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Takagi–Sugeno fuzzy inference system for modeling stage–discharge relationship

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Summary Direct measurement of discharge in a stream is not only difficult and time consuming but also expensive. Therefore, the discharge in a stream is related to the stage through a number of carefully measured discharge values. A relationship between stages and corresponding measured discharges is usually derived using various graphical and analytical methods. As the relationship between stages and measured discharges is not linear, conventional methods based on least squares regression analysis for fitting a relationship are unable to model the non-linearity in the relationship and spatially in the cases when hysteresis is present in the data. The aim of the present study is to investigate the potential of Takagi–Sugeno (TS) fuzzy inference system for modeling stage–discharge relationships and the investigations are illustrated by application of the model to observed gauge and discharges of various gauging stations in Narmada river system, India. A step by step procedure for developing TS fuzzy model is also presented. The results show that the TS fuzzy modeling approach is superior than the conventional and artificial neural network (ANN) based approaches. Comparison of the models on hypothetical data set also reveals that the fuzzy logic based approach is also able to model the hysteresis effect (loop rating curve) more accurately than the ANN approach.

Introduction

Stream flow information is important for effective and reliable planning and management of various water resources activities and the assessment, management and control of water resources can be effective if accurate and continuous information on river-flow is available. Generally a network of river gauging stations provides continuous information on river stage and sparse information of corresponding discharges. Thus, the continuous discharge data corresponding to observed gauge can be obtained by developing a stage discharge relationship and using this relationship to convert the recorded stages into corresponding discharges. This

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relationship is determined by correlating measurements of discharge with the corresponding observations of stage (Maidment, 1992). However, under certain conditions (flatter gradients and constricted channels) the discharge for a flood on a rising stage differs from that on the falling stage. This phenomenon is called hysteresis and results in a looped stage–discharge curve (Tawfik et al., 1997) for floods with different stage–discharge relations for rising and falling water stages. Rating curve development approaches can be categorized into three main groups: the single curve approach, the raising and falling approach, and the Jone's approach (Tawfik et al., 1997). DeGagne et al. (1996) documented the process of developing a decision support system for the analysis and use of stage–discharge rating curve while other possible models have been proposed by Gawne and Simonovic (1994) and Yu (2000).

The functional relationship between stage and discharge is complex and can not always be captured by these traditional modeling techniques (Bhattacharya and Solomatine, 2005). In the real world, stage and discharge relationship do not exhibit simple structure and are difficult to analyze and model accurately. Therefore, it seems necessary that soft computing methods e.g. artificial neural network (ANN) and fuzzy logic, which are suited to complex non-linear models, be used for the analysis. There are several applications of ANNs in stage–discharge modeling. Jain and Chalisgaonkar (2000) used three layer feed forward ANNs to establish stage–discharge relationship. Bhattacharya and Solomatine (2005) have found that the predictive accuracy of ANN model is superior than the traditional rating curves. The effectiveness of an ANN with a radial basis function was explored by Sudheer and Jain (2003). The ANN based approaches have also provided promising results in modeling loop rating curves (Jain and Chalisgaonkar, 2000; Sudheer and Jain, 2003).

The purpose of this study is to investigate and explore the potential of an alternate soft computing technique for stage discharge modeling based on fuzzy logic. The ability of fuzzy logic to model nonlinear events makes it even more important to investigate its ability to model stage discharge relationship. Uncertainty in conventional gauge–discharge rating curves involves a variety of components such as measurement noise, inadequacy of the model, insufficiency of river flow conditions, etc. Fuzzy logic based modeling approach has a significant potential to tackle the uncertainty problem in this field and to model nonlinear functions of arbitrary complexity. Other advantage of fuzzy logic is its flexibility and tolerance to imprecise data (Zadeh, 1999). Fuzzy rule based modeling is a qualitative modeling scheme where the system behavior is described using a natural language (Sugeno and Yasukawa, 1993). The transparency of the fuzzy rules provides explicit qualitative and quantitative insights into the physical behavior of the system (Coppola et al., 2002). The application of fuzzy logic as a modeling tool in the field of water resources is a relatively new concept although some studies have been carried out to a limited extent and these studies have generated considerable enthusiasm. Fuzzy rule based modeling has been attempted in water resources management, reservoir operation, flood forecasting and other areas of water resources analysis (Bardossy and Duckstein, 2002; Fontane et al., 1997; Kindler, 1992; Mujumdar and Sasikumar, 2002; Panigrahi and Mujumdar, 2000; Sasikumar and Mujumdar, 1998; Deka and Chandramouli, 2003; Lohani et al., 2005). This paper is concerned with the application of an emerging, powerful soft computing technique fuzzy logic to stage discharge rating curves.

Overview of fuzzy logic

The classical theory of crisp sets can describe only the membership or non-membership of an item to a set. While, fuzzy logic is based on the theory of fuzzy sets which relates to classes of objects with unsharp boundaries in which membership is a matter of degree. In this approach, the classical notion of binary membership in a set has been modified to include partial membership ranging between 0 and 1 (Zadeh, 1965). The membership function is described by an arbitrary curve suitable from the point of view of simplicity, convenience, speed, and efficiency. A sharp set is a sub set of a fuzzy set where the membership function can take only the values 0 and 1.

The range of the model input values, which are judged necessary for the description of the situation, can be portioned into fuzzy sets. The process of formulating the mapping from a given input to an output using fuzzy logic is called the fuzzy inference (Jang, 1993). The basic structure of any fuzzy inference system is a model that maps characteristics of input data to input membership functions, input membership function to rules, rules to a set of output characteristics, output characteristics to output membership functions, and the output membership function to a single-valued output or a decision associated with the output (Jang et al., 2002). In rulebased fuzzy systems, the relationships between variables are represented by means of fuzzy if-then rules e.g. ''*If* antecedent proposition *then* consequent proposition''. Depending on the particular structure of the consequent proposition, three main types of fuzzy models are distinguished as: (1) Linguistic (Mamdani Type) fuzzy model (Zadeh, 1973; Mamdani, 1977) (2) Fuzzy relational model (Pedrycz, 1984; Yi and Chung, 1993) (3) Takagi–Sugeno (TS) fuzzy model (Takagi and Sugeno, 1985). A major distinction can be made between the linguistic model, which has fuzzy sets in both antecedents and consequents of the rules, and the TS model, where the consequents are (crisp) functions of the input variables. Fuzzy relational models can be regarded as an extension of linguistic models, which allow for different degrees of association between the antecedent and the consequent linguistic terms. In this work, the TS fuzzy model is employed to develop stage discharge rating curve. These models are relatively easy to identify and their structure can be readily analyzed (Lohani et al., 2005).

Takagi–Sugeno fuzzy inference system

A fuzzy rule-based model suitable for the approximation of many systems and functions is the Takagi–Sugeno (TS) fuzzy model (Takagi and Sugeno, 1985). In the TS fuzzy model, the rule consequents are usually taken to be either crisp numbers or linear functions of the inputs

$$R_i: \text{ IF } x \text{ is } A_i \text{ THEN } y_i = a_i^T x + b_i, \quad i = 1, 2, \ldots M \tag{1}$$

where $x \in \Re^n$ is the antecedent and $y_i \in \Re$ is the consequent of the ith rule. In the consequent, a_i is the parameter vector and b_i is the scalar offset. The number of rules is denoted by M and A_i is the (multivariate) antecedent fuzzy set of the ith rule defined by the membership function

$$\mu_i(x) : \Re^n \rightarrow [0, 1] \quad (2)$$

The fuzzy antecedent in the TS fuzzy model is normally defined as an and-conjunction by means of the product operator

$$\mu_i(x) = \prod_{j=1}^{p} \mu_{ij}(x_j) \quad (3)$$

where x_j is the jth input variable in the p dimensional input data space, and μ_{ij} the membership degree of x_j to the fuzzy set describing the jth premise part of the ith rule. $\mu_i(x)$ is the overall truth value of the ith rule.

For the input x the total output y of the TS model is computed by aggregating the individual rules contributions:

$$y = \sum_{i=1}^{M} u_i(x) \cdot y_i \quad (4)$$

where u_i is the normalized degree of fulfillment of the antecedent clause of rule R_i

$$u_i(x) = \frac{\mu_i(x)}{\sum_{i'=1}^{M} \mu_{i'}(x)} \quad (5)$$

The y_is are called consequent functions of the M rules and are defined by:

$$y_i = w_{i0} + w_{i1}x_1 + w_{i2}x_2 + \cdots + w_{ip}x_p \quad (6)$$

where w_{ij} are the linear weights for the ith rule consequent function.

TS fuzzy models are quasi linear in nature. They result in smooth transition between linear sub-models (Fig. 1), which are responsible for separate sub-space of states. This property of the TS fuzzy model allows separating the identification problem into two sub-problems: (i) appropriate partitioning of the state space of interest by clustering and; (ii) parameter identification of the consequent part.

Generation of TS fuzzy model

In general, TS fuzzy modeling involves structure identification and parameter identification. The structure identification consists in initial rule generation after elimination of insignificant variables, in the form of IF-THEN rules and their fuzzy sets. Parameter identification includes consequent parameter identification based on certain objective criteria. In many situations, such rules are difficult to identify by manual inspection and therefore are usually derived from observed data using techniques known collectively as fuzzy clustering. The basic purpose of fuzzy, clustering is to identify natural grouping of the data from a large data set, producing a concise representation of a systems behaviour, similar to traditional clustering procedures, a user can specify the expected number of clusters or let the system ''find'' the likely number of clusters from input data. Various method have been developed in the literature, such as fuzzy C-means clustering (Bezdek, 1973; Jang et al., 2002), mountain clustering (Yager and Filev, 1994), Gaustafron–Kessel (GK) fuzzy clustering (Gustafson and Kessel, 1979), and subtractive clustering (Chiu, 1994).

Fuzzy C-means clustering method is an iterative technique that starts with a set of cluster centers and generates membership grades, used to induce new cluster centers. In this approach number of iterations depends on the initial values of the clusters' centers. Mountain clustering method is based on grid of data space that computes the potential value (mountain function) for each point on the grid on the basis of its distance to the actual data point. The greatest potential point (one of the grid vertices) represents the first cluster (highest point on the mountain). Subsequently, the potential for each grid point is adjusted, allowing for the determination of all remaining clusters. Subtractive clustering method (Chiu, 1994) is an extension of the mountain clustering method, where data points (not grid points) are considered as the potential candidates for cluster centers. As a result, clusters are elected from the system training data according to their potential. Therefore, in subtractive clustering method, the computation is simply proportional to the number of data points and independent of the dimension of the problem under consideration.

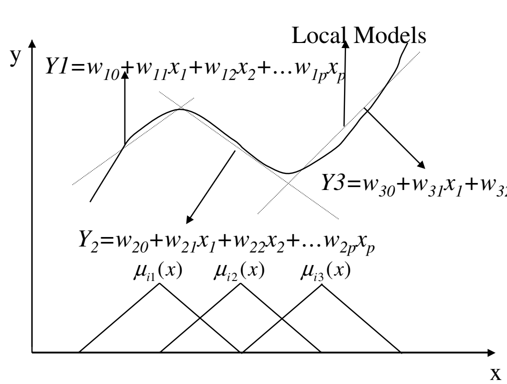

Figure 1 Takagi–Sugeno fuzzy model as a smooth piece-wise linear approximation of non-linear function.

Subtractive clustering

The subtractive clustering approach is used to determine the number of rules and antecedent membership functions by considering each cluster center (D_i) as a fuzzy rule. In this approach each data point of a set of N data points $\{x_1, \ldots, x_N\}$ in a p-dimensional space is considered as the candidate for cluster centers. After normalizing and scaling data points in each direction, a density measure at data point x_i is computed on the basis of its location with respect to other data points and expressed as:

$$D_i = \sum_{j=1}^{N} \exp\left(-\left(\frac{2}{r_a}\right)^2 . \|x_i - x_j\|^2\right) \quad (7)$$

where r_a is a positive constant called cluster radius.

A data point is considered as a cluster center when more data points are closer to it. Therefore, the data point (x_1^*) with highest density measure (D_1^*) is considered as first cluster center. Now excluding the influence of the first cluster center, the density measure of all other data points is recalculated as

$$D_i = D_i - D_1^* \cdot \mu(x_i^*) \quad (8)$$

$$\mu(x_i^*) = \exp\left(-\frac{\|x_i - x_i^*\|^2}{(r_b/2)^2}\right) \quad (9)$$

where r_b ($r_b > r_a$) is a positive constant that results in a measurable reduction in density measures of neighborhood data points so as to avoid closely spaced cluster centers (Chiu, 1994).

After the density measure for each data point is revised (Eq. 8), the data point with the highest remaining density measure is obtained and set as the next cluster center x_2^* and all of the density measures for data points are revised again. The process is repeated and the density measures of remaining data points after computation of kth cluster center is revised by substituting the location (x_k^*) and density measure (D_k^*) of the kth cluster center in Eq. (8). This process is stopped when a sufficient number of cluster centers are generated. A sophisticated stopping criterion for automatically determining the number of clusters is suggested by Chiu (1994, 1996).

The cluster center derived using the subtractive clustering approach represents a prototype that exhibits certain characteristic of the system to be modeled. These

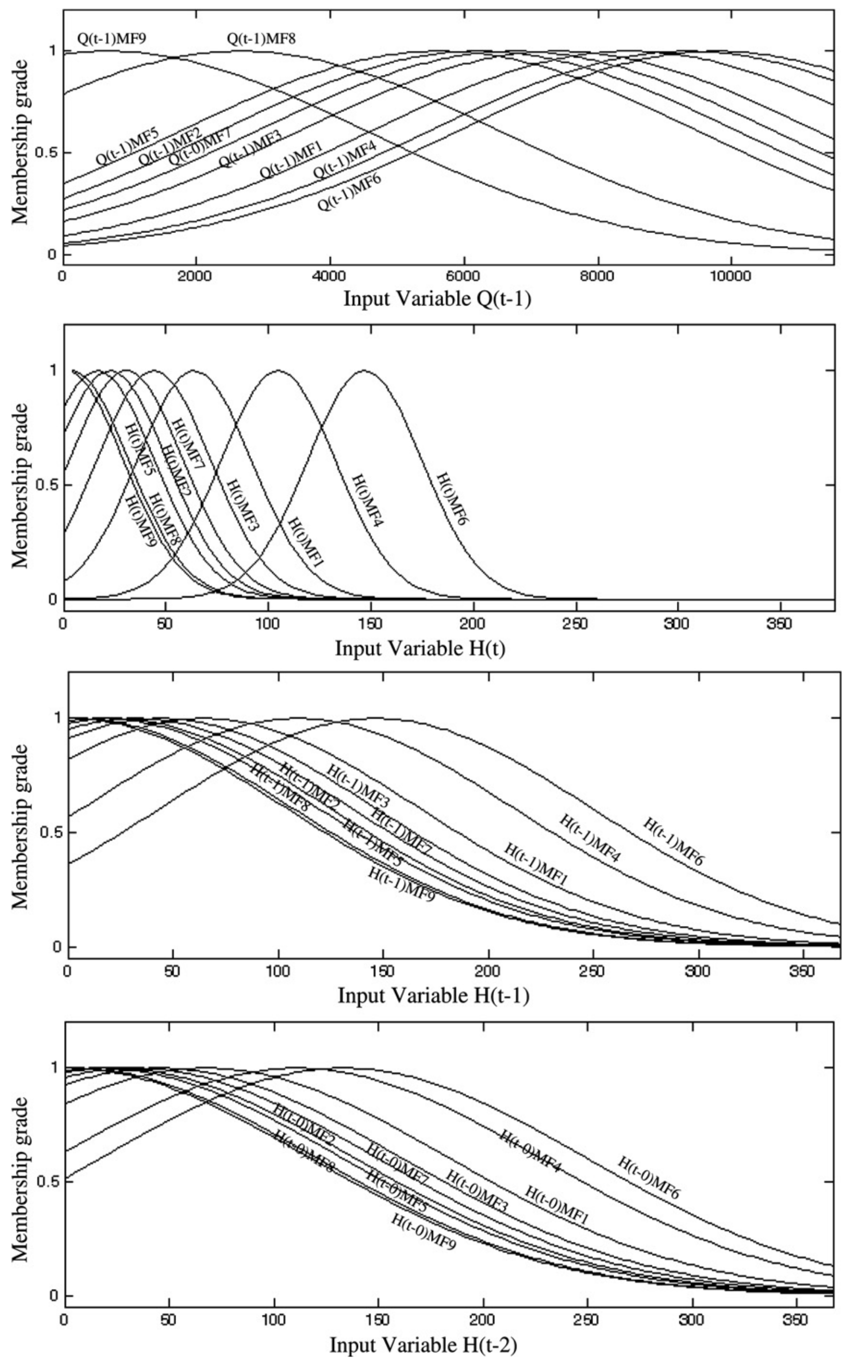

Figure 2 Membership functions of input variables for fuzzy model $Q_t = f(H_t, H_{t-1}, H_{t-2}, Q_{t-1})$ at Jamtara site.

Table 1 Fuzzy rules for fuzzy model $Q_t = f(H_t,H_{t-1},H_{t-2},Q_{t-1})$ at Jamtara site

Rule	Rule description
1	If $(H(t-2)$ is $H(t-2)mf1)$ and $(H(t-1)$ is $H(t-1)mf1)$ and $(H(t)$ is $H(t)mf1)$ and $(Q(t-1)$ is $Q(t-1)mf1)$ then $Q(t)$ is $65.86*H(t-2) - 61.85*H(t-1) + 983.2*H(t) + 1146*Q(t-1) + 1952$
2	If $(H(t-2)$ is $H(t-2)mf2)$ and $(H(t-1)$ is $H(t-1)mf2)$ and $(H(t)$ is $H(t)mf2)$ and $(Q(t-1)$ is $Q(t-1)mf2)$ then $Q(t)$ is $268.4*H(t-2) + 492.7*H(t-1) + 3589*H(t) + 3730*Q(t-1) - 1357$
3	If $(H(t-2)$ is $H(t-2)mf3)$ and $(H(t-1)$ is $H(t-1)mf3)$ and $(H(t)$ is $H(t)mf3)$ and $(Q(t-1)$ is $Q(t-1)mf3)$ then $Q(t)$ is $4887*H(t-2) + 5995*H(t-1) + 96{,}900*H(t) + 17{,}400*Q(t-1) + 53{,}380$
4	If $(H(t-2)$ is $H(t-2)mf4)$ and $(H(t-1)$ is $H(t-1)mf4)$ and $(H(t)$ is $H(t)mf4)$ and $(Q(t-1)$ is $Q(t-1)mf4)$ then $Q(t)$ is $9047*H(t-2) + 10{,}590*H(t-1) + 145{,}400*H(t) + 19{,}540*Q(t-1) - 76{,}790$
5	If $(H(t-2)$ is $H(t-2)mf5)$ and $(H(t-1)$ is $H(t-1)mf5)$ and $(H(t)$ is $H(t)mf5)$ and $(Q(t-1)$ is $Q(t-1)mf5)$ then $Q(t)$ is $785.2*H(t-2) + 1270*H(t-1) + 6190*H(t) + 485.6*Q(t-1) - 13{,}410$
6	If $(H(t-2)$ is $H(t-2)mf6)$ and $(H(t-1)$ is $H(t-1)mf6)$ and $(H(t)$ is $H(t)mf6)$ and $(Q(t-1)$ is $Q(t-1)mf6)$ then $Q(t)$ is $-74.47*H(t-2) - 196.7*H(t-1) -2137*H(t) - 158.1*Q(t-1) + 1085$
7	If $(H(t-2)$ is $H(t-2)mf7)$ and $(H(t-1)$ is $H(t-1)mf7)$ and $(H(t)$ is $H(t)mf7)$ and $(Q(t-1)$ is $Q(t-1)mf7)$ then $Q(t)$ is $-16.5*H(t-2) - 7.963*H(t-1) - 140.7*H(t) - 4.747*Q(t-1) + 76.07$
8	If $(H(t-2)$ is $H(t-2)mf8)$ and $(H(t-1)$ is $H(t-1)mf8)$ and $(H(t)$ is $H(t)mf8)$ and $(Q(t-1)$ is $Q(t-1)mf8)$ then $Q(t)$ is $0.1015*H(t-2) - 0.2973*H(t-1) - 2.965*H(t) - 0.8626*Q(t-1) + 2.92$
9	If $(H(t-2)$ is $H(t-2)mf9)$ and $(H(t-1)$ is $H(t-1)mf9)$ and $(H(t)$ is $H(t)mf9)$ and $(Q(t-1)$ is $Q(t-1)mf9)$ then $Q(t)$ is $0.001763*H(t-2) - 0.1055*H(t-1) + 0.2319*H(t) + 0.3554*Q(t-1) + 0.5115$

cluster centers (x_i^*) can be reasonably used as the centers for the fuzzy rules' premise and antecedent membership function that describes the system behaviour. For jth variable of the input data vector x, the degree to which rule i is fulfilled is defined by Gaussian membership functions:

Table 2 Values of performance indices and error functions for fuzzy, ANN and conventional models — calibration and validation data of Jamtara site

Fuzzy/ANN model (number of rules/nodes in hidden layer)	Calibration data		Validation data	
	(a) Coefficient of correlation (b) RMSE (m^3/s) (c) Efficiency (%)	(a) AE (m^3/s) (b) RE (%)	(a) Coefficient of correlation (b) RMSE (m^3/s) (c) Efficiency (%)	(a) AE (m^3/s) (b) RE (%)
$Q_t = f(H_t)$ (5/7)	(a) 0.994(0.985) (b) 125.6(135.7) (c) 91.20(89.72)	(a) 39.81(43.48) (b) 4.85(5.59)	(a) 0.996(0.995) (b) 91.40(99.91) (c) 92.00(90.44)	(a) 42.62(47.72) (b) 6.20(7.41)
$Q_t = f(H_t,H_{t-1})$ (7/5)	(a) 0.996(0.987) (b) 84.68(98.25) (c) 95.99(94.61)	(a) 35.32(37.23) (b) 4.42(5.12)	(a) 0.996(0.996) (b) 89.74(97.04) (c) 92.28(90.98)	(a) 36.28(41.39) (b) 5.96(6.23)
$Q_t = f(H_t,Q_{t-1})$ (7/6)	(a) 0.996(0.991) (b) 82.67(93.71) (c) 96.18(95.1)	(a) 29.49(33.41) (b) 4.24(5.11)	(a) 0.996(0.996) (b) 83.72(93.45) (c) 93.28(91.63)	(a) 34.98(37.33) (b) 4.81(5.83)
$Q_t = f(H_t,H_{t-1},Q_{t-1})$ (8/6)	(a) 0.997(0.994) (b) 65.02(75.44) (c) 97.64(96.82)	(a) 29.22(31.12) (b) 4.22(4.91)	(a) 0.996(0.993) (b) 82.13(89.32) (c) 93.54(92.36)	(a) 31.72(35.32) (b) 4.54(4.82)
$Q_t = f(H_t,H_{t-1},H_{t-2},Q_{t-1})$ (9/8)	(a) 0.998(0.996) (b) 66.38(73.52) (c) 97.54(96.98)	(a) 28.15(29.33) (b) 4.18(4.90)	(a) 0.997(0.991) (b) 78.87(87.19) (c) 94.04(92.72)	(a) 26.28(32.76) (b) 4.72(4.81)
$Q_t = f(H_t,H_{t-1},H_{t-2},Q_{t-1},Q_{t-2})$ (12/8)	(a) 0.998(0.996) (b) 72.71(76.05) (c) 97.05(95.77)	(a) 30.84(30.97) (b) 5.09(5.21)	(a) 0.995(0.989) (b) 80.17(88.97) (c) 93.84(92.41)	(a) 28.55(34.21) (b) 6.26(6.25)
Curve fitting	(a) 0.993 (b) 147.4 (c) 81.45	(a) 67.39 (b) 11.32	(a) 0.989 (b) 117.57 (c) 86.76	(a) 74.22 (b) 17.39

Table 3 Values of performance indices and error functions for fuzzy, ANN and conventional models – calibration and validation data of Manot site

Fuzzy/ANN model (number of rules/nodes in hidden layer)	Calibration data					Validation data				
	(a) Coefficient of correlation	(b) RMSE (m^3/s)	(c) Efficiency (%)	(a) AE (m^3/s)	(b) RE (%)	(a) Coefficient of correlation	(b) RMSE (m^3/s)	(c) Efficiency (%)	(a) AE (m^3/s)	(b) RE (%)
$Q_t = f(H_t)$ (4/6)	0.961(0.946)	84.12(90.40)	92.2(90.99)	6.82(7.41)	10.74(10.76)	0.962(0.945)	44.76(57.83)	85.4(75.62)	7.49(8.11)	13.59(13.63)
$Q_t = f(H_t,H_{t-1})$ (7/6)	0.963(0.954)	74.38(87.88)	93.9(91.48)	6.54(7.10)	9.22(9.14)	0.964(0.955)	42.35(51.32)	86.92(80.81)	7.23(7.83)	11.63(11.67)
$Q_t = f(H_t,Q_{t-1})$ (8/7)	0.979(0.961)	73.09(78.14)	94.11(93.26)	5.57(6.44)	9.12(9.02)	0.975(0.960)	39.04(50.12)	88.89(81.69)	6.25(6.88)	9.15(9.39)
$Q_t = f(H_t,H_{t-1},Q_{t-1})$ (8/6)	0.988(0.962)	70.18(76.18)	94.57(93.60)	4.26(5.31)	8.23(8.89)	0.987(0.962)	37.47(49.86)	89.76(81.88)	4.65(5.23)	8.25(8.27)
$Q_t = f(H_t,H_{t-1},H_{t-2},Q_{t-1})$ (9/8)	0.989(0.960)	70.14(80.26)	94.57(92.89)	3.07(3.49)	7.88(12.23)	0.988(0.964)	36.36(47.39)	90.37(83.63)	4.46(5.12)	7.59(15.13)
$Q_t = f(H_t,H_{t-1},H_{t-2},Q_{t-1},Q_{t-2})$ (14/8)	0.986(0.961)	72.96(79.67)	94.13(93.00)	4.40(4.67)	8.10(12.51)	0.983(0.958)	38.61(48.03)	89.13(83.18)	4.89(5.03)	8.17(14.78)
Curve fitting	0.955	92.21	90.62	18.97	15.33	0.951	64.57	79.61	17.32	14.27

$$\mu_{ij}(x_i) = \exp\left(\frac{(x_i - x_i^*)}{(r_a/2)^2}\right) \tag{10}$$

For every unique input vector a membership degree to each fuzzy set greater than 0 is computed, and therefore every rule in the rule base fires. This leads to the possibility of generating only a couple of rules for describing the accurate relationship between input and output data.

Estimation of linear consequent parameters

All linear consequent parameters can be estimated simultaneously using global estimation approach. In the case of global estimation approach the regression matrix X for N data samples becomes:

$$X = [X_1 X_2 \cdots X_j \cdots X_M] \tag{11}$$

where M are number of rules in the system and

$$X_i = \begin{bmatrix} u_i(x(1)) & x_1(1)\cdot u_i(x(1)) & x_2(1)\cdot u_i(x(1)) & x_3(1)\cdot u_i(x(1)) & x_p(1)\cdot u_i(x(1)) \\ u_i(x(2)) & x_1(2)\cdot u_i(x(2)) & x_2(2)\cdot u_i(x(2)) & x_3(2)\cdot u_i(x(2)) & x_p(2)\cdot u_i(x(2)) \\ u_i(x(3)) & x_1(3)\cdot u_i(x(3)) & x_2(3)\cdot u_i(x(3)) & x_3(3)\cdot u_i(x(3)) & x_p(3)\cdot u_i(x(3)) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ u_i(x(N)) & x_1(N)\cdot u_i(x(N)) & x_2(N)\cdot u_i(x(N)) & x_3(N)\cdot u_i(x(N)) & x_p(N)\cdot u_i(x(N)) \end{bmatrix} \tag{12}$$

where u_j is the basis function defined in Eq. (5), $x_j(k)$ is the value of the jth variable in the kth data point. With this the model output as defined in Eq. (4) becomes:

$$y = X \cdot w \tag{13}$$

where w is the parameter vector from Eq. (6) containing only the linear weights w_{ij} obtained from the membership degree through the multidimensional membership functions. These linear parameters can be estimated by least square method.

Rating curve analysis

Rating curve is a useful tool to transfer observed stage to corresponding river discharge. It simplifies the need for costly and time consuming discharge measurements. The river discharge is crucial in flood management, water yield computation and hydrologic design. Rating curves are mostly of the form:

Table 4 Values of performance indices and error functions for fuzzy, ANN and conventional models — calibration and validation data of Satrana site

Fuzzy/ANN model (number of rules/nodes in hidden layer)	Calibration data		Validation data	
	(a) Coefficient of correlation (b) RMSE (m^3/s) (c) Efficiency (%)	(a) AE (m^3/s) (b) RE (%)	(a) Coefficient of correlation (b) RMSE (m^3/s) (c) Efficiency (%)	(a) AE (m^3/s) (b) RE (%)
$Q_t = f(H_t)$ (4/5)	(a) 0.982(0.991) (b) 41.14(32.21) (c) 91.20(94.60)	(a) 2.97(1.11) (b) 12.96(13.03)	(a) 0.981(0.989) (b) 24.03(23.91) (c) 92.06(92.07)	(a) 3.98(3.81) (b) 13.15(13.87)
$Q_t = f(H_t,H_{t-1})$ (6/8)	(a) 0.993(0.992) (b) 27.06(29.23) (c) 96.19(95.55)	(a) 2.09(2.31) (b) 10.66(11.91)	(a) 0.992(0.989) (b) 19.48(22.48) (c) 94.74(92.99)	(a) 2.56(3.01) (b) 11.03(11.72)
$Q_t = f(H_t,Q_{t-1})$ (6/5)	(a) 0.994(0.993) (b) 26.29(30.01) (c) 96.40(95.31)	(a) 1.83(1.89) (b) 9.46(9.75)	(a) 0.992(0.992) (b) 19.12(21.19) (c) 94.93(93.77)	(a) 1.92(2.01) (b) 9.86(9.02)
$Q_t = f(H_t,H_{t-1},Q_{t-1})$ (8/6)	(a) 0.996(0.994) (b) 23.73(27.48) (c) 97.04(96.07)	(a) 1.76(1.76) (b) 8.86(9.01)	(a) 0.993(0.992) (b) 18.89(20.17) (c) 95.05(94.36)	(a) 1.73(1.77) (b) 8.29(8.34)
$Q_t = f(H_t,H_{t-1},H_{t-2},Q_{t-1})$ (9/8)	(a) 0.997(0.995) (b) 22.08(27.41) (c) 97.46(96.09)	(a) 1.75(2.10) (b) 8.06(8.26)	(a) 0.995(0.994) (b) 18.23(20.17) (c) 95.39(94.35)	(a) 1.77(2.01) (b) 8.05(8.21)
$Q_t = f(H_t,H_{t-1},H_{t-2},Q_{t-1},Q_{t-2})$ (12/10)	(a) 0.996(0.995) (b) 25.91(28.21) (c) 96.50(95.86)	(a) 1.86(2.05) (b) 8.44(8.91)	(a) 0.994(0.993) (b) 18.24(20.94) (c) 95.39(93.93)	(a) 1.93(2.17) (b) 8.20(8.32)
Curve fitting	(a) 0.965 (b) 52.36 (c) 81.45	(a) 4.57 (b) 16.39	(a) 0.978 (b) 52.60 (c) 61.66	(a) 5.32 (b) 16.44

$$Q = a(H - H_0)^b \tag{14}$$

where Q is the discharge (m^3/s); H is the river stage (m); a and b are constant; and H_0 is the stage (m) at which discharge is nil. A first estimate of H_0 is usually chosen after examining the characteristics of the historical stage data and then by trial and error the final value of H_0 is chosen which gives the best fit i.e. the minimum sum of squares of errors (SSE). In a flood event, the relationship between stage and discharge is not unique. During the rising flood, the flood wave receives less hindrance in propagation than in a falling flood. For the same stage this causes higher discharge during the rising flood than during a falling flood. This effect is known as hysteresis; it results in a loop-rating curve and justifies the premise that the relationship between stage and discharge is not a one-to-one mapping, but has the dependency of discharge on past stage and discharge values. Sometimes the stage data is divided into rising and falling stages separately to take care the hysteresis effect (Bhattacharya and Solomatine, 2005). Then separate regression models (Eq. 14) are developed for each set. This approach is not without limitations as data separation is often subjective and therefore the use of rating curves need expertise and is prone to errors. Artificial neural network (ANN) is a powerful procedure for non linear function mapping. Various attempts have been made to establish the applicability of ANN for gauge—discharge rating curve modeling (Jain and Chalisgaonkar, 2000; Sudheer and Jain, 2003; Bhattacharya and Solomatine, 2005).

Study area and data used

Data from discharge measuring stations in the upper catchment of river Narmada in central India have been considered. The data considered for model calibration were a mix of dry and wet periods, and the validation period was an average year. The data used for analysis consisted of following six sets:

1. The daily stage and discharge record of the Jamtara gauging site on the Narmada River. The catchment area at this site is 17157 km^2. Here 1281 pairs of gauge and discharge were available. In this case, the first 900 pairs of data were used to fit the conventional rating curve and to calibrate ANN and fuzzy based models, and the remaining 301 for validation.
2. The daily stage and discharge record of the Narmada River at Manot gauging site. The catchment area at this site is 4980 km^2. Here 506 pairs of gauge and discharge were available. In this case, the first 330 pairs of data were used to fit the conventional rating curve and to calibrate ANN and fuzzy based models, and the remaining 176 for validation.

Table 5 Values of performance indices and error functions for fuzzy, ANN and conventional models — calibration and validation data of Hriday Nagar site (Case-I)

Fuzzy/ANN model (number of rules/nodes in hidden layer)	Calibration data: (a) Coefficient of correlation (b) RMSE (m^3/s) (c) Efficiency (%)	Calibration data: (a) AE (m^3/s) (b) RE (%)	Validation data: (a) Coefficient of correlation (b) RMSE (m^3/s) (c) Efficiency (%)	Validation data: (a) AE (m^3/s) (b) RE(%)
$Q_t = f(H_t)$ (4/5)	(a) 0.988(0.964) (b) 40.37(49.27) (c) 94.8(92.25)	(a) 3.57(4.32) (b) 10.05(10.11)	(a) 0.986(0.961) (b) 70.98(81.21) (c) 85.40(75.62)	(a) 4.26(4.74) (b) 11.12(12.03)
$Q_t = f(H_t,H_{t-1})$ (6/8)	(a) 0.989(0.964) (b) 36.83(47.78) (c) 95.67(92.71)	(a) 3.09(3.30) (b) 10.00(10.12)	(a) 0.981(0.960) (b) 76.62(84.56) (c) 82.98(80.81)	(a) 3.83(4.06) (b) 10.89(11.10)
$Q_t = f(H_t,Q_{t-1})$ (6/5)	(a) 0.990(0.974) (b) 40.03(49.01) (c) 94.88(92.64)	(a) 3.04(3.05) (b) 8.15(9.27)	(a) 0.985(0.972) (b) 67.13(78.26) (c) 86.94(82.25)	(a) 2.67(2.89) (b) 8.03(9.01)
$Q_t = f(H_t,H_{t-1},Q_{t-1})$ (8/6)	(a) 0.992(0.989) (b) 33.76(37.77) (c) 96.37(95.44)	(a) 2.95(3.48) (b) 7.19(7.33)	(a) 0.988(0.982) (b) 63.18(73.95) (c) 88.43(84.15)	(a) 2.16(2.82) (b) 7.72(7.17)
$Q_t = f(H_t,H_{t-1},H_{t-2},Q_{t-1})$ (9/8)	(a) 0.992(0.990) (b) 30.11(36.32) (c) 97.11(95.79)	(a) 2.94(3.03) (b) 6.99(6.21)	(a) 0.989(0.983) (b) 57.32(69.41) (c) 90.47(86.04)	(a) 1.98(2.23) (b) 6.53(6.02)
$Q_t = f(H_t,H_{t-1},H_{t-2},Q_{t-1},Q_{t-2})$ (12/10)	(a) 0.991(0.989) (b) 32.82(36.97) (c) 96.56(95.63)	(a) 3.06(3.71) (b) 7.01(7.39)	(a) 0.980(0.980) (b) 59.28(72.55) (c) 89.81(84.75)	(a) 1.77(2.13) (b) 6.74(6.77)
Curve fitting	(a) 0.961 (b) 76.23 (c) 81.45	(a) 6.31 (b) 12.45	(a) 0.960 (b) 93.4 (c) 74.72	(a) 8.47 (b) 12.81

3. The daily stage and discharge record of the Kolar River (a tributary of the Narmada River) at Satarana gauging site. The catchment area at this site is 820 km^2. Here 768 pairs of gauge and discharge were available. In this case, the first 600 pairs of data were used to fit the conventional rating curve and to calibrate ANN and fuzzy based models, and the remaining 168 for validation.
4. The daily stage and discharge record of the Banjar River (a tributary of the Narmada River) at Hriday Nagar gauging site. The catchment area at this site is 1110 km^2. Here 1494 pairs of gauge and discharge were available. In this case (Case-I), the first 900 pairs of data were used to fit the conventional rating curve and to calibrate ANN and fuzzy based models, and the remaining 594 for validation.
5. A data set of 120 pairs of gauge and discharge data of Hirday Nagar gauging site covering both very low and very high values of flow were used to verify suitability of model for substantially less data. In this case (Case-II), the first 80 pairs of data were used to fit the conventional rating curve and to calibrate ANN and fuzzy based models, and the remaining 40 for validation.
6. A hypothetical data set (200 pairs) wherein the rating curve exhibits hysteresis. The hypothetical data sets of serial number 1, 3, 5, ... were used to calibrate the model and data sets at serial number 2, 4, 6, ... were used to validated the model. The hypothetical data set considered in this study is similar to one considered by Jain and Chalisgaonkar (2000).

Results and discussion

For each data set a rating curve given by Eq. (14) was fitted using the stage and discharge data selected for model calibration. Observed zero of the gauge values have been used for computing the water depths and subsequently the rating curves are developed by least square method linearly relating the log of the water depth with the log of the flow values. However, the current discharge can be better mapped by considering, in addition to the current value of river stage, the stage and the discharge at the previous time steps (Jain and Chalisgaonkar, 2000). Accordingly, the current study analyzed different combinations of antecedent gauge and discharge values. Six TS fuzzy models were developed during the analysis with the corresponding input vectors as follows:

Model 1: $Q_t = f(H_t)$
Model 2: $Q_t = f(H_t,H_{t-1})$
Model 3: $Q_t = f(H_t,Q_{t-1})$
Model 4: $Q_t = f(H_t,H_{t-1},Q_{t-1})$
Model 5: $Q_t = f(H_t,H_{t-1},H_{t-2},Q_{t-1})$
Model 6: $Q_t = f(H_t,H_{t-1},H_{t-2},Q_{t-1},Q_{t-2})$

where Q_t and H_t corresponds to the river flow and gauge at time t.

For the six input—output data vectors ANN models were also developed. The feed-forward back propagation ANN network used in this study consists of input neurons (gauge and discharge of previous time steps) in the input layer and a single output neuron (discharge) in the output layer with one hidden layer. The number of neurons in the hidden layer of the network was finalized after many trials. The newff function available in the neural network toolbox of MATLAB was used to create ANN model. During training the weights and biases of the network were adjusted using gradient descent with momentum weight and bias learning function. The default performance function for feed forward networks is mean square error (the average squared error between the network outputs and the target outputs).

The most significant factors, identified in the previous sections, were used to develop a fuzzy model. This was carried out into two steps: (i) determining cluster centers and hence the number of fuzzy rules and their associated membership functions; and (ii) optimizing the model utilizing TS fuzzy technique based on least square estimation (LSE). The clustering partitions a data set into a number of groups such that the similarity within a group is large than that among groups. Most similarity matrices are sensitive to the range of elements in the input vectors, therefore the data set under consideration has been normalized within the unit hypercube. The FIS of TS fuzzy model is generated using subtractive clustering. In order to find out optimum cluster centers and thus the optimum fuzzy model, the cluster radius (r_a) of subtractive clustering algorithm was varied between 0.1 and 1 with steps of 0.02. The cluster centers and thus the Gaussian membership function obtained for each case were used to compute consequent parameters through a linear least square method and a model was built. Using the global model performance indices such as root mean square error (RMSE) between the computed and observed discharge, the correlation coefficient and model efficiency (Nash and Sutcliffe, 1970), the optimal parameter combination of the model was sought. Once the optimization process is finished, the optimized membership functions for each input variable and consequent parameters are defined for an optimized TS fuzzy model. The process is repeated for each of the six data sets. Fig. 2 shows the membership functions for TS fuzzy model 5 developed for Jamtara site. Nine optimized fuzzy rules with nine membership functions for each variable are extracted (Table 1). It is evident from Table 1 that the output of the TS fuzzy model for each rule is in the form of linear equation. For every input vector a membership degree to each fuzzy set greater than 0 is computed from the Gaussian membership function. Therefore, all the rules fires simultaneously for each combination inputs and thus provides a crisp output value for a given input data vector using Eq. (4). The model

Table 6 Values of performance indices and error functions for fuzzy, ANN and conventional models — calibration and validation data of Hriday Nagar site (Case-II)

Fuzzy/ANN model (number of rules/nodes in hidden layer)	Calibration data fuzzy (ANN)		Validation data fuzzy (ANN)	
	(a) Coefficient of correlation (b) RMSE (m^3/s) (c) Efficiency (%)	(a) AE (m^3/s) (b) RE (%)	(a) Coefficient of correlation (b) RMSE (m^3/s) (c) Efficiency (%)	(a) AE (m^3/s) (b) RE (%)
$Q_t = f(H_t)$ (4/4)	(a) 0.991(0.981) (b) 38.72(41.76) (c) 94.15(93.07)	(a) 5.045(5.67) (b) 12.35(12.29)	(a) 0.979(0.979) (b) 94.35(94.62) (c) 84.40(87.32)	(a) 6.32(7.83) (b) 12.19(12.55)
$Q_t = f(H_t,H_{t-1})$ (4/4)	(a) 0.984(0.982) (b) 31.39(37.82) (c) 96.15(94.42)	(a) 5.02(5.45) (b) 11.21(11.33)	(a) 0.979(0.978) (b) 92.11(93.31) (c) 87.99(87.67)	(a) 6.08(6.17) (b) 11.01(11.82)
$Q_t = f(H_t,Q_{t-1})$ (5/4)	(a) 0.987(0.984) (b) 27.22(35.19) (c) 97.15(95.19)	(a) 4.94(5.07) (b) 9.78(9.02)	(a) 0.982(0.980) (b) 89.39(92.88) (c) 88.68(87.79)	(a) 5.71(5.77) (b) 9.51(9.47)
$Q_t = f(H_t,H_{t-1},Q_{t-1})$ (5/4)	(a) 0.987(0.985) (b) 22.88(35.11) (c) 97.95(95.19)	(a) 3.87(4.98) (b) 8.73(8.84)	(a) 0.984(0.981) (b) 85.11(91.23) (c) 89.74(88.22)	(a) 5.13(4.79) (b) 8.65(19.02)
$Q_t = f(H_t,H_{t-1},H_{t-2},Q_{t-1})$ (5/4)	(a) 0.989(0.986) (b) 20.03(35.02) (c) 98.43(95.21)	(a) 3.09(4.43) (b) 8.21(8.32)	(a) 0.984(0.982) (b) 84.97(88.47) (c) 89.78(88.92)	(a) 4.73(4.67) (b) 8.46(8.54)
$Q_t = f(H_t,H_{t-1},H_{t-2},Q_{t-1},Q_{t-2})$ (5/5)	(a) 0.989(0.986) (b) 20.49(35.89) (c) 98.36(94.97)	(a) 3.71(4.51) (b) 8.33(8.03)	(a) 0.981(0.981) (b) 85.97(88.46) (c) 89.83(88.92)	(a) 4.79(5.02) (b) 8.83(8.08)
Curve fitting	(a) 0.965 (b) 94.5 (c) 86.60	(a) 24.62 (b) 14.81	(a) 0.964 (b) 111.63 (c) 82.36	(a) 27.77 (b) 13.78

performance indices and error values (absolute and relative errors) were used to check the ability of the models during the calibration and validation stage.

Rating curves can be more accurately modeled by adding more input information in the form of stage as well as discharge values at previous time steps (Jain and Chalisgaonkar, 2000) as represented by models 3–6. In practice the discharge values of previous time steps are not readily available. Therefore, the model calibrated using the known pairs of gauge—discharge data sets was validated using computed discharge values in input data vector. In this process one/two known values of discharges of previous time steps were used to compute next one/two discharges as the case may be (one for models 3, 4, 5 and two for model 6). From the next step onwards the computed discharge values were used as input for computation of discharge from Models 3 to 6 following the procedure suggested by Sudheer and Jain (2003). Using computed discharge as input to these models may cause the error to carry over from one step to another. However, such carry over errors may not have any signifi-

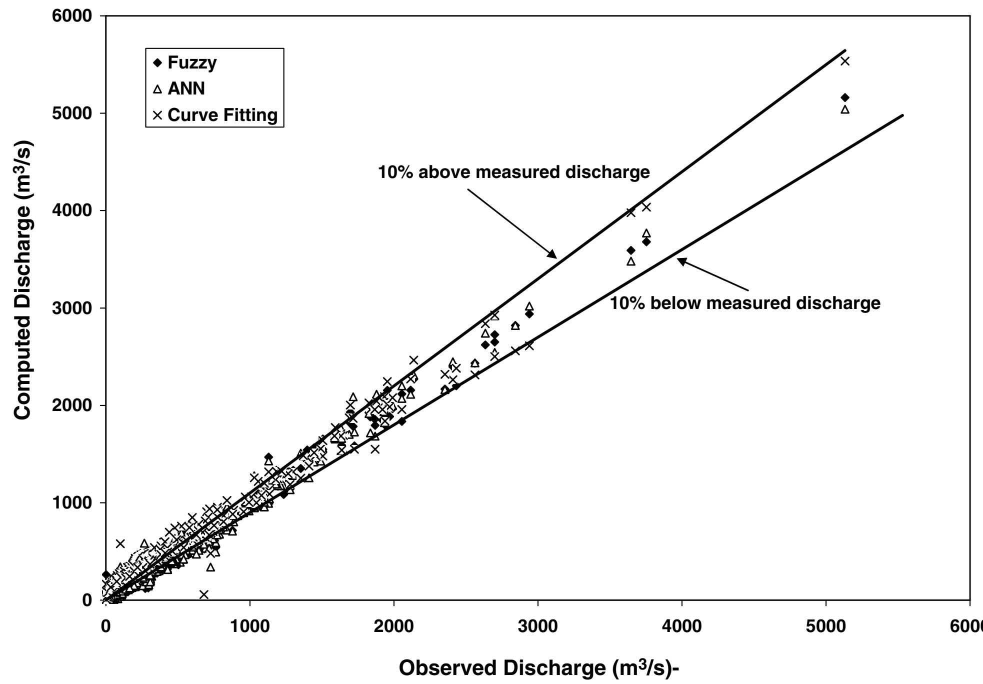

Figure 3 A scatter plot of computed and observed discharge data — Jamtara site.

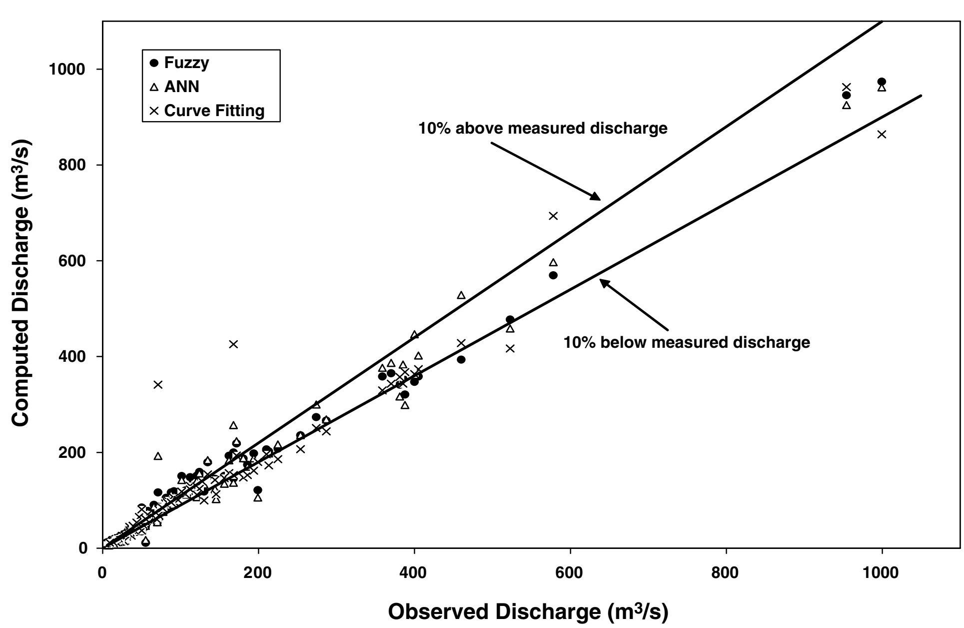

Figure 4 A scatter plot of computed and observed discharge data — Manot site.

cant affect due to higher accuracy of fuzzy and ANN model (Sudheer and Jain, 2003).

The fuzzy models are compared with the back propagation ANN model and conventional curve fitting approaches using all the six data sets (Tables 2—6). It can be seen from these tables that the RMSE values are significantly higher for the ANN and curve fitting approaches than the fuzzy approach. The fuzzy models also show higher coefficient of correlation. However, a clear cut picture is not evident using only coefficient of correlation as a performance measure. Therefore the model efficiency was also used to evaluate the performance of various model structures. In general the curve fitting approach shows very poor model efficiency. While, the fuzzy model shows better model efficiency in comparison to an equivalent ANN model. For the practical point of view, the accurate estimation of

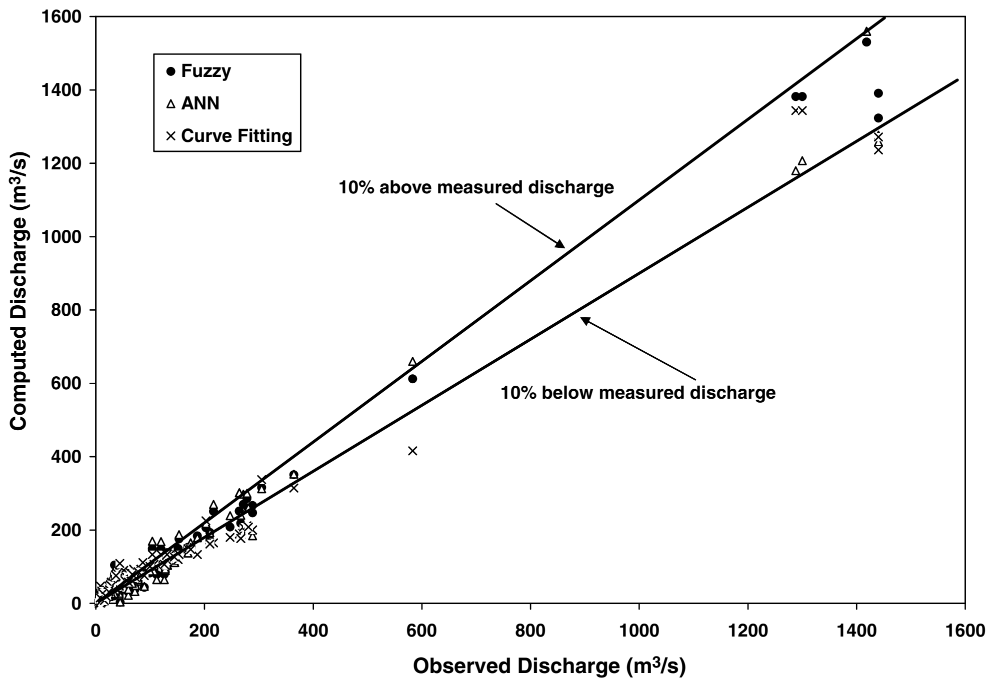

Figure 5 A scatter plot of computed and observed discharge data — Satrana site.

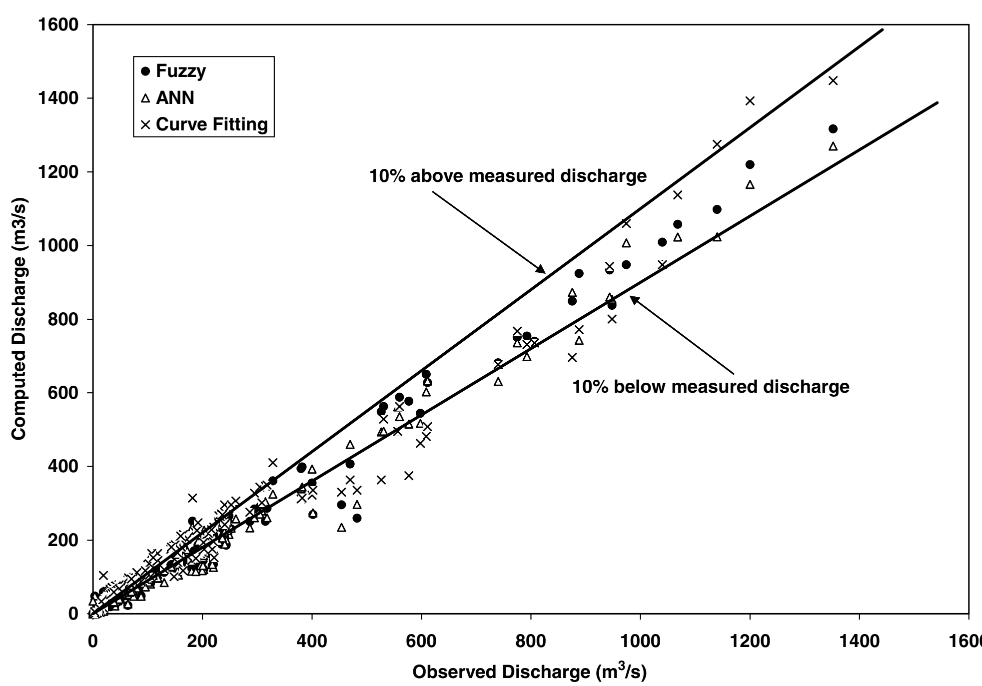

Figure 6 A scatter plot of computed and observed discharge data — Hridya Nagar site (Case-I).

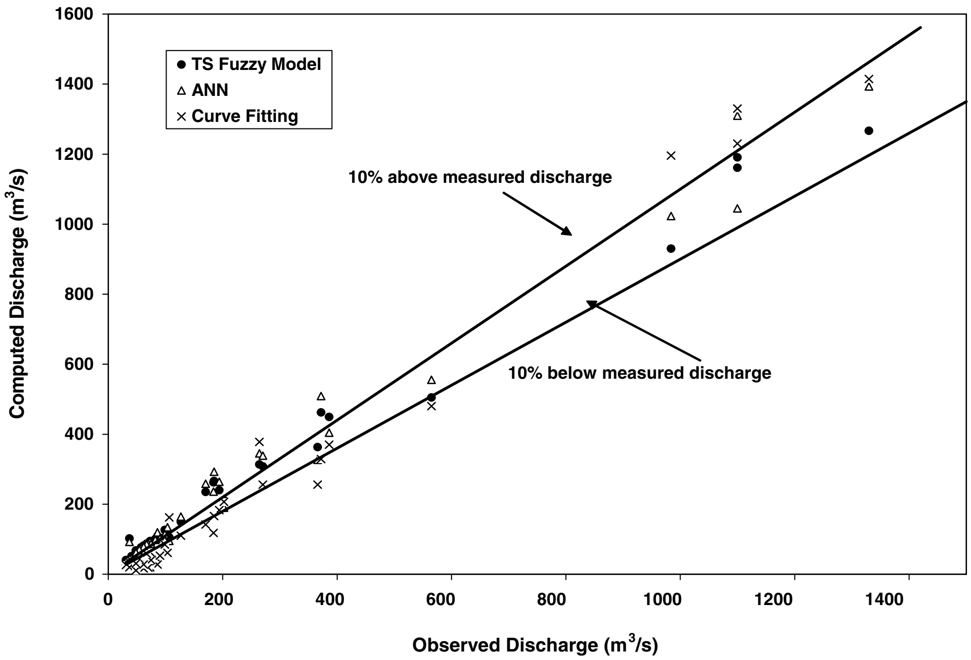

Figure 7 A scatter plot of computed and observed discharge data — Hridya Nagar site (Case-II).

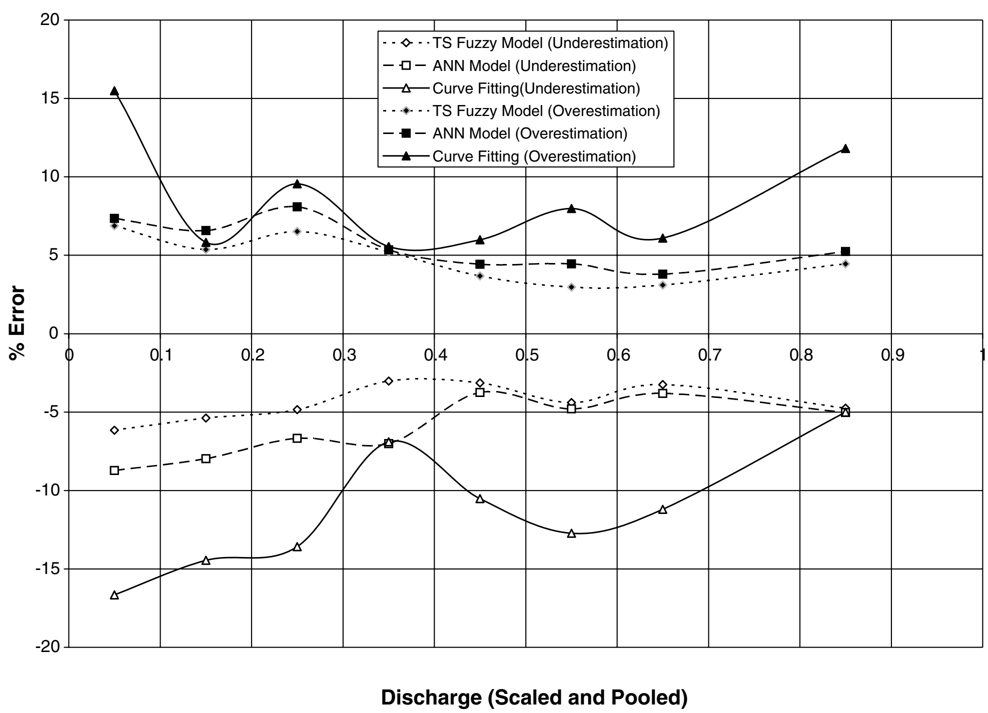

Figure 8 Variation of over and under estimation error with discharge for different model.

discharge is in particular important for the low and high flows. Therefore, the results were also compared using average absolute error (AAE) between observed and computed low flows and relative error (ARE) between observed and computed high flows. Keeping in view the flow pattern in the selected rivers, top 10% flow values were used to compute average relative errors and rest of the flow values were used to compute average absolute errors. It may be noted from the results (Tables 2—6) that the ANN performs better than the conventional curve fitting models and the fuzzy model outperforms both ANN and curve fitting models in both low flow and high flow region of rating curve. It is also evident from computed errors that fuzzy logic models provide proper discharge estimates in situations where functional relationship between stage and discharge is complex and can not be modeled very accurately by

Table 7 Values of performance indices and error functions for fuzzy, ANN and conventional models — calibration and validation data of hysteresis (hypothetical case)

Fuzzy/ANN model (number of rules/nodes in hidden layer)	Calibration data: (a) Coefficient of correlation (b) RMSE (m^3/s) (c) Efficiency (%)	Calibration data: (a) AE (m^3/s) (b) RE (%)	Validation data: (a) Coefficient of correlation (b) RMSE(m^3/s) (c) Efficiency (%)	Validation data: (a) AE (m^3/s) (b) RE (%)
$Q_t = f(H_t)$ (4/4)	(a) 0.939(0.926) (b) 141.1(155.3) (c) 91.21(89.35)	(a) 132.08(141.31) (b) 9.7(9.9)	(a) 0.936(0.919) (b) 144.2(160.7) (c) 90.37(89.67)	(a) 130.13(132.23) (b) 9.1(10.16)
$Q_t = f(H_t, H_{t-1})$ (5/2)	(a) 0.991(0.981) (b) 122.32(150.69) (c) 93.39(89.97)	(a) 90.42(93.21) (b) 7.50(7.90)	(a) 0.987(0.979) (b) 134.5(155.1) (c) 92.32(89.06)	(a) 110.43(111.56) (b) 7.65(8.23)
$Q_t = f(H_t, Q_{t-1})$ (6/3)	(a) 0.997(0.995) (b) 81.54(98.61) (c) 97.06(95.70)	(a) 85.21(89.10) (b) 6.19(6.62)	(a) 0.997(0.996) (b) 86.39(95.23) (c) 95.09(93.77)	(a) 97.56 (100.53) (b) 6.21(7.25)
$Q_t = f(H_t, H_{t-1}, Q_{t-1})$ (6/8)	(a) 0.998(0.998) (b) 26.81(33.51) (c) 97.11(95.81)	(a) 73.10(75.31) (b) 6.14(6.52)	(a) 0.998(0.997) (b) 27.44(32.29) (c) 97.38(95.70)	(a) 74.78(80.00) (b) 6.10(6.87)
$Q_t = f(H_t, H_{t-1}, H_{t-2}, Q_{t-1})$ (8/8)	(a) 0.999(0.999) (b) 14.76(19.93) (c) 98.81(96.70)	(a) 71.25(71.89) (b) 6.08(6.28)	(a) 0.999(0.999) (b) 15.83(21.25) (c) 98.78(96.69)	(a) 72.46(73.58) (b) 6.05(6.88)
$Q_t = f(H_t, H_{t-1}, H_{t-2}, Q_{t-1}, Q_{t-2})$ (8/8)	(a) 0.999(0.999) (b) 14.98(20.19) (c) 98.24(95.89)	(a) 72.38(73.1) (b) 6.74(6.80)	(a) 0.999(0.998) (b) 15.57(21.48) (c) 98.07(95.61)	(a) 75.15(75.34) (b) 6.35(6.81)
Curve fitting	(a) 0.843 (b) 161.21 (c) 85.52	(a) 139.42 (b) 7.24	(a) 0.841 (b) 164.93 (c) 84.35	(a) 139.41 (b) 7.24

conventional rating curves. As more and more information (input) is added to the model, generally the coefficient of correlation improves and RMSE is reduced. This may be due to auto correlation and cross correlation structure in the input data vector. Increase in inputs beyond a certain limit in the model cause reduction in model performance.

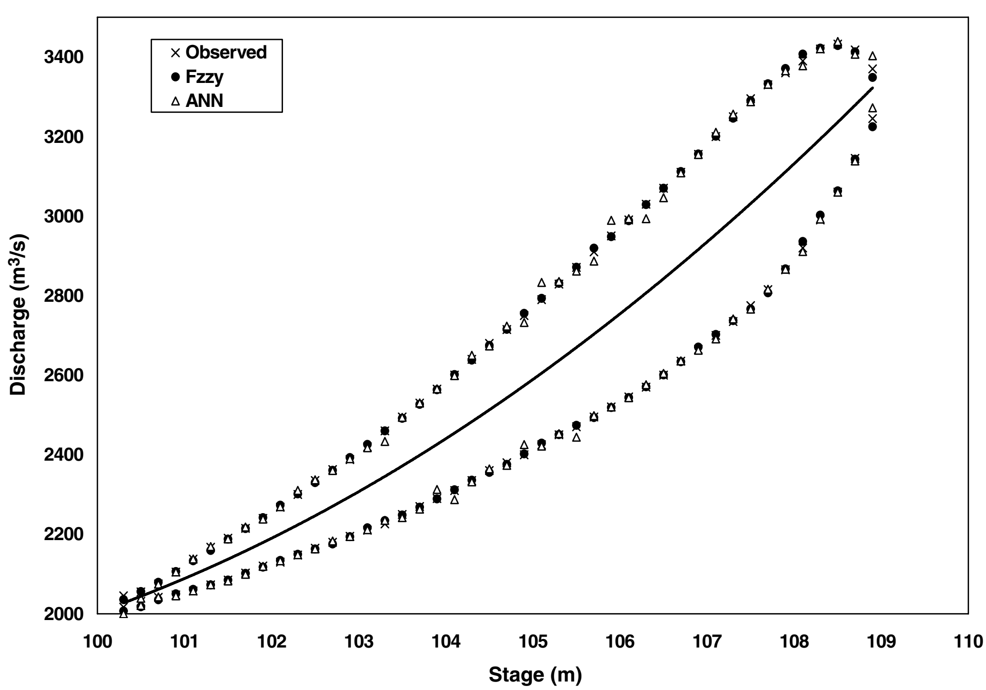

Figure 9 Comparison of computed discharges at various gauges using curve fitting, ANN and fuzzy logic — Hypothetical data.

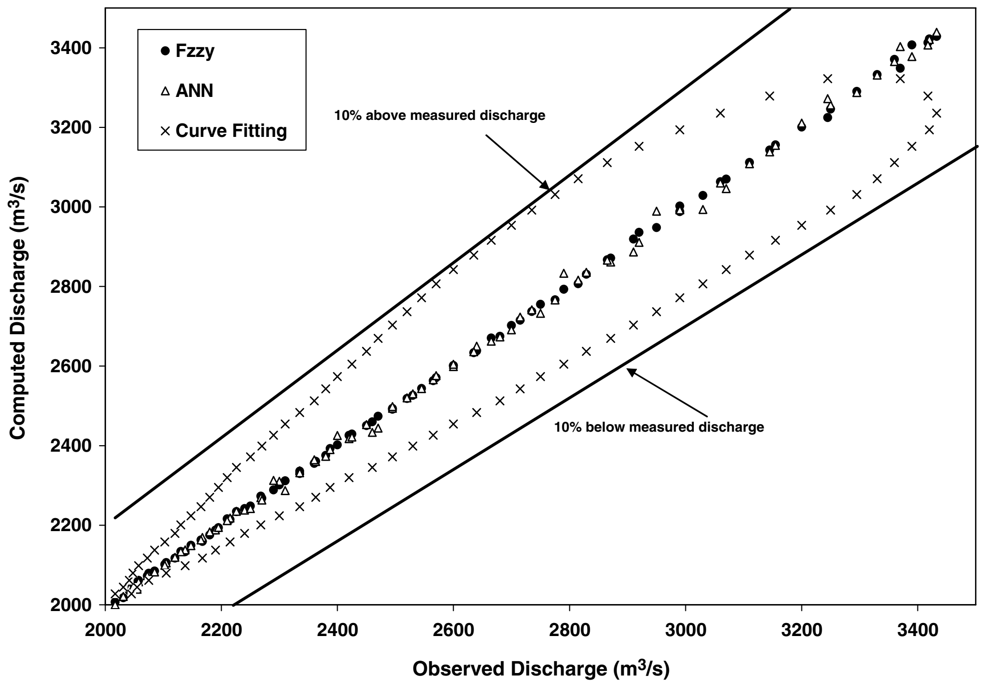

Figure 10 A scatter plot of computed and observed discharge data — Hypothetical data.

This is due to reduction in auto correlation and cross correlation in the input data vector. From Tables 2—6, it is also apparent that model 5, which consists of three antecedent gauges and one discharge in input, showed the highest coefficient of correlation, minimum RMSE and maximum model efficiency, and it is selected as the best fit model for describing gauge—discharge relationship of all the selected gauging sites. Figs. 3—7, illustrate that the estimated discharge values from fuzzy and ANN models are generally falls within ±10% of observed discharges. However, the discharge computed by conventional curve fitting approach in general showed more than 10% variation from observed discharges. Furthermore, Figs. 3—7 illustrate that the discharges estimated by fuzzy logic model are very close to observed discharge values. All the three (conventional rating curve, fuzzy model and ANN model) models developed for data sets 1 to 5 are based on varying data lengths from 80 to 900 gauge—discharge pairs. This indicates that the irrespective of data length the fuzzy models are always more accurate than the equivalent ANN or conventional rating curve models developed for selected gauging sites.

A model with a minimum RMSE may not be sufficient to eliminate the uncertainty in the model choice. In order to estimate bias of the Fuzzy, ANN and curve fitting models for different output ranges the testing data sets of four gauging sites were scaled so as to lie in the range of zero to one and than pooled together. Further, the scaled and pooled data set was divided into two sets: (i) data set with the data points for which models generally underestimated discharge; and (ii) data set containing the points for which discharge was overestimated. The average underestimation and overestimation errors were computed and plotted for the following eight discharge ranges: 0—0.1, 0.1—0.2, 0.2—0.3, 0.3—0.4, 0.4—0.5, 0.5—0.6, 0.6—0.7, 0.7—1.0 (Fig. 8). It is seen that in each range the overestimation and underestimation error of fuzzy model is more or less same and also generally less than the ANN and curve fitting models. This indicates the absence of bias in fuzzy model in all ranges of discharges. The underestimation error in the low flow is more for curve fitting approach and in high flow region all the three models are very close to each other. However, the over estimation error is in the same range for all the three models in low flow situation. While, in high flow situation the over estimation error is very small for fuzzy model. In general fuzzy model gave very accurate estimation for all the flow situations.

Modeling hysteresis

Using the curve fitting approach the modeling of hysteresis is not possible as it fits an average or steady state curve. Jain and Chalisgaonkar (2000) applied ANN to model hysteresis considering a hypothetical case. In this study almost a similar loop curve is modeled using both ANN and fuzzy logic based approach and the results are presented in Table 7. The results indicate that the fuzzy model performs better than the ANN approach for modeling hysteresis. Further, it is seen from this table that the models utilizing more information of previous time periods are useful in estimating the hysteresis more accurately. Here, in this case models 5 and 6 showed highest correlation and minimum RMSE in comparison to other model structures. Fig. 9 shows the estimated discharges at various stages using curve fitting, ANN and fuzzy models. The results clearly indicate that the conventional curve fitting approach failed to model the hysteresis present in the data. Fig. 9 shows a slight difference in the middle and upper part of the estimated and hypothetical loops when ANN modeling approach was applied. However, the fuzzy logic based modeling approach models the loop rating curve more accurately throughout entire data range and this can be better observed from Fig. 10.

Conclusions

Any attempt to improve the modeling of gauge—discharge rating curves is significantly important for rainfall-runoff modeling and flood forecasting. In this study, a fuzzy logic algorithm is developed to estimate the discharge from measured gauge data. The satisfactory estimation of the discharge by the proposed fuzzy models from the six different data sets indicate that gauge—discharge modeling can reliably employ the fuzzy model. Also, in this study, the fuzzy model is tested against both the ANN and conventional curve fitting approach. The fuzzy models estimate the discharge more accurately both in case of actual observed data sets of various lengths and hypothetical data set. The ANNs can be synthesized without making use of the detailed and explicit knowledge of the underlying physics and a fuzzy logic algorithm has the ability to describe the knowledge in a descriptive human-like manner in the form of simple rules using linguistic variables. The fuzzy model proposed herein may be concluded to be appropriate for modeling of rating curves.

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