Uber Rocket Calculus Problem

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Section Q

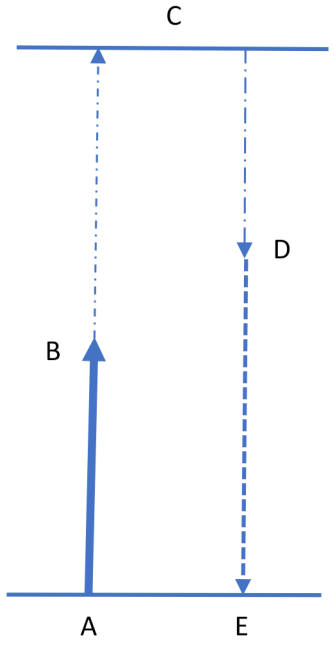
Description

One calm afternoon Calculus Cam decides to launch Hamster Huey into the air using a model rocket. The rocket is launched straight up off the ground, from rest. The rocket engine is designed to burn for specified time while producing non-constant net acceleration given by the equations below. After the engine stops the rocket continues upward in free-fall. A parachute opens after the rocket falls a specified vertical distance from its maximum height. When the parachute opens, assume the rocket instantly stops, and then increases speed to a terminal velocity given by the equation below. Assume the air resistance affects the rocket only during the parachute stage.

Strategy

To find total time, we divide the problem into 4 components as shown on the diagram. Each component is defined by 2 points, which are labeled from A-E. To standardize the notation, we introduce two rules. (1) Variables subscripted with a single letter will refer to its instantaneous value at the point indicated by the subscript. (2) Variables subscripted with two letters will refer to the component(s) bounded by the two points. Variables with two-letter subscripts have an implied Δ, except for the constants a_{BC} and a_{CD} defined in the givens. For example, y_{CD} can be written as Δy_{CD} or $y_D - y_C$.
At each component, the objective is to calculate the Δt of that step, as well as all necessary values needed for the following steps. This can be done in an iterative manner until the value of t_{DE} is calculated. Finally, the Δt of every component is added in a summation step to obtain the total time, t_{AE}.

Diagram



Givens

$$a_{AB}[t] = -1.4t^2 + 23$$
$$t_{AB} = 4.3\ s$$
$$a_{BC} = -9.8\ m/s^2$$
$$a_{CD} = -9.8\ m/s^2$$
$$y_{CD} = -76\ m$$
$$v_{DE}[t] = -18\left(1 - e^{\frac{-t}{6}}\right)$$

Component AB

$$v_{AB}[t] = \int_0^t a_{AB}[t]\,dt + v_A$$
$$v_{AB}[t] = \int_0^t (-1.4t^2 + 23)\,dt + 0$$
$$\underline{v_{AB}[t] = -\frac{7}{15}t^3 + 23t}$$

$$v_B = v_{AB}[t_{AB}]$$
$$v_B = -\frac{7}{15}(4.3)^3 + 23(4.3)$$
$$\underline{v_B = 61.80\ m/s}$$

$$y_{AB}[t] = \int_0^t v_{AB}[t]\,dt + y_A$$
$$y_{AB}[t] = \int_0^t \left(-\frac{7}{15}t^3 + 23t\right)dt + 0$$
$$\underline{y_{AB}[t] = -\frac{7}{60}x^4 + \frac{23}{2}x^2}$$

$$y_B = y_{AB}[t_{AB}]$$
$$y_B = -\frac{7}{60}(4.3)^4 + \frac{23}{2}(4.3)^2$$
$$\underline{y_B = 172.75\ m}$$

Component BC

$$v_C^2 = v_B^2 + 2a_{BC}y_{BC}$$
$$0 = (61.80)^2 + 2(-9.8)y_{BC}$$
$$\underline{y_{BC} = 194.86\ m}$$

$$y_{BC} = y_C - y_B$$
$$194.86 = y_C - 172.75$$
$$\underline{y_C = 367.61\ m}$$

$$y_{BC} = \frac{1}{2}(v_c + v_B)t_{BC}$$
$$194.86 = \frac{1}{2}(0 + 61.80)t_{BC}$$
$$\underline{t_{BC} = 6.3062\ s}$$

Component CD

$$y_{CD} = \frac{1}{2}a_{CD}t_{CD}^2 + v_C t_{CD}$$
$$-76 = \frac{1}{2}(-9.8)t_{CD}^2 + 0$$
$$\underline{t_{CD} = 3.9383\ s}$$

$$y_{CD} = y_D - y_C$$
$$-76 = y_D - 367.61$$
$$\underline{y_D = 291.61\ m}$$

Component DE

$$y_{DE} = y_E - y_D$$
$$y_{DE} = 0 - 291.61$$
$$\underline{y_{DE} = -291.61\ m}$$

$$y_{DE} = \int_0^{t_{DE}} v_{DE}[t]\, dt$$

$$y_{DE} = \int_0^{t_{DE}} -18\left(1 - e^{\frac{-t}{6}}\right) dt$$

$$-291.61 = -18e^{\frac{-t_{DE}}{6}}\left((t_{DE} - 6)\cdot e^{\frac{t_{DE}}{6}} + 6\right)$$

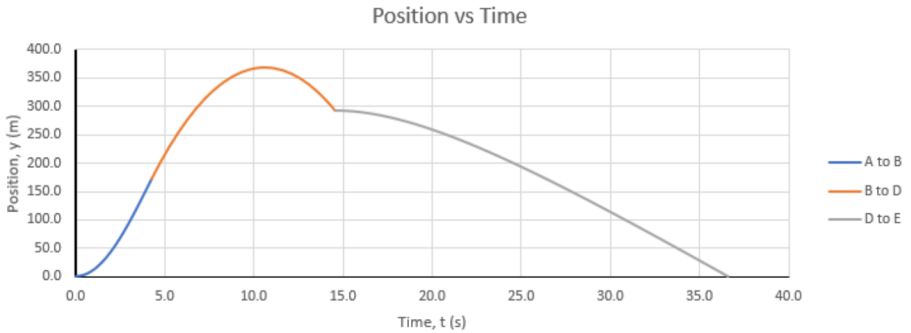
$$\underline{t_{DE} = 22.048\ s}$$

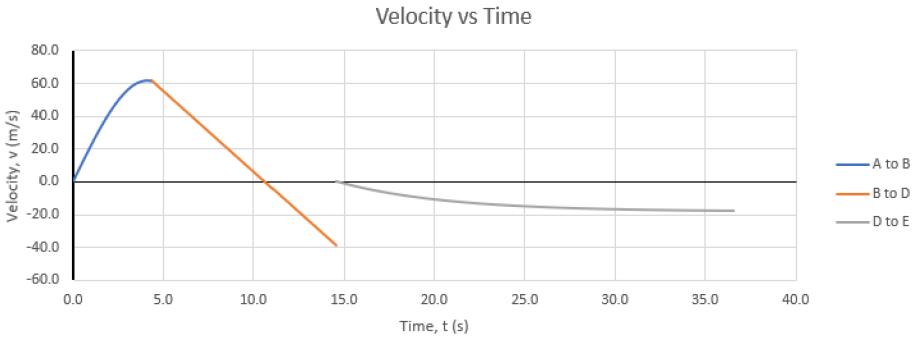
Summation

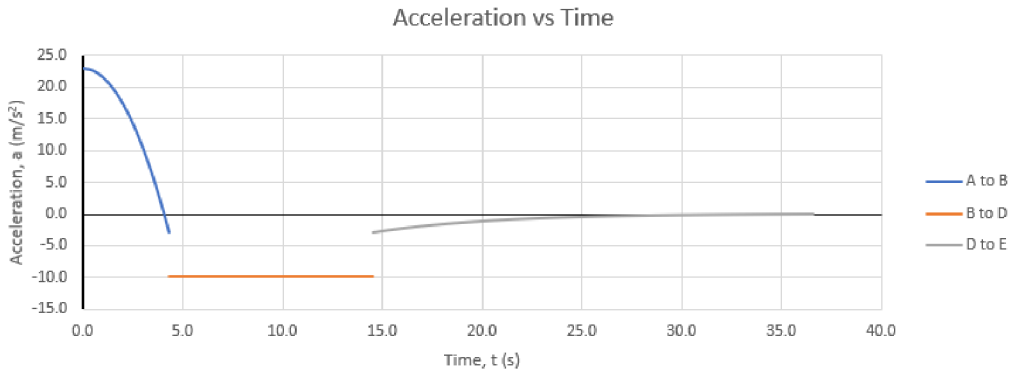
$$t_{AE} = t_{AB} + t_{BC} + t_{CD} + t_{DE}$$

$$t_{AE} = 4.3 + 6.3062 + 3.9383 + 22.048$$

$$\boxed{t_{AE} = 36.59\ s}$$







	t (s)	a (m/s^2)	v (m/s)	x (m)
A	0.0	23.0	0.0	0.0
	0.5	22.7	11.4	2.9
	1.0	21.6	22.5	11.4
	1.5	19.9	32.9	25.3
	2.0	17.4	42.3	44.1
	2.5	14.3	50.2	67.3
	3.0	10.4	56.4	94.1
	3.5	5.9	60.5	123.4
	4.0	0.6	62.1	154.1
	4.3	-2.9	61.8	172.7
B	4.3	-9.8	61.8	172.7
	4.8	-9.8	56.9	202.4
	5.3	-9.8	52.0	229.6
	5.8	-9.8	47.1	254.4
	6.3	-9.8	42.2	276.7
	6.8	-9.8	37.3	296.6
	7.3	-9.8	32.4	314.0
	7.8	-9.8	27.5	329.0
	8.3	-9.8	22.6	341.5
	8.8	-9.8	17.7	351.6
	9.3	-9.8	12.8	359.2
	9.8	-9.8	7.9	364.4
	10.3	-9.8	3.0	367.1
	10.6	-9.8	0.1	367.6
C	10.6	-9.8	0.1	367.6
	11.1	-9.8	-4.8	366.4
	11.6	-9.8	-9.7	362.7
	12.1	-9.8	-14.6	356.6
	12.6	-9.8	-19.5	348.1
	13.1	-9.8	-24.4	337.1
	13.6	-9.8	-29.3	323.7
	14.1	-9.8	-34.2	307.8
	14.5	-9.8	-38.6	291.7

	t (s)	a (m/s^2)	v (m/s)	x (m)
D	14.5	-3.0	0.0	291.7
	15.0	-2.8	-1.4	291.4
	15.5	-2.5	-2.8	290.3
	16.0	-2.3	-4.0	288.6
	16.5	-2.1	-5.1	286.4
	17.0	-2.0	-6.1	283.5
	17.5	-1.8	-7.1	280.2
	18.0	-1.7	-8.0	276.5
	18.5	-1.5	-8.8	272.3
	19.0	-1.4	-9.5	267.7
	19.5	-1.3	-10.2	262.8
	20.0	-1.2	-10.8	257.6
	20.5	-1.1	-11.4	252.0
	21.0	-1.0	-11.9	246.2
	21.5	-0.9	-12.4	240.1
	22.0	-0.9	-12.8	233.8
	22.5	-0.8	-13.3	227.3
	23.0	-0.7	-13.6	220.6
	23.5	-0.7	-14.0	213.6
	24.0	-0.6	-14.3	206.6
	24.5	-0.6	-14.6	199.3
	25.0	-0.5	-14.9	192.0
	25.5	-0.5	-15.1	184.5
	26.0	-0.4	-15.4	176.9
	26.5	-0.4	-15.6	169.1
	27.0	-0.4	-15.8	161.3
	27.5	-0.3	-15.9	153.4
	28.0	-0.3	-16.1	145.4
	28.5	-0.3	-16.3	137.3
	29.0	-0.3	-16.4	129.1
	29.5	-0.2	-16.5	120.9
	30.0	-0.2	-16.6	112.6
	30.5	-0.2	-16.7	104.2

	t (s)	a (m/s^2)	v (m/s)	x (m)
D	31.0	-0.2	-16.8	95.8
	31.5	-0.2	-16.9	87.4
	32.0	-0.2	-17.0	78.9
	32.5	-0.1	-17.1	70.4
	33.0	-0.1	-17.2	61.8
	33.5	-0.1	-17.2	53.2
	34.0	-0.1	-17.3	44.6
	34.5	-0.1	-17.4	35.9
	35.0	-0.1	-17.4	27.2
	35.5	-0.1	-17.5	18.5
	36.0	-0.1	-17.5	9.7
	36.6	-0.1	-17.5	0.0