An Improved Trilateral Filter for Image Denoising using an Effective Impulse Detector

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***Abstract*— In this paper, we improve the trilateral filter for removing the mix of impulse noise and Gaussian noise by adopting a new local image statistic to identify noise pixels in image corrupted with impulse noise, new statistic is called the Rank-ordered Absolute Differences statistics based on Extremum Compression (EC-ROAD). The EC-ROAD can effectively characterize each image pixel as either an impulse or an unaffected pixel, then we incorporate it into the bilateral filter by adding a third component to the weighting function. A switching mechanism is adopted to smooth the impulse noise and Gaussian noise. To improve the performance of our method we investigate the question of selecting optimal parameters. Our method has demonstrated superior performance in suppressing impulse noise and mixed noise, and outperformed other competitive methods both in terms of quantitative measures of image restoration and qualitative judgments of image quality.**

Keywords- Trilateral filter; EC-ROAD; impulse noise; mixed noise; parameter selection;

I. INTRODUCTION

Image noise is inevitably present in the image acquisition and transmission processes. A fundamental problem of image processing is to remove noise from an image while preserving its features as much as possible. The nature of the problem depends on the type of noise added to the image. Commonly, two noise models can adequately represent most noise added to the image: additive Gaussian noise and impulse noise.

Additive Gaussian noise is characterized by adding to each image pixel a value from a zero-mean Gaussian distribution (AWGN, Additive White Gaussian Noise). Such noise is usually introduced during image acquisition. The zero-mean property of the distribution allows such noise to be removed by averaging pixel values locally. Many denoising methods have been developed to remove AWGN over the years. Classical linear filters, such as Gaussian filter, smooth noise efficiently but blur edges significantly. To solve this problem, nonlinear methods is used to preserve edges, such as anisotropic diffusion [1], bilateral filter [2]. The essence of these methods is to use local measures of an image to quantitatively detect edges and smooth them less than the rest of the image. Impulse noise is characterized by replacing a portion of an image's pixel values with random values, leaving the remainder unchanged. It can be introduced due to transmission errors. The intensities of the corrupted pixels are much different from their neighbors. The Gaussian noise removal methods cannot adequately impulse noise because they treat the noise pixels as details to be preserved. For this reason, many nonlinear filters, such as the extensions of the median filter [3], or otherwise use rank statistics [4], have been developed for the removal of impulse noise. The common idea among these filters is to detect the impulse pixels and replace them with estimated values, while leaving the remaining pixels unchanged. Obviously, when applied to images corrupted with Gaussian noise, such filters are ineffective and leave noisy pixels unchanged.

Removing the mixture of Gaussian and impulse noise (mixed noise) is difficult and there has not been much work carried out on building these filters. Peng and Lucke suggested a fuzzy filter designed for mixed noise [5]. Abreu et al. [6] proposed the effective method to remove impulse noise, i.e., median-based SD-ROM filter. They also demonstrated the SD-ROM filter's ability to remove Gaussian noise as well as mixed noise. In 2005, Garnett et al. proposed a universal noise removal algorithm with an impulse detector [7]. This algorithm (trilateral filter) combined bilateral filter with an impulse weight based Rank-ordered Absolute Differences (ROAD) statistic, which is employed to detect and filter impulse noise. Garnett et al. point out the proposed trilateral filter can effectively remove impulse, Gaussian, and mixed noise, and outperform the previous methods. However, we find the robustness of the ROAD drops very fast with the increase of impulse noise levels.

In this paper, an improved trilateral filter for the removal of impulse, Gaussian, and mixed noise is present. Our work mainly includes: 1) an new image statistic (EC-ROAD) for impulse noise detection is proposed and 2) investigate the selection question of optimal parameters. The behavior of our method can be adaptively changed to remove impulses while retaining the ability to smooth Gaussian noise. Additionally, our method can be easily adapted to remove mixed noise.

The remainder of the paper is organized as follows. In Section II, we introduce Rank-ordered Absolute Differences (ROAD) statistic and Extremum Compression ROAD (EC-ROAD) statistic for detecting impulses. In Section III, we briefly explain the bilateral filter and describe how to incorporate the EC-ROAD statistic into the bilateral filter to create a trilateral filter. In Section IV, experimental results are given to demonstrate our method's effectiveness. Finally, conclusions are provided in Section V.

II. EXTREMUM COMPRESSION ROAD STATISCIS

A. Noise Models

We define the noisy image corrupted by additive Gaussian noise as

$$u(i,j) = u^0(i,j) + n(i,j) \tag{1}$$

where $u^0(i,j)$ is the original pixel intensity value of image u^0, $u(i,j)$ is the corrupted pixel value, and $n(i,j)$ is the Gaussian noise at the pixel location (i,j)

For an image of 8-bit grayscale pixel resolution, the pixel intensities lie in the dynamic range $[L_{\min}, L_{\max}]$, where $L_{\min}$ and $L_{\max}$ are the lowest and highest intensities, respectively. Regardless of its origin, impulse noise exhibits nonstationary statistical characteristics [8] and only a portion of pixels in the image is contaminated by impulse noise. Based on this fact, the model for impulse noise with probability p is defined as

$$u(i,j) = \begin{cases} n(i,j) & with \ \ probability \quad p \\ u^0(i,j) & with \ \ probability \ \ 1-p \end{cases} \tag{2}$$

where n denotes impulse noise again.

B. Rank-ordered Absolute Differences (ROAD) Statistic

Garnett et al. [6] proposed a local image statistics called Rank-ordered Absolute Differences (ROAD) statistics to detect impulsive pixels in an image. It provides a new measure of how close a pixel value is to its m most similar neighbors.

Let $x = (x_1, x_2)$ be the location of the pixel under consideration, and let Ω_x denote a $r \times r$ neighborhood centered at x. For each pixel $y \in \Omega_x$, define $d(x,y)$ as the absolute difference in intensity of the pixels between x and y

$$d(x,y) = | u(x) - u(y) | \tag{3}$$

let $Dist(d)$ denote the vector whose elements are the sorted d values in ascending order, and define

$$ROAD_m(x) = \sum_{k=1}^{m} \alpha_k(x) \tag{4}$$

where m is the number of most similar neighbors with x or the index of $Dist(d)$, and $\alpha_k(x) = kth$ smallest $d(x,y)$ in $Dist(d)$. The ROAD values are very high for impulse noise pixels and much lower for uncorrupted pixels. In general, the ROAD statistic can output appropriate values for the lower levels of impulse noise, but the situation is unchanged when the impulsive noise level is higher, the ROAD statistic often fails to detect the impulse noise pixel.

An example of the ROAD statistic is illustrated in Fig. 1(a). Test image is the 512×512 8-bit grayscale Lena image and corrupted with salt-and-pepper noise $(p = 50\%)$. We only consider the case of $m = 4$ in a 3×3 neighborhood of center at x. We select a neighborhood centered at $(100,100)$ from the corrupted Lena image, namely, $\Omega_{100,100}$, $u(100,100)$ is a pixel corrupted by impulse noise, as illustrated in Fig. 1.

As illustrated in Figure 1 (a), ROAD(100,100)=0. According to [7], the pixel $u(100,100)$ is deemed to be uncorrupted pixel. Result is the ROAD statistic fails to detect the impulse noise pixel. False detection results lead to the invalidation of the following switching mechanism. This situation often happens because impulses often "clump" together in the noisy image with high noise levels. It is difficult for the ROAD statistic to characterize each image pixel as either an impulse or an unaffected pixel.

$$\Omega_{100,100} = \begin{pmatrix} 255 & 99 & 255 \\ 255 & 255 & 104 \\ 102 & 255 & 106 \end{pmatrix}$$

(a)

$$\begin{pmatrix} 0 & 99 & 0 \\ 0 & - & 104 \\ 102 & 0 & 106 \end{pmatrix}$$

$$Dist(d) = \{0,0,0,0,99,102,104,106\}$$

$$a_1 = a_2 = 0$$
$$a_3 = a_4 = 0$$

The four smallest absolute differences

$$\text{ROAD}(100,100) = \sum_{k=1}^{m} a_k = 0$$

Final calculation of ROAD

(b)

$$V_{100,100}^{EC} = \{99,102,104,106,255\}$$

Extremum Compression result

$$Dist(d') = \{0,149,151,153,156\}$$

$$b_1 = 0, b_2 = 149$$
$$b_3 = 151$$

The three smallest absolute differences

$$\text{EC-ROAD}(100,100) = \sum_{k=1}^{m'} b_k = 300$$

Final calculation of EC-ROAD

Figure 1. Demonstrating how to calculate ROAD and EC-ROAD

C. Extremum Compression ROAD Statiscis (EC-ROAD)

To cope with the deficiency of ROAD statistic, we propose a new statistic method called Extremum Compression ROAD (EC-ROAD) statistic. The EC-ROAD includes four steps:

1) The pixel values in Ω_x, except for central pixel x, are rearranged in a vectorized format. Let V_x denote the sorted vector in ascending order.

2) Compress the maximum or the minimum in V_x, and only reserve one maximum or minimum (denoted by V_x^{EC}).

3) Calculate $d'(x,y') = | v(x) - V_x^{EC}(y') |$, y' is the index of reserved pixel, let $Dist(d')$ denote the sortee $d'(x,y')$ in ascending order.

4) Let $m' = Round(Dim(V_x^{EC})/2)$ denote the index of V_x^{EC}, $Dim(\cdot)$ function gets the dimension of the vector V_x^{EC}, $Round(\cdot)$ represents the round function, We define.

$$EC - ROAD_{m'}(x) = \sum_{k=1}^{m'} b_k(x) \quad (5)$$

and $b_k(x) = kth$ smallest $d'(x, y')$ in $Dist(d')$.

As illustrated in Figure 1 (b), the EC-ROAD statistic has a very high value. it does very well on the detection of impulse noise. In the experiments, we have found that the trilateral filter [7] based on ROAD statistic often remains some unremoved impulses when $p \geq 20\%$. For EC-ROAD statistic, this happens when $p \geq 40\%$. In addition, EC-ROAD statistic has the nearly same value with ROAD statistic for images only corrupted by additive Gaussian noise.

III. THE IMPROVED TRILATERAL FILTER

Garnett et al. extended the bilateral filter [2] to create a nonlinear filter capable of removing impulse, Gaussian, and mixed noise from images via integrating impulsive properties. The new nonlinear filter is called the trilateral filter.

A. Bilateral Filter

The bilateral filter is a nonlinear filter proposed by Tomasi and Manduchi to smooth images. It has been successfully used in removing AWGN while preserving edges. The bilateral filter takes a weighted sum of the pixels in a local neighborhood, the weights depend on both the spatial distance and the intensity distance. Let $N(x)$ be a spatial neighborhood of x, The weight of each pixel $y \in N(x)$ is the product of two components, one spatial weight component and one intensity weight component

$$w(x, y) = w_S(x, y) w_R(x, y) \quad (6)$$

where

$$w_S(x, y) = \exp\left(-|x - y|^2 \big/ 2\sigma_S^2\right) \quad (7)$$

and

$$w_R(x, y) = \exp\left(-|u(x) - u(y)|^2 \big/ 2\sigma_R^2\right) \quad (8)$$

Finally, the restored pixel $\widetilde{u}(x)$ is calculated as follows

$$\widetilde{u}(x) = \frac{\sum_{y \in N(x)} w(x, y) u(y)}{\sum_{y \in N(x)} w(x, y)} \quad (9)$$

The w_S weighting function gives higher weight to pixel y that are spatially close to the central pixel x, and the w_R weighting function is used to penalize the difference of intensities between x and y.

In weighting functions, the parameter σ_S and σ_R control the behavior of the bilateral filter. σ_S and σ_R characterizes the spatial and intensity domain behaviors. In case of image denoising applications, the selection of optimal parameter values has not a universal rule, Zhang and Gunturk [9] pointed out the intensity domain parameter σ_R is more critical than the spatial domain parameter σ_S

B. Trilateral Filter Based on EC-ROAD Statistic

As analyzed above, The ROAD statistic fails to measure each image pixel as either an impulse or an unaffected pixel. In this paper, we use the EC-ROAD statistic instead of the ROAD statistic, and incorporate the EC-ROAD statistic into the bilateral filter by introducing a weighting function influenced by impulse noise. The "impulsive" weight w_I at a point x is given by

$$w_I(x) = \exp\left(-ROAD_{EC}(x)^2 \big/ 2\sigma_I^2\right) \quad (10)$$

we set $ROAD_{EC}(x)$=EC-ROAD(x) in (10) and (11) for simplicity. The parameter σ_I is to penalize high EC-ROAD values.

To add the impulsive weight while still retaining the intensity component of the bilateral filter. A switching mechanism is proposed to determinate how much to use the intensity component in the presence of impulse noise, a "joint impulsivity" J of $y \in N(x)$ with respect to x is defined as:

$$J(x, y) = \exp\left\{-\left(\frac{ROAD_{EC}(x) + ROAD_{EC}(y)}{2}\right)^2 \Bigg/ 2\sigma_J^2\right\} \quad (11)$$

The $J(x, y)$ function assumes values in [0,1]. The σ_J parameter controls the shape of the function. If at least one of x or y is impulse noise, and has a high EC-ROAD value, then $J(x, y) \approx 0$. If neither pixel is impulse noise, and neither has a high EC-ROAD value, then $J(x, y) \approx 1$.

Finally, we can integrate the impulsive component into a nonlinear filter designed to weight pixels based on their spatial, intensity, and impulsive properties. The combined nonlinear filter is called as the trilateral filter. The trilateral weight of y with respect to the central point x is defined as

$$w(x, y) = w_S(x, y) w_R(x, y)^{J(x,y)} w_I(y)^{1 - J(x,y)} \quad (12)$$

Equation (12) has the following properties: the intensity weight is more heavily when $J(x, y) \approx 1$ to smooth regions without impulses and less heavily when $J(x, y) \approx 0$, because if either pixel is an impulse, the intensity weight fails to function correctly as illustrated above. Conversely, the impulsive weight is less heavily when $J(x, y) \approx 1$ and more heavily when $J(x, y) \approx 0$. By using this switching mechanism, the appropriate weight is applied on each pixel in $N(x)$.

The improved trilateral filter works very well to remove impulse noise without reducing the bilateral filter's ability to remove Gaussian noise. For images with no impulse noise, and, thus, few points with high EC-ROAD values, $J(x, y)$ term in (12) effectively shuts off the impulsive component of the weight and only uses the intensity and spatial weights.

C. Selection of Parameters

There are four filtering parameters control the behavior of our method. Through numerous experimentations, we observe that the following optimal values of parameters can obtain better results.

For the removal of Gaussian noise, the trilateral filter reverts to the bilateral filter as described above. A good range for σ_S is roughly $[1,3]$, this setting not only can effectively average out the noise, but also preserve edges. The optimal σ_R value changes significantly as the noise standard deviation changes, it is set to be about twice or triple the standard deviation of the added AWGN for automatically selecting σ_R . In addition, in this paper we assume that the standard deviation σ of AWGN in an image is known as a priori knowledge.

For suppressing impulse noise, the smaller values of σ_S work better, such as $\sigma_S \in [0.3,0.8]$, the smaller σ_S values give higher weights to pixels that are spatially close to the center pixel, edges of image can better preserved. σ_R also has a great influence on filtering results, if σ_R is smaller, noisy data could remain isolated and untouched, the optimal range of σ_R is roughly $[40,70]$.

To remove mixed noise. A good range for σ_S is roughly $[0.9,1.5]$, σ_R is set to be about fourfold the standard deviation of AWGN. This setting can work very well on the removal of mixed noise.

We also have found through experimentations that a proper value for σ_I is 50. In general, σ_I does well to remove impulse, Gaussian and mixed noise in the interval $[40,60]$. Experimental results have shown that the optimal value of σ_J for most images is around 60, but any value in the interval $[50,70]$ work well on the removal of noise.

IV. EXPERIMENTAL RESULTS

To validate the noise removal capabilities of our method, we performed various experiments and compared the results with several existing methods. In the experiments, neighborhood size is set to 5×5 , we used grayscale Lena and Barbara images of size 512×512 and 8-bit resolution as test images. Our method produced results superior to the compared methods in terms of visual quality and quantitative measures.

In this paper, we used the peak signal-to-noise ratio (PSNR) to give quantitative measures of restored image by different methods. If u^0 is the original image of size $m\times n$ and $\tilde{u}$ is a restored image of u^0 , the PSNR is given by

$$PSNR(\tilde{u}) = 10\log_{10}\left(\frac{\sum_{i,j=1}^{m,n} 255^2}{\sum_{i,j=1}^{m,n}(\tilde{u}(i,j)-u^0(i,j))^2}\right) \tag{13}$$

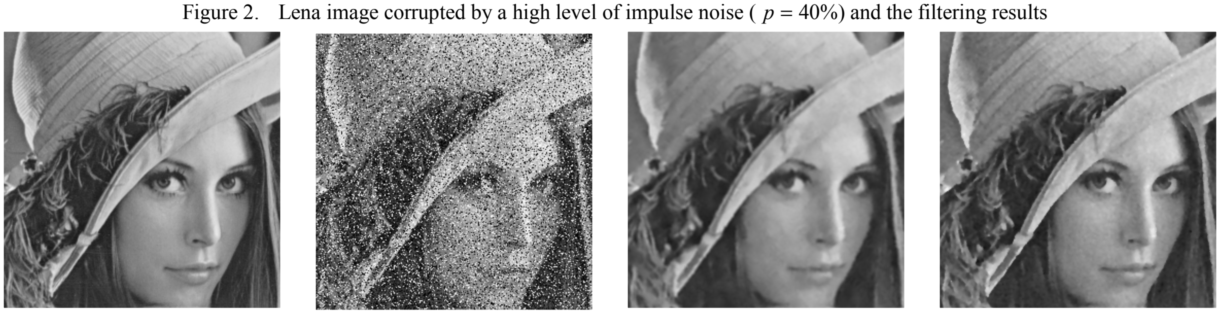
Larger PSNR values signify better image restoration. We tested each method on impulse noise levels from $p = 10\%$ to 50% in steps of 10%. The compared methods include the 5×5 standard median filter (SMF), the adaptive median filter (AMF), and the trilateral filter. For the trilateral filter, the iteration must be implemented for $p \geq 20\%$. For high levels of noise, applying several iterations avoids residual impulse noise, but increasing the number of iteration results in over-smoothing.

We compared the performance of our method with the performance of SMF, AMF, and the trilateral filter on images corrupted with impulse noise and mixed noise. For each method tested, we varied its parameters exhaustively to obtain the best possible result. As mentioned above, EC-ROAD and ROAD statistic have the nearly same value for AWGN. For the removal of AWGN, our method and the trilateral filter has the nearly same result. In general, the result of both our method and the trilateral filter in terms of PSNR is slightly lower than the result of bilateral filter for removing AWGN.

Fig. 2 shows the Lena image corrupted with 40% impulse noise and compared the output of our method with the outputs of AMF and the trilateral filter. Our method removed well the noise while preserving edges, and produced the best visual effects. The output of AMF appeared plenty of artifacts and the staircasing effects in edges, such as hair. The trilateral filter blurs details in the image.

(a) Corrupted by impulse noise (b) AMF (c) Trilateral filter [7] (d) Our method

Figure 2. Lena image corrupted by a high level of impulse noise (p = 40%) and the filtering results



(a) Original image (b) Corrupted by missed noise (c) Trilateral filter [7] (d) Our method

Figure 3. Lena image corrupted by mixed noise (σ = 10, p = 20%) and the filtering results

TABLE I. PSNR VALUES AFTER APPLYING VARIOUS METHODS TO IMAGES CORRUPTED WITH IMPULSE NOISE

Method	Lena image					Barbara image				
	$p = 10\%$	$p = 20\%$	$p = 30\%$	$p = 40\%$	$p = 50\%$	$p = 10\%$	$p = 20\%$	$p = 30\%$	$p = 40\%$	$p = 50\%$
Currupted	15.42	12.43	10.68	9.45	8.46	15.29	10.27	10.48	9.22	8.26
SMF(5×5)	30.87	30.31	29.56	28.41	27.77	24.02	23.81	23.52	23.04	22.68
AMF	38.03	35.86	33.59	31.99	29.89	28.83	27.62	26.41	25.25	24.05
Trilateral Filter [7]	37.82	35.13	33.12	31.26	29.44	28.65	27.35	26.06	24.75	23.34
Our method	39.46	36.63	33.98	32.35	30.17	29.51	28.27	26.79	25.46	24.18

Trilateral filter is implemented iteratively for $p \geq 20\%$, the number of iteration is respectively 1, 1, 3, and 4 for $p = 20\%$, 30%,40%, and 50%.

Our method need not iteration for $p < 35\%$, only one iteration for $p \leq 50\%$.

TABLE II.
PSNR VALUES OF NOISY IMAGES RESTORED
WITH VARIOUS METHODS

Method	Mixed Noise			
	σ =10, p =20%		σ =10, p =30%	
	Lena	Barbara	Lena	Barbara
Corrupted	12.37	12.18	10.60	10.42
Trilateral Filter [7]	31.64	30.41	30.07	28.95
Our method	32.26	31.18	30.96	29.69

The trilateral filter and our method can effectively remove mixed noise. Fig. 3 shows the simulation results for Lena image corrupted by mixed noise, AWGN of standard deviation 10 and impulse noise with $p = 20\%$. The output of our method was superior to the trilateral filter in visual quality.

Once the visual quality of images restored by our method had been confirmed, we concentrated on the quantitative measures of the restored image. We tested each method on impulse noise levels. Table I shows the PSNR values of the restored Lena and Barbara images. Our method provides results with higher PSNR values than the results of the trilateral filter tested for each level of noise.

We also compared the performance of our method with the performance of the trilateral filter on images corrupted with mixed noise. As seen in the PSNR results of Table II, our method is better than the trilateral filter.

V. CONCLUSION

Removing the mixture of Gaussian and impulse noise is very difficult, there has not been much work carried out on building this kind of filters. Many algorithms for removing Gaussian noise, such as the bilateral filter, tend to treat impulse noise as edge pixels, and output unsatisfactory results. Furthermore, algorithms for the removal of impulse noise, such as median filter, only use one noisy pixel value instead of another noisy pixel, and fail to remove Gaussian noise

In order to process mixed noise, we propose a new statistic method called Extremum Compression ROAD statistic (EC-ROAD), then incorporate the EC-ROAD statistic into the bilateral filter by adding a third component to the weighting function. A switch based on the EC-ROAD statistic is adopted to adjust weight distribution between the intensity and impulsive components. Meanwhile, we investigate the setting of filtering parameters, and select optimal parameters to improve the performance of our method further. Compared to other nonlinear filters, our method consistently performs well in removing the impulse noise and mixed noise.

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