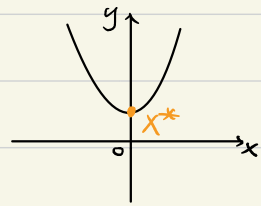
Lab 2

Sep 1^{st}, 2023

Optimization Review

Objective function:

- $f(x)$ single variable ex: $y = 3x^2+1$



① find $f'(x)$, set it to 0

② solve for x^*

③ check $f''(x)$ $\begin{cases} \text{if positive} \rightarrow \text{minimum} \\ \text{if negative} \rightarrow \text{maximum} \end{cases}$

- $f(x)$ multiple variable

ex: $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_p X_p + \varepsilon$

↖ Multiple Linear Regression

β_i : coefficients

β_0 : intercept

express in matrix form: $y = X\beta + \varepsilon_i$

① find $\nabla f(x)$ set it to 0

② solve for x^*

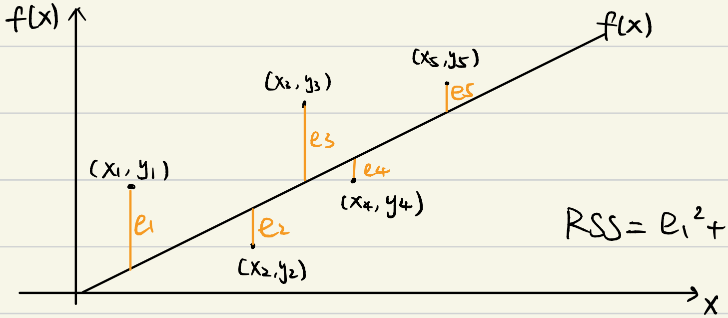
③ check Hessian $H(x)$ $\begin{cases} \text{if positive} \rightarrow \text{minimum} \\ \text{if negative} \rightarrow \text{maximum} \end{cases}$

$*$ $\nabla f(x) = \left(\frac{\partial f}{\partial x_1} \cdots \frac{\partial f}{\partial x_n}\right)$

$$H(x) = \begin{bmatrix} 0 & \frac{\partial f}{\partial x_1} & \cdots & \cdots & \frac{\partial f}{\partial x_n} \\ \frac{\partial f}{\partial x_1} & \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} & \cdots & \frac{\partial^2 f}{\partial x_1 \partial x_n} \\ \vdots & \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial^2 x_2} & \cdots & \frac{\partial^2 f}{\partial x_2 \partial x_n} \\ \frac{\partial f}{\partial x_n} & \cdots & \cdots & \cdots & \frac{\partial^2 f}{\partial^2 x_n} \end{bmatrix}$$

To find best β (fit equation the best):

$\min RSS(\beta)$ RSS: residual sum of squares



$$RSS = e_1^2 + e_2^2 + e_3^2 + e_4^2 + e_5^2$$

$$RSS(\beta) = \sum_{i=1}^{n} (y_i - \beta_0 - \beta_1 x_{i1} - \cdots - \beta_p x_{ip})^2$$

$$= \| y - X\beta \|_2^2$$

$$= (y - X\beta)^T (y - X\beta)$$

$$= y^T y - 2\beta^T (X^T y) + \beta X^T X \beta$$

$$\nabla RSS(\beta) = -2X^T y + 2X^T X\beta$$

$$= -2X^T (y - X\beta) \rightarrow \text{set } = 0$$

$$-2X^T (y - X\beta) = 0$$

$$2X^T X\beta = 2X^T y$$

$$\vec{\beta} = (X^T X)^{-1} X^T y$$