Solution for exercise MTH 10102

1. Find the value of $\int\left(\frac{e^{\sqrt{x}}}{\sqrt{x}}+2^{x+1}\right)dx$

Solution $$\int\left(\frac{e^{\sqrt{x}}}{\sqrt{x}}+2^{x+1}\right)dx=\int\frac{e^{\sqrt{x}}}{\sqrt{x}}dx+\int 2^{x+1}\,dx$$

Let $u=\sqrt{x}$

Then $du=\frac{1}{2\sqrt{x}}dx$ or $2du=\frac{1}{\sqrt{x}}dx$

Thus,

$$\int\left(\frac{e^{\sqrt{x}}}{\sqrt{x}}+2^{x+1}\right)dx=\int 2e^{u}\,du+\int 2^{x+1}\,dx$$

$$=2e^{u}+\frac{2^{x+1}}{\ln 2}+c$$

$$=2e^{\sqrt{x}}+\frac{2^{x+1}}{\ln 2}+c$$

Therefore, $\int\left(\frac{e^{\sqrt{x}}}{\sqrt{x}}+2^{x+1}\right)dx=2e^{\sqrt{x}}+\frac{2^{x+1}}{\ln 2}+c$

2. Find the value of $\int\sin^{6}x\cos^{3}x\,dx$

Solution $$\int\sin^{6}x\cos^{3}x\,dx=\int\left(\sin^{6}x\right)\left(\cos^{2}x\right)\left(\cos x\right)dx$$

$$.=\int\left(\sin^{6}x\right)\left(1-\sin^{2}x\right)\left(\cos x\right)dx\,.$$

$$=\int\left(\sin^{6}x-\sin^{8}x\right)\left(\cos x\right)dx$$

$$=\int\left(\sin^{6}x-\sin^{8}x\right)d\sin x$$

$$=\frac{\sin^{7}x}{7}-\frac{\sin^{9}x}{9}+C$$

3. Find the value of $\int \cot^5 x \csc^3 x\, dx$

Solution $\quad \int \cot^5 x \csc^3 x\; dx \quad = \int \cot^4 x \csc^2 x \left(\cot x\ \cos c\, x\right) dx$

$$= \int \left(\csc^2 x - 1\right)^2 \csc^2 x \left(\cot x\ \csc x\right) dx$$

$$= \int \left(\csc^4 x - 2\csc^2 x + 1\right) \csc^2 x \left(\cot x\ \csc x\right) dx$$

$$= \int \left(\csc^6 x - 2\csc^4 x + \csc^2 x\right)\left(\cot x\ \csc x\right) dx$$

$$= -\int \left(\csc^6 x - 2\csc^4 x + \csc^2 x\right) d\left(\csc x\right)$$

$$= -\left(\frac{\csc^7 x}{7} - \frac{2\csc^5 x}{5} + \frac{\csc^3 x}{3}\right) + C$$

4. Find the value of $\int_0^5 |2x-5|\, dx$

Solution $\quad$ From $\quad f(x) = |2x-5|$

Then $\quad f(x) = \begin{cases} 2x-5 & , \quad x \geq \frac{5}{2} \\ -2x+5 & , \quad x < \frac{5}{2} \end{cases}$

Therefore, $\quad \int_0^5 |2x-5|\, dx = \int_0^{\frac{5}{2}} (-2x+5)\, dx + \int_{\frac{5}{2}}^5 (2x-5)\, dx$

$$= \left(\frac{-2x^2}{2} + 5x\right)\Bigg|_{x=0}^{x=\frac{5}{2}} + \left(\frac{2x^2}{2} - 5x\right)\Bigg|_{x=\frac{5}{2}}^{x=5}$$

$$= \left[-\left(\frac{5}{2}\right)^2 + (5)\left(\frac{5}{2}\right) - 0\right] + \left[\left((5)^2 - (5)(5)\right) - \left(\left(\frac{5}{2}\right)^2 - (5)(\frac{5}{2})\right)\right]$$

$$= \left[-\frac{25}{4} + \frac{25}{2}\right] + \left[25 - 25 - \frac{25}{4} + \frac{25}{2}\right]$$

$$= -\frac{50}{4} + \frac{50}{2}$$

$$= \frac{25}{2}$$

5. Find the value of $\int x^3 \cos\left(x^4 - 10\right) dx$

Solution Given $u = x^4 - 10$

$du = 4x^3 dx$ or $\frac{1}{4} du = x^3 dx$

Then
$$\int x^3 \cos\left(x^4 - 10\right) dx = \int \frac{1}{4} \cos(u)\, du$$
$$= \frac{1}{4} \int \cos(u)\, du$$
$$= \frac{1}{4}\left(\sin(u) + c\right)$$
$$= \frac{1}{4} \sin(x^4 - 10) + C$$

6. Find the value of $\int \sqrt{1 + \sqrt{1 + x}}\, dx$

Solution Let $u = 1 + \sqrt{1 + x}$ or $\sqrt{1 + x} = u - 1$

$du = \frac{1}{2\sqrt{1 + x}} dx$ or $2du = \frac{1}{\sqrt{1 + x}} dx$

Then
$$\int \sqrt{1 + \sqrt{1 + x}}\, dx = \int 2\sqrt{u}\left(u - 1\right) du$$
$$= 2\int \left(u^{\frac{3}{2}} - u^{\frac{1}{2}}\right) du$$
$$= 2\left(\frac{2}{5} u^{\frac{5}{2}} - \frac{2}{3} u^{\frac{3}{2}}\right) + C$$
$$= \frac{4}{5}\left(1 + \sqrt{1 + x}\right)^{\frac{5}{2}} - \frac{4}{3}\left(1 + \sqrt{1 + x}\right)^{\frac{3}{2}} + C$$

7. Find the value of $\int \frac{x}{x^2-8x+20}dx$

Solution $$\int \frac{x}{x^2-8x+20}dx = \int \frac{x}{x^2-8x+16-16+20}dx$$

$$= \int \frac{x-4+4}{(x-4)^2+4}dx$$

$$= \int \frac{x-4}{(x-4)^2+4}dx + \int \frac{4}{(x-4)^2+4}dx$$

Consider $\int \frac{x-4}{(x-4)^2+4}dx$

Let $u = x-4$

Then $du = dx$

So, $\int \frac{x-4}{(x-4)^2+4}dx = \int \frac{u}{u^2+4}du$

Let $z = u^2+4$

Then $dz = 2udu$ or $\frac{1}{2}dz = udu$

Thus, $\int \frac{u}{u^2+4}du = \int \frac{1}{2z}dz$

$$= \frac{1}{2}\ln|z| + C$$

$$= \frac{1}{2}\ln|u^2+4| + C$$

Therefore $\int \frac{x-4}{(x-4)^2+4}dx = \frac{1}{2}\ln\left|(x-4)^2+4\right| + C$

Consider $\int \frac{4}{(x-4)^2+4}dx$

Given $u = x-4$

Then $du = dx$

Thus, $\int \frac{4}{(x-4)^2+4}dx = \int \frac{4}{u^2+4}du$

$$= 2\arctan\left(\frac{u}{2}\right) + C$$

$$= 2\arctan\left(\frac{x-4}{2}\right) + C$$

Therefore $\displaystyle\int \frac{x}{x^2 - 8x + 20}\,dx = \frac{1}{2}\ln\left|(x-4)^2 + 4\right| + 2\arctan\left(\frac{x-4}{2}\right) + C$

8. Find the value of c such that $\int_{-1}^{c} \left|2-x^2\right| dx = \frac{1+8\sqrt{2}}{3}$, where $c > \sqrt{2}$

Solution From $f(x) = \left|2-x^2\right|$

We will get $f(x) = \begin{cases} -\left(2-x^2\right) & , \quad x \geq \sqrt{2} \\ 2-x^2 & , \quad -\sqrt{2} < x < \sqrt{2} \\ -\left(2-x^2\right) & , \quad x \leq -\sqrt{2} \end{cases}$

Hence, $\int_{-1}^{c} \left|2-x^2\right| dx = \int_{-1}^{\sqrt{2}} \left(2-x^2\right) dx - \int_{\sqrt{2}}^{c} \left(2-x^2\right) dx$

$$= \left(2x - \frac{x^3}{3}\right)\Bigg|_{x=-1}^{x=\sqrt{2}} - \left(2x - \frac{x^3}{3}\right)\Bigg|_{x=\sqrt{2}}^{x=c}$$

$$= \left(\left(2\sqrt{2} - \frac{2\sqrt{2}}{3}\right) - \left(-2 + \frac{1}{3}\right)\right) - \left(\left(2c - \frac{c^3}{3}\right) - \left(2\sqrt{2} - \frac{2\sqrt{2}}{3}\right)\right)$$

$$= \frac{8\sqrt{2}}{3} + \frac{5}{3} - 2c + \frac{c^3}{3}$$

Since $\int_{-1}^{c} \left|2-x^2\right| dx = \frac{1+8\sqrt{2}}{3}$

So $\frac{1+8\sqrt{2}}{3} = \frac{8\sqrt{2}}{3} + \frac{5}{3} - 2c + \frac{c^3}{3}$

$c^3 - 6c + 4 = 0$

$(c-2)\left(c+1+\sqrt{3}\right)\left(c+1-\sqrt{3}\right)$

$c \in \left\{-1-\sqrt{3}, -1+\sqrt{3}, 2\right\}$

Since $c > \sqrt{2}$ we will get $c = 2$

Therefore, the value of c that make $\int_{-1}^{c} \left|2-x^2\right| dx = \frac{1+8\sqrt{2}}{3}$ is $c = 2$

9. Find the value of $\int_{0}^{15} \frac{1}{1+\sqrt[4]{x+1}}\,dx$

Solution Given $u = 1+\sqrt[4]{x+1}$ or $\sqrt[4]{x+1} = u-1$

then $du = \frac{1}{4}\left(x+1\right)^{-\frac{3}{4}}dx$ or $4du = \left(x+1\right)^{-\frac{3}{4}}dx$

Substitute $x = 15$ into $u = 1+\sqrt[4]{x+1}$, we will get

$$u = 3$$

Substitute $x = 0$ into $u = 1+\sqrt[4]{x+1}$, we will get

$$u = 2$$

Thus,
$$\begin{aligned}
\int_{0}^{15} \frac{1}{1+\sqrt[4]{x+1}}\,dx &= \int_{2}^{3} \frac{4\left(u-1\right)^{3}}{u}\,du \\
&= 4\int_{2}^{3} \frac{u^{3}-3u^{2}+3u-1}{u}\,du \\
&= 4\int_{2}^{3} \left(u^{2}-3u+3-\frac{1}{u}\right)du \\
&= 4\left(\frac{u^{3}}{3}-\frac{3u^{2}}{2}+3u-\ln\left|u\right|\right)\Bigg|_{u=2}^{u=3} \\
&= 4\left(\left(9-\frac{27}{2}+9-\ln\left|3\right|\right)-\left(\frac{8}{3}-6+6-\ln\left|2\right|\right)\right) \\
&= 4\left(18-\frac{27}{2}-\ln 3-\frac{8}{3}+\ln 2\right) \\
&= \frac{22}{3}+4\ln\frac{2}{3}
\end{aligned}$$

10. Find the value of $\int 2x\sin(2x)\,dx$

Solution Use integration by parts $\int u\,dv = uv - \int v\,du$

$u = 2x$ $\qquad dv = \sin(2x)\,dx$

$du = 2dx$ $\qquad v = \int \sin(2x)\,dx = -\dfrac{\cos(2x)}{2}$

We will get $\int 2x\sin(2x)\,dx = -x\cos(2x) - \int\left(-\dfrac{\cos(2x)}{2}\right)\cdot 2dx$

$$= -x\cos(2x) + \int \cos(2x)\frac{d(2x)}{2}$$

$$= -x\cos(2x) + \frac{1}{2}\sin(2x) + C$$

11. Find the value of $\int \cos(\ln x)\,dx$

Solution Use integration by parts $\int u\,dv = uv - \int v\,du$

$u = \cos(\ln x)$ $dv = dx$

$du = -\frac{1}{x}\sin(\ln x)\,dx$ $v = \int dv = \int dx = x$

Then $\int \cos(\ln x)\,dx = x\cos(\ln x) - \int x \cdot \left(-\frac{1}{x}\sin(\ln x)\right)dx$

$= x\cos(\ln x) + \int \sin(\ln x)\,dx + c_1$ (1)

Evaluate $\int \sin(\ln x)\,dx$ by using integration by parts $\int u\,dv = uv - \int v\,du$

Let $u = \sin(\ln x)$ and $dv = dx$

$du = \frac{1}{x}\cos(\ln x)\,dx$ and $v = \int dv = \int dx = x$

Then, $\int \sin(\ln x)\,dx = x\sin(\ln x) - \int x \cdot \frac{1}{x}\cos(\ln x)\,dx$

$= x\sin(\ln x) - \int \cos(\ln x)\,dx + c_2$ (2)

Substitute equation 2 into equation 1

$$\int \cos(\ln x)\,dx = x\cos(\ln x) + \int \sin(\ln x)\,dx + c_1$$

$$\int \cos(\ln x)\,dx = x\cos(\ln x) + \left[x\sin(\ln x) - \int \cos(\ln x)\,dx + c_2\right] + c_1$$

$$\int \cos(\ln x)\,dx = x\cos(\ln x) + x\sin(\ln x) - \int \cos(\ln x)\,dx + (c_1 + c_2)$$

$$2\int \cos(\ln x)\,dx = x\cos(\ln x) + x\sin(\ln x) + C$$, where $C = c_1 + c_2$

$$\int \cos(\ln x)\,dx = \frac{x\cos(\ln x) + x\sin(\ln x)}{2} + C$$

12. Find the value of $\int \frac{3x+5}{x^3-x^2-x+1}dx$

Solution Factorize the denominator of $\frac{3x+5}{x^3-x^2-x+1}$ we will get

$$\frac{3x+5}{x^3-x^2-x+1}=\frac{3x+5}{(x-1)^2(x+1)}$$

Consider $\frac{3x+5}{(x-1)^2(x+1)}$, we obtain

$$\frac{3x+5}{(x-1)^2(x+1)}=\frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{C}{x+1}$$

$$3x+5=A(x-1)(x+1)+B(x+1)+C(x-1)^2 \qquad \underline{\qquad\qquad}(1)$$

Substitute $x=1$ into equation 1, we obtain

$$B=4$$

Substitute $x=-1$ into equation 1, we obtain

$$C=\frac{1}{2}$$

Substitute $x=0$, $B=4$ and $C=\frac{1}{2}$ into equation 1, we will get

$$A=-\frac{1}{2}$$

Therefore, $\frac{3x+5}{(x-1)^2(x+1)}=-\frac{1}{2(x-1)}+\frac{4}{(x-1)^2}+\frac{1}{2(x+1)}$

$$\therefore \quad \int \frac{3x+5}{x^3-x^2-x+1}dx=\int\left(-\frac{1}{2(x-1)}+\frac{4}{(x-1)^2}+\frac{1}{2(x+1)}\right)dx$$

$$=-\frac{1}{2}\ln|x-1|-\frac{4}{x-1}+\frac{1}{2}\ln|x+1|+C$$

13. Find the value of $\int \frac{8x^2-10x+3}{(x-4)\left(x^2+1\right)}dx$

Solution From $\frac{8x^2-10x-3}{(x-4)(x^2+1)} = \frac{A}{x-4} + \frac{Bx+C}{x^2+1}$

We will get $8x^2-10x-3 = (x-4)(x^2+1)\left(\frac{A}{x-4} + \frac{Bx+C}{x^2+1}\right)$

$$8x^2-10x-3 = A(x^2+1) + (Bx+C)(x-4)$$

$$8x^2-10x-3 = Ax^2 + A + Bx^2 - 4Bx + Cx - 4C$$

$$8x^2-10x-3 = (A+B)x^2 + (-4B+C)x + (A-4C)$$

Comparing Coefficients of x, we get that

$$\begin{aligned} A+B &= 8 && (1)\\ -4B+C &= -10 && (2)\\ A-4C &= -3 && (3) \end{aligned}$$

From equation 3, $A = 4C-3$ substitute into equation 1, we obtain

$$B+4C=11$$

$$B=11-4C$$

Substitute $B=11-4C$ into equation 1, we will get

$$-4(11-4C)+C=-10$$

$$C=2$$

Substitute $C=2$ into equation 2 and 3, we get

$$A=5 \text{ and } B=3$$

Therefore, $\int \frac{8x^2-10x-3}{(x-4)(x^2+1)}dx = \int \frac{5}{x-4} + \frac{3x+2}{x^2+1}dx = \int \frac{5}{x-4} + \frac{3x}{x^2+1} + \frac{2}{x^2+1}dx$

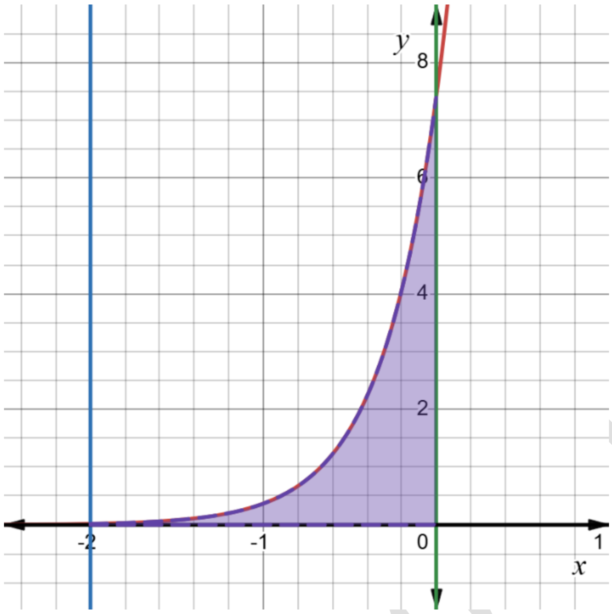
$$= \int \frac{5}{x-4}dx + \int \frac{3x}{x^2+1}dx + \int \frac{2}{x^2+1}dx$$

$$= 5\ln|x-4| + 3\int \frac{x}{x^2+1}dx + 2\int \frac{1}{x^2+1}dx$$

$$= 5\ln|x-4| + \frac{3}{2}\ln\left|x^2+1\right| + 2\arctan(x) + C$$

14. Find the area enclosed by the curve $y = e^{3x+2}$, $x = -2$, $x - axis$ and $y - axis$

Solution the area enclosed by the curve $y = e^{3x+2}$, $x = -2$, $x - axis$ and $y - axis$ is show in the following figure.



Therefore, the area enclosed by all curves is $A = \int_{-2}^{0} e^{3x+2}\, dx$

Let $u = 3x + 2$

Then $du = 3dx$

Find the value of u where $x = -2$, we obtain

$$u = -4$$

Find the value of u where $x = 0$, we obtain

$$u = 2$$

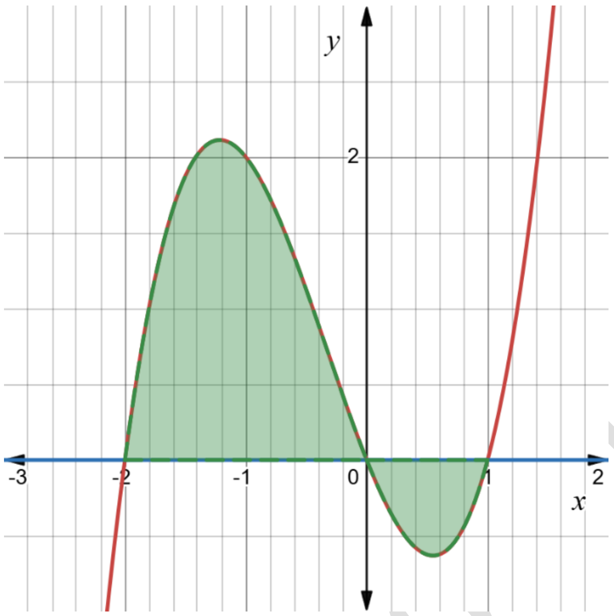
So, $A = \int_{-2}^{0} e^{3x+2}\, dx = \frac{1}{3}\int_{-4}^{2} e^{u}\, dx$

$$= \frac{1}{3}e^{u}\Big|_{u=-4}^{u=2}$$

$$= \frac{e^{2} - e^{-4}}{3} \quad \text{unit square}$$

15. Find the area enclosed by the curve $y = x^3 + x^2 - 2x$ and $x - axis$

Solution the area enclosed by the curve $y = x^3 + x^2 - 2x$ and $x - axis$ is show in the following figure.



Find the intersection points between a curve $y = x^3 + x^2 - 2x$ and $x - axis$, we get

$$x^3 + x^2 - 2x = 0$$

$$x\left(x^2 + x - 2\right) = 0$$

$$x(x-1)(x+2) = 0$$

$$\therefore \quad x \in \{-2, 0, 1\}$$

Therefore, the intersection points between a curve $y = x^3 + x^2 - 2x$ and $x - axis$ are $(-2, 0)$, $(0, 0)$ and $(1, 0)$

Find the area enclosed by all curve, we get

$$A = \int_{-2}^{0}\left(x^3 + x^2 - 2x\right)dx + \int_{0}^{1}\left(0 - \left(x^3 + x^2 - 2x\right)\right)dx$$

$$= \left(\frac{1}{4}x^4 + \frac{1}{3}x^3 - x^2\right)\Bigg|_{x=-2}^{x=0} - \left(\frac{1}{4}x^4 + \frac{1}{3}x^3 - x^2\right)\Bigg|_{x=-0}^{x=1}$$

$$= \left(0 - \left(\frac{(-2)^4}{4} + \frac{(-2)^3}{3} - (-2)^2\right)\right) - \left(\left(\frac{(1)^4}{4} + \frac{(1)^3}{3} - (1)^2\right) - 0\right)$$

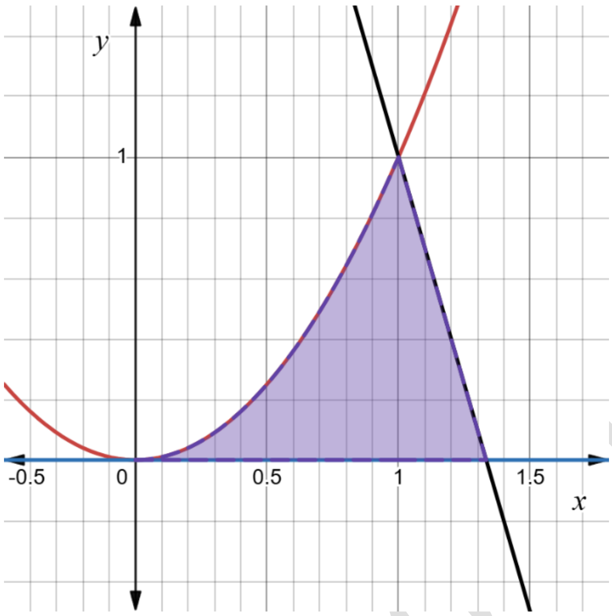
$$=\left(-4+\frac{8}{3}+4\right)-\left(\frac{1}{4}+\frac{1}{3}-1\right)$$

$$=\frac{8}{3}-\left(-\frac{5}{12}\right)$$

$$=\frac{37}{12}\text{ unit square}$$

16. Find the area enclosed by the curve $y = x^2$, $y = -3x + 4$ and $x - axis$

Solution the area enclosed by the curve $y = x^2$, $y = -3x + 4$ and $x - axis$ as a following figure.



Find the intersection point between a curve $y = -3x + 4$ and $x - axis$, we will get

$$-3x + 4 = 0$$

$$x = \frac{4}{3}$$

Therefore, the intersection point between a curve $y = -3x + 4$ and $x - axis$ is $\left(\frac{4}{3}, 0\right)$

Find the area enclosed by all curve, we get

$$A = \int_0^1 x^2\,dx + \int_1^{\frac{4}{3}} (-3x + 4)\,dx$$

$$= \left.\frac{x^3}{3}\right|_{x=0}^{x=1} + \left.\left(-\frac{3x^2}{2} + 4x\right)\right|_{x=1}^{x=\frac{4}{3}}$$

$$= \left(\frac{1}{3} - 0\right) + \left(\left(-\frac{3}{2} \times \left(\frac{4}{3}\right)^2 + 4 \times \left(\frac{4}{3}\right)\right) - \left(-\frac{3}{2} + 4\right)\right)$$

$$= \frac{1}{2} \text{ unit square}$$

17. Find the value of $\int_0^2 \frac{x}{(x^2-1)^2}dx$ and consider whether it is a convergent or divergent integral. If it converges, what value does it converge to?

Solution Since $\int_0^2 \frac{x}{(x^2-1)^2}dx = \int_0^1 \frac{x}{(x^2-1)^2}dx + \int_1^2 \frac{x}{(x^2-1)^2}dx$

Consider $\int \frac{x}{(x^2-1)^2}dx$

Let $u = x^2-1$ then $du = 2xdx$ or $\frac{1}{2}du = xdx$

So.
$$\begin{aligned}\int \frac{x}{(x^2-1)^2}dx &= \int \frac{1}{2u^2}du \\ &= -\frac{1}{2u}+C \\ &= -\frac{1}{2\left(x^2-1\right)}+C\end{aligned}$$

Thus,
$$\int_0^1 \frac{x}{(x^2-1)^2}dx = \lim_{b\to 1^-}\int_0^b \frac{x}{(x^2-1)^2}dx = -\frac{1}{2}\lim_{b\to 1^-}\left.\frac{1}{x^2-1}\right|_{x=0}^{x=b} = -\frac{1}{2}\lim_{b\to 1^-}\left(\frac{1}{b^2-1}+1\right) = +\infty$$

and
$$\int_1^2 \frac{x}{(x^2-1)^2}dx = \lim_{a\to 1^+}\int_a^2 \frac{x}{(x^2-1)^2}dx = -\frac{1}{2}\lim_{a\to 1^+}\left.\frac{1}{x^2-1}\right|_{x=a}^{x=2} = -\frac{1}{2}\lim_{a\to 1^+}\left(\frac{1}{3}-\frac{1}{a^2-1}\right) = +\infty$$

Therefore, $\int_0^2 \frac{x}{(x^2-1)^2}dx$ is diverges

18. Find the value of $\int_0^{+\infty} \frac{1}{\sqrt{x+2x^2+x^3}}dx$ and consider whether it is a convergent or divergent integral. If it converges, what value does it converge to?

Solution Since $\int_0^{+\infty} \frac{1}{\sqrt{x+2x^2+x^3}}dx = \int_0^{1} \frac{1}{\sqrt{x+2x^2+x^3}}dx + \int_1^{+\infty} \frac{1}{\sqrt{x+2x^2+x^3}}dx.$

Consider $\int \frac{1}{\sqrt{x+2x^2+x^3}}dx = \int \frac{1}{\sqrt{x(1+2x+x^2)}}dx = \int \frac{1}{\sqrt{x(1+x)^2}}dx = \int \frac{1}{\sqrt{x}\left|1+x\right|}dx$

Let $u = \sqrt{x}$, so, we get $du = \frac{1}{2\sqrt{x}}dx$ or $dx = 2\sqrt{x}du$

Thus,

$$\int \frac{1}{\sqrt{x+2x^2+x^3}}dx = \int \frac{1}{\sqrt{x}\left|1+x\right|}dx$$

$$= \int \frac{2}{(1+u^2)}du$$

$$= 2\arctan(u)$$

$$= 2\arctan(\sqrt{x}) + C$$

We obtain

$$\int_0^{1} \frac{1}{\sqrt{x+2x^2+x^3}}dx = \lim_{a\to 0^+}\int_a^{1} \frac{1}{\sqrt{x+2x^2+x^3}}dx = \lim_{a\to 0^+} 2\left|\arctan(\sqrt{x})\right|_{x=a}^{x=1}$$

$$= 2\left(\frac{\pi}{4}\right) - 2(0) = \frac{\pi}{2},$$

and

$$\int_1^{+\infty} \frac{1}{\sqrt{x+2x^2+x^3}}dx = \lim_{b\to +\infty}\int_1^{b} \frac{1}{\sqrt{x+2x^2+x^3}}dx = \lim_{b\to +\infty} 2\left|\arctan(\sqrt{x})\right|_{x=1}^{x=b}$$

$$= 2\left(\frac{\pi}{2}\right) - 2\left(\frac{\pi}{4}\right) = \frac{\pi}{2}.$$

Therefore, $\int_0^{+\infty} \frac{1}{\sqrt{x+2x^2+x^3}}dx$ is converges to π .

19. Use the trapezoidal rule to estimate the value of $\int_{0.0}^{2.0} f(x)\,dx$ with $n=4$, given than $f(x)$ as shown in the following table.

x	0.0	0.5	1.0	1.5	2.0
$f(x)$	1.8	2.4	1.6	1.2	0.4

Solution From the problem, we have $a=0,\ b=2,\ n=4.$

So, we get $h = \dfrac{b-a}{n} = \dfrac{2-0}{4} = \dfrac{2}{4} = 0.5,$

and $x_0 = 0,\ x_1 = 0.5,\ x_2 = 1,\ x_3 = 1.5,\ x_4 = 2.$

From the trapezoid sum:

$$\int_a^b f(x)\,dx \approx T_n = \frac{h}{2}\left[y_0 + 2y_1 + 2y_2 + 2y_3 + 2y_4 + \ldots + 2y_{n-2} + 2y_{n-1} + y_n\right]$$

Therefore, we obtain

$$\begin{aligned}\int_0^2 f(x)\,dx \approx T_4 &= \frac{0.5}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + f(x_4)\right]\\ &= \frac{0.5}{2}\left[1.8 + 2(2.4) + 2(1.6) + 2(1.2) + 0.4\right]\\ &= \frac{0.5}{2}\left[1.8 + 4.8 + 3.2 + 2.4 + 0.4\right]\\ &= \frac{0.5}{2}\left[12.6\right]\\ &= \frac{1}{4}\left[12.6\right]\\ &= 3.15.\end{aligned}$$

20. Use Simpson's rule to estimate the value of $\int_{0.0}^{3.2} xf(x)\,dx$ with $n=4$, given that $f(x)$ as shown in the following table.

x	0.0	0.4	0.8	1.2	1.6	2.0	2.4	2.8	3.2
$f(x)$	2.8	2.5	2.3	2.4	2.1	3.6	0.4	0.8	0.0

Solution From the problem, we have $a=0,\ b=3.2,\ n=4.$

So, we get $h = \dfrac{b-a}{n} = \dfrac{3.2-0}{4} = \dfrac{3.2}{4} = 0.8,$

and $x_0 = 0,\ x_1 = 0.8,\ x_2 = 1.6,\ x_3 = 2.4,\ x_4 = 3.2.$

From the Simpson's rule:

$$\int_a^b f(x)\,dx \approx S_n = \frac{h}{3}\left[y_0 + 4y_1 + 2y_2 + 4y_3 + 2y_4 + \ldots + 2y_{n-2} + 4y_{n-1} + y_n\right]$$

Therefore, we obtain

$$\begin{aligned}
\int_{0.0}^{3.2} xf(x)\,dx \approx S_4 &= \frac{0.8}{3}\left[x_0 f(x_0) + 4x_1 f(x_1) + 2x_2 f(x_2) + 4x_3 f(x_3) + x_4 f(x_4)\right] \\
&= \frac{0.8}{3}\left[(0)(2.8) + 4(0.8)(2.3) + 2(1.6)(2.1) + 4(2.4)(0.4) + (3.2)(0)\right] \\
&= \frac{0.8}{3}\left[\ 4(0.8)(2.3) + 2(1.6)(2.1) + 4(2.4)(0.4)\right] \\
&= \frac{0.8}{3}\left[\ 7.36 + 6.72 + 3.84\right] \\
&= \frac{0.8}{3}\left[17.92\right] \\
&= 4.7787.
\end{aligned}$$