IEOR142 Lab 1

Fall 2023

August 25, 2023

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Outline

Course Information

- Lecture: Tue and Thu 12:30 – 2:00pm, Genetics&Plant Bio 100
- Discussion Labs:

  - Fri 2:00 – 3:00, Lewis 9,

  - 3:00 – 4:00, 4:00 – 5:00 Etcheverry 3106
- GSIs Office Hours:
  – Qiran: TBD
  – Serena: TBD
  – Francis: TBD

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Suggestions

- Start early with your assignments
- Check bCourses and Ed regularly
- Read the announcements on bCourses and ask questions on Ed

Vectors

- A vector $a \in \mathbb{R}^n$ is a collection of n real numbers, $a_1, a_2, \cdots, a_n$, as a single point in a n-dimensional space arranged in a column or a row.
- We usually write vectors in column format

$$a = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} \tag{1}$$

Matrices I

Column view

- Matrices can be viewed as a collection of (column) vectors of same size, that is, as a collection of points in a multi-dimensional space.
- Given n vectors $a_1, \cdots, a_n \in \mathbb{R}^m$, we can define the $m \times n$ matrix A with a_j as columns

$$A = \left(a_1, \cdots, a_n\right) \tag{2}$$

- Geometrically, $A \in \mathbb{R}^{m \times n}$ represents n points in a m-dimensional space.

$$A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ a_{21} & \cdots & a_{2n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix} \tag{3}$$

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Matrices II

Row view

- We can describe a matrix in row-wise. Given m vectors $b_1, \cdots, b_m \in \mathbb{R}^n$, we can define $m \times n$ matrix B with the transposed vectors b_i^T as rows

$$B = \begin{bmatrix} b_1^T \\ b_2^T \\ \vdots \\ b_m^T \end{bmatrix} \tag{4}$$

- Geometrically, B represents m points in a n-dimensional space.

Matrix Properties I

- **Transpose**. If $A \in \mathbb{R}^{m \times n}$, then $A^T \in \mathbb{R}^{n \times m}$

$$(A^T)_{ij} = A_{ji} \quad A = \begin{bmatrix} a_{11} & \cdots & a_{m1} \\ a_{12} & \cdots & a_{m2} \\ \vdots & \ddots & \vdots \\ a_{1n} & \cdots & a_{mn} \end{bmatrix} \tag{5}$$

- **Symmetric**. A matrix is symmetric if $A = A^T$
- **Inverse**. Given a **square matrix** $A \in \mathbb{R}^{n \times n}$, the inverse of A is denoted as A^{-1} such that

$$AA^{-1} = A^{-1}A = I_n \tag{6}$$

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Matrix Properties II

Other useful properties

- $(A^T)^T = A$
- $(A + B)^T = A^T + B^T$
- $(\alpha A)^T = \alpha A^T$
- $(AB)^T = B^T A^T$
- $(AB)^{-1} = B^{-1} A^{-1}$

Matrix Operations

- **Inner Product.** Given two vectors $a, b \in \mathbb{R}^n$, the inner product of a, b is

$$\langle a, b \rangle = a^T b = \sum_{i}^{n} a_i b_i \tag{7}$$

- **Matrix Multiplication.** $A \in \mathbb{R}^{m \times n}, B \in \mathbb{R}^{n \times p}$ then $C = AB$

$$C \in \mathbb{R}^{m \times p} \quad C_{ij} = \sum_{k=1}^{n} A_{ik} B_{kj} \tag{8}$$

Gradient

- Gradients generalize derivatives to scalar functions of several variables. The gradient of $f : \mathbb{R}^d \to \mathbb{R}$, denoted ∇f, is given by

$$\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \vdots \\ \frac{\partial f}{\partial x_n} \end{bmatrix} \quad \left[\nabla f\right]_i = \frac{\partial f}{\partial x_i} \tag{9}$$

- Two important rules

$$\nabla_x(a^T x) = \nabla_x(x^T a) = a \tag{10}$$

$$\nabla_x(x^T A x) = (A + A^T)x, \quad A \in \mathbb{R}^{n \times n} \tag{11}$$

If A is symmetric, we can simplify the result to $2Ax$

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Multiple Linear Regression

- The regression coefficient estimates $\hat{\beta} = (\hat{\beta}_1, \cdots, \hat{\beta}_p)$ are chosen to minimize RSS(β)

$$\text{RSS}(\beta) = \|y - X\beta\|_2^2 \tag{12}$$

- The gradient of RSS(β) is

$$\|y - X\beta\|_2^2 = (y - X\beta)^T(y - X\beta) = y^Ty - 2\beta^T(X^Ty) + \beta X^TX\beta \tag{13}$$

$$\nabla \text{RSS}(\beta) = -2X^Ty + 2X^TX\beta = -2X^T(y - X\beta) \tag{14}$$

- Set the gradient to 0 and we can get the solution of $\min_{\beta \in \mathbb{R}^p} \text{RSS}(\beta)$

$$-2X^T(y - X\beta) = 0 \to 2X^TX\beta = 2X^Ty \tag{15}$$

$$\hat{\beta} = (X^TX)^{-1}X^Ty \tag{16}$$

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Math Reference

- Linear algebra refresh: https://www.khanacademy.org/math/linear-algebra
- Multivariable calculus refresh: https://www.khanacademy.org/math/multivariable-calculus
- Math for ML: https://gwthomas.github.io/docs/math4ml.pdf
- The Matrix Cookbook: https://www.math.uwaterloo.ca/~hwolkowi/matrixcookbook.pdf

Python, Jupyter and Anaconda

- **Python** is a general-purpose programming language.
- **Jupyter** is a browser-based interactive development environment for notebooks, code and data. It allows users to configure and arrange workflows in data science and scientific computing.
- **Anaconda** is a distribution of the Python and R programming languages for scientific computing, that aims to simplify package management and deployment.
- Instructions on installing Python and Jupyter can be found on Ed
- **Packages** are collections of Python functions compiled in a well-defined format. Packages need to be loaded into the session to be used.