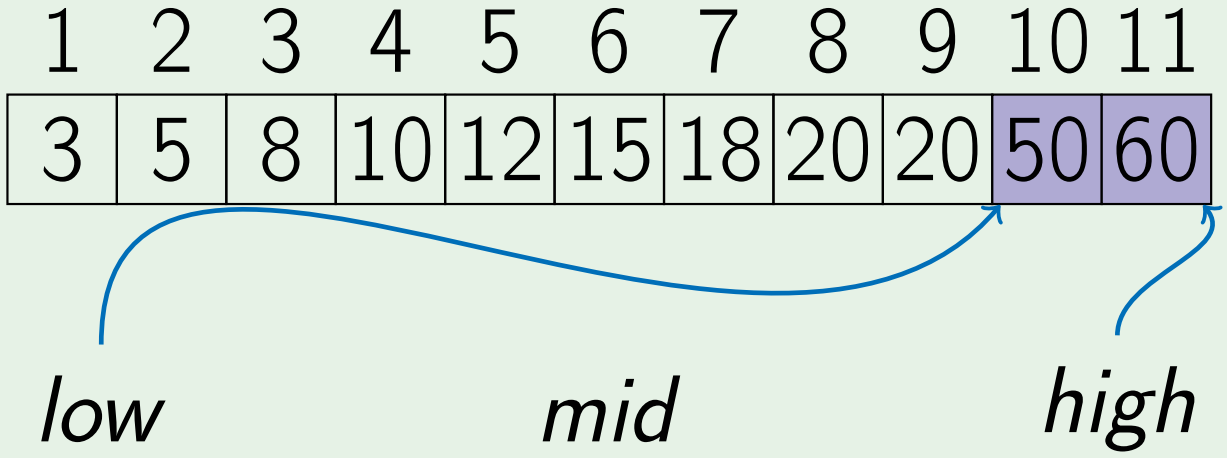
$\texttt{BinarySearch}(A, 1, 11, 50)$
$\texttt{BinarySearch}(A, 7, 11, 50)$
$\texttt{BinarySearch}(A, 10, 11, 50)$



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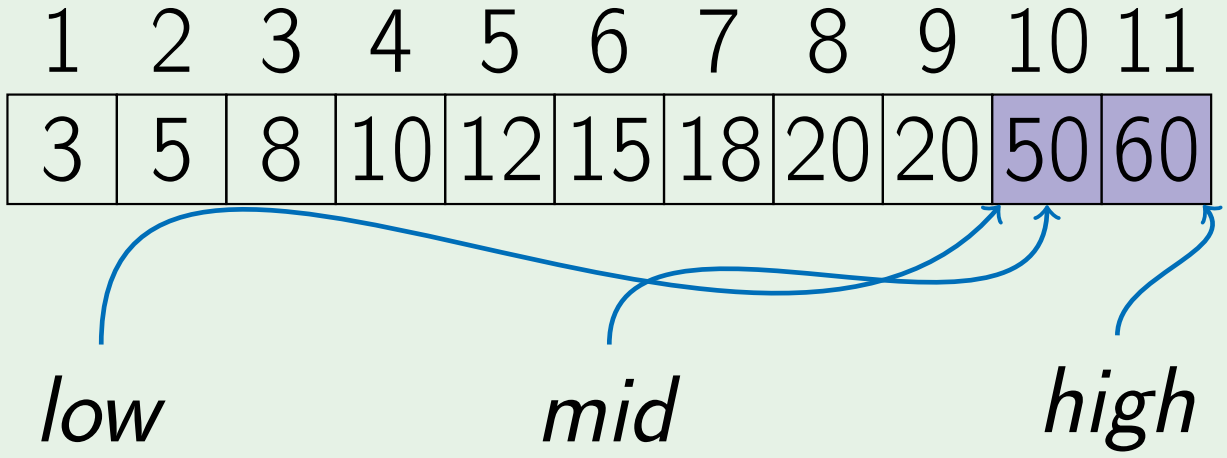

# Example: Searching for the key 50

$\texttt{BinarySearch}(A, 1, 11, 50)$
$\texttt{BinarySearch}(A, 7, 11, 50)$
$\texttt{BinarySearch}(A, 10, 11, 50) \rightarrow 10$

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|
| 3 | 5 | 8 | 10 | 12 | 15 | 18 | 20 | 20 | 50 | 60 |

# Summary

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- Break problem into non-overlapping subproblems of the same type.

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- Recursively solve those subproblems.

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- Break problem into non-overlapping subproblems of the same type.
- Recursively solve those subproblems.
- Combine results of subproblems.

# BinarySearch($A$, $low$, $high$, $key$)

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if high < low:
    return low − 1
mid ← ⌊low + (high − low)/2⌋
if key = A[mid]:
    return mid
else if key < A[mid]:
    return BinarySearch(A, low, mid − 1, key)
else:
    return BinarySearch(A, mid + 1, high, key)
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# Binary Search Recurrence Relation

$$T(n) = T\left(\left\lfloor \frac{n}{2} \right\rfloor\right) + c$$

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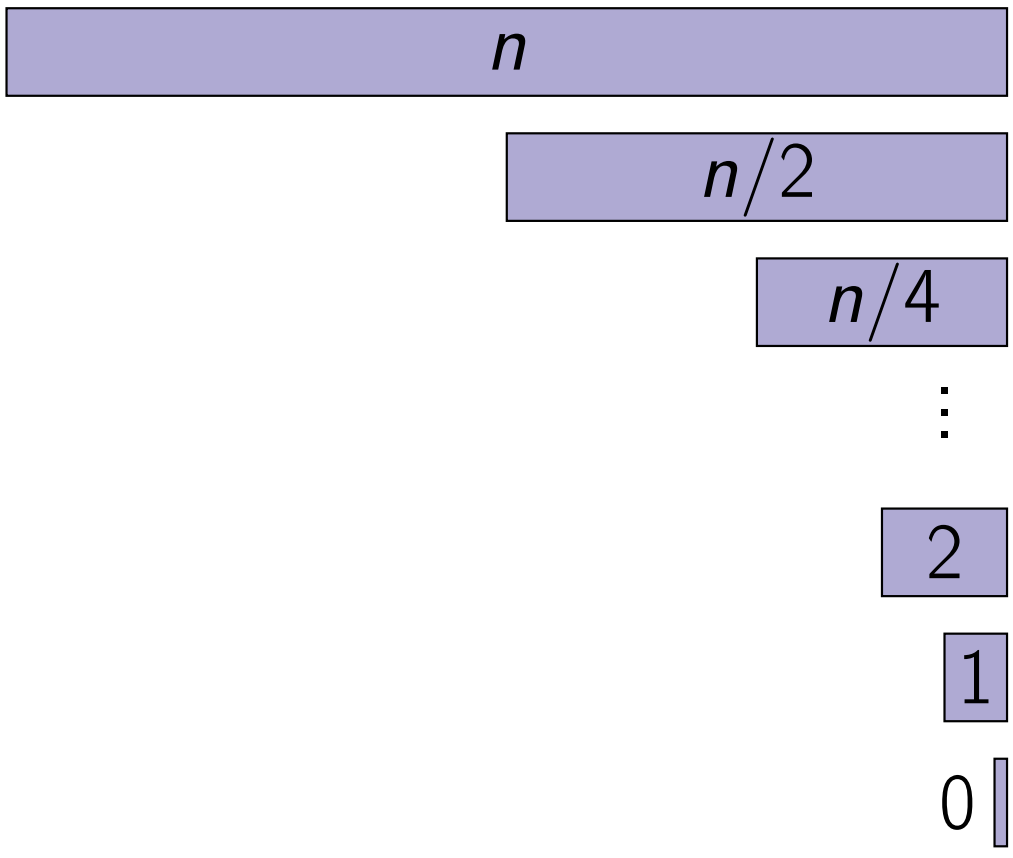
$$T(0) = c$$

# Runtime of Binary Search



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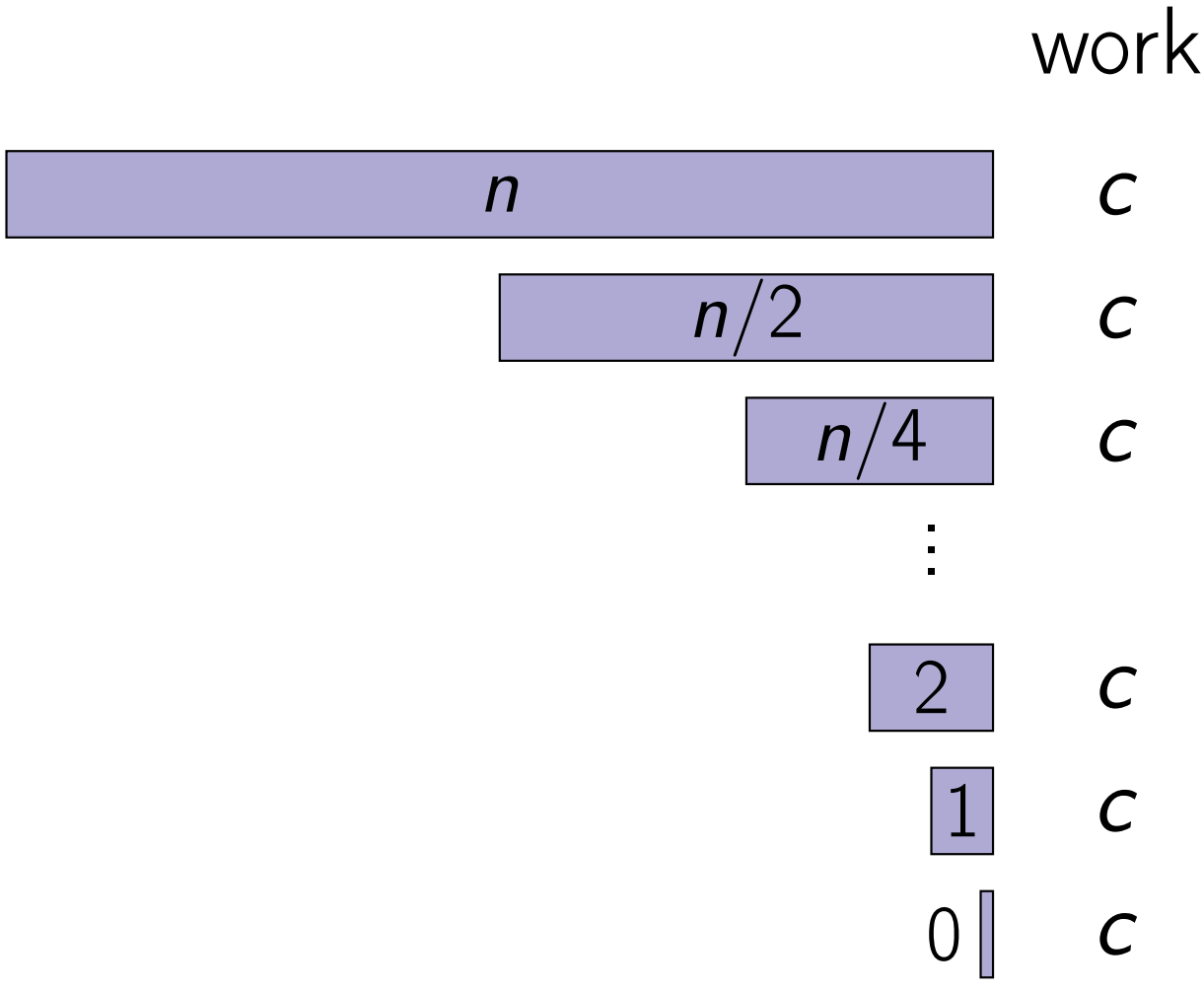

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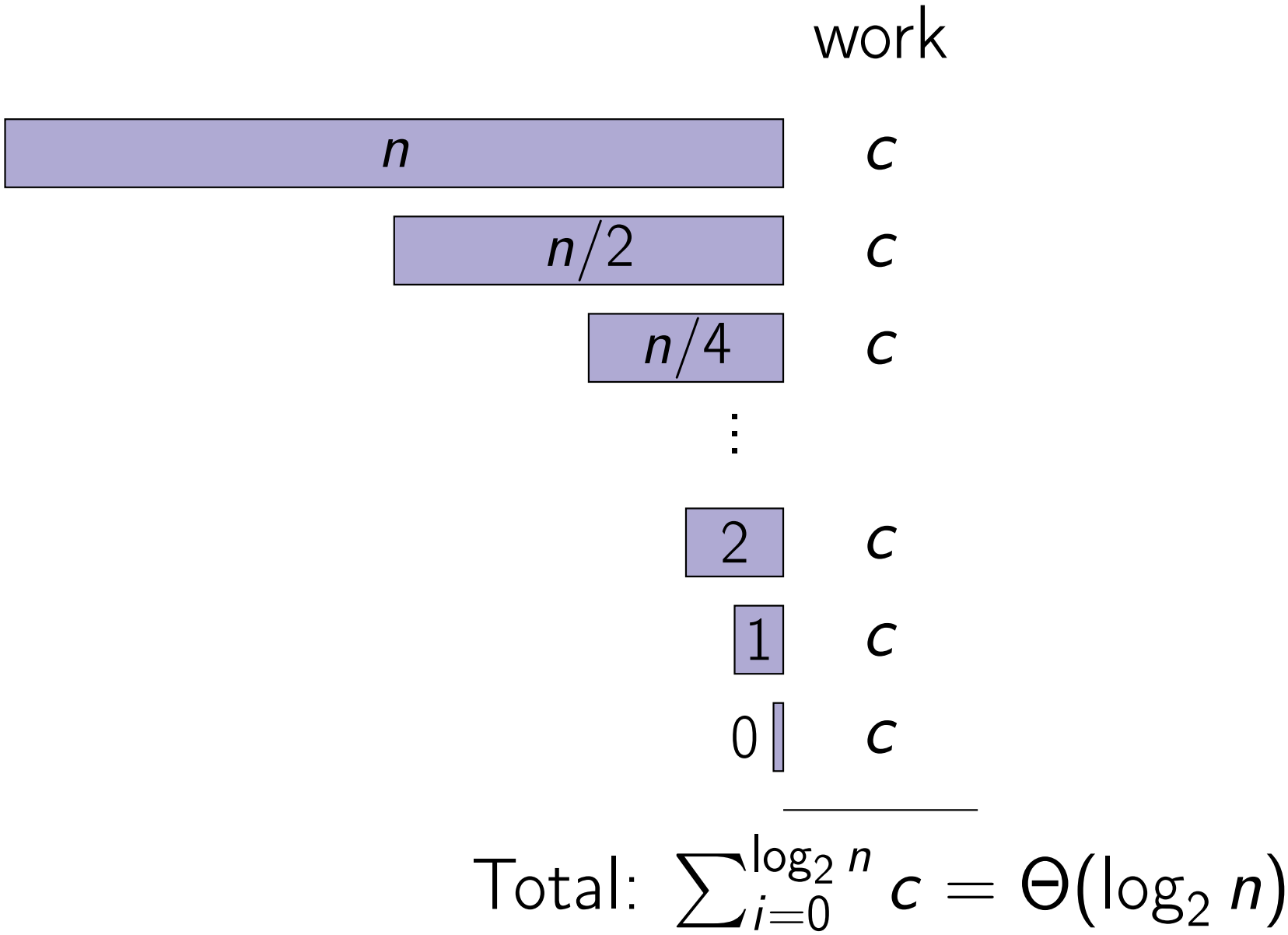


Total: $\sum_{i=0}^{\log_2 n} c = \Theta(\log_2 n)$

# Iterative Version

`BinarySearchIt`$(A, low, high, key)$

`while` $low \leq high$:

$mid \leftarrow \left\lfloor low + \frac{high - low}{2} \right\rfloor$

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while low ≤ high:
    mid ← ⌊low + (high−low)/2⌋
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```

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`BinarySearchIt`$(A, low, high, key)$

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while low ≤ high:
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    else if key < A[mid]:
        high = mid − 1
```

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while low ≤ high:
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    else if key < A[mid]:
        high = mid − 1
    else:
        low = mid + 1
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# Iterative Version

**BinarySearchIt($A$, $low$, $high$, $key$)**

```
while low ≤ high:
    mid ← ⌊low + (high − low)/2⌋
    if key = A[mid]:
        return mid
    else if key < A[mid]:
        high = mid − 1
    else:
        low = mid + 1
return low − 1
```

# Real-life Example

| **english** | **french** | **italian** | **german** | **spanish** |
|---|---|---|---|---|
| house | maison | casa | Haus | casa |
| chair | chaise | sedia | Sessel | silla |
| pimple | bouton | foruncolo | Pickel | espenilla |

# Real-life Example

| **english** (sorted) | **french** (sorted) | **italian** (sorted) | **german** (sorted) | **spanish** (sorted) |
|---|---|---|---|---|
| chair | chaise | casa | Haus | casa |
| house | bouton | foruncolo | Pickel | espenilla |
| pimple | maison | sedia | Sessel | silla |

# Real-life Example

| **english** | **french** | **italian** | **german** | **spanish** |
|---|---|---|---|---|
| house | maison | casa | Haus | casa |
| chair | chaise | sedia | Sessel | silla |
| pimple | bouton | foruncolo | Pickel | espenilla |

**english**
sorted

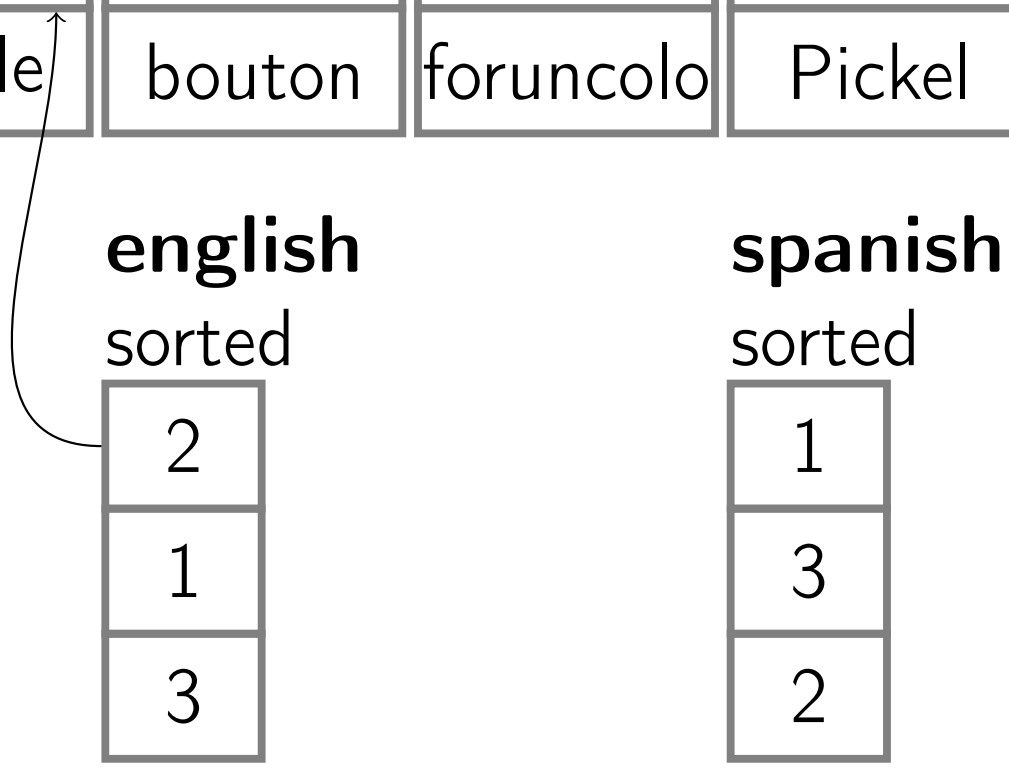
| |
|---|
| 2 |
| 1 |
| 3 |

**spanish**
sorted

| |
|---|
| 1 |
| 3 |
| 2 |

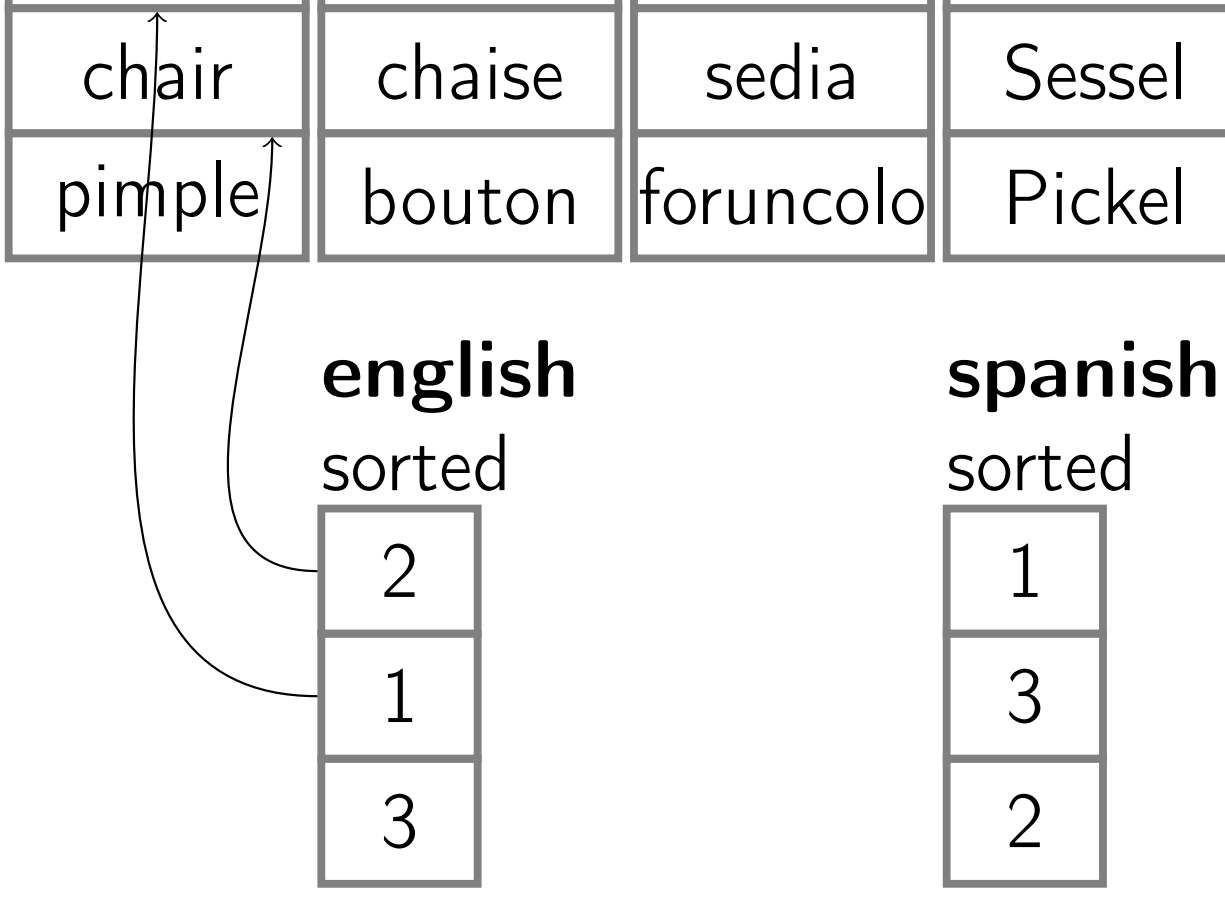
# Real-life Example

| **english** | **french** | **italian** | **german** | **spanish** |
|---|---|---|---|---|
| house | maison | casa | Haus | casa |
| chair | chaise | sedia | Sessel | silla |
| pimple | bouton | foruncolo | Pickel | espenilla |



# Real-life Example

| **english** | **french** | **italian** | **german** | **spanish** |
|---|---|---|---|---|
| house | maison | casa | Haus | casa |
| chair | chaise | sedia | Sessel | silla |
| pimple | bouton | foruncolo | Pickel | espenilla |



# Real-life Example

| **english** | **french** | **italian** | **german** | **spanish** |
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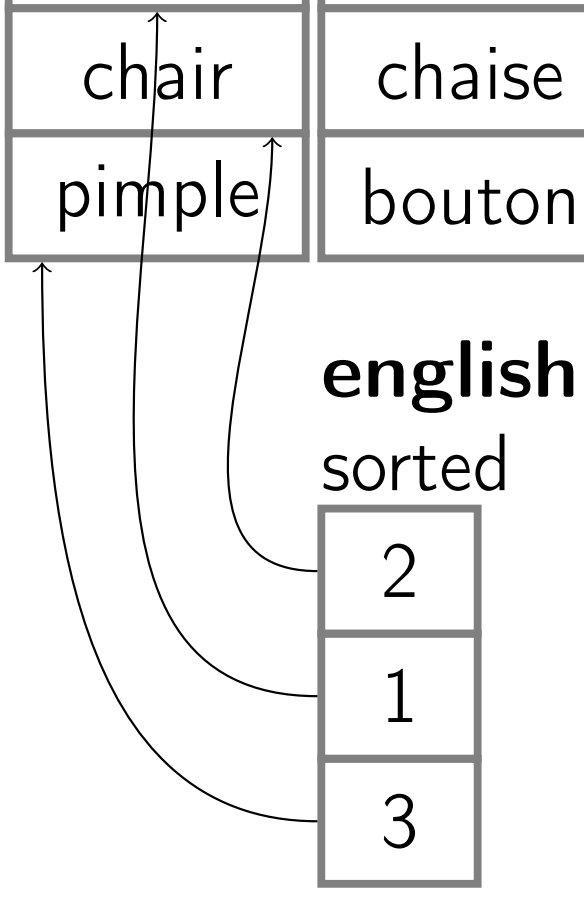


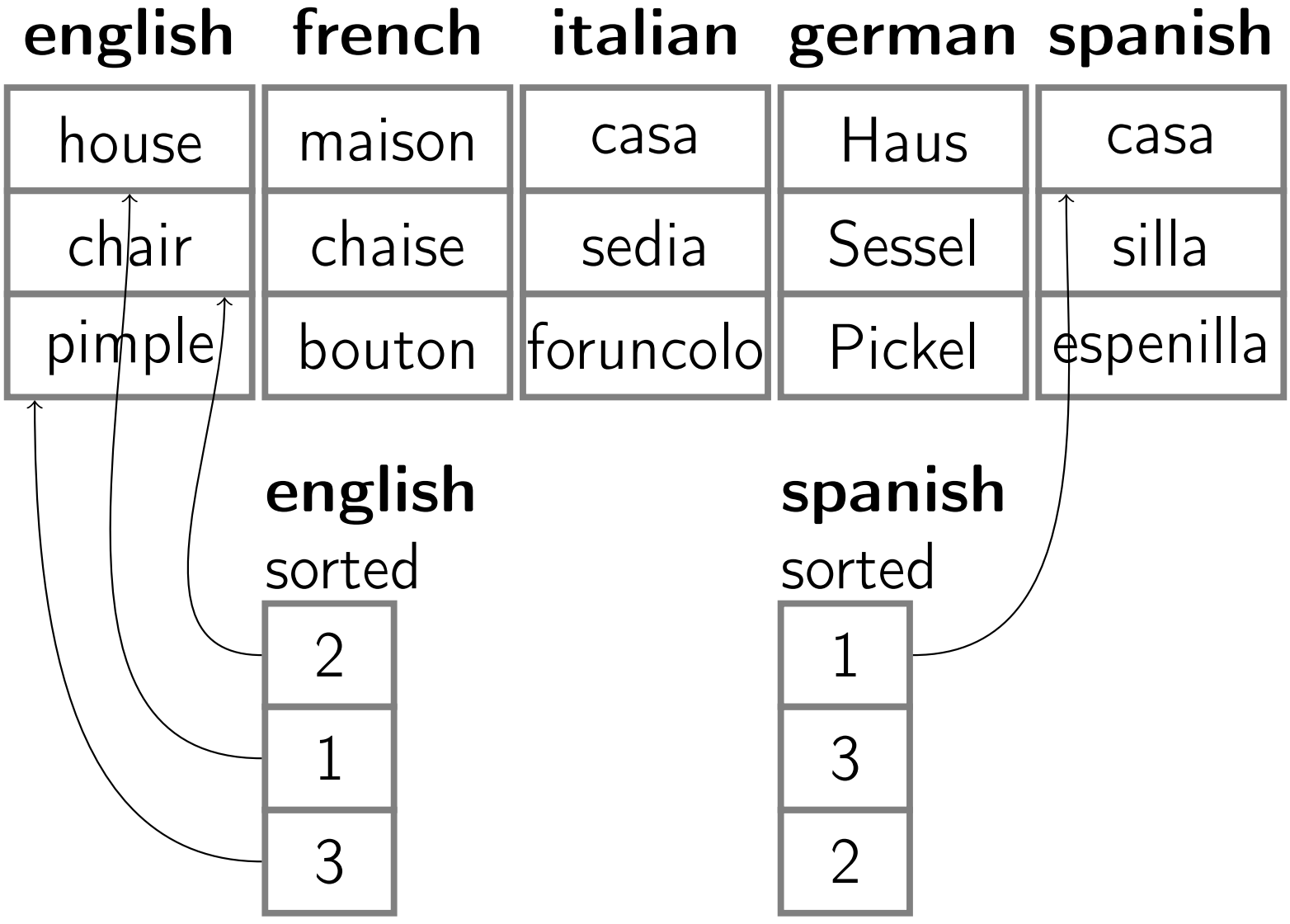
**english**
sorted

| |
|---|
| 2 |
| 1 |
| 3 |

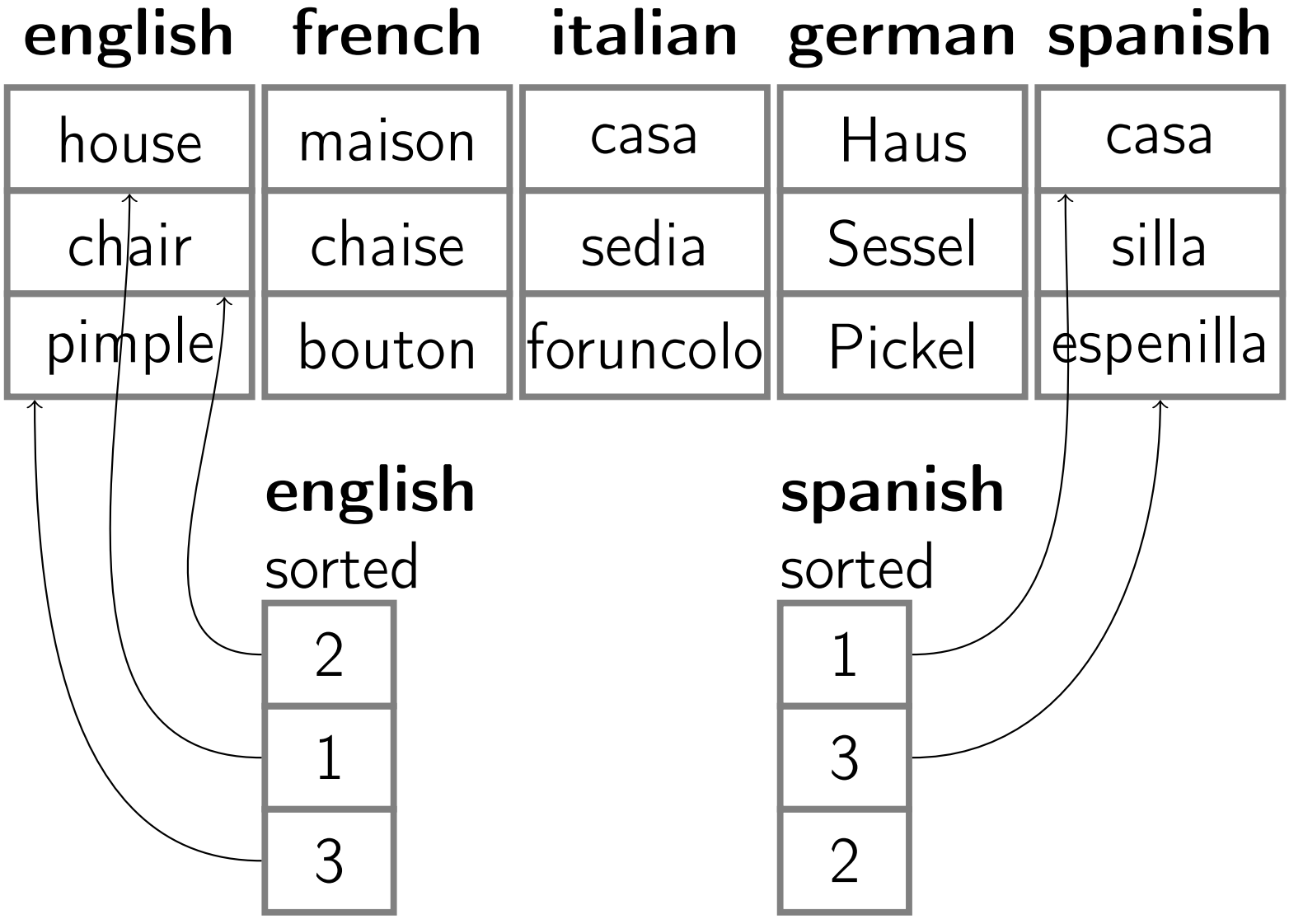
**spanish**
sorted

| |
|---|
| 1 |
| 3 |
| 2 |

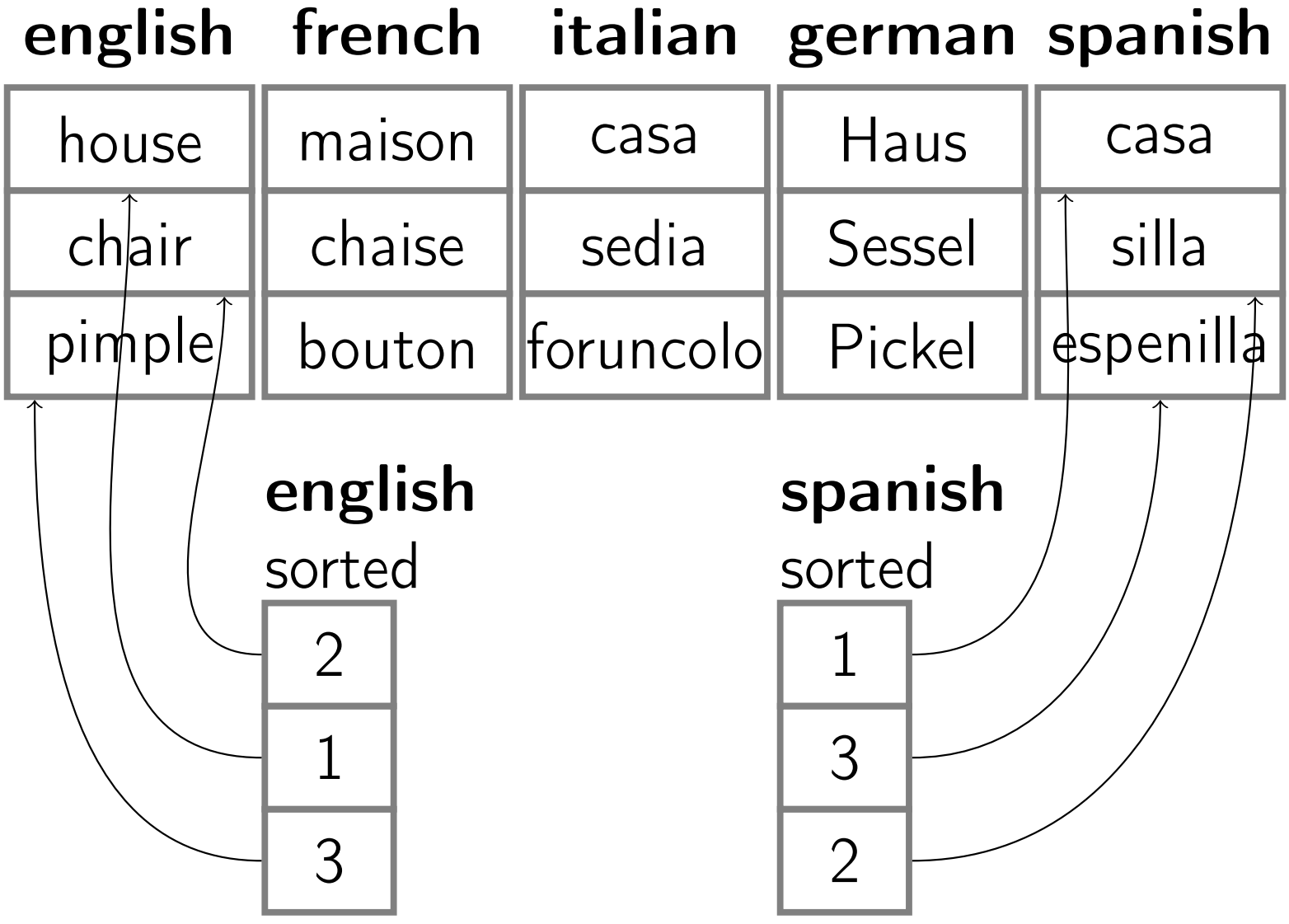
# Real-life Example



# Real-life Example



# Real-life Example



# Summary

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The runtime of binary search is $\Theta(\log n)$.