CSCI-UA 472 Artificial Intelligence

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Homework 03

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Problem 1

Let Ω be a domain consisting of Facebook posts, people, and topics. Let $\mathcal{L}$ be the first-order language containing the following non-logical symbols:

- Predicates:
  - $F(p, q)$ — Person q is a friend of person p.
  - $W(p, u)$ — Person p wrote post u.
  - $S(p, u)$ — Person p sees post u.
  - $L(p, u)$ — Person p likes post u.
  - $A(u, t)$ — Post u is about topic t.
- Constants:
  B — Barbara. T — Tom. G — George. H — The topic of whales.

Express the following statements in $\mathcal{L}$. In all of these, ignore the difference between past tense and present tense. As always in first-order logic, interpret "some" as meaning "one or more."

(a) Everyone always sees their own posts.

(b) Barbara always likes any post about whales that she sees.

(c) Barbara sees all of the posts that her friends write.

(d) Each of Tom's friends has written some posts about whales.

(e) Some of Tom's friends are also friends of Barbara's.

(f) Some of Tom's friends have written some posts that Barbara has liked.

(g) All of George's posts are about whales.

(h) Every friend of Barbara's has written some posts.

(i) Barbara has never liked any post of George.

Solution to Problem 1

(a) $\forall_{p,u}\ W(p,u) \Rightarrow S(p,u)$

(b) $\forall_u\ [A(u,H) \wedge S(B,u)] \Rightarrow L(B,u)$

(c) $\forall_{p,u}\ [F(B,p) \wedge W(p,u)] \Rightarrow S(B,u)$

(d) $\forall_p\ F(T,p) \Rightarrow \exists_u\ W(p,u) \wedge A(u,H)$

(e) $\exists_p\ F(T,p) \wedge F(B,p)$

(f) $\exists_{p,u}\ F(T,p) \wedge W(p,u) \wedge L(B,u)$

(g) $\forall_u\ W(G,u) \Rightarrow T(u,H)$

(h) $\forall_p\ F(B,p) \Rightarrow \exists_u\ W(p,u)$

(i) $\neg\exists_u\ W(G,u) \wedge L(B,u)$

Problem 2

One can carry out prediction in the peg puzzle world in Datalog; that is, you can write axioms that, given the starting state and the sequence of jumps, you can infer the sequence of state.

Let Ω be the set of holes and the set of points in time. Let $\mathcal{L}$ be a Datalog language with the following predicates:

- $P(h,t)$. Hole h has a peg in it at time t.
- $E(h,t)$. Hole h is empty at time t.
- $J(ha,hb,hc,tx,ty)$. Starting at time tx and ending at time ty, the peg in ha is jumped to hc over hb.
- $N(ha,hb)$. Holes ha and hb are not equal.

For the four hole situation in programming assignment 2, you could then assert the following axioms, using constants H1, H2, H3, H4, T1, T2, T3 with the obvious meanings.

Causal axioms:
1. J(ha,hb,hc,tx,ty) $\Rightarrow$ P(hc,ty).

2. J(ha,hb,hc,tx,ty) $\Rightarrow$ E(ha,ty).
3. J(ha,hb,hc,tx,ty) $\Rightarrow$ E(hb,ty).

Frame axioms:
4. J(ha,hb,hc,tx,ty) $\wedge$ N(hd,ha) $\wedge$ N(hd,hb) $\wedge$ N(hd,hc) $\wedge$ P(hd,tx) $\Rightarrow$ P(hd,ty).
5. J(ha,hb,hc,tx,ty) $\wedge$ N(hd,ha) $\wedge$ N(hd,hb) $\wedge$ N(hd,hc) $\wedge$ E(hd,tx) $\Rightarrow$ E(hd,ty).

Inequality is symmetric:
6. N(ha,hb) $\Rightarrow$ N(hb,ha).

Starting state:
7. E(H1,T1).
8. P(H2,T1).
9. P(H3,T1).
10. P(H4,T1).

Jumps executed:
11. J(H3,H2,H1,T1,T2).
12. J(H4,H1,H2,T2,T3).

Unique names:
13. N(H1,H2).
14. N(H1,H3).
15. N(H1,H4).
16. N(H2,H3).
17. N(H2,H4).
18. N(H3,H4).

(The "one action at a time" axioms from the propositional encoding cannot be stated in Datalog. The precondition axioms can be stated, but have no value in inference, since the jumps carried out have to be stated explicitly anyway.)

Show the result of doing forward chaining on these axioms, in the style of the handout on Datalog.

Solution to Problem 2

To answer this question, we need to combine the rules with the facts to perform forward chaining on the given axioms. In the given question, we have rules from 1 to 6, and facts from 7 to 18.

Combining rule (6) i.e. **inequality is symmetric** with unique names (13 - 18), we infer:

- Combining rule (6) with fact (13) infer (19), given the substitution
  N(ha $\rightarrow$ H1, hb $\rightarrow$H2) $\Rightarrow$ N(H2,H1)
- Combining rule (6) with fact (14) infer (20), given the substitution
  N(ha $\rightarrow$ H1, hb $\rightarrow$H3) $\Rightarrow$ N(H3,H1)
- Combining rule (6) with fact (15) infer (21), given the substitution
  N(ha $\rightarrow$ H1, hb $\rightarrow$H4) $\Rightarrow$ N(H4,H1)
- Combining rule (6) with fact (16) infer (22), given the substitution
  N(ha $\rightarrow$ H2, hb $\rightarrow$H3) $\Rightarrow$ N(H3,H2)
- Combining rule (6) with fact (17) infer (23), given the substitution
  N(ha $\rightarrow$ H2, hb $\rightarrow$H4) $\Rightarrow$ N(H4,H2)
- Combining rule (6) with fact (18) infer (24), given the substitution
  N(ha $\rightarrow$ H3, hb $\rightarrow$H4) $\Rightarrow$ N(H4,H3)

Deriving facts from first jump (11):

- Combining rule (1) with fact (11) to infer (25) with substitution

  J(ha$\rightarrow$H3,hb$\rightarrow$H2,hc$\rightarrow$H1,tx$\rightarrow$T1,ty$\rightarrow$T2)$\Rightarrow$ P(H1, T2)
- Combining rule (2) with fact (11) to infer (26) with substitution

  J(ha$\rightarrow$H3,hb$\rightarrow$H2,hc$\rightarrow$H1,tx$\rightarrow$T1,ty$\rightarrow$T2) $\Rightarrow$ E(H3, T2)
- Combining rule (3) with fact (11) to infer (27) with substitution

  J(ha$\rightarrow$H3,hb$\rightarrow$H2,hc$\rightarrow$H1,tx$\rightarrow$T1,ty$\rightarrow$T2) $\Rightarrow$ E(H2, T2)

Deriving facts from second jump (12):

- Combining rule (1) with fact (12) to infer (28) with substitution

  J(ha$\rightarrow$H4,hb$\rightarrow$H1,hc$\rightarrow$H2,tx$\rightarrow$T2,ty$\rightarrow$T3)$\Rightarrow$ P(H2, T3)

- Combining rule (2) with fact (12) to infer (29) with substitution

  J(ha→H4,hb→H1,hc→H2,tx→T2,ty→T3)⇒ E(H4, T3)

- Combining rule (3) with fact (12) to infer (30) with substitution

  J(ha→H4,hb→H1,hc→H2,tx→T2,ty→T3)⇒ E(H1, T3)

Frame axioms:

- Using the rule (4) with facts (10, 11, 21, 23, 24) infer (31), given the substitution

  J(ha→H3,hb→H2,hc→H1,tx→T1,ty→T2) ⇒ P(H4, T2)

- Using the rule (5) with facts (12, 18, 20, 22, 26) infer (32), given the substitution

  J(ha→H4,hb→H1,hc→H2,tx→T2,ty→T3)⇒ E(H3, T3)

Problem 3

Suppose that you additionally want to keep track of individual pegs. For instance in four hole examples, the pegs are colored red, white, and blue, and are named PR, PW, and PB. The domain Ω is expanded to include pegs as well.

Rewrite the language (i.e. the set of predicates) and restate the axioms to deal correctly with the behaviors of the individual pegs.

Solution to Problem 3

In this problem, we now have to track individual pegs that are of different colors. Thus, new predicates should be drived as such:

New Predicates:

- $P(p, h, t)$. Peg p is in hole h at time t.
- $E(h, t)$. Hole h is empty at time t
- $J(p, ha, hb, hc, tx, ty)$. Starting at time tx and ending at time ty, the peg p in ha has jumped to hc over hb.
- $N(ha, hb)$. Holes ha and hb are not equal.

These new predicates urge to restate the axioms to deal with behavior of individual pegs.

New axioms:
Rules:
Causal Axioms:
1. J(pa, ha,hb,hc,tx,ty) $\Rightarrow$ P(pa, hc,ty).
2. J(pa, ha,hb,hc,tx,ty) $\Rightarrow$ E(ha,ty).
3. J(pa, ha,hb,hc,tx,ty) $\Rightarrow$ E(hb,ty).

Frame axioms:
4. J(pa, ha,hb,hc,tx,ty) $\wedge$ N(hd,ha) $\wedge$ N(hd,hb) $\wedge$ N(hd,hc) $\wedge$ P(pa,hd,tx) $\Rightarrow$ P(pd,hd,ty).
5. J(pa, ha,hb,hc,tx,ty) $\wedge$ N(hd,ha) $\wedge$ N(hd,hb) $\wedge$ N(hd,hc) $\wedge$ E(hd,tx) $\Rightarrow$ E(hd,ty).

Inequality is symmetric:
6. N(ha,hb) $\Rightarrow$ N(hb,ha).

Facts:
Starting state:
7. E(H1,T1).
8. P(PR,H2,T1).
9. P(PW,H3,T1).
10. P(PB,H4,T1).

Jumps executed:
11. J(PW, H3,H2,H1,T1,T2).
12. J(PB, H4,H1,H2,T2,T3).

Unique names:
13. N(H1,H2).
14. N(H1,H3).
15. N(H1,H4).
16. N(H2,H3).
17. N(H2,H4).
18. N(H3,H4).

End of Assignment. Thank you!