Recurrence Relation

A **recurrence relation** is an equation that recursively defines a sequence, where each term of the sequence is defined as a function of its preceding terms.

General Form

A recurrence relation for a sequence a_n can be expressed as:

$$a_n = f(a_{n-1}, a_{n-2}, \ldots, a_{n-k}, n)$$

where:

- n is the index, usually starting from 0 or 1.
- k is the number of previous terms on which the current term depends.
- f is a function involving the previous terms.

Example: Fibonacci Sequence

The **Fibonacci sequence** is a classic example of a recurrence relation:

$$F_n = F_{n-1} + F_{n-2} \quad \text{for } n \geq 2$$

with the initial conditions:

$$F_0 = 0, \quad F_1 = 1$$

Explanation of Fibonacci Recurrence Relation

- F_0 and F_1 are the starting points of the sequence. - For $n \geq 2$, each term F_n is calculated by adding the previous term F_{n-1} and the term before that F_{n-2}.

Example Calculation

Calculating the first few terms of the Fibonacci sequence using the recurrence relation:

$$\begin{aligned}
F_0 &= 0 \\
F_1 &= 1 \\
F_2 &= F_1 + F_0 = 1 + 0 = 1 \\
F_3 &= F_2 + F_1 = 1 + 1 = 2 \\
F_4 &= F_3 + F_2 = 2 + 1 = 3 \\
F_5 &= F_4 + F_3 = 3 + 2 = 5 \\
F_6 &= F_5 + F_4 = 5 + 3 = 8
\end{aligned}$$

Types of Recurrence Relations

1. **Linear vs. Non-linear**: If the function f is linear (e.g., $a_n = 2a_{n-1} + 3a_{n-2}$), it is a linear recurrence relation. If it is non-linear (e.g., $a_n = a_{n-1}^2 + a_{n-2}$), it is a non-linear recurrence relation.

2. **Homogeneous vs. Non-homogeneous**: Homogeneous recurrence relations have the form $a_n = f(a_{n-1}, \ldots, a_{n-k})$ without additional terms. Non-homogeneous relations include extra terms (e.g., $a_n = a_{n-1} + 2^n$).

3. **Order**: The order of a recurrence relation is determined by the number of preceding terms it depends on. For example, a_n depending on a_{n-1} and a_{n-2} is a second-order recurrence relation.