Fibonacci Sequence Presentation

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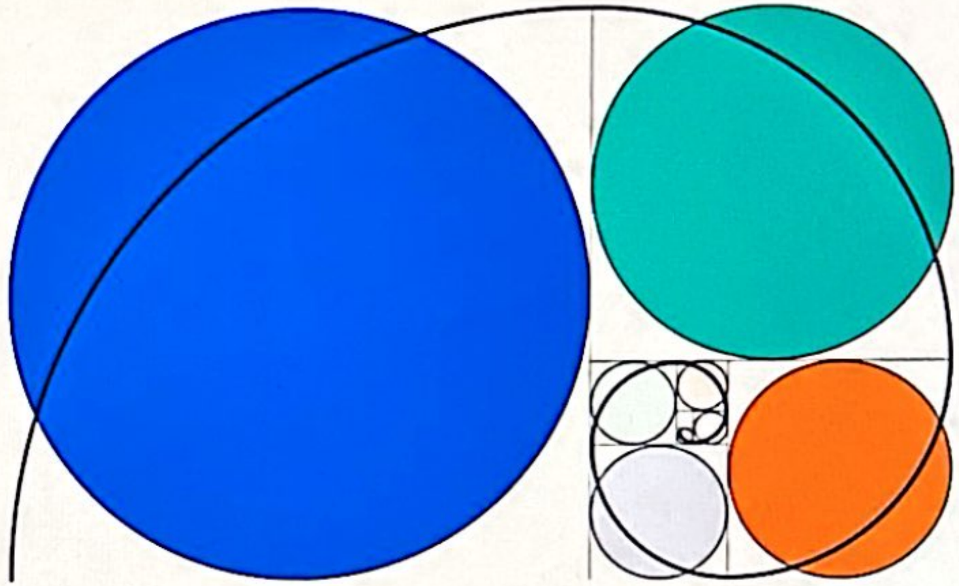
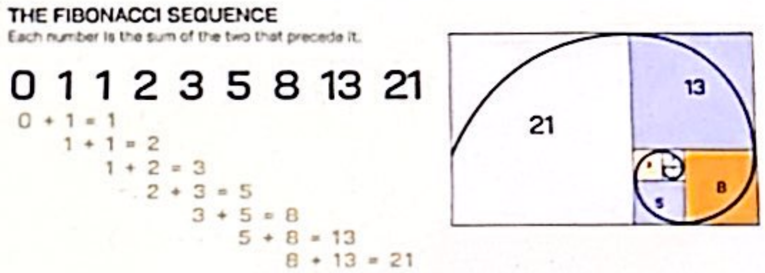


Introduction to the Fibonacci Sequence

- The Fibonacci sequence is a series of numbers where each number (known as a Fibonacci number) is the sum of the two preceding ones. It typically starts with 0 and 1 or 0 and 0.
- The sequence goes:
- 0,1,1,2,3,5,8,13,21,34,55,...

Formula:-

- Fn = Fn-1 + Fn-2
- Fn-1 = the previous term;
- Fn-2 = the term before that;
- Example:- 5 = 3 + 2



Definition 5 The Fibonacci sequence, $f_0, f_1, f_2, \ldots$, is defined by the initial conditions $f_0 = 0, f_1 = 1$, and the recurrence relation

$$f_n = f_{n-1} + f_{n-2}$$

for $n = 2, 3, 4, \ldots$.

Recurrence Relation.

- "A recurrence relation is an equation that recursively defines a sequence of values based on previous terms. In the case of Fibonacci Sequence, each number is the sum of two preceding ones, which is essentially a Recurrence Relation."
- This type of recurrence relation is Linear and homogeneous.
- **Linear:** Term is a combination of preceding terms.
- **Homogeneous:** The Sequence depends only on previous terms
- **Non-Linear:** A relation where the terms are combinations in a non-linear way i.e., through multiplications or exponents. Example: Factorial Sequence : F(n) = n. F(n-1), F(0) =1.
- **Non-Homogeneous :** *A Sequence that* not only depends on the previous terms but also has any additional *independent terms.* Example: Arithmetic Sequence with a Linear term *an = 2an-1 +n* , a(0)=1

The initial conditions are $a_1 = 2$, because both bit strings of length one, 0 and 1 do not have consecutive 0s, and $a_2 = 3$, because the valid bit strings of length two are 01, 10, and 11. To obtain a_5, we use the recurrence relation three times to find that

$$a_3 = a_2 + a_1 = 3 + 2 = 5,$$
$$a_4 = a_3 + a_2 = 5 + 3 = 8,$$
$$a_5 = a_4 + a_3 = 8 + 5 = 13.$$

Remark: Note that $\{a_n\}$ satisfies the same recurrence relation as the Fibonacci sequence. Because $a_1 = f_3$ and $a_2 = f_4$ it follows that $a_n = f_{n+2}$.

Historical Background

Links

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FIBONACCI (1170-1250) Fibonacci (short for *filius Bonacci*, or "son of Bonacci") was also known as Leonardo of Pisa. He was born in the Italian commercial center of Pisa. Fibonacci was a merchant who traveled extensively throughout the Mideast, where he came into contact with Arabian mathematics. In his book *Liber Abaci*, Fibonacci introduced the European world to Arabic notation for numerals and algorithms for arithmetic. It was in this book that his well known rabbit problem (described in Section 8.1) appeared. Fibonacci also wrote books on geometry and trigonometry and on Diophantine equations, which involve finding integer solutions to equations.

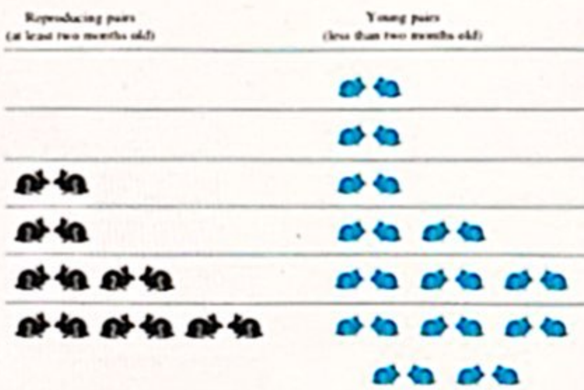
- The Sanskrit grammarian 'Pingala' mentioned a sequence resembling Fibonacci's in texts dating back to the 5th century BC and the 3rd century AD.

Rabbit Population Problem

- Denote by fn the number of pairs of rabbits after n months. We will show that fn, n =1,2,3,…, are the terms of the Fibonacci sequence.
- The rabbit population can be modeled using a recurrence relation. At the end of the first month, the number of pairs of rabbits on the island is f1 = 1. Because this pair does not breed during the second month, f2 = 1 also. To find the number of pairs after n months, add the number on the island the previous month, fn−1, and the number of newborn pairs, which equals fn−2, because each newborn pair comes from a pair at least 2 months old.
- Consequently, the sequence {fn} satisfies the recurrence relation
- fn = fn−1 + fn−2
- for n ≥ 3together with the initial conditions f1 = 1 and f2 = 1. Because this recurrence relation and the initial conditions uniquely determine this sequence, the number of pairs of rabbits on the island after n months is given by the nth Fibonacci number

EXAMPLE 1 **Rabbits and the Fibonacci Numbers** Consider this problem, which was originally posed by Leonardo Pisano, also known as Fibonacci, in the thirteenth century in his book *Liber abaci*. A young pair of rabbits (one of each sex) is placed on an island. A pair of rabbits does not breed until they are 2 months old. After they are 2 months old, each pair of rabbits produces another pair each month, as shown in Figure 1. Find a recurrence relation for the number of pairs of rabbits on the island after *n* months, assuming that no rabbits ever die.

Links



Month	Reproducing pairs	Young pairs	Total pairs
1	0	1	1
2	0	1	1
3	1	1	2
4	1	2	3
5	2	3	5
6	3	5	8