The Fibonacci Sequence

Zayn Ali

November 2024

1 Introduction to the Fibonacci Sequence

What Fibonacci sequence is?

1.1 Definition:-

The Fibonacci sequence is a series of numbers where each number (known as a Fibonacci number) is the sum of the two preceding ones. It typically starts with 0 and 1 or 0 and 0. The sequence goes: 0,1,1,2,3,5,8,13,21,34,55,...

1.2 Significance:-

The Fibonacci sequence not only holds mathematical significance but also reveals profound connections within nature and practical applications in fields like finance. Its beauty lies in its simplicity and its ability to describe complex patterns found throughout our world. Whether you're studying mathematics or observing nature's design, the Fibonacci sequence offers endless fascination and insight. The Fibonacci sequence is not just a mathematical curiosity; it represents fundamental principles that govern growth patterns in nature and art. Its connections with various fields, from biology to finance, demonstrate its universal relevance. By understanding the properties and applications of this remarkable sequence, we gain insight into both mathematical theory and real-world phenomena. As we continue exploring mathematics and its applications, let us appreciate how concepts like the Fibonacci sequence enrich our understanding of the world around us. Whether through its aesthetic beauty or practical utility, the Fibonacci sequence remains a captivating subject worthy of study and admiration.

1.3 Recursive Nature:-

The recursive formula defining the Fibonacci sequence means that each term relies on its predecessors. This property makes it an excellent example of recursion in mathematics.

1.4 Formula:-

- Fn = Fn-1 + Fn-2

- Fn-1 = the previous term;

- Fn-2 = the term before that;

Example:- 5 = 3 + 2;

1.5 The most important phenomenon of the Fibonacci sequence:-

The Golden Ratio - The Divine Proportion How the ratio of successive Fibonacci numbers approximates the golden ratio? = 1.61803. As the sequence progresses, the ratio between consecutive Fibonacci numbers gets closer to this golden ratio. The ratio as we divide two consecutive terms of the Fibonacci sequence is the most beautiful and important part of this sequence. The human body itself exhibits Fibonacci proportions. The ratio of the length of the forearm to the length of the hand, for example, is often close to the Golden Ratio. Many composers have used the sequence to structure their compositions and create harmonious melodies.

1.6 Historical Background

The sequence was introduced to the Western world by Leonardo of Pisa, known as Fibonacci, in his 1202 book "Liber Abaci" (The Book of Calculation). Fibonacci used the sequence to illustrate the growth pattern of a population of rabbits, starting from a single pair, assuming that every month each pair of mature rabbits produces a new pair, which becomes productive from the second month on. The Book of Calculations by Liber Abaci explains the power of the Fibonacci sequence. How it emerges in the world.

1.7 Fibonacci sequence First discovery

Fibonacci was not discovered by Liber Abaci but in ancient India 1000s of years ago. The Sanskrit grammarian 'Pingala' mentioned a sequence resembling Fibonacci's in texts dating back to the 5th century BC and the 3rd century AD. Liber Abaci's contribution was that he introduced it to the Western world and showed growth patterns through it. The literal meaning of Fibonacci:- The literal meaning of Fibonacci is 'son of the Bonacci clan'. What was the Fibonacci New Year? The Fibonacci new year was 1/1/23. Do you know how many kilo-meters are in a mile? There are 1.6 miles in 1 km. 1.6 is the value of the Golden ratio. If you want to know how many miles in a km you can just see the Fibonacci sequence. 8 miles = 13km, And 90 km round down to 89km = 55 miles.

1.8   The Great Pyramid of Giza:

- Some theories suggest that the dimensions of the Great Pyramid incorporate the Golden Ratio.

- The ratio of the height to half the base length is close to the Golden Ratio.

1.9   Fibonacci - A basis for e^x:-

Fibonacci number can also give the basis for a function whose derivative is also itself.