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O.R. Applications

A linear programming approach for aircraft boarding strategy

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Abstract

Airlines are going through very difficult times financially which has been triggered by fierce competition and current high fuel prices. To stay competitive, it is absolutely essential that they achieve high efficiency in the areas that have control over. Airlines start generating revenues while their aircraft are flying. Reducing aircraft turn-around times is an important goal with passenger boarding being a major metric.

This paper examines the interferences among the passengers that cause delays in boarding times for a single aisle aircraft. It offers a new mixed integer linear program to minimize these interferences. We then apply this mathematical model to an Airbus-320 aircraft. Alternative efficient solutions are generated based on the rate of passenger boarding. A simulation model is adopted to identify appropriate boarding patterns. The recommended boarding patterns provide a lower number of interferences than other strategies as well as accommodating neighboring passengers to board together.

Keywords: OR in airlines; Integer programming; Transportation; Boarding strategy

1. Introduction

The airlines are currently undergoing difficult times financially. The increase in fuel prices, competition from low cost carriers and operations inefficiency has resulted in bankruptcies or major losses for airlines all over the world. The second, third and fourth largest carriers in the US have all filed for bankruptcy. It is therefore extremely important for airlines to be efficient in the areas that they have control over. Bazargan (2004) provides descriptions and mathematical models commonly adopted by the airlines for scheduling and improved efficiency.

Airlines start generating revenue by utilizing and flying their aircraft; they do not generate any revenue while their aircraft are on the ground. As a result, the turn-around time is a major metric for an airline's operations (Van de Briel et al., 2005). The time from the arrival of the aircraft until its next departure constitutes

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turn-around time. To have a higher utilization of their aircraft, airlines attempt to minimize the turn-around times (Ferrari and Nagel, 2004; Van Landeghem and Beuselinck, 2002; Lewis and Lieber, 2005). The components in turn-around times include taxi-times, passenger/baggage deplaning, maintenance checks, fueling and passenger/baggage boarding. Taxi-times are influenced by air/ground congestions at the airports. Therefore, airlines flying short-haul flights typically select airports (within the same region) with low air/ground congestions (Bazargan and Vasigh, 2003).

Van Landeghem and Beuselinck (2002) identify that the typical aircraft turn-around (coming to and leaving the gate) is approximately 30–60 minutes. They do note, however, that this time is almost always in excess of 30 minutes. A major component in this metric is the passenger boarding time. Airlines seem to adopt different aircraft boarding strategies based on airline culture and service level (Van de Briel et al., 2005). Some airlines do not impose any strategy and let the passengers board randomly. Others arrange the passengers into groups, zones or call-offs based on specific boarding strategy adopted by the airline. Each of these groups is then called to board the aircraft in sequence.

The most common traditional boarding aircraft strategies are back-to-front (BF) and window-to-aisle (out–in or OI). In both strategies, passengers are typically assigned to groups. First-class and special need passengers are boarded first. In the back-to-front strategy, as the name implies, passengers start filling up the aircraft from back to the front. The passengers are divided into groups based on the row of their seats from the back of the aircraft to the front. Each group is then called in sequence to board the aircraft (see Figs. 2–4 when $\alpha = 1$). In the out–in (OI) strategy, the emphasis is based on the locations of the seat, whether it is a window, middle or aisle seat (see Fig. 2 when $\alpha = 0$). Different airlines either adopt one of these two strategies or a hybrid of the two.

Surprisingly, the aircraft boarding strategy has received little attention from both academia and the industry. The few reported research and published works on this topic focus on modeling the problem using computer-based simulation (see for example Ferrari and Nagel, 2004; Ferrari, 2005; Van Landeghem and Beuselinck, 2002). While these methods provide a good understanding of existing boarding strategies and enable us to evaluate various known strategies and conduct what-if scenarios, they do not help us find the best and possibly unknown alternatives (Van de Briel et al., 2005). Analytical approaches can help achieve these alternatives. Unfortunately, the number of analytical approaches to boarding strategy is even scarcer.

A recent and very interesting analytical approach to aircraft boarding strategy is by Van de Briel et al. (2005). They model the aircraft boarding strategy using a non-linear assignment model with quadratic and cubic terms. The model attempts to minimize the total interferences among the passengers (discussed later in Section 3). The non-linear problem belonging to NP-hard complexity class is solved and verified using simulation modeling. The final recommended '*reverse pyramid*' boarding strategy was implemented at America West Airline.

This paper presents a new mixed integer linear program approach to generate efficient boarding strategies. As indicated earlier, analytical approaches on this topic are very limited. To the best of our knowledge, no linear programming approach has been adopted to study the boarding strategies. The mathematical model attempts to minimize the total interferences among the passengers which are major causes for boarding delays, subject to operational and side constraints. The model provides the flexibility to optimize the boarding times for various aircraft with different seat capacities. The model also allows us to examine the impact of number of boarding groups and the speed that these groups are called on the overall adopted boarding strategy. This paper then adopts the mathematical model to a single aisle Airbus-320 aircraft. This specific type of aircraft was selected because it is one of the most common types of aircraft adopted by many airlines. Another reason was that it enables us to provide a comparison between the performance of our model and those reported by the non-linear model developed by Van de Briel et al. (2005), who adopted a similar aircraft in their analysis. A simulation model is included to provide some guidelines on the speed of boarding the passengers and how it affects the efficient patterns for boarding the aircraft.

Sections 2 and 3 define interferences and how they are formulated in our model. Section 4 presents the mixed linear integer model. Sections 5 and 6 examine the parameters and solutions generated by the mathematical model. Sections 7 and 8 present the simulation model and recommendations finally Section 9 concludes this paper.

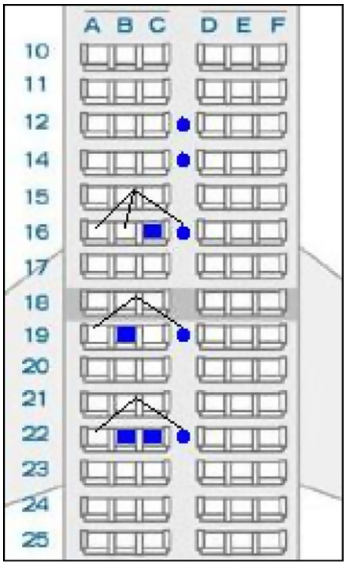

Fig. 1. Seat and aisle interferences (A & F – window, B & E – middle, and C & D – aisle seats).

2. Interferences

Boarding interferences occur when a passenger blocks another passenger to proceed to his or her seat. Two types of interferences, seat and aisle, may occur. Seat interferences happen when a passenger blocks another passenger assigned to the same row (see Fig. 1).

Aisle interferences happen when a lower row passenger is in front of higher row passengers when boarding the aircraft. In this case, the passenger in the lower row will block all the passengers behind him/her to stow the baggage in the overhead bin (if any) and be seated (see Fig. 1). The following sections describe the mathematical models for each type of interferences.

3. Model description

Our focus in this section is to develop a mathematical model which captures the behavior of passengers boarding the aircraft. We attempt to minimize the total number of interferences subject to operational and side constraints. Note that this model assumes a single aisle aircraft where passengers board through a single door.

Each seat in this aircraft is represented by (i,j) where i $(i = 1,\ldots,N)$ is the row and j $(j = \mathrm{A},\mathrm{B},\ldots,\mathrm{F})$ is the location of the seat within row i as shown in Fig. 1.

The mathematical model assigns each passenger in seat (i,j) to a group k $(k = 1,\ldots,G)$.

The following binary decision variable is adopted for our mixed integer linear programming model. Let

$$x_{i,j,k} = \begin{cases} 1 & \text{if seat } j \text{ in row } i \text{ is assigned to group } k, \\ 0 & \text{otherwise.} \end{cases}$$

Our objective is to assign seats (i,j) to groups $(k \in G)$ so that the total number of interferences with penalties attached to them (as described later) is minimized.

3.1. Seat interferences

There might be two types of seat interferences. Between groups and within groups seat interferences as follows.

3.1.1. Between groups seat interferences

This type of seat interference occurs when a passenger from a previous group blocks another passenger in a later group. For example, in Fig. 1 if the passenger in seat 16C (aisle seat) boarded in group 2 and passenger 16B (middle seat) boarded in a later group, then the passenger in 16C is blocking passenger in seat 16B. In this case, the passenger in seat 16C needs to exit and let the passenger in 16B first, thus blocking the flow of other passengers in the aisle. More seat interferences occur if the passenger in seat 16A (window seat) boards after these two passengers.

Considering all the possible combinations, four types of seat interferences between different groups can occur. The first is when the aisle seat blocks middle seat. The second is when the aisle seat blocks the passenger in window seat. The third type is when the middle seat block window seat, and finally, the fourth is when both aisle and middle seats block the passenger in the window seat. We examine the mathematical model for each case separately.

3.1.1a. Aisle and middle seat interferences

First we develop the seat interferences on the left hand side of the aircraft according to Fig. 1 for seats B and C. We define $SB_{i,\mathrm{BC},k}$ as a binary variable representing number of seat interference between the aisle (seat C) and middle seat (seat B) in row i who boarded in group k. $SB_{i,\mathrm{BC},k}$ takes a value of 1 if an interference occurs and 0 otherwise.

Mathematically, $SB_{i,\mathrm{BC},k}$ can be expressed as the following constraint in our mathematical model:

$$SB_{i,\mathrm{BC},k} \geqslant x_{i,\mathrm{B},k} + \sum_{l=1}^{k-1} x_{i,\mathrm{C},l} - 1 \quad \forall i, \text{ and } k > 1. \tag{3.1}$$

On the right hand side of this constraint, we have $x_{i,\mathrm{B},k}$ which represents if the passenger in the middle seat (B) boards in group k and $\sum_{l=1}^{k-1} x_{i,\mathrm{C},l}$ indicates that if the passenger sitting in the aisle seat C has boarded in all previous groups (before k). The term (-1) in the above constraint is added to set the value of $SB_{i,\mathrm{BC},k}$ equal to 1 if there is an interference and 0 otherwise. Table 1 clarifies this further and examine the values of $SB_{i,\mathrm{BC},k}$ for a given row i.

Note that in the constraint (3.1) we use greater or equal sign ($\geqslant$). This is to insure that $SB_{i,\mathrm{BC},k}$ takes a value of 0 for the last case in Table 1 if both terms on the right hand side are 0.

Similarly we write the constraint for aisle and middle seat interferences (seats E and D) on the right hand side of the aircraft

$$SB_{i,\mathrm{ED},k} \geqslant x_{i,\mathrm{E},k} + \sum_{l=1}^{k-1} x_{i,\mathrm{D},l} - 1 \quad \forall i, \text{ and } k > 1 \tag{3.2}$$

3.1.1b. Aisle and window seat interferences

We adopt a similar approach as 3.1.1a to express the number of seat interferences and its corresponding constraints between aisle and window seats among different groups for both sides of the aircraft.

$$SB_{i,\mathrm{AC},k} \geqslant x_{i,\mathrm{A},k} + \sum_{l=1}^{k-1} x_{i,\mathrm{C},l} - 1 \quad \forall i, \text{ and } k > 1, \tag{3.3}$$

$$SB_{i,\mathrm{FD},k} \geqslant x_{i,\mathrm{F},k} + \sum_{l=1}^{k-1} x_{i,\mathrm{D},l} - 1 \quad \forall i, \text{ and } k > 1. \tag{3.4}$$

Table 1
Examining aisle and middle seat interference

$x_{i,\mathrm{B},k}$	$\sum_{l=1}^{k-1} x_{i,\mathrm{C},l}$	$SB_{i,\mathrm{BC},k} \geqslant$	Comments
1	1	1	Interference occurs
0	1	0	No interference
1	0	0	No interference
0	0	–1	No interference

3.1.1c. Window and middle seat interferences

Similarly the number of seat interference and constraint between window and middle seats among different groups for both sides of the aircraft are as follows:

$$SB_{i,\mathrm{AB},k} \geqslant x_{i,\mathrm{A},k} + \sum_{l=1}^{k-1} x_{i,\mathrm{B},l} - 1 \quad \forall i, \text{ and } k > 1, \tag{3.5}$$

$$SB_{i,\mathrm{FE},k} \geqslant x_{i,\mathrm{F},k} + \sum_{l=1}^{k-1} x_{i,\mathrm{E},l} - 1 \quad \forall i, \text{ and } k > 1. \tag{3.6}$$

3.1.1d. Window with both aisle and middle seats interferences

Having established the above seat interferences, we do not need to express a specific set of constraints for a window seat when both middle and aisle seats have already boarded. This type of interference has already been addressed in the form of two separate constraints (interferences) in this section. These two interferences are window with middle and window with aisle seats. For example, consider a case when a passenger in seat A boards just after both passengers in seats B and C. In this case, according to constraints (3.3) and (3.5) above both $SB_{i,\mathrm{AC},k}$ and $SB_{i,\mathrm{AB},k}$ will take a value of 1 implying that the passenger sitting in seat A (window) will have a total of 2 seat interferences with aisle and middle seats.

3.1.1e. Total seat interferences among different groups

We can now express the total number of seat interferences between different groups by adding all the seat interferences discussed above. Let TSB represent the total seat interferences between groups, then

$$TSB = \sum_{i=1}^{N} \sum_{k=2}^{G} (SB_{i,\mathrm{BC},k} + SB_{i,\mathrm{ED},k} + SB_{i,\mathrm{AC},k} + SB_{i,\mathrm{FD},k} + SB_{i,\mathrm{AB},k} + SB_{i,\mathrm{FE},k}). \tag{3.7}$$

3.1.1f. Within groups seat interferences

This type of interference occurs among the passengers boarding in the same group. We assume that the sequence that the passengers within a group board the aircraft is random. For example, suppose that passengers in seats 16A and 16B are boarding in the same group. When their group is called then passenger 16A may board first and will be in front of 16B in their respected group or vice versa. In the former case when the passenger in seat 16A boards before 16B, no interference will occur. However, in the latter case when the passenger in 16B boards before 16A, we have one seat interference.

Adopting the same argument as Section 3.1.1, we denote the binary variable $SW_{i,\mathrm{BC},k}$ to represent the seat interference between the aisle seat C and middle seat B who board in the same group. We can write the following constraint for this variable:

$$SW_{i,\mathrm{BC},k} \geqslant x_{i,\mathrm{B},k} + x_{i,\mathrm{C},k} - 1 \quad \forall i, k. \tag{3.8}$$

Similar to Section 3.1.1a, $SW_{i,\mathrm{BC},k}$ can take a value of 1 or 0 depending on values of $x_{i,\mathrm{B},k}$ and $x_{i,\mathrm{C},k}$. If both passengers in seats B and C in row i are boarding in the same group k, then the constraint (3.8) returns a value of 1 for $SW_{i,\mathrm{BC},k}$ otherwise it will be 0. However, as indicated before the order of these two passengers is random. Therefore the expected number of seat interferences between passengers in seats B and C within the same group is

$$\frac{1}{2} SW_{i,\mathrm{BC},k}. \tag{3.9}$$

Similarly, we can express constraints for other seat interferences within the same group as follows:

$$SW_{i,\mathrm{AC},k} \geqslant x_{i,\mathrm{A},k} + x_{i,\mathrm{C},k} - 1 \quad \forall i, k, \tag{3.10}$$

$$SW_{i,\mathrm{AB},k} \geqslant x_{i,\mathrm{A},k} + x_{i,\mathrm{B},k} - 1 \quad \forall i, k, \tag{3.11}$$

$$SW_{i,\mathrm{ED},k} \geqslant x_{i,\mathrm{E},k} + x_{i,\mathrm{D},k} - 1 \quad \forall i, k, \tag{3.12}$$
$$SW_{i,\mathrm{FD},k} \geqslant x_{i,\mathrm{F},k} + x_{i,\mathrm{D},k} - 1 \quad \forall i, k, \tag{3.13}$$
$$SW_{i,\mathrm{FE},k} \geqslant x_{i,\mathrm{F},k} + x_{i,\mathrm{E},k} - 1 \quad \forall i, k. \tag{3.14}$$

Similar to expression (3.9), the expected number of seat interferences within each group is 1/2 each of the above SW. Again we do not need to add a new constraint for the case when all three neighboring passengers are in the same group. This is because the variables (SW) in the constraints relating to every pair of these three passengers returns a value of 1.

To clarify this, let us examine this using our constraints described in this section. If neighboring passengers in seats A, B and C are all in the same group, then all three corresponding SW in constraints (3.8), (3.10) and (3.11) take a value of 1. The expected number of seat interferences is 1/2 for each of the corresponding SW. Therefore the expected number of interferences is 1.5 which is obtained by adding $\frac{1}{2} + \frac{1}{2} + \frac{1}{2}$.

Now let us consider all possible combinations for boarding these 3 passengers. Table 2 shows the sequence and the number of interferences for each combination. As we can see, there are 6 combinations with a total of 9 interferences. Therefore, on average we have 9/6 or 1.5 interferences, which is the same as was implied by the constraints discussed above.

The total seat interferences within the same groups (TSW) is therefore obtained by

$$TSW = \frac{1}{2}\sum_{i=1}^{N}\sum_{k=1}^{G}(SW_{i,\mathrm{BC},k} + SW_{i,\mathrm{ED},k} + SW_{i,\mathrm{AC},k} + SW_{i,\mathrm{FD},k} + SW_{i,\mathrm{AB},k} + SW_{i,\mathrm{FE},k}). \tag{3.15}$$

3.2. Aisle interferences

In this section, we formulate the aisle interferences. Similar to seat interferences, there are two types of aisle interferences, within groups and between groups.

3.2.1. Within groups aisle interferences

This common type of aisle interference relates to cases where passengers assigned to a group block each other. This occurs when a passenger in a lower row blocks other passengers behind him/her in order to be seated. The problem becomes compounded when the passenger has multiple bags to store in the overhead bins (Van Landeghem and Beuselinck, 2002). We further break down these within group aisle interferences into interferences with lower rows and interferences with same rows.

3.2.1a. Within groups aisle interferences with lower rows

Let the integer variable $AW1_{i,j,k}$ represent the (maximum) number of aisle interference for passenger in seat (i,j) assigned to group k with lower row passengers in the same group. Similar to the previous section, we can write the constraint for $AW1_{i,j,k}$ as follows:

$$AW1_{i,j,k} \geqslant 6(i-1)x_{i,j,k} + \sum_{u=1}^{i-1}\sum_{v=1}^{6}x_{u,v,k} - 6(i-1) \quad \forall i > 1, j, k. \tag{3.16}$$

Table 2
Seat interferences for neighboring passengers boarding in the same group

Order	Number of interferences
..A..B..C..	3
..A..C..B..	2
..B..C..A..	1
..B..A..C..	2
..C..B..A..	0
..C..A..B..	1
Total	9

On the right hand side of this constraint, the first term takes a value of $6(i-1)$ if the passenger in seat (i,j) is assigned to group k and zero otherwise. The second term, adds up all the passengers in the same group k, who have a lower row seat assignment than i. The third term on the right hand side is adopted to provide the correct count on aisle interferences for $AW1_{i,j,k}$.

Note that to determine $AW1_{i,j,k}$ we added-up all the passengers in lower rows than row i. This implies the worst case where the passenger in seat (i,j) boards after all passengers in lower rows in the same group. Therefore, $AW1_{i,j,k}$ represents the maximum number of aisle interference for passenger in seat (i,j) assigned to group k.

The lowest number of aisle interferences for the passenger in seat (i,j) assigned to group k, occurs if this passenger boards before all the passengers in lower rows in that group. In this case since this passenger moves all the way down the aisle to his/her designated row without being blocked by anyone within this group, then there are no within group aisle interferences. Therefore, the expected number of aisle interferences for passenger in seat (i,j) assigned to group k is

$$\text{expected number of aisle inteferences} = \frac{\min + \max}{2} = \frac{0 + AW1_{i,j,k}}{2} = \frac{1}{2} AW1_{i,j,k}. \tag{3.17}$$

The total expected number of aisle interferences for all passengers with their lower row passengers presented by AWL, is therefore determined by

$$AWL = \frac{1}{2} \sum_{i=2}^{N} \sum_{j=1}^{6} \sum_{k=1}^{G} AW1_{i,j,k}. \tag{3.18}$$

3.2.1b. Within groups aisle interferences in the same row

In Section 3.2.1a above, we did not consider possible aisle interferences among the passengers in the same row and same group. This section addresses the expected number of aisle interferences for these passengers. We define integer variable $AW2_{i,j,k}$ to represent the (maximum) number of aisle interferences between the passenger in seat (i,j) and all other passengers in the same row i, all boarding in group k. We write the following constraint for $AW2_{i,j,k}$:

$$AW2_{i,j,k} \geqslant 5x_{i,j,k} + \sum_{u=1, u\neq j}^{6} x_{i,u,k} - 5 \quad \forall i,j,k. \tag{3.19}$$

The first term on the right hand side takes a value of 5 or zero depending on if the passenger in seat (i,j) is in group k or not. The second term on the RHS adds up the number of all the passengers in row i and group k, except the passenger seating in (i,j). The third term is again used to provide the right number of counts for aisle interferences. Similar to the previous section, the minimum number of interferences for passenger in seat (i,j) boarding in group k is 0 and the maximum is $AW2_{i,j,k}$. Therefore the expected number of aisle interferences in the same row within the same group for this passenger is $1/2AW2_{i,j,k}$.

We define AWS, to represent the total expected number of same row aisle interferences for all passengers which is given by

$$AWS = \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{6} \sum_{k=1}^{G} AW2_{i,j,k}. \tag{3.20}$$

3.2.2. Between groups aisle interferences

This type of aisle interference, as many of us have experienced, occurs when a group of passengers are called to board the aircraft while some or all of the passengers in the previous group are still in the jet-way (staircase) or aircraft door trying to be seated. These interferences occur and get worse as the time between boarding the passengers and groups decrease. We define the integer variable $AB_{i,j,k}$ to represent the maximum number of aisle interferences for passenger in seat (i,j) who boards in group k $(k > 1)$ with all the passengers in the previous group $(k-1)$. We write the following constraint for $AB_{i,j,k}$:

$$AB_{i,j,k} \geqslant 6(i)x_{i,j,k} + \sum_{i=1}^{i}\sum_{u=1}^{6} x_{i,u,k-1} - 6(i) \quad \forall i,j,k > 1. \tag{3.21}$$

The first term on the right hand side of this constraint takes a value of $6i$ if passenger in seat (i,j) is assigned to group k and 0 otherwise. The second term adds up all the passengers who boarded in group $(k-1)$. Again the third term is used to provide the correct count.

This constraint for $AB_{i,j,k}$ assumes that none of the passengers in the previous group is seated when the passenger in seat (i,j) assigned to group k boards the aircraft. Of course, the expected number of aisle interferences for this passenger depends on how quickly each group of passengers is called to board the aircraft. We will assume that for any passenger in group $k(k>1)$ boarding the aircraft, there is a fraction of passengers from the previous group $(k-1)$ still in the jet-way trying to reach to their rows and seats. We call this fraction α $(0 \leqslant \alpha \leqslant 1)$. Therefore, the expected number of aisle interferences between the passenger in seat (i,j) assigned to group k with passengers in group $(k-1)$ is $\alpha AB_{i,j,k}$.

When α is 0, no aisle interferences occur between groups. This occurs when a new group of passengers is called to board the aircraft and all passengers in the previous group are fully seated. On the other extreme, when α is equal to 1, the time between calling groups to board is so short that the passengers in each group line up behind the previous group in the aisle or jet-way. In our later analyses, we examine various values for α and its impact on boarding pattern and strategy.

Let ABG represent the total aisle interferences between groups for all passengers boarding in group k $(k>1)$. ABG is therefore determined by

$$ABG = \alpha \sum_{i=1}^{N}\sum_{j=1}^{6}\sum_{k=2}^{G} AB_{i,j,k}. \tag{3.22}$$

To keep the model simple at this stage and without loss of generality, we only included aisle interferences between passengers in group k $(k>1)$ with passengers in group $(k-1)$. It is of course possible to mathematically include the aisle interferences between passengers in group k and groups $(k-2$, for $k>2)$ or $(k-3$ for $k>3)$, etc. Our simulation study, discussed later, also confirmed that for realistic times between passengers to board the aircraft, these second or third level interferences are relatively very low compared to the first level that is considered in the model.

4. Mathematical model

We can now summarize our mixed integer linear program as follows:

$$\text{Minimize} \quad p_1 TSB + p_2 TSW + p_3 AWL + p_4 AWS + p_5 ABG \tag{4.1}$$

$$\text{Subject to:} \quad SB_{i,\mathrm{BC},k} \geqslant x_{i,\mathrm{B},k} + \sum_{l=1}^{k-1} x_{i,\mathrm{C},l} - 1 \quad \forall i, \text{ and } k > 1, \tag{4.2}$$

$$SB_{i,\mathrm{ED},k} \geqslant x_{i,\mathrm{E},k} + \sum_{l=1}^{k-1} x_{i,\mathrm{D},l} - 1 \quad \forall i, \text{ and } k > 1, \tag{4.3}$$

$$SB_{i,\mathrm{AC},k} \geqslant x_{i,\mathrm{A},k} + \sum_{l=1}^{k-1} x_{i,\mathrm{C},l} - 1 \quad \forall i, \text{ and } k > 1, \tag{4.4}$$

$$SB_{i,\mathrm{FD},k} \geqslant x_{i,\mathrm{F},k} + \sum_{l=1}^{k-1} x_{i,\mathrm{D},l} - 1 \quad \forall i, \text{ and } k > 1, \tag{4.5}$$

$$SB_{i,\mathrm{AB},k} \geqslant x_{i,\mathrm{A},k} + \sum_{l=1}^{k-1} x_{i,\mathrm{B},l} - 1 \quad \forall i, \text{ and } k > 1, \tag{4.6}$$

$$SB_{i,\mathrm{FE},k} \geqslant x_{i,\mathrm{F},k} + \sum_{l=1}^{k-1} x_{i,\mathrm{E},l} - 1 \quad \forall i, \text{ and } k > 1, \tag{4.7}$$

$$SW_{i,\mathrm{BC},k} \geqslant x_{i,\mathrm{B},k} + x_{i,\mathrm{C},k} - 1 \quad \forall i,k, \tag{4.8}$$
$$SW_{i,\mathrm{AC},k} \geqslant x_{i,\mathrm{A},k} + x_{i,\mathrm{C},k} - 1 \quad \forall i,k, \tag{4.9}$$
$$SW_{i,\mathrm{AB},k} \geqslant x_{i,\mathrm{A},k} + x_{i,\mathrm{B},k} - 1 \quad \forall i,k, \tag{4.10}$$
$$SW_{i,\mathrm{ED},k} \geqslant x_{i,\mathrm{E},k} + x_{i,\mathrm{D},k} - 1 \quad \forall i,k, \tag{4.11}$$
$$SW_{i,\mathrm{FD},k} \geqslant x_{i,\mathrm{F},k} + x_{i,\mathrm{D},k} - 1 \quad \forall i,k, \tag{4.12}$$
$$SW_{i,\mathrm{FE},k} \geqslant x_{i,\mathrm{F},k} + x_{i,\mathrm{E},k} - 1 \quad \forall i,k, \tag{4.13}$$
$$AW1_{i,j,k} \geqslant 6(i-1)x_{i,j,k} + \sum_{u=1}^{i-1}\sum_{v=1}^{6} x_{u,v,k} - 6(i-1) \quad \forall i>1,j,k, \tag{4.14}$$
$$AW2_{i,j,k} \geqslant 5x_{i,j,k} + \sum_{u=1,u\neq j}^{6} x_{i,u,k} - 5 \quad \forall i,j,k, \tag{4.15}$$
$$AB_{i,j,k} \geqslant 6(i)x_{i,j,k} + \sum_{i=1}^{i}\sum_{u=1}^{6} x_{i,u,k-1} - 6(i) \quad \forall i,j,k>1, \tag{4.16}$$
$$TSB = \sum_{i=1}^{N}\sum_{k=2}^{G}(SB_{i,\mathrm{BC},k} + SB_{i,\mathrm{ED},k} + SB_{i,\mathrm{AC},k} + SB_{i,\mathrm{FD},k} + SB_{i,\mathrm{AB},k} + SB_{i,\mathrm{FE},k}), \tag{4.17}$$
$$TSW = \frac{1}{2}\sum_{i=1}^{N}\sum_{k=1}^{G}(SW_{i,\mathrm{BC},k} + SW_{i,\mathrm{ED},k} + SW_{i,\mathrm{AC},k} + SW_{i,\mathrm{FD},k} + SW_{i,\mathrm{AB},k} + SW_{i,\mathrm{FE},k}), \tag{4.18}$$
$$AWS = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{6}\sum_{k=1}^{G} AW2_{i,j,k}, \tag{4.19}$$
$$AWL = \frac{1}{2}\sum_{i=2}^{N}\sum_{j=1}^{6}\sum_{k=1}^{G} AW1_{i,j,k}, \tag{4.20}$$
$$ABG = \alpha\sum_{i=1}^{N}\sum_{j=1}^{6}\sum_{k=2}^{G} AB_{i,j,k}, \tag{4.21}$$
$$\sum_{k=1}^{G} x_{i,j,k} = 1 \quad \forall i,j, \tag{4.22}$$
$$\sum_{i=1}^{N}\sum_{j=1}^{6} x_{i,j,k} \geqslant \min\_pax \quad \forall k, \tag{4.23}$$
$$\sum_{i=1}^{N}\sum_{j=1}^{6} x_{i,j,k} \leqslant \max\_pax \quad \forall k, \tag{4.24}$$
$$x_{i,j,k} \in \{0,1\} \quad \forall i,j,k. \tag{4.25}$$

The objective function (4.1) attempts to minimize the total expected number of seat and aisle interferences. $p_1, p_2, \ldots, p_5$ represent the penalties assigned to different types of interferences. The values of these penalties will be discussed later. The set of constraints (4.2) to (4.21) were explained extensively in Section 3. Constraint (4.22) ensures that each seat is assigned to only one group. The constraints (4.23) and (4.24) assign a minimum (min_*pax*) and maximum (max_*pax*) number of passengers to each group and finally (4.25) indicates that the decision variables are binary.

5. Model parameters

As indicated before, $p_1, p_2, \ldots, p_5$ are adopted to assign different weights to different seat and aisle interferences. The literature adopting simulation for boarding strategy mainly use triangular distributions (Kelton et al., 2002) to model the times for seat and aisle interferences. Van Landeghem and Beuselinck (2002) use triangular distributions (3, 3.6, 4.2) and (1.8, 2.4, 3) seconds in their models for seat and aisle interferences respectively. Similar time parameters are used in the simulation study by Ferrari and Nagel (2004). We adopted the mean of these distributions to represent the penalties. These distributions have a mean of 3.6 and 2.4 for seat and aisle interferences respectively. Without loss of generality, we assign the same weight to seat (*TSB* and *TSW*) and same weight to aisle (*AWL*, *AWS*, *ABG*) interferences as follows:

$$p_1 = p_2 = 3.6,$$
$$p_3 = p_4 = p_5 = 2.4.$$

In Section 3, we assumed that for passengers in group k ($k > 1$) boarding the aircraft, there is a fraction of passengers from the previous group ($k - 1$) still in the jet-way trying to reach their seats. We called this fraction α ($0 \leqslant \alpha \leqslant 1$). The two extreme cases when α is 0 or 1, representing situations when there is 0% or 100% between group interferences, were discussed earlier. To identify the impact of α on boarding strategy, however, we considered various values for this parameter. We solved the above mathematical model for the following values of α:

$$\alpha \in \{0, .1, .3, .5, .7, .9, 1\}.$$

The airlines typically adopt 4, 5 or 6 groups to board the passengers (Van de Briel et al., 2005). To provide a better understanding of boarding strategy as the number of groups change and to make a comparison with results reported in the literature, we also solved our mixed integer model with 4, 5 and 6 groups, i.e.

$$G \in \{4, 5, 6\}.$$

Typically the airlines favor a balanced number of passengers among different groups. In our model, we set *min_pax* and *max_pax*, to allow a maximum of 20% fluctuations around the mean as follows:

$$\min\_pax = \left\lceil \frac{N}{G} \times .8 \right\rceil,$$
$$\max\_pax = \left\lceil \frac{N}{G} \times 1.2 \right\rceil,$$

where $\lceil \bullet \rceil$ denotes the integer value of '•'. N represents the number of rows.

Similar to Van de Briel et al. (2005), we consider an Airbus-320 airplane with 26 rows. The first three rows (with 4 seats in each row) are assigned to first/business class (group 1) and always board first. In our model we study all other passengers who are assigned to the other 23 rows ($N = 23$), with six seats in each row, who must be allocated to different groups.

6. Computation and implementation

Considering the possible values for parameters, we have 21 (3 groups (G) and 7 values of α for each group) mixed integer linear models. The 4, 5 and 6 group linear integer models have 414, 552 and 690 binary decision variables and 2131, 2886 and 3641 constraints respectively. These models were solved using CPLEX (www.i-log.com) solver.

Figs. 2–4 present the different boarding patterns based on the solutions generated for 4, 5 and 6 groups and each values of α respectively. In this paper, we refer to these solutions as '*efficient solutions*'. It is of interest to see how various values of α impact the boarding pattern. As these figures imply, the patterns shift from *reverse pyramid* (*RP*) to *back-to-front* (*BF*) as α increases. Specifically, we see that for $\alpha \geqslant .5$ the pattern rapidly starts to converge to *back-to-front* strategy. This occurs when the time between boarding the passengers into the aircraft is so short that each group lines-up behind at least 50% of passengers from the previous group.

Tables 3–5 present the number of seat, aisle, total interferences and the values of objective functions for *efficient solutions* for each group and each value of α. Column '*eff sol*' represents each of the interferences

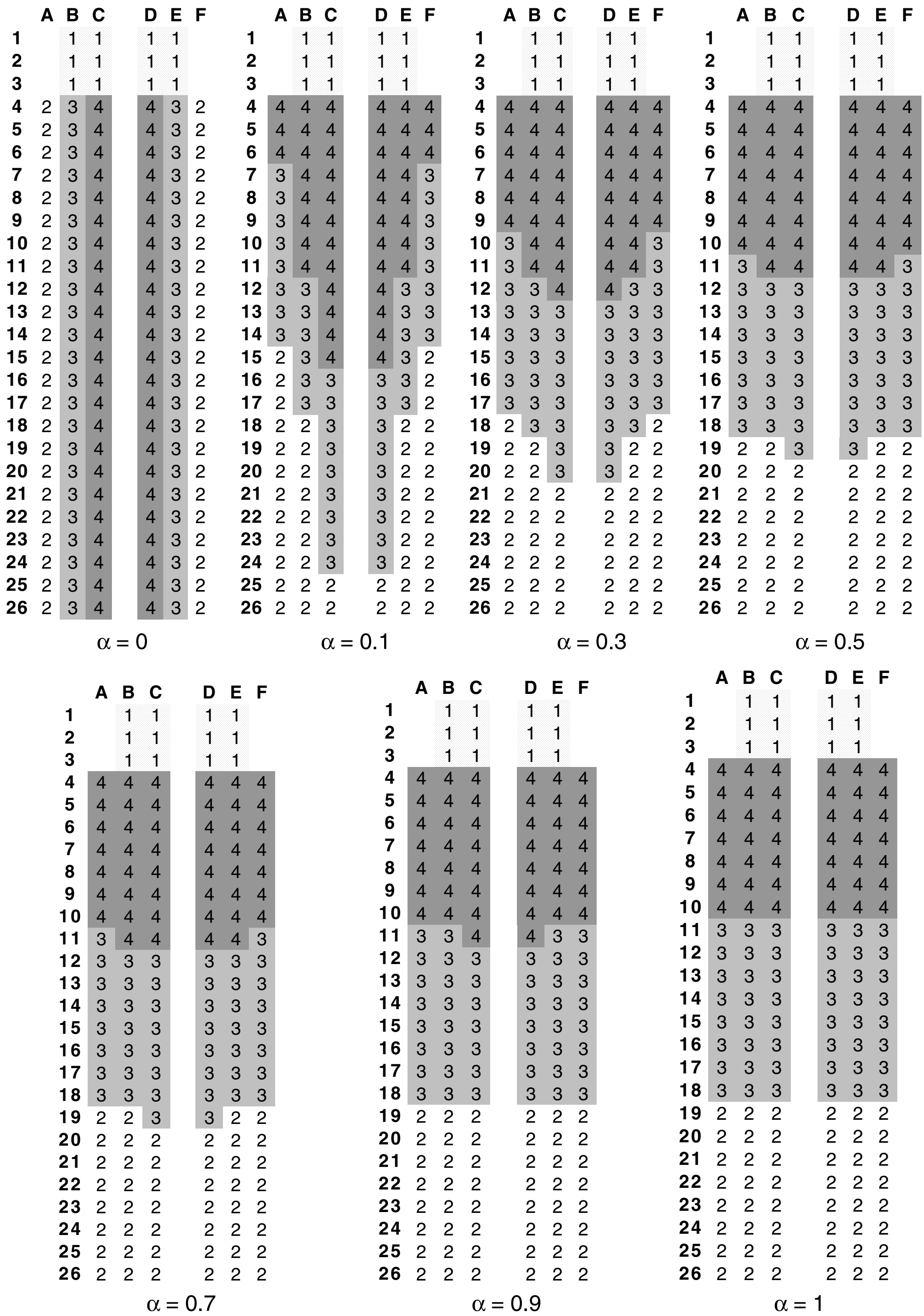

Fig. 2. *Efficient solutions* for boarding patterns for 4 groups and different values of α.

$\alpha = 0$

	A	B	C	D	E	F
1		1	1	1	1	
2		1	1	1	1	
3		1	1	1	1	
4	3	4	5	5	4	3
5	3	4	5	5	4	3
6	3	4	5	5	4	3
7	3	4	5	5	4	3
8	3	4	5	5	4	3
9	2	4	5	5	4	2
10	2	4	5	5	4	2
11	2	4	5	5	4	2
12	2	4	5	5	4	2
13	2	4	5	5	4	2
14	2	4	5	5	4	2
15	2	3	5	5	3	2
16	2	3	5	5	3	2
17	2	3	5	5	3	2
18	2	3	5	5	3	2
19	2	3	5	5	3	2
20	2	3	5	5	3	2
21	2	3	4	4	3	2
22	2	3	4	4	3	2
23	2	3	4	4	3	2
24	2	3	4	4	3	2
25	2	3	4	4	3	2
26	2	3	4	4	3	2

$\alpha = 0.1$

	A	B	C	D	E	F
1		1	1	1	1	
2		1	1	1	1	
3		1	1	1	1	
4	5	5	5	5	5	5
5	4	5	5	5	5	4
6	4	5	5	5	5	4
7	4	5	5	5	5	4
8	4	5	5	5	5	4
9	4	5	5	5	5	4
10	4	4	5	5	4	4
11	4	4	5	5	4	4
12	3	4	5	5	4	3
13	3	4	5	5	4	3
14	3	4	5	5	4	3
15	3	3	4	4	3	3
16	2	3	4	4	3	2
17	2	3	4	4	3	2
18	2	3	4	4	3	2
19	2	3	4	4	3	2
20	2	3	4	4	3	2
21	2	3	3	3	3	2
22	2	2	3	3	2	2
23	2	2	3	3	2	2
24	2	2	3	3	2	2
25	2	2	3	3	2	2
26	2	2	3	3	2	2

$\alpha = 0.3$

	A	B	C	D	E	F
1		1	1	1	1	
2		1	1	1	1	
3		1	1	1	1	
4	5	5	5	5	5	5
5	5	5	5	5	5	5
6	5	5	5	5	5	5
7	5	5	5	5	5	5
8	4	5	5	5	5	4
9	4	5	5	5	5	4
10	4	4	5	5	4	4
11	4	4	4	4	4	4
12	4	4	4	4	4	4
13	4	4	4	4	4	4
14	3	4	4	4	4	3
15	3	4	4	4	4	3
16	3	3	4	4	3	3
17	3	3	3	3	3	3
18	3	3	3	3	3	3
19	3	3	3	3	3	3
20	2	3	3	3	3	2
21	2	2	3	3	2	2
22	2	2	3	3	2	2
23	2	2	3	3	2	2
24	2	2	3	3	2	2
25	2	2	3	3	2	2
26	2	2	3	3	2	2

$\alpha = 0.5$

	A	B	C	D	E	F
1		1	1	1	1	
2		1	1	1	1	
3		1	1	1	1	
4	5	5	5	5	5	5
5	5	5	5	5	5	5
6	5	5	5	5	5	5
7	5	5	5	5	5	5
8	5	5	5	5	5	5
9	4	5	5	5	5	4
10	4	4	4	4	4	4
11	4	4	4	4	4	4
12	4	4	4	4	4	4
13	4	4	4	4	4	4
14	4	4	4	4	4	4
15	3	3	4	4	3	3
16	3	3	3	3	3	3
17	3	3	3	3	3	3
18	3	3	3	3	3	3
19	3	3	3	3	3	3
20	3	3	3	3	3	3
21	2	2	3	3	2	2
22	2	2	2	2	2	2
23	2	2	2	2	2	2
24	2	2	2	2	2	2
25	2	2	2	2	2	2
26	2	2	2	2	2	2

$\alpha = 0.7$

	A	B	C	D	E	F
1		1	1	1	1	
2		1	1	1	1	
3		1	1	1	1	
4	5	5	5	5	5	5
5	5	5	5	5	5	5
6	5	5	5	5	5	5
7	5	5	5	5	5	5
8	5	5	5	5	5	5
9	4	5	5	5	5	4
10	4	4	4	4	4	4
11	4	4	4	4	4	4
12	4	4	4	4	4	4
13	4	4	4	4	4	4
14	4	4	4	4	4	4
15	3	3	4	4	3	3
16	3	3	3	3	3	3
17	3	3	3	3	3	3
18	3	3	3	3	3	3
19	3	3	3	3	3	3
20	3	3	3	3	3	3
21	2	2	3	3	2	2
22	2	2	2	2	2	2
23	2	2	2	2	2	2
24	2	2	2	2	2	2
25	2	2	2	2	2	2
26	2	2	2	2	2	2

$\alpha = 0.9$

	A	B	C	D	E	F
1		1	1	1	1	
2		1	1	1	1	
3		1	1	1	1	
4	5	5	5	5	5	5
5	5	5	5	5	5	5
6	5	5	5	5	5	5
7	5	5	5	5	5	5
8	5	5	5	5	5	5
9	5	5	5	5	5	5
10	4	4	4	4	4	4
11	4	4	4	4	4	4
12	4	4	4	4	4	4
13	4	4	4	4	4	4
14	4	4	4	4	4	4
15	3	3	3	3	3	3
16	3	3	3	3	3	3
17	3	3	3	3	3	3
18	3	3	3	3	3	3
19	3	3	3	3	3	3
20	3	3	3	3	3	3
21	2	2	2	2	2	2
22	2	2	2	2	2	2
23	2	2	2	2	2	2
24	2	2	2	2	2	2
25	2	2	2	2	2	2
26	2	2	2	2	2	2

$\alpha = 1$

	A	B	C	D	E	F
1		1	1	1	1	
2		1	1	1	1	
3		1	1	1	1	
4	5	5	5	5	5	5
5	5	5	5	5	5	5
6	5	5	5	5	5	5
7	5	5	5	5	5	5
8	5	5	5	5	5	5
9	4	4	4	4	4	4
10	4	4	4	4	4	4
11	4	4	4	4	4	4
12	4	4	4	4	4	4
13	4	4	4	4	4	4
14	4	4	4	4	4	4
15	3	3	3	3	3	3
16	3	3	3	3	3	3
17	3	3	3	3	3	3
18	3	3	3	3	3	3
19	3	3	3	3	3	3
20	3	3	3	3	3	3
21	2	2	2	2	2	2
22	2	2	2	2	2	2
23	2	2	2	2	2	2
24	2	2	2	2	2	2
25	2	2	2	2	2	2
26	2	2	2	2	2	2

Fig. 3. *Efficient solutions* for boarding patterns for 5 groups and different values of α.

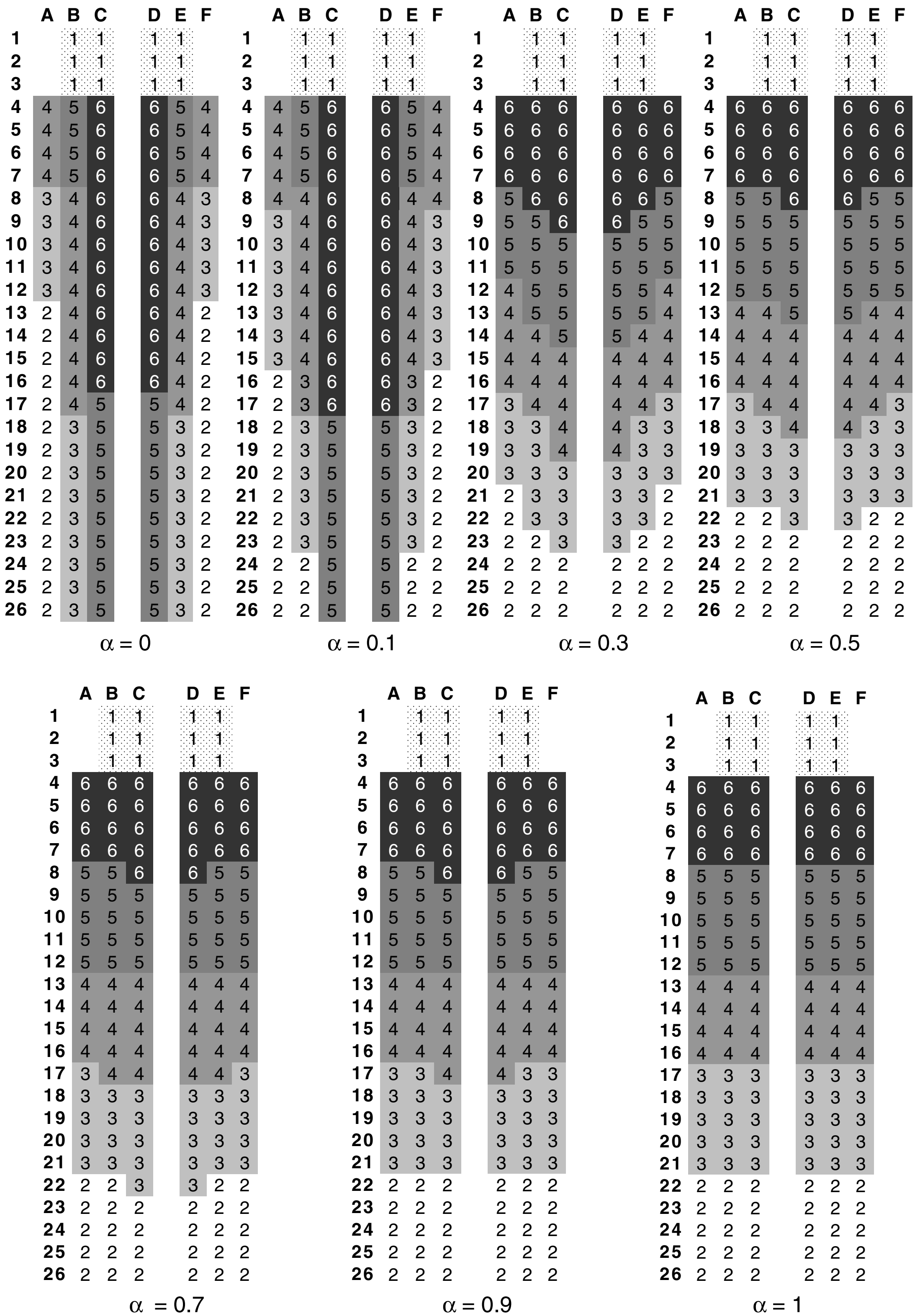

Fig. 4. *Efficient solutions* for boarding patterns for 6 groups and different values of α.

Table 3
Seat, aisle and total interferences for 4 groups for *efficient* and *reverse pyramid solutions*

α	Interference	Eff sol	Rev Pyr	Sum	Eff sol	Rev Pyr
0	TSB	0	0	Total seat interferences	0	0
	TSW	0	0			
	AWL	1518	1518	Total aisle interferences	1587	1587
	AWS	69	69			
	ABG	0	0			
	Obj. Func.	3808.8	3808.8	Total Int.	1587	1587
0.1	TSB	0	0	Total seat interferences	32	0
	TSW	32	0			
	AWL	1454	1518	Total aisle interferences	1713.8	1807.8
	AWS	197	69			
	ABG	62.8	220.8			
	Obj. Func.	4228.3	4338.7	Total Int.	1745.8	1807.8
0.3	TSB	0	0	Total seat interferences	57	0
	TSW	57	0			
	AWL	1404	1518	Total aisle interferences	1725	2249.4
	AWS	297	69			
	ABG	24	662.4			
	Obj. Func.	4345.2	5398.6	Total Int.	1782	2249.4
0.5	TSB	0	0	Total seat interferences	65	0
	TSW	65	0			
	AWL	1388	1518	Total aisle interferences	1725	2691
	AWS	329	69			
	ABG	8	1104			
	Obj. Func.	4374	6458.4	Total Int.	1790	2691
0.7	TSB	0	0	Total seat interferences	65	0
	TSW	65	0			
	AWL	1388	1518	Total aisle interferences	1728.2	3132.6
	AWS	329	69			
	ABG	11.2	1545.6			
	Obj. Func.	4381.7	7518.2	Total Int.	1793.2	3132.6
0.9	TSB	0	0	Total seat interferences	67	0
	TSW	67	0			
	AWL	1386	1518	Total aisle interferences	1730.2	3574.2
	AWS	337	69			
	ABG	7.2	1987.2			
	Obj. Func.	4393.7	8578.1	Total Int.	1797.2	3574.2
1	TSB	0	0	Total seat interferences	67	0
	TSW	67	0			
	AWL	1723	1518	Total aisle interferences	2068	3795
	AWS	337	69			
	ABG	8	2208			
	Obj. Func.	4395.6	9108	Total Int.	2135	3795

and their totals discussed in Section 3. Note that the value of objective functions are based on penalties $p_1, p_2, \ldots, p_5$ discussed earlier.

The only comparable mathematical model and solution which we are aware of is a paper by Van de Briel et al. (2005). Their solutions (referred to as *reverse pyramid*) for 4, 5 and 6 groups are similar to ours when $\alpha = 0$. Despite the fact that they included between groups interferences, they report only very minimal (about 2 or 3) such interferences in their paper for six group boarding strategy. To provide a comparison between the

Table 4
Seat, aisle and total interferences for 5 groups for *efficient* and *reverse pyramid solutions*

α	Interference	Eff sol	Rev Pyr	Sum	Eff sol	Rev Pyr
0	*TSB*	0	0	Total Seat Interferences	0	0
	TSW	0	0			
	AWL	1122	1122	Total Aisle Interferences	1191	1191
	AWS	69	69			
	ABG	0	0			
	Obj. Func.	2858.4	2858.4	Total Int.	1191	1191
0.1	*TSB*	0	0	Total Seat Interferences	17	0
	TSW	17	0			
	AWL	1090	1122	Total Aisle Interferences	1313.8	1356.6
	AWS	137	69			
	ABG	86.8	165.6			
	Obj. Func.	3214.3	3255.8	Total Int.	1330.8	1356.6
0.3	*TSB*	0	0	Total Seat Interferences	51	0
	TSW	51	0			
	AWL	1020	1122	Total Aisle Interferences	1329	1687.8
	AWS	273	69			
	ABG	36	496.8			
	Obj. Func.	3373.2	4050.7	Total Int.	1380	1687.8
0.5	*TSB*	0	0	Total Seat Interferences	63	0
	TSW	63	0			
	AWL	996	1122	Total Aisle Interferences	1329	2019
	AWS	321	69			
	ABG	12	828			
	Obj. Func.	3416.4	4845.6	Total Int.	1392	2019
0.7	*TSB*	0	0	Total Seat Interferences	63	0
	TSW	63	0			
	AWL	996	1122	Total Aisle Interferences	1333.8	2350.2
	AWS	321	69			
	ABG	16.8	1159.2			
	Obj. Func.	3427.9	5640.5	Total Int.	1396.8	2350.2
0.9	*TSB*	0	0	Total Seat Interferences	69	0
	TSW	69	0			
	AWL	990	1122	Total Aisle Interferences	1335	2681.4
	AWS	345	69			
	ABG	0	1490.4			
	Obj. Func.	3452.4	6435.4	Total Int.	1404	2681.4
1	*TSB*	0	0	Total Seat Interferences	69	0
	TSW	69	0			
	AWL	990	1122	Total Aisle Interferences	1335	2847
	AWS	345	69			
	ABG	0	1656			
	Obj. Func.	3452.4	6832.8	Total Int.	1404	2847

two models, we evaluated the performance of *reverse pyramid* models under different values of α. In Tables 3–5, column '*rev pyr*' represents the performance of this solution as α changes. As the table implies, *reverse pyramid* strategy does not perform satisfactorily as α increases.

Another major difficulty with *reverse pyramid* boarding strategy (see Figs. 2–4 when $\alpha = 0$) is that no two neighboring seats have the same boarding groups. This means that passengers in parties of 2 or more who will sit next to each other board the aircraft separately and at different times. This may not appeal to either passengers or airlines.

Table 5
Seat, aisle and total interferences for 6 groups for *efficient* and *reverse pyramid solutions*

α	Interference	Eff sol	Rev Pyr	Sum	Eff sol	Rev Pyr
0	*TSB*	0	0	Total Seat Interferences	0	0
	TSW	0	0			
	AWL	884	884	Total Aisle Interferences	953	953
	AWS	69	69			
	ABG	0	0			
	Obj. Func.	2287.2	2287.2	Total Int.	953	953
0.1	*TSB*	0	0	Total Seat Interferences	4	0
	TSW	4	0			
	AWL	878	884	Total Aisle Interferences	1059.4	1344.2
	AWS	85	69			
	ABG	96.4	391.2			
	Obj. Func.	2556.96	2600.2	Total Int.	1063.4	1344.2
0.3	*TSB*	0	0	Total Seat Interferences	47	0
	TSW	47	0			
	AWL	792	884	Total Aisle Interferences	1091	1344.2
	AWS	257	69			
	ABG	42	391.2			
	Obj. Func.	2787.6	3226.1	Total Int.	1138	1344.2
0.5	*TSB*	0	0	Total Seat Interferences	59	0
	TSW	59	0			
	AWL	788	884	Total Aisle Interferences	1115	1605
	AWS	305	69			
	ABG	22	652			
	Obj. Func.	2835.6	3852	Total Int.	1174	1605
0.7	*TSB*	0	0	Total Seat Interferences	63	0
	TSW	63	0			
	AWL	758	884	Total Aisle Interferences	1095.8	1865.8
	AWS	321	69			
	ABG	16.8	912.8			
	Obj. Func.	2856.7	4477.9	Total Int.	1158.8	1865.8
0.9	*TSB*	0	0	Total seat interferences	65	0
	TSW	65	0			
	AWL	756	884	Total aisle interferences	1099.4	2126.6
	AWS	329	69			
	ABG	14.4	1173.6			
	Obj. Func.	2872.6	5103.8	Total Int.	1164.4	2126.6
1	*TSB*	0	0	Total seat interferences	69	0
	TSW	69	0			
	AWL	756	884	Total aisle interferences	1101	2257
	AWS	345	69			
	ABG	0	1304			
	Obj. Func.	2890.8	5416.8	Total Int.	1170	2257

7. Simulation model

In the previous sections we provided extensive analyses on boarding strategy and boarding patterns as α varies. In this section, we attempt to identity and evaluate realistic values for α and examine how the rate of boarding the passengers affects α and in turn the total interferences.

In order to study and determine the values of α for different boarding times, we developed a simulation model in Arena Simulation Modeling Software (Kelton et al., 2002). The main focus of this simulation study

Table 6
Expected number of passengers and values of α for different groups and inter-arrival times

		Time between arrivals (seconds)							
		3	4	5	6	7	8	9	10
4 groups Avg pax per econ class group: 46	# Pax	43.6	24.8	14.1	8.8	4.67	0.67	0	0
	α	0.95	0.54	0.31	0.19	0.10	0.01	0.00	0.00
5 groups Avg pax per econ class group: 34.5	# Pax	32.6	18.9	10.9	6.63	3.88	0	0	0
	α	0.95	0.55	0.32	0.19	0.11	0.00	0.00	0.00
6 groups Avg pax per econ class group: 27.6	# Pax	25.9	15.8	9	5.76	3.24	0	0	0
	α	0.94	0.57	0.33	0.21	0.12	0.00	0.00	0.00
Average α		0.94	0.55	0.32	0.20	0.11	0.00	0.00	0.00

is to identify how many passengers from a previous group will be in front of the new group thus providing some guideline for values of α.

The simulation model is similar to those reported by Van Landeghem and Beuselinck (2002) and Ferrari and Nagel (2004). We used similar aircraft load factor and time distributions for aisle and seat interferences. We do not duplicate the details of the simulation model here and refer the interested readers to these papers.

We ran the simulation models for the inter-arrival times of passengers for boarding to change from 3 seconds to 10 seconds or 20 to 6 passengers per minute. Our main task of measuring performance in this study was to identify the number of passengers at the door, where new passengers will line-up behind them for different times between passengers boarding. Enough replications for each scenario were run to reduce the 95% confidence interval half-width for number of waiting passengers in the previous group to less than .5. We ran our simulation model for different boarding strategies including *back-to-front*, *out–in*, *reverse pyramid* and *efficient solutions* for 4, 5 and 6 groups.

Table 6 provides the average number of passengers (# pax), waiting at the door when the new groups are called in to board for different boarding times and different groups. Note that these numbers only apply to economy class passengers (groups 2 or higher). As an example for 4 groups, according to Table 6, on average we have more than 32 passengers waiting at the door when the boarding time between passengers is 3 seconds. This number falls to 0 for inter-arrival times of 9 seconds and more.

In a 4 group boarding strategy, we have one group for business/first-class passengers and three groups assigned to economy class passengers (see Fig. 2). There are 138 (23 rows $\times$ 6 seats per row) economy seats. Therefore on average we have 46 passengers (138 economy seats divided by 3 economy groups) assigned to each economy group. To get α, the proportion of passengers from the previous group who block the new group, we divide the number of waiting passengers (# pax in Table 6) by the average number of economy passengers per group (46 passengers for 4 groups). Therefore as we see in Table 6, for 3 seconds inter-arrival time, we get .95 for α. As inter-arrival times increase, α starts to decrease where it reaches to 0 for inter-arrival times of 9 seconds or higher. Similarly, Table 6, presents values of α for different groups and inter-arrival times. The last row, titled *average* α, presents the average values of α for different groups and inter-arrival times.

8. Recommendation

For average passenger arrival times, Van de Briel et al. (2005) considered 7 seconds with 1 gate agent and 5 seconds with 2 gate agents (rounded to nearest seconds) and Van Landeghem and Beuslinck (2002) considered 6–7 seconds in their simulation models. These times are based on actual observations of passenger boarding times at different airlines and different airports. Using these inter-arrival times as realistic passenger boarding times and referring back to Table 6, we see that the realistic values for α range from 0.32 for 5 seconds to 0.11 for 7 seconds inter-arrival times. Therefore, those *efficient solutions* reported, for α taking values 0.1 and 0.3 in Section 6 of this paper with their patterns presented in Figs. 2–4, seem to be appropriate for boarding strategies. These solutions not only provide a lower number of interferences and objective functions than *reverse* *pyramid* but also they are more appealing to both the airlines and the passengers as they can accommodate neighboring passengers traveling in groups to board together.

Since 6 group boarding strategy results in the lowest number of total interferences, we recommend the boarding patterns in Fig. 4 with $\alpha = 0.3$ for two gate agents and $\alpha = 0.1$ with one gate agent as efficient boarding strategies for an Airbus-320 aircraft.

9. Conclusion

This paper introduced a new mixed integer linear program to minimize the total number of passenger interferences which cause delays in aircraft boarding. The operational and side constraints for this mathematical model were examined. The model was then applied to an Airbus-320 aircraft which is commonly used by many airlines. Alternative *efficient solutions* were generated based on the speed of boarding the passengers. We examined and compared the performance of other boarding strategies with these efficient solutions. A simulation model was adopted to identify appropriate boarding patterns as the speed of boarding the passengers change. The recommended solutions not only provide a lower number of interferences among passengers, they are more appealing to both the airlines and the passengers as they can accommodate neighboring passengers to board together.

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