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O.R. Applications

Evaluating the best main battle tank using fuzzy decision theory with linguistic criteria evaluation

Ching-Hsue Cheng [a,*], Yin Lin [b]

[a] *Department of Information Management, National Yunlin University of Science & Technology, Touliu Yunlin 640, Taiwan*
[b] *Department of Management Science, Chinese Military Academy, Kaohsiung 830, Taiwan*

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Abstract

To face the reality of practical multiple criteria problems usually possessing characters of fuzziness, and to consider group decision making with various subjective–objective backgrounds usually participating in decision-making process. In this paper, the experts' opinions are described by linguistic terms which can be expressed in trapezoidal (or triangular) fuzzy numbers. To make the consensus of the experts consistent, we utilize fuzzy Delphi method to adjust the fuzzy rating of every expert to achieve the consensus condition. For the aggregate of many experts' opinions, we take the operation of fuzzy numbers to get the mean of fuzzy rating, $\tilde{x}_{ij}$ and the mean of weight, $\tilde{w}_{\bullet j}$. In multi-alternatives and multi-attributes cases, the fuzzy decision matrix $\tilde{X} = [\tilde{x}_{ij}]_{m\times n}$ is constructed by the mean of the fuzzy rating, $\tilde{x}_{ij}$. Then, we can derive the aggregate fuzzy numbers by multiplying the fuzzy decision matrix with the corresponding fuzzy attribute weights. The final results become a problem of ranking fuzzy numbers. We also propose an easy procedure of using fuzzy numbers to rank aggregate fuzzy numbers $\tilde{A}_i$. In this way, we can obtain the best selection for evaluating system. For practical application, we propose an algorithm for evaluating the best main battle tank by fuzzy decision theory and compare it with other method.

Keywords: Military application; Fuzzy group decision making (FGDM); Ranking fuzzy numbers; Linguistic variables

1. Introduction

Conflict always occurs in group decision making since members in a group generally do not reach the same decision. Resolving conflicts becomes an important issue in group decision making. For group decision making, the main approach is collective individual decision making [9]. Based on group benefit considerations, group decision-making problems may be defined as follows:

* Corresponding author. Tel.: +886-5-5342601 ext 5312; fax: +886-5-534-2601 ext 5312.
E-mail address: chcheng@mis4k.mis.yuntech.edu.tw (C.-H. Cheng).

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(i) there are two or more individuals, each of them characterized by his or her own perceptions, attitudes, motivations, and personalities,
(ii) who recognize the existence of a common problem, and
(iii) attempt to reach a collective decision.

Frequently, real world decision-making problems are ill defined, i.e., their objectives and parameters are not precisely known. These obstacles, which are due to a lack of precision, have been dealt with by others. But, due to the fact that the requirements on the data and on the environment are very high and that many real world problems are fuzzy by nature and not random, the probability applications have not been very satisfactory in a lot of cases. On the other hand, the application of fuzzy set theory in real world decision-making problems has given very good results. Its main feature is that it provides a more flexible framework, where it is possible to redress satisfactorily many of the obstacles of due to a lack of precision.

One of the most commonly used qualitative forecasting techniques is the Delphi method [1]. It was developed in the 1960s by the Rand Corporation at Santa Monica, CA. The name comes from the ancient Greek oracles of Delphi who were famous for forecasting the future. This technique attempts to develop forecasts through "group consensus". The Delphi method can be described as follows:

1. This approach uses a panel of experts who do not meet. It is normally used for long run forecasting.
2. The individuals normally fax/mail or telephone their responses which greatly increase the speed of generating the forecast.
3. The method is based on an iterative approach, which involves two or three rounds of iterations.
4. In the first round all individuals are asked a series of questions and the results are then gathered together. These aggregate results are then given back to each member of the panel for the second round. They are then asked if they wish to change their forecast. This process continues until either no individual change his or her forecast or a level of general agreement exists.

Long range forecasting is responsible for introducing problems related to imprecise and incomplete data information. Also the decisions made by the experts rely on their individual competence and are subjective. Therefore, it is more appropriate to present the data by fuzzy numbers instead of crisp numbers. That is, fuzzy Delphi method more suitable than Delphi method in real world. The fuzzy Delphi method was introduced by Kaufman and Gupta [14].

In many fuzzy multiple criteria decision-making problems, the final scores of alternatives are represented in terms of fuzzy numbers. In order to choose a best alternative, we need a method for building a crisp total ordering from fuzzy numbers. To resolve the task of comparing fuzzy numbers, many authors have proposed fuzzy ranking methods that yield a totally ordered set or ranking. These methods range from the trivial to the complex, from including one fuzzy number attribute to including many fuzzy number attributes. A review and comparison of these existing methods can be found in [5,16,19].

Usually, experts express their opinions by means of numerical values (numerical setting). When experts are not able to give exact numerical value to express their opinions, then a more realistic alternative option is using linguistic assessments instead of numerical values [6–8,10,11]. In such a situation, for each variable in the problem domain, an appropriate linguistic label set is chosen and used by individuals who participate in the decision-making process to express their opinions. This setting is known as the linguistic setting.

In this paper, the experts' opinions are described by linguistic terms which can be expressed in trapezoidal (or triangular) fuzzy numbers. To make the consensus of the experts consistent, we use fuzzy Delphi method to adjust the fuzzy rating of every expert to achieve the consensus condition. When fuzzy decision matrices are constructed, the aggregate fuzzy numbers can be obtained by multiplying the fuzzy decision matrix with the corresponding fuzzy attribute weight. The final results become ranking fuzzy numbers problem. We also propose an easy procedure of ranking fuzzy numbers to rank aggregate fuzzy numbers. Therefore, we can get the best selection for evaluating system by our proposed method.

For application to the evaluating of the best main battle tank, we also present an algorithm for evaluating the best main battle tank by a general and easy fuzzy group decision making. That is, we establish four linguistic criteria for evaluating the best main battle tank, the four linguistic criteria are "mobility capability", "communication and control capability", "self-defense capability", and "attack capability". Then, we can calculate the aggregate fuzzy numbers and rank its orderings by our proposed method.

2. The basic concepts of fuzzy numbers

A fuzzy number is a fuzzy set $\tilde{A}$ on R which possesses as the following three properties:

(i) $\tilde{A}$ is a normal fuzzy set;
(ii) The α-cut $\tilde{A}^{\alpha}$ of $\tilde{A}$ is a closed interval for every $\alpha \in (0, 1]$;
(iii) The support of $\tilde{A}$ is bounded.

Special cases of fuzzy numbers include crisp real number and intervals of real numbers. Although there are many shapes of fuzzy numbers, the triangular and trapezoidal shapes are used most often for representing fuzzy numbers. The following describes and definitions show that membership function of triangular and trapezoidal fuzzy number, and its operations.

Definition 2.1. A fuzzy number $\tilde{A}$ is convex [15,18], if

$$\mu_{\tilde{A}}[\lambda x_1 + (1-\lambda)x_2] \geqslant \min[\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)], \quad x_1, x_2 \in X, \ \ \lambda \in [0, 1]. \tag{1}$$

Alternatively, a fuzzy set is convex if all α-level sets are convex.

Definition 2.2. A fuzzy set $\tilde{A}$ in the universe of discourse X is normal if [14,17]

$$\sup_x \mu_{\tilde{A}}(x) = 1. \tag{2}$$

A nonempty fuzzy set $\tilde{A}$ can always be normalized by $\mu_{\tilde{A}}(x)/\sup_x \mu_{\tilde{A}}(x)$.

Definition 2.3. A fuzzy number is a fuzzy subset in the universe of discourse X that is both convex and normal.

One of the most important concepts of fuzzy sets is the concept of an α-cut and its variant. It is a bridge from well-defined structure to fuzzy environment.

Definition 2.4. For a fuzzy set $\tilde{A}$ defined on X and for any number $\alpha \in [0, 1]$, the α-cut, $\tilde{A}^{\alpha}$, and the strong α-cut, $\tilde{A}^{\alpha+}$ are defined as:

$$\begin{aligned} \tilde{A}^{\alpha} &= \{x \mid \mu_{\tilde{A}}(x) \geqslant \alpha\}, \\ \tilde{A}^{\alpha+} &= \{x \mid \mu_{\tilde{A}}(x) > \alpha\}. \end{aligned} \tag{3}$$

That is, the α-cut (or the strong α-cut) of a fuzzy set $\tilde{A}$ is the crisp set $\tilde{A}^{\alpha}$ (or the crisp set $\tilde{A}^{\alpha+}$) that contains all the elements of the universal set X whose membership grades in $\tilde{A}$ are greater than or equal to (or only greater than) the specified value of α.

Definition 2.5. A triangular fuzzy number can define as a triplet (a_1, a_2, a_3). Its membership function is defined as

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < a_1, \\ (x - a_1)/a_2 - a_1, & a_1 \leqslant x \leqslant a_2, \\ (a_3 - x)/a_3 - a_2, & a_2 \leqslant x \leqslant a_3, \\ 0, & x > a_3. \end{cases} \quad (4)$$

Let $\tilde{A}$ and $\tilde{B}$ be two fuzzy numbers parameterized by the triplet (a_1, a_2, a_3) and (b_1, b_2, b_3), respectively. Then the operations of triangular fuzzy numbers are expressed as [5]:

$$\begin{aligned} \tilde{A}(+)\tilde{B} &= (a_1, a_2, a_3)(+)(b_1, b_2, b_3) = (a_1 + b_1, a_2 + b_2, a_3 + b_3), \\ \tilde{A}(-)\tilde{B} &= (a_1, a_2, a_3)(-)(b_1, b_2, b_3) = (a_1 - b_3, a_2 - b_2, a_3 - b_1), \\ \tilde{A}(\times)\tilde{B} &= (a_1, a_2, a_3)(\times)(b_1, b_2, b_3) = (a_1 b_1, a_2 b_2, a_3 b_3), \\ \tilde{A}(\div)\tilde{B} &= (a_1, a_2, a_3)(\div)(b_1, b_2, b_3) = (a_1/b_3, a_2/b_2, a_3/b_1). \end{aligned} \quad (5)$$

Definition 2.6. A trapezoidal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$, $a_1 \leqslant a_2 \leqslant a_3 \leqslant a_4$, (if $a_2 = a_3$, $\tilde{A}$ is a triangular fuzzy number) its membership function is defined by

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x < a_1, \\ (x - a_1)/(a_2 - a_1), & a_1 \leqslant x \leqslant a_2, \\ 1, & a_2 \leqslant x \leqslant a_3, \\ (x - a_4)/(a_3 - a_4), & a_3 \leqslant x \leqslant a_4, \\ 0, & x > a_4. \end{cases} \quad (6)$$

Let $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ be any two positive trapezoidal fuzzy numbers. Then the operations $\{+, -, \times, \div\}$ are defined by [4]:

$$\begin{aligned} \tilde{A}(+)\tilde{B} &= (a_1, a_2, a_3, a_4)(+)(b_1, b_2, b_3, b_4) = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4), \\ \tilde{A}(-)\tilde{B} &= (a_1, a_2, a_3, a_4)(-)(b_1, b_2, b_3, b_4) = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1), \\ \tilde{A}(\times)\tilde{B} &= (a_1, a_2, a_3, a_4)(\times)(b_1, b_2, b_3, b_4) = (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4), \\ \tilde{A}(\div)\tilde{B} &= (a_1, a_2, a_3, a_4)(\div)(b_1, b_2, b_3, b_4) = (a_1/b_4, a_2/b_3, a_3/b_2, a_4/b_1). \end{aligned} \quad (7)$$

3. A easy and simple fuzzy group decision-making method

In this section, we will summarily introduce fuzzy Delphi method [2], ranking fuzzy numbers procedure, and the defuzzification value of the trapezoidal fuzzy number. At last, we will propose an algorithm to state our proposed method how to solve fuzzy group decision-making problem.

3.1. Fuzzy Delphi method

The fuzzy Delphi method consists of the following steps.

Step 1. Experts E_i, $i = 1, \ldots, n$, provide the possible realization time (or rating) of a certain event: the earliest time (or the pessimistic rating) $a_1^{(i)}$, the most plausible time (or rating) $(a_2^{(i)}, a_3^{(i)})$, and the latest time (or the optimistic rating) $a_4^{(i)}$. The time given by each expert E_i are presented in the form of a trapezoidal fuzzy number

$$\tilde{A}^{(i)} = \left(a_1^{(i)}, a_2^{(i)}, a_3^{(i)}, a_4^{(i)}\right), \quad i = 1, \ldots, n. \quad (8)$$

Step 2. First, the average (mean) $\tilde{A}_m$ of all $\tilde{A}^{(i)}$ is computed. This requires computation of the average of all $a_1^{(i)}, a_2^{(i)}, a_3^{(i)}$, and $a_4^{(i)}$, $i = 1, \ldots, n$. Hence

$$\tilde{A}_m = (a_{m1}, a_{m2}, a_{m3}, a_{m4}) = \left(\frac{1}{n}\sum_{i=1}^{n} a_1^{(i)}, \frac{1}{n}\sum_{i=1}^{n} a_2^{(i)}, \frac{1}{n}\sum_{i=1}^{n} a_3^{(i)}, \frac{1}{n}\sum_{i=1}^{n} a_4^{(i)}\right). \tag{9}$$

Then for each expert E_i the differences

$$\left(a_{m1} - a_1^{(i)}, a_{m2} - a_2^{(i)}, a_{m3} - a_3^{(i)}, a_{m4} - a_4^{(i)}\right) = \left(\frac{1}{n}\sum_{i=1}^{n} a_1^{(i)} - a_1^{(i)}, \frac{1}{n}\sum_{i=1}^{n} a_2^{(i)} - a_2^{(i)}, \frac{1}{n}\sum_{i=1}^{n} a_3^{(i)} - a_3^{(i)}, \frac{1}{n}\sum_{i=1}^{n} a_4^{(i)} - a_4^{(i)}\right) \tag{10}$$

are found and sent back to the expert E_i for reexamination.

Step 3. Each expert E_i presents a revised trapezoidal fuzzy number

$$\tilde{B}^{(i)} = \left(b_1^{(i)}, b_2^{(i)}, b_3^{(i)}, b_4^{(i)}\right), \quad i = 1, \ldots, n. \tag{11}$$

This process starting with Step 2 is repeated. The average $\tilde{B}_m$ is calculated by formula (9) with the differences that now $a_1^{(i)}, a_2^{(i)}, a_3^{(i)}, a_4^{(i)}$ are substituted correspondingly by $b_1^{(i)}, b_2^{(i)}, b_3^{(i)}, b_4^{(i)}$. If it still necessary new trapezoidal fuzzy numbers $\tilde{C}^{(i)} = (c_1^{(i)}, c_2^{(i)}, c_3^{(i)}, c_4^{(i)})$ are presented, and their average $\tilde{C}_m$ is calculated. The process could be repeated again and again until to successive means $\tilde{A}_m, \tilde{B}_m, \tilde{C}_m, \ldots$ become reasonably close (we can define the distance of two fuzzy numbers, $d_i \leqslant 0.2$).

Step 4. At a later time, the same process may reexamine the ratings, if there is important information available due to new discoveries.

3.2. Ranking fuzzy numbers procedure

Many methods for ranking of fuzzy numbers have been suggested. Each method appears to have some advantages as well as disadvantage [5]. In fuzzy multiple criteria decision-making problem, many triangular fuzzy numbers can intuitively be rank ordered by drawing their curves. If its ordering cannot be ranked by its figures, we can use other methods to rank fuzzy numbers. In this paper, we emphasize an easy and simple method. Therefore, we proposed a fuzzy numbers ranking procedure. This procedure can be introduced as follows:

(1) *Intuition ranking method*. From membership function curves of fuzzy numbers, many fuzzy numbers can easily rank its orderings by intuition ranking method. Due to Lee and Li [16] pointed out that human intuition would favor a fuzzy number with the following characteristics: higher mean value and at the same time lower spread.

(2) *If its ordering cannot rank by intuition ranking method*. We can rank fuzzy numbers by α-cut method [17], fuzzy mean and spread [16], or other methods. In this paper, we use the defuzzification value of the trapezoidal fuzzy number to do the necessary rank orderings. We take trapezoidal fuzzy number to represent experts' opinion.

3.3. The defuzzification value of the trapezoidal fuzzy number

Definition 3.1. For a trapezoidal fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$, its defuzzification value [3,13,14] is defined to be

$$c = (a_1 + a_2 + a_3 + a_4)/4. \tag{12}$$

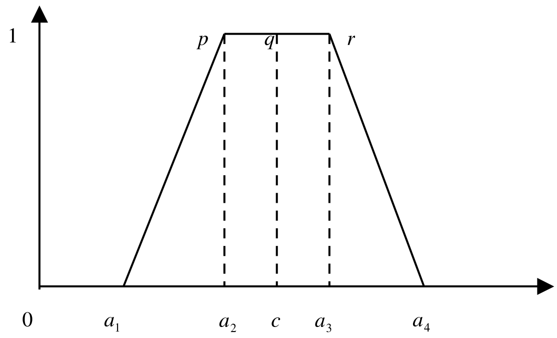

Fig. 1. The defuzzification value of the trapezoidal fuzzy number.

From Fig. 1, we can see that if the left area $\triangle a_1pa_2 + \square a_2pqc$ is equal to the right area $\square cqra_3 + \triangle a_3ra_4$, then

$$(1)(a_2 - a_1)/2 + (c - a_2)(1) = (a_3 - c)(1) + (1)(a_4 - a_3)/2 \quad \Rightarrow \quad c = (a_1 + a_2 + a_3 + a_4)/4.$$

Therefore, we obtain the defuzzification value of the trapezoidal fuzzy number is $c = (a_1 + a_2 + a_3 + a_4)/4$.

3.4. An algorithm for our proposed method

1. *Fuzzy Delphi method to adjust the consensus condition.* The decision-makers use the linguistic weighting variables to assess the importance of the criteria, and utilize the linguistic rating variables to evaluate the rating of alternatives with respect to each qualitative criterion. The linguistic weighting variables and the linguistic rating variables are shown in Tables 2 and 3, respectively. Then, we use fuzzy Delphi method to adjust the fuzzy rating and weighting by every expert to achieve the consensus condition, which obtain the importance weight of the linguistic criteria and the rating of the three decision-makers under linguistic criteria.
2. Calculate the mean of fuzzy rating and weighting, and transfer linguistic terms to positive trapezoidal fuzzy numbers.
3. Construct fuzzy decision matrix by normalizing the mean of fuzzy rating for three alternatives.
4. Aggregate the fuzzy evaluations by $\tilde{A}_i = [\tilde{x}_{ij}] \cdot [\tilde{w}_j]^{\mathrm{t}}$, $i = 1, \ldots, m$, $j = 1, \ldots, n$, i.e.,

$$\begin{bmatrix} \tilde{A}_1 \\ \vdots \\ \tilde{A}_m \end{bmatrix} = \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \ldots & \tilde{x}_{1n} \\ \tilde{x}_{21} & \tilde{x}_{22} & \ldots & \tilde{x}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{x}_{m1} & \tilde{x}_{m2} & \ldots & \tilde{x}_{mn} \end{bmatrix} \cdot \begin{bmatrix} \tilde{w}_1 \\ \tilde{w}_2 \\ \vdots \\ \tilde{w}_n \end{bmatrix} = \begin{bmatrix} \tilde{x}_{11} \cdot \tilde{w}_1 + \tilde{x}_{12} \cdot \tilde{w}_2 + \cdots + \tilde{x}_{1n} \cdot \tilde{w}_n \\ \tilde{x}_{21} \cdot \tilde{w}_1 + \tilde{x}_{22} \cdot \tilde{w}_2 + \cdots + \tilde{x}_{2n} \cdot \tilde{w}_n \\ \vdots \\ \tilde{x}_{m1} \cdot \tilde{w}_1 + \tilde{x}_{m2} \cdot \tilde{w}_2 + \cdots + \tilde{x}_{mn} \cdot \tilde{w}_n \end{bmatrix}, \tag{13}$$

   where $\tilde{x}_{ij}$ denote fuzzy rating of i alternative at j criteria, $\tilde{w}_j$ denote fuzzy weighting of j criteria.
5. Ranking fuzzy number $\tilde{A}_i$, $i = 1, 2, \ldots, m$.

If its ordering cannot rank by intuition ranking method, we can rank fuzzy numbers by the defuzzification value of the trapezoidal fuzzy number to rank it orderings.

4. Evaluating the best main battle tank

In evaluating process of weapon system, the expert not only must refer basic performance of weapon system, but also need to consider subjective factors, expectations, and some important information.

However, these information usually cannot concretely quantify and with fuzziness. Therefore, evaluating weapon system by fuzzy group decision making is considerately. Next, we construct a practical example for evaluating best main battle tank. We propose that the best main battle tank can be evaluated by four linguistic criteria, the four linguistic criteria are: "mobility capability", "communication and control capability", "self-defense capability", and "attack capability". In Jane [20], we select M1A1 (USA), Challenger 2 (UK), and Leopard 2 (Germany) as our evaluating entity; it basic performance data are listed in Table 1.

A committee of three decision-makers D_1, D_2, and D_3 has been formed to select the most suitable main battle tank. First, we construct hierarchical structure of this decision problem; it is shown as Fig. 2.

Table 1
Basic performance data for three types of main battle tank

Item	Type		
	M1A1 (USA)	Challenger (UK)	Leopard II (Germany)
Armament	1×120 mm gun 2×7.62 mm MG 1×12.7 mm MG	1×120 mm L30 gun 2×7.62 mm MG	1×120 mm gun 2×7.62 mm MG3
Ammunition	40 1000 11400	Up to 50 projectile stowage positions (7.62 mm) 4000	42 4750
Smoke grenade dischargers	2×6	2×5	2×8
Power to weight ratio	27 hp/t	19.2 hp/t	25.12 hp/t
Max. road speed	72 km	56 km/h	72 km
Max. range	498 km	450 km	500 km
Fording	1.219 m	1.07 m	1 m
Gradient	60%	60%	60%
Vertical obstacles	1.244 m	0.9 m	1.1 m
Trench	2.743 m	2.43 m	3.00 m
Armour protection	Good	Excellent	Fair
Acclimatization	Good	Fair	Good
Communication	Fair	Fair	Fair
Scout	Medium	Medium	Medium

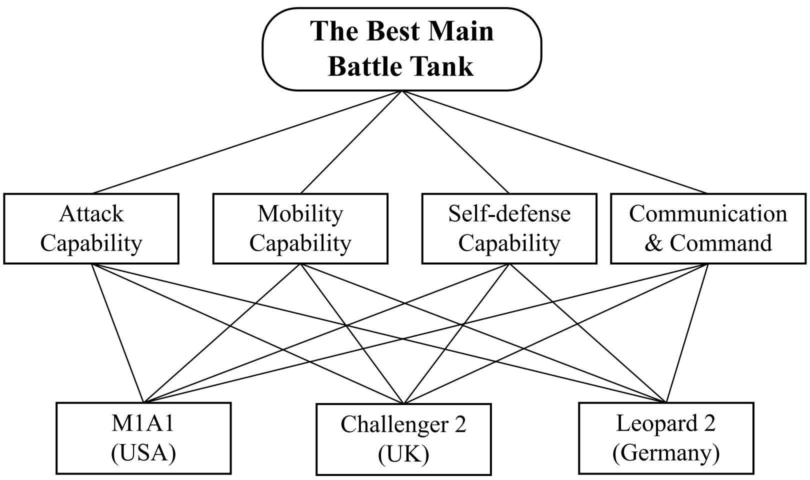

Fig. 2. The hierarchical structure of evaluating three types of main battle tank.

4.1. The evaluation process using our proposed method

We use our proposed method to solve this problem and the computational procedure is summarized as follows:

Step 1. Fuzzy Delphi method to adjust the consensus condition. The decision-makers use the linguistic weighting variables to assess the importance of the criteria, and utilize the linguistic rating variables to evaluate the rating of alternatives with respect to each qualitative criterion. The linguistic weighting variables and the linguistic rating variables are shown in Tables 2 and 3, respectively. We use fuzzy Delphi method to adjust the fuzzy rating and weighting by every expert to achieve the consensus condition, which obtain the importance weight of the linguistic criteria and the rating of the three decision-makers under linguistic criteria, it is shown as Tables 4 and 5, respectively.

Step 2. Calculate the mean of fuzzy rating and weighting, and transfer linguistic terms to positive trapezoidal fuzzy numbers; it is listed in the last column of Tables 4 and 5.

Step 3. Construct fuzzy decision matrix by normalizing the mean of fuzzy rating for three alternatives, it is shown in Table 6.

Table 2
Linguistic variables of the importance weight

Very low (VL)	(0, 0, 0.1, 0.2)
Low (L)	(0.1, 0.2, 0.2, 0.3)
Medium low (ML)	(0.2, 0.3, 0.4, 0.5)
Medium (M)	(0.4, 0.5, 0.5, 0.6)
Medium high (MH)	(0.5, 0.6, 0.7, 0.8)
High (H)	(0.7, 0.8, 0.8, 0.9)
Very high (VH)	(0.8, 0.9, 1, 1)

Table 3
Linguistic variables for the ratings

Very poor (VP)	(0, 0, 1, 2)
Poor (P)	(1, 2, 2, 3)
Medium poor (MP)	(2, 3, 4, 5)
Fair (F)	(4, 5, 5, 6)
Medium good (MG)	(5, 6, 7, 8)
Good (G)	(7, 8, 8, 9)
Very good (VG)	(8, 9, 10, 10)

Table 4
The importance weight of the linguistic criteria and its mean

	D1	D2	D3	Mean
Attack	VH	H	H	(0.73, 0.83, 0.86, 0.93)
Mobility	VH	H	VH	(0.76, 0.86, 0.93, 0.96)
Self-defense	M	VH	MH	(0.56, 0.66, 0.73, 0.80)
Communication and command	M	M	M	(0.40, 0.50, 0.50, 0.60)

Table 5
The ratings of attribute performance for three type of main battle tank

Criteria	Item	Type		
		M1A1 (USA)	Challenger 2 (UK)	Leopard 2 (Germany)
Attack	Armament	MG	G	G
	Ammunition	VG	MG	MG
	Smoke grenade dischargers	G	MG	VG
	Mean	(6.66, 7.66, 8.33, 9.00)	(5.66, 6.66, 7.33, 8.33)	(6.66, 7.66, 8.33, 9.00)
Mobility	Power to weight ratio	G	F	G
	Max. road speed	G	F	G
	Max. range	G	MG	G
	Passing trench/ obstacle	G	MG	MG
	Mean	(7.00, 8.00, 8.00, 9.00)	(4.50, 5.50, 6.00, 7.00)	(6.50, 7.50, 7.77, 8.77)
Self-defense	Armour protection	MG	G	F
	Acclimatization	MG	F	MG
	Mean	(5.00, 6.00, 7.00, 8.00)	(5.50, 6.50, 6.50, 7.50)	(4.50, 5.50, 6.00, 7.00)
Communication and command	Communication	G	G	G
	Scout	MG	MG	MG
	Mean	(6.00, 7.00, 7.50, 8.50)	(6.00, 7.00, 7.50, 8.50)	(6.00, 7.00, 7.50, 8.50)

Table 6
Normalizing the ratings of attribute performance for three main battle tank

	Attack	Mobility	Self-defense	Communication and control
M1A1 (USA)	(0.66, 0.76, 0.83, 0.90)	(0.70, 0.80, 0.80, 0.90)	(0.50, 0.60, 0.70, 0.80)	(0.60, 0.70, 0.75, 0.85)
Challenger 2 (UK)	(0.56, 0.66, 0.73, 0.83)	(0.45, 0.55, 0.60, 0.70)	(0.55, 0.65, 0.65, 0.75)	(0.60, 0.70, 0.75, 0.85)
Leopard 2 (Germany)	(0.66, 0.76, 0.83, 0.90)	(0.65, 0.75, 0.77, 0.87)	(0.45, 0.55, 0.60, 0.70)	(0.60, 0.70, 0.75, 0.85)

From Tables 4 and 6, we get fuzzy weight matrix $\tilde{W}$ and fuzzy decision matrix $\tilde{X}$, respectively,

$$\tilde{X} = \begin{bmatrix} (0.66, 0.76, 0.83, 0.90) & (0.70, 0.80, 0.80, 0.90) & (0.50, 0.60, 0.70, 0.80) & (0.60, 0.70, 0.75, 0.85) \\ (0.56, 0.66, 0.73, 0.83) & (0.45, 0.55, 0.60, 0.70) & (0.55, 0.65, 0.65, 0.75) & (0.60, 0.70, 0.75, 0.85) \\ (0.66, 0.76, 0.83, 0.90) & (0.65, 0.75, 0.77, 0.87) & (0.45, 0.55, 0.60, 0.70) & (0.60, 0.70, 0.75, 0.85) \end{bmatrix},$$

$$\tilde{W} = \begin{bmatrix} (0.73, 0.83, 0.86, 0.93) \\ (0.76, 0.86, 0.93, 0.96) \\ (0.56, 0.66, 0.73, 0.80) \\ (0.40, 0.50, 0.50, 0.60) \end{bmatrix}.$$

Step 4. Aggregate the fuzzy evaluations by $\tilde{A}_i = [\tilde{x}_{ij}] \cdot [\tilde{w}_j]^{\mathrm{t}},\ i = 1, \ldots, m,\ j = 1, \ldots, n$, where $\cdot$ denote composition operation of fuzzy numbers.

$$\begin{bmatrix} \tilde{A}_{\text{USA}} \\ \tilde{A}_{\text{UK}} \\ \tilde{A}_{\text{Germany}} \end{bmatrix} = \begin{bmatrix} (0.66, 0.76, 0.83, 0.90) & (0.70, 0.80, 0.80, 0.90) & (0.50, 0.60, 0.70, 0.80) & (0.60, 0.70, 0.75, 0.85) \\ (0.56, 0.66, 0.73, 0.83) & (0.45, 0.55, 0.60, 0.70) & (0.55, 0.65, 0.65, 0.75) & (0.60, 0.70, 0.75, 0.85) \\ (0.66, 0.76, 0.83, 0.90) & (0.65, 0.75, 0.77, 0.87) & (0.45, 0.55, 0.60, 0.70) & (0.60, 0.70, 0.75, 0.85) \end{bmatrix}$$

$$\otimes \begin{bmatrix} (0.73, 0.83, 0.86, 0.93) \\ (0.76, 0.86, 0.93, 0.96) \\ (0.56, 0.66, 0.73, 0.80) \\ (0.40, 0.50, 0.50, 0.60) \end{bmatrix}$$

$$= \begin{bmatrix} (1.53, 2.06, 2.34, 2.85) \\ (1.30, 1.80, 2.04, 2.55) \\ (1.48, 2.03, 2.24, 2.74) \end{bmatrix}.$$

Therefore,

$$\tilde{A}_{\text{USA}} = (1.53, 2.06, 2.34, 2.85),$$

$$\tilde{A}_{\text{UK}} = (1.30, 1.80, 2.03, \text{ and } 2.55),$$

$$\tilde{A}_{\text{Germany}} = (1.48, 2.03, 2.24, \text{ and } 2.74),$$

where $\tilde{A}_{\text{USA}}$ is Aggregate fuzzy number for M1A1 (USA), $\tilde{A}_{\text{UK}}$ is Aggregate fuzzy number for Challenger 2 (UK), $\tilde{A}_{\text{Germany}}$ is Aggregate fuzzy number for Leopard 2 (Germany).

Step 5. Ranking aggregate fuzzy number $\tilde{A}_i$, $i = \text{USA}, \text{UK}, \text{Germany}$, and determining the best alternatives.

In Section 3, we have proposed a ranking fuzzy numbers procedure. Firstly, we can intuitively rank its ordering by drawing its curves, as in Fig. 3. The best main battle tank is M1A1 (USA). Secondly, we can also defuzzify the values of the trapezoidal fuzzy numbers. The aggregate fuzzy evaluations are aggregate trapezoidal fuzzy numbers $\tilde{A}_i$, which are characterized by a quadruple (a_1, a_2, a_3, a_4). Then the defuzzification value of the trapezoidal fuzzy number is $c = (a_1 + a_2 + a_3 + a_4)/4$ (Eq. 12).

Hence, we can defuzzify $\tilde{A}_{\text{USA}}$, $\tilde{A}_{\text{UK}}$, and $\tilde{A}_{\text{Germany}}$ as in the following:

$$\tilde{A}_{\text{USA}} = (1.53 + 2.06 + 2.34 + 2.85)/4 = 2.20,$$

$$\tilde{A}_{\text{UK}} = (1.30 + 1.80 + 2.03 + 2.55)/4 = 1.92,$$

$$\tilde{A}_{\text{Germany}} = (1.48 + 2.03 + 2.24 + 2.73)/4 = 2.12.$$

Therefore, the ordering of the main battle tank is

$$\tilde{A}_{\text{USA}} > \tilde{A}_{\text{Germany}} > \tilde{A}_{\text{UK}}.$$

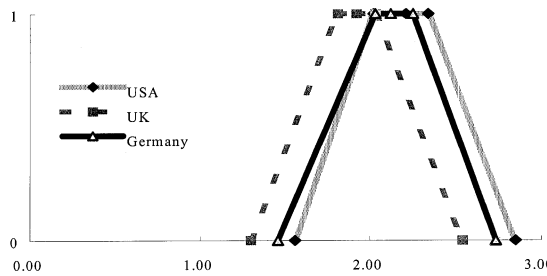

Fig. 3. The aggregated trapezoidal fuzzy numbers for three alternatives.

From above two methods of ranking fuzzy numbers, M1A1 (USA) is the best choice for main battle tank.

4.2. The evaluation process using synthetic evaluation method

Here we use a simple non-fuzzy additive weighting approach to resolve the evaluation problem. From Tables 2–5, we defuzzify trapezoidal fuzzy numbers by Eq. (12), and normalize defuzzification values of the importance weight (Table 4). We can obtain the crisp judgment matrix X (whose elements are the ratings of attribute performance) and the weighting vector W in the following:

$$X = \begin{bmatrix} 7.92 & 8 & 6.5 & 7.25 \\ 7 & 5.75 & 6.5 & 7.25 \\ 7.92 & 6.72 & 5.75 & 7.25 \end{bmatrix}, \quad W = [0.2885 \quad 0.3023 \quad 0.2369 \quad 0.1723].$$

Therefore, the aggregate matrix is

$$Y = X \cdot W^{\mathrm{t}} = \begin{bmatrix} 7.49_{\mathrm{USA}} \\ 5.55_{\mathrm{UK}} \\ 7.20_{\mathrm{Germany}} \end{bmatrix},$$

where "·" denotes multiplication and "t" denotes transposition.

From these calculated aggregate values, the ordering of the main battle tank is

$$\mathrm{M1A1}_{\mathrm{USA}} > \mathrm{Challenger}_{\mathrm{Germany}} > \mathrm{Leopard}_{\mathrm{UK}}.$$

The result is same as our proposed method in Section 4.1, M1A1 (USA) is the best choice for main battle tank.

4.3. The evaluation process using Herrera and Martinez's method

Herrera and Martinez [12] have proposed a 2-type fuzzy linguistic representation model for computing with words, which use label indexes (i.e., by means of linguistic 2-tuples) to represent linguistic terms. The linguistic 2-tuple (s, α) is composed by a linguistic term, s and a numeric value, α representing the symbolic translation. The 2-tuple linguistic aggregation operators is defined as

$$\bar{x}^e = \Delta\left(\frac{\sum_{i=0}^{g} \beta_i w_i}{\sum_{i=0}^{g} w_i}\right), \tag{14}$$

where Δ function is $\Delta : [0, g] \to S \times [-0.5, 0.5)$, $\beta_i \in [0, g], i = 0, \ldots, g$, are the result of an aggregation of the indexes of a set of labels assessed in a linguistic term set $S = \{s_0, s_1, \ldots, s_g\}$ (i.e., S have $g+1$ linguistic terms), and their associated weights $w_i \in [0, 1]$, $\sum_{i=0}^{g} w_i = 1$.

Here, we shall use Eq. (14), to evaluate the best main battle tank. To do this, we simplify the computing process as follows.

1. From Tables 2 and 3, two set of seven terms S could be given as

$$S - \text{weight} = \{s_0 : VL, s_1 : L, s_2 : ML, s_3 : M, s_4 : MH, s_5 : H, s_6 : VH\},$$
$$S - \text{rating} = \{s_0 : VP, s_1 : P, s_2 : MP, s_3 : F, s_4 : MG, s_5 : G, s_6 : VG\}.$$

2. Calculating Tables 4 and 5 by Eq. (14), we obtain

$$X = \begin{bmatrix} 5 & 5 & 4 & 4.5 \\ 4.33 & 3.5 & 4 & 4.5 \\ 5 & 4.75 & 3.5 & 4.5 \end{bmatrix}, \quad W - \text{not normalize} = [5.33 \quad 5.67 \quad 4.33 \quad 3].$$

So, the aggregate matrix is

$$Y = X \cdot W^{\mathrm{t}} = \begin{bmatrix} 85.83_{\mathrm{USA}} \\ 73.76_{\mathrm{UK}} \\ 82.25_{\mathrm{Germany}} \end{bmatrix}.$$

Its result is same as other two methods in Sections 4.1 and 4.2, the M1A1 (USA) is the best choice for main battle tank.

5. Conclusion

In this paper, we have constructed a general and easy fuzzy group decision-making method, and construct a practical example of selecting the best main battle tank to illustrate our proposed method. The experts' opinions are described by linguistic terms that can be expressed in trapezoidal fuzzy numbers. For making uniformly consensus of the experts, we utilize fuzzy Delphi method to adjust the fuzzy rating by every expert to achieve the consensus condition. In multi-alternatives and multi-attribute, each expert give the corresponding fuzzy rating and fuzzy attribute weight by fuzzy Delphi method, and use the operation of fuzzy number to get all mean of fuzzy rating and the mean of weight. At last, we have proposed an algorithm to select the best main battle tank by our proposed method and compare it with synthetic method, Herrera & Martinez's method. In future, we will use our proposed method to evaluate other decision making.

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