Golden Ration

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1 What is golden ratio?

1.1 Introduction :-

We know that the Fibonacci sequence is given as 0,1,1,2,3,5,8,13,21,......
Now, if we divide any term with it's previous term, We keep on getting closer to a number which is called the Golden ratio and it's approximated value is 1.618.
Golden Ratio which is denoted by a Greek symbol Phi (ϕ) is the MOST IRRATIONAL NUMBER ever i.e It is impossible to find it's exact value. Its value is given by:
$\phi \approx \frac{1+\sqrt{5}}{2}$

1.2 Predicate logic:-

$\forall(n)(n >= 2- > F(n) = F(n-1) + F(n-2), n \epsilon R)$
$\phi = F_{n+1}/F_n$ where n approaches ∞

1.3 Finding the value of Golden ratio :-

The formula to find the exact value of golden ratio is
$\phi = Lim_{n->\infty} F_{n+1}/F_n$.
By this formula we can conclude that it is impossible to find the exact value of the Golden ratio.

The time complexity to compute the golden ratio to arbitrary precision depends on the desired number of decimal places. However, it is impossible to compute the golden ratio with 100 percent accuracy because it is an irrational number. A program to show a really precise value of the Golden Ratio using Fibonacci Series:

2 THE DIVINE PROPORTION :-

The golden ratio is found in natural phenomena, including the arrangement of leaves, the patterns of shells, the spirals of galaxies, and even the proportions of the human body that is why it is called the DIVINE PROPORTION.

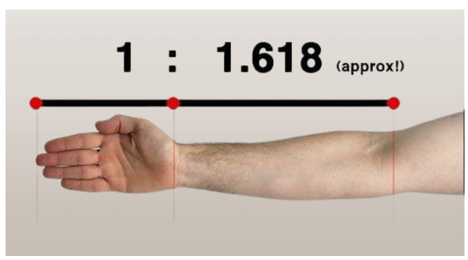

Figure 1: For example in human body,the ratio of your arm to the ratio of your hand is 1.618.

Even the ratio of your finger to your hand and the ratio of your arm to the ratio of your shoulder is 1.618. There are many other relations of Golden ratio to Nature, which is why some people call it Nature's Number.

3 THE GOLDEN SPIRAL:-

The infinite spiral associated with the golden ratio is called the golden spiral. Golden spiral is a logarithmic spiral whose growth factor is the golden ratio (ϕ).
As we know Fibonacci Sequence is given by 1,1,2,3,5,8,... now if we take a 1 by 1, 2 by 2, 3 by 3, 5 by 5 squares just like the Fibonacci Series We get:- The golden spiral

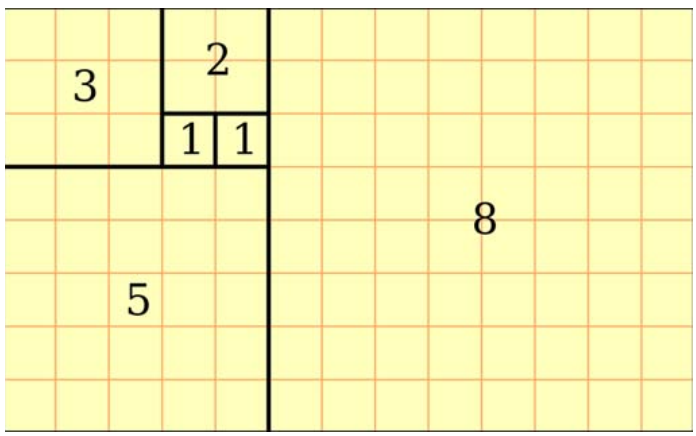

Figure 2: Squares by Fibonacci Sequence.

is constructed such that the sizes of squares drawn in its progression follow the Fibonacci sequence, and the ratio of their sides approaches (Φ) as the sequence extends.
This gives us something like:-

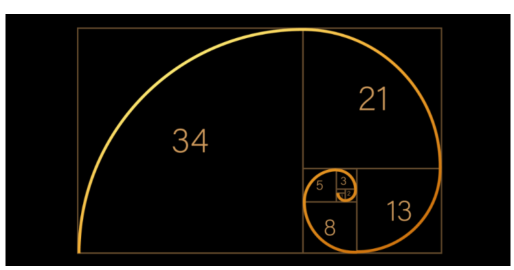

Figure 3: The Golden Spiral

From the above Predicate Logic that We constructed, We can tell than Fibonacci Series is an infinite Series so the Golden Spiral is also Infinite.
The Golden Spiral has a great significance in the designs of Nature and it also has some practical applications in Architecture(Esp for designing spiral staircases),Economics(Market Trends),Logo Designing, Arts and Photos (to align the picture within the spiral for visual appeal).

3.1 Golden Spiral in Nature

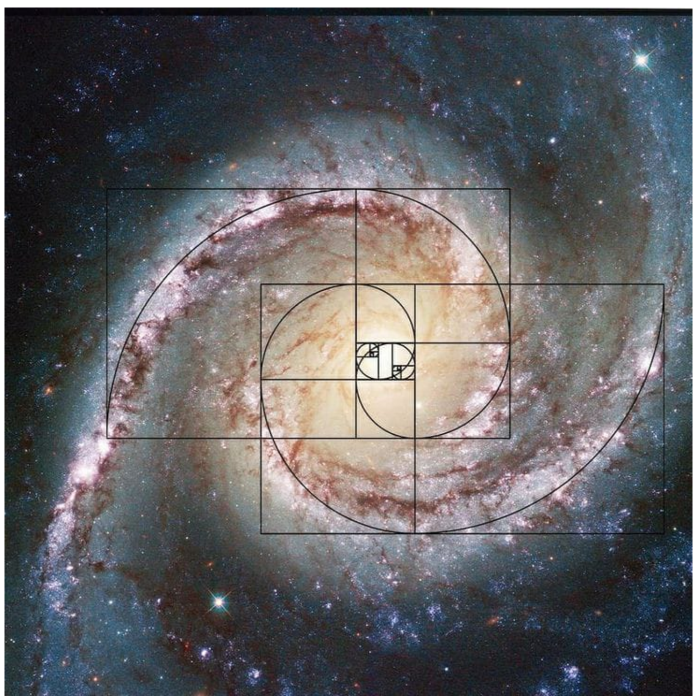

Figure 4: Golden Spiral in Galaxies

There are many other Examples like the shape of ears, the pattern in flowers etc.

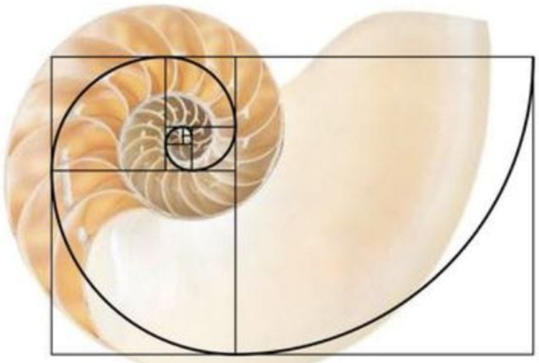

Figure 5: Golden Ratio in shells

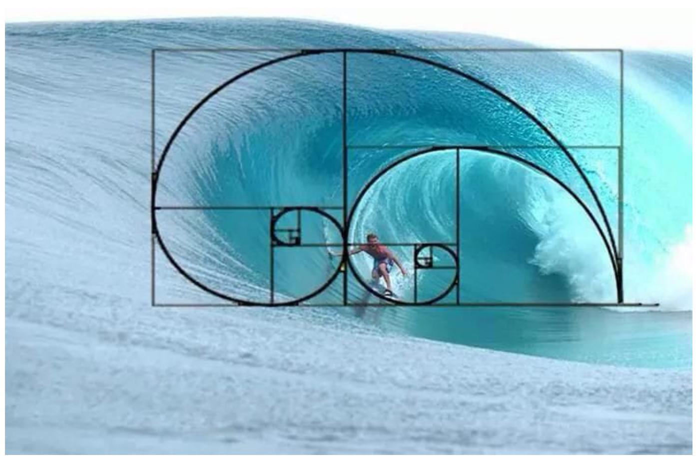

Figure 6: Golden Spiral in Hurricanes and Waves

4 Golden Ratio and Discrete Mathematics:-

4.1 Golden ratio by Recursion Method:-

We know that in Fibonacci Series $F_0 = 0$ and $F_1 = 1$, also:
$F_n = F_{n-1} + F_{n-2}$ for $n >= 2$.
By putting n = 2, $F_2 = F_1 + F_2 = 1 + 0 = 1$
Similarly, when n = 3, $F_3 = F_2 + F_1 = 1 + 1 = 2$
when n = 4, $F_4 = F_3 + F_2 = 2 + 1 = 3$
when n = 5, $F_5 = F_4 + F_3 = 3 + 2 = 5$
when n = 6, $F_6 = F_5 + F_4 = 5 + 3 = 8$
when n = 7, $F_7 = F_6 + F_5 = 8 + 5 = 13$
when n = 8, $F_8 = F_7 + f_6 = 13 + 8 = 21$
By this we can conclude that the growth of Fibonacci Series is exponential so lets assume,
$F_n = r^n$, where r is a constant
So,
$r^n = r^{n-1} + r^{n-2}$
Dividing by r^{n-2}, Assuming r is not equal to Zero
This gives us, $r^2 - r - 1 = 0$
After using Quadratic Equation to solve this we get,

$r = \frac{1+\sqrt{5}}{2} \wedge \frac{1-\sqrt{5}}{2}$

We know that, $\frac{1+\sqrt{5}}{2} \approx \phi \approx 1.618$
And, $\frac{1-\sqrt{5}}{2} \approx 1 - \phi = \neg\phi$, 1 - ϕ is the conjugate of Golden Ratio

4.2 Binary Trees:-

In the study of binary trees and recursive structures, the golden ratio often appears when analyzing the growth rate of certain tree-related algorithms.
For e.g For example, the number of nodes in certain types of binary trees grows exponentially according to the golden ratio, much like the Fibonacci sequence.

4.3 Golden Ratio in Growth models:-

In discrete growth models (like population models or branching processes), the golden ratio can emerge as a limiting growth factor. In these models, the ratio of successive states or stages of growth approaches ϕ as the process continues.

In certain types of graphs or paths, the golden ratio can appear as a limiting ratio of path lengths or distances as the number of steps increases.

4.4 Algorithm Designs:-

Algorithms like binary search trees and certain divide-and-conquer algorithms can exhibit performance improvements related to the golden ratio due to their optimal growth behavior. So it can help in developing efficient algorithms.