Assignment # 1

Operations Research

By:

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6-B

21L-7565

Question 1

(A)

x_1: bookshelves

x_2: coffee table

we have the Objective function as follows

$$\text{Maximize } z = 120x_1 + 80x_2$$

Subject to:

$2x_1 + x_2 \leq 6$ => wood constraint

$7x_1 + 8x_2 \leq 28$ => time constraint

$x_1, x_2 \geq 0$ => non-negativity constraint

-> we plot the above system graphically using geogebra (graph attached with assignment) and get the feasable solution space.

-> we have the following corner points

A: $(2.22, 1.55)$

B: $(0, 3.5)$

C: $(3, 0)$

D: $(0, 0)$

now substituting these corner points in our objective function we get the following values.

$z = 120(2.22) + 80(1.55)$, $z = 391.1$ => optimal

$z = 120(0) + 80(3.5)$, $z = 280$

$z = 120(3) + 80(0) = 360$

$z = 120(0) + 80(0) = 0$

(A)

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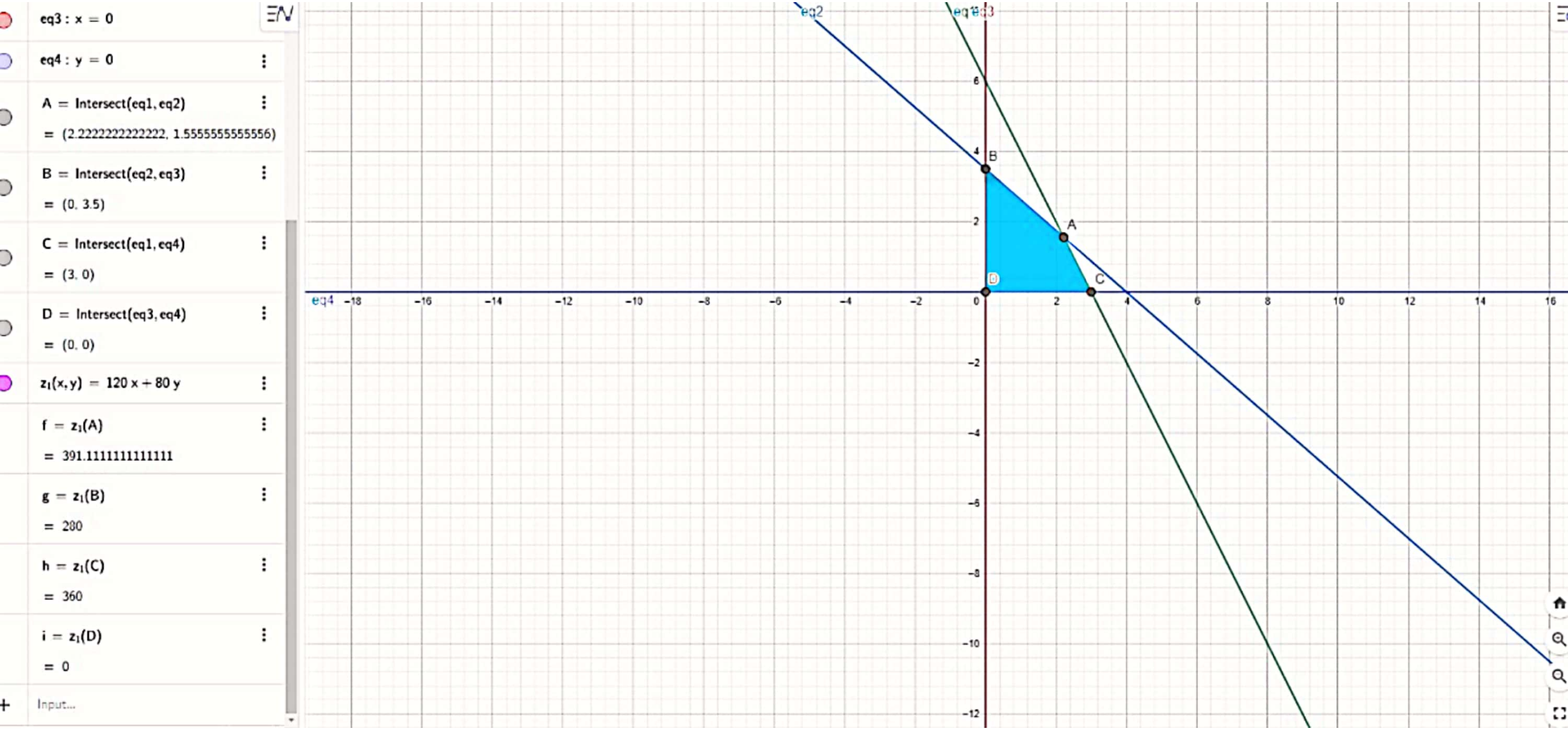
$z = 120(3) + 80(0) = 360$

$z = 120(0) + 80(0) = 0$



eq3 : x = 0
eq4 : y = 0
A = Intersect(eq1, eq2)
= (2.2222222222222, 1.5555555555556)
B = Intersect(eq2, eq3)
= (0, 3.5)
C = Intersect(eq1, eq4)
= (3, 0)
D = Intersect(eq3, eq4)
= (0, 0)
z1(x, y) = 120 x + 80 y
f = z1(A)
= 391.1111111111111
g = z1(B)
= 280
h = z1(C)
= 360
i = z1(D)
= 0
Input...
eq2
eq1
eq3
eq4
A
B
C
D

Date: Day:

Since our problem is to maximize we pick the highest value of z i.e. 391.11.

So $z = 391.11$ at $x_1 = 2.22 \approx 2$ bookshelves and $x_2 = 1.55 \approx 1$ coffeetables.

~~Question2~~

(B)

Let

x_1: no. of Type I decorative screens

x_2: no. of Type II decorative screens

we have our objective function as:

$$\text{Maximize } z = 20x_1 + 30x_2$$

Subject to:

$2x_1 + 3x_2 \le 40 \Rightarrow$ labour cost

$5x_1 + 7x_2 \le 100 \Rightarrow$ time constraint

$\left.\begin{array}{l} x_1 \ge 10 \\ x_2 \ge 5 \end{array}\right\} \Rightarrow$ demand constraint.

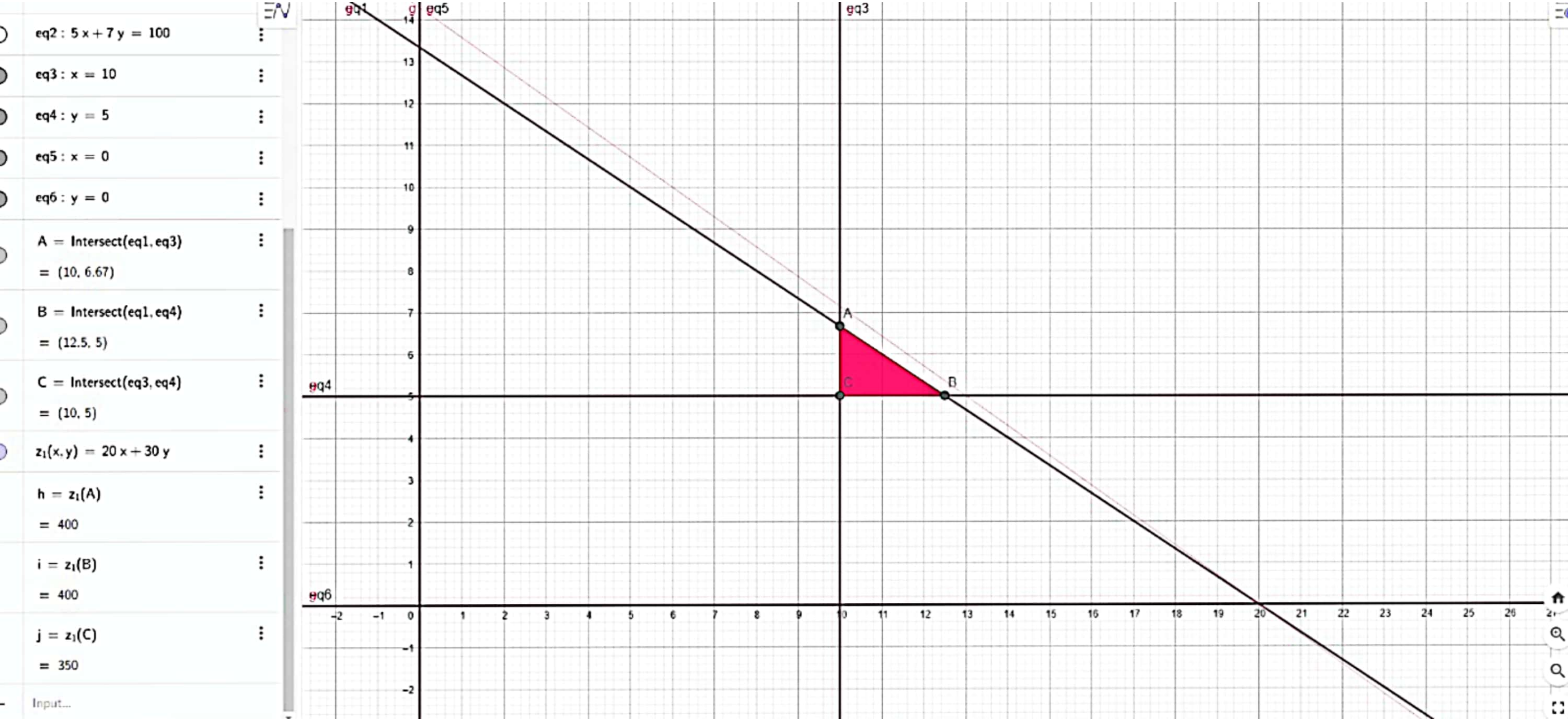
$x_1, x_2 \ge 0 \Rightarrow$ non-negativity constraint

again we plot the system graphically using geogebra (graph attached with assignment). we get the feasible solution space and the following corner points

A: (10, 6.66)

B: (12.5, 5)

C: (10, 5)


eq2 : 5 x + 7 y = 100
eq3 : x = 10
eq4 : y = 5
eq5 : x = 0
eq6 : y = 0
A = Intersect(eq1, eq3)
= (10, 6.67)
B = Intersect(eq1, eq4)
= (12.5, 5)
C = Intersect(eq3, eq4)
= (10, 5)
z1(x, y) = 20 x + 30 y
h = z1(A)
= 400
i = z1(B)
= 400
j = z1(C)
= 350
Input...
eq1
g
eq5
eq3
eq4
eq6
A
B
C

Date: Day:

now substituting the corner points in our objective function we get :

$z = 20(10) + 30(6.66) = 400$ } optimal
$z = 20(12.5) + 30(5) = 400$
$z = 20(10) + 30(5) = 350$

Since our problem is to maximize the profit so we have 2 optimal solutions one at (10, 6.66) and the other at (12.5, 5).

So we can say that
$z = 400$ for 12 decorative type I screen and 5 decorative type II screen
OR
10 type I and 6 type II screens.

(c)

Maximize $z = 1170x + 1110y$

subject to:

$9x + 5y \geq 500$
$7x + 9y \geq 300$
$5x + 3y \leq 1500$
$7x + 9y \leq 1900$
$2x + 4y \leq 1000$
$x, y \geq 0$

lets solve it using geogebra

Date: Day:

after getting our feasable solution space we get the following intersection points

$A = (0, 100)$
$B = (0, 211.11)$
$C = (271.43, 0)$
$D = (55.56, 0)$

now let's substitute these point in our objective function. we get the following values.

$z = 1170(0) + 1110(100) = 111{,}000$

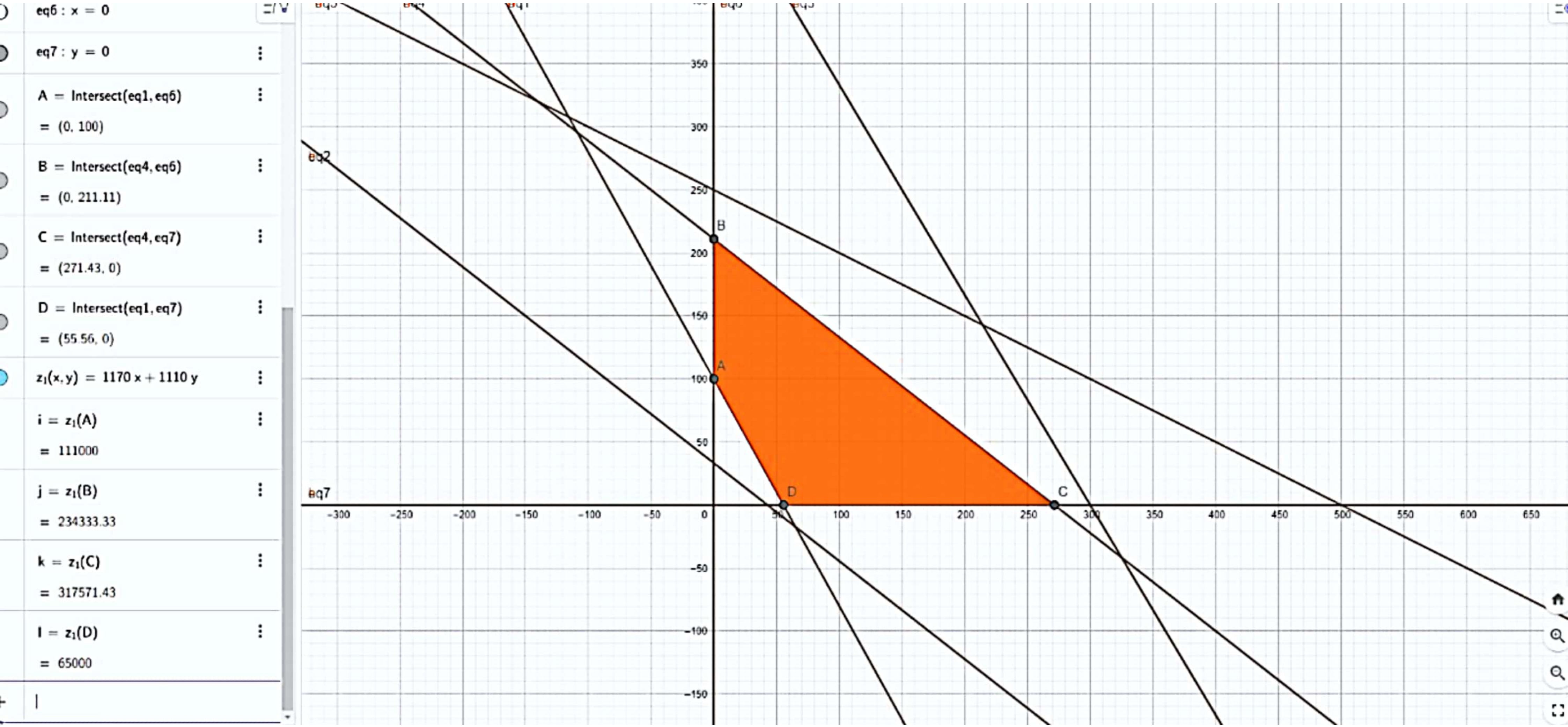
$z = 1170(0) + 1110(211.11) = 234333.33$

$z = 1170(271.43) + 1110(0) = 317571.43$

$z = 1170(55.56) + 1110(0) = 65000$

So optimal value of $z = 317571.43$

at $x_1 = 271.43$ and $x_2 = 0$


eq6 : x = 0
eq7 : y = 0
A = Intersect(eq1, eq6)
= (0, 100)
B = Intersect(eq4, eq6)
= (0, 211.11)
C = Intersect(eq4, eq7)
= (271.43, 0)
D = Intersect(eq1, eq7)
= (55.56, 0)
$z_1(x, y) = 1170x + 1110y$
$i = z_1(A)$
= 111000
$j = z_1(B)$
= 234333.33
$k = z_1(C)$
= 317571.43
$l = z_1(D)$
= 65000
eq2
eq7
A
B
C
D

Day:

Question 2

~~Maximize z=~~

Maximize $z = x_1 + 3x_2$

Subject to:

$x_1 + x_2 \le 2$

$-x_1 + x_2 \le 4$

x_1 unrestricted

$x_2 \ge 0$.

to convert our system of inequalities into a system of equations we need to introduce extra variables

$x_1 + x_2 + x_3 = 2$

$-x_1 + x_2 + x_4 = 4$

x_1 unrestricted

$x_2, x_3, x_4 \ge 0$

n = no. of variable = 4

m = no. of equation = 2

Setting n-m variables to zer

4-2 = 2

~~so we set x_1 and $x_2 = 0$~~

~~then we have values of $x_3 = 2$ and $x_4 = 4$~~

Date: Day:

So now we will form all the possible pairs of zero and non-zero variables

zero ↓ non-basic; non-zero variables ↓ basic variable

Combination (zero)	Basic variables	Basic solution	z
(x_1, x_2)	(x_3, x_4)	(2, 4) feasible	0
(x_1, x_3)	(x_2, x_4)	(2, 2) feasible	6
(x_1, x_4)	(x_2, x_3)	(4, -2) non-feasible	–
(x_2, x_3)	(x_1, x_4)	(2, 6) feasible	2
(x_2, x_4)	(x_1, x_3)	(-4, 6) feasible	-4
(x_3, x_4)	(x_1, x_2)	(-1, 3) feasible	$\boxed{8}$ optimum

$x_1 + x_2 = 2$

$-x_1 + x_2 = 4$

$2x_2 = 6$

$x_2 = 3$

now we substitute the Basic solution in our objective function

~~$z = 2 + 3(4) = 2 + 12 = 14$~~

Maximize $z = x_1 + 3x_2 + 0x_3 + 0x_4$

$z = \cancel{2+3}\ 0 + 3(0) + 0(2) + 0(4) = 0$ (x_1, x_2)

$z = 0 + 3(2) + 0(0) + 0(2) = 6$ (x_1, x_3)

$z = 2 + 0 + 0 + 0(6) = 2$ (x_2, x_3)

$z = -4 + 0 + 0(6) + 0(0) = -4$ (x_2, x_4)

$z = -1 + 3(3) + 0 + 0 = 8$ (x_3, x_4)

Question 2

Maximize $z = x_1 + 3x_2$

Subject to:

$$x_1 + x_2 \le 2$$
$$-x_1 + x_2 \le 4$$

Since our problem is to maximize so our optimal solution is $z = 8$ at $x_1 = -1$ and $x_2 = 3$

Question 3

Food	Protein	Carbs	Fats	Cost $ (oz)
A	20	50	4	2
B	30	30	9	3
C	40	20	11	5
D	40	25	10	6
E	45	50	9	8
F	30	20	10	8

let $x_1, x_2, x_3, x_4, x_5, x_6$ represent the 6 types of Feed

we have our objective function as

$$z = 2x_1 + 3x_2 + 5x_3 + 6x_4 + 8x_5 + 8x_6 \text{ (Minimize)}$$

Subject to:

$$20x_1 + 30x_2 + 40x_3 + 40x_4 + 45x_5 + 30x_6 \ge 70$$
$$50x_1 + 30x_2 + 20x_3 + 25x_4 + 50x_5 + 20x_6 \ge 100$$
$$4x_1 + 9x_2 + 11x_3 + 10x_4 + 9x_5 + 10x_6 \ge 20$$

ariables				
1	0.909091			
2	1.818182			
3	0			
4	0			
5	0			
6	0			
bj. func.				
nin	7.272727			
ontraints				
1	72.72727		>=	70
2	100		>=	100
3	20		>=	20

Question 4

part a:

$$\text{Min } z = x_1 + 2x_2 - 3x_3 - 2x_4$$

Subject to:

$$x_1 + 2x_2 - 3x_3 + x_4 = 4$$

$$x_1 + 2x_2 + x_3 + 2x_4 = 4$$

$$x_1, x_2, x_3, x_4 \geq 0$$

$m = 2$, $n = 4$, $n - m = 4 - 2 = 2$

we will use the algebraic method ~~were~~ where $n-m$ variables are set to zero (Nonbasic variables) and then solving the equations for the remaining m variables ~~to fin~~ (Basic variables) to find the basic solutions.

found by solving above equations

obtained by substituting values in obj function

Non-basic Variable	Basic variable	Basic solution	Feasible	Objective value, z
(x_1, x_2)	(x_3, x_4)	$(-4/7, 16/7)$	NO	—
(x_1, x_3)	(x_2, x_4)	$(2, 0)$	YES	4
(x_1, x_4)	(x_2, x_3)	$(2, 0)$	YES	4
(x_2, x_3)	(x_1, x_4)	$(4, 0)$	YES	4
(x_2, x_4)	(x_1, x_3)	$(4, 0)$	YES	4
(x_3, x_4)	(x_1, x_2)	∞	—	—

optimum

① $z = \cancel{0+0}\ 0 + 2(2) - 0 - 0 = 2 = 4$

② $z = 0 + 2(2) - 0 - 0 = 4$

③ $= 4 + 0 + 0 - 2(0) = 4$

④ $= \cancel{0+}\ 4 + 0 + 3(0) + 0 = 4$

So we have the optimum value of z(min) at two points

1) $\boxed{x_1 = 4,\ x_2 = 0,\ x_3 = 0,\ x_4 = 0}$

2) $\boxed{x_1 = 0,\ x_2 = 2,\ x_3 = 0,\ x_4 = 0}$

part b

$$\text{Max } z = 10x_1 + 11x_2$$

Subject to:

$$x_1 + 2x_2 \leq 150$$
$$3x_1 + 4x_2 \leq 200$$
$$6x_1 + x_2 \leq 175$$
$$x_1, x_2 \geq 0$$

To solve it using simplex method we first need to convert it into standard form:

$$\text{Max } z = 10x_1 + 11x_2 + 0S_1 + 0S_2 + 0S_3$$

subject to:

$$x_1 + 2x_2 + S_1 = 150$$
$$3x_1 + 4x_2 + S_2 = 200$$
$$6x_1 + x_2 + S_3 = 175$$
$$x_1, x_2, S_1, S_2, S_3 \geq 0$$

$m=3,\ n=5,\ n-m=2$

$$z - 10x_1 - 11x_2 - 0S_1 - 0S_2 - 0S_3$$

→ pivot column

Basic	z	x_1	x_2	S_1	S_2	S_3	Solution	
z	1	-10	-11	0	0	0	0	
S_1	0	1	2	1	0	0	150	(150:2 = 75)
pivot row ← S_2	0	3	4	0	1	0	200	(200:4 = 50) minimum
S_3	0	6	1	0	0	1	175	(175:1 = 175)

Entering variable = x_2, leaving variable = S_2 pivot element = 6

→ pivot column

Basic	z	x_1	x_2	s_1	s_2	s_3	Sol	
z	1	$-7/4$	0	0	$11/4$	0	550	
s_1	0	$-1/2$	0	1	$-1/2$	0	50	$(50 : -1/2)$ ~~= 66.66~~ = –
x_2	0	$3/4$	1	0	$1/4$	0	50	$(50 : 3/4) = 66.66$
s_3	0	$21/4$	0	0	$-1/4$	1	125	$(125 : 21/4) = 23.809$ (minimum)

pivot row

New pivot row = $\dfrac{\text{current pivot row}}{\text{pivot element}} = \dfrac{0 \quad 3 \quad 4 \quad 0 \quad 1 \quad 0 \quad 200}{4}$

New row = current row − (pivot column coefficient) x (New pivot row)

z-row = 1 −10 −11 0 0 0 0

$$
\begin{array}{rrrrrrrr}
 & 1 & -10 & -11 & 0 & 0 & 0 & 0 \\
- & 0 & -\frac{33}{4} & -11 & 0 & -11/4 & 0 & -550 \\
\hline
 & 1 & -7/4 & 0 & 0 & 11/4 & 0 & 550
\end{array}
$$

s_1-row = 0 1 2 1 0 0 150

$$
\begin{array}{rrrrrrrr}
 & 0 & 1 & 2 & 1 & 0 & 0 & 150 \\
- & 0 & 6/4 & 2 & 0 & 1/2 & 0 & 100 \\
\hline
 & 0 & -1/2 & 0 & 1 & -1/2 & 0 & 50
\end{array}
$$

s_3-row = ~~0 6 1 0 0 1 175~~

~~− 0 15/4 6 0 6/4 0 300~~

~~0 3/2 −5~~

$$
\begin{array}{rrrrrrrr}
 & 0 & 6 & 1 & 0 & 0 & 1 & 175 \\
- & 0 & 3/4 & 1 & 0 & 1/4 & 0 & 50 \\
\hline
 & 0 & 21/4 & 0 & 0 & -1/4 & 1 & 125
\end{array}
$$

Entring variable = x_1 leaving variable = s_3 pivot element = $21/4$

Basic	z	x_1	x_2	s_1	s_2	s_3	Solution
z	1	0	0	0	8/3	1/3	1775/3
s_1	0	0	0	1	-11/21	2/21	1300/21
x_2	0	0	1	0	2/7	-1/7	225/7
x_1	0	1	0	0	-1/21	4/21	$\frac{500}{21}$

new pivot row = $\dfrac{0 \quad 21/4 \quad 0 \quad 0 \quad -1/4 \quad 1 \quad 125}{21/4}$

new z-row = 1   -7/4   0   0   11/4   0   550

- 0   -7/4   0   0   1/12   -1/3 $-\frac{125}{3}$

1   0   0   0   8/3   1/3   1775/3

new s_1-row = 0   -1/2   0   1   -1/2   0   50

- 0   -1/2   0   0   1/42   -4/42 $-\frac{500}{42}$

0   0   0   1   -11/21   4/42   1300/21

new x_2-row = 0   3/4   1   0   1/4   0   50

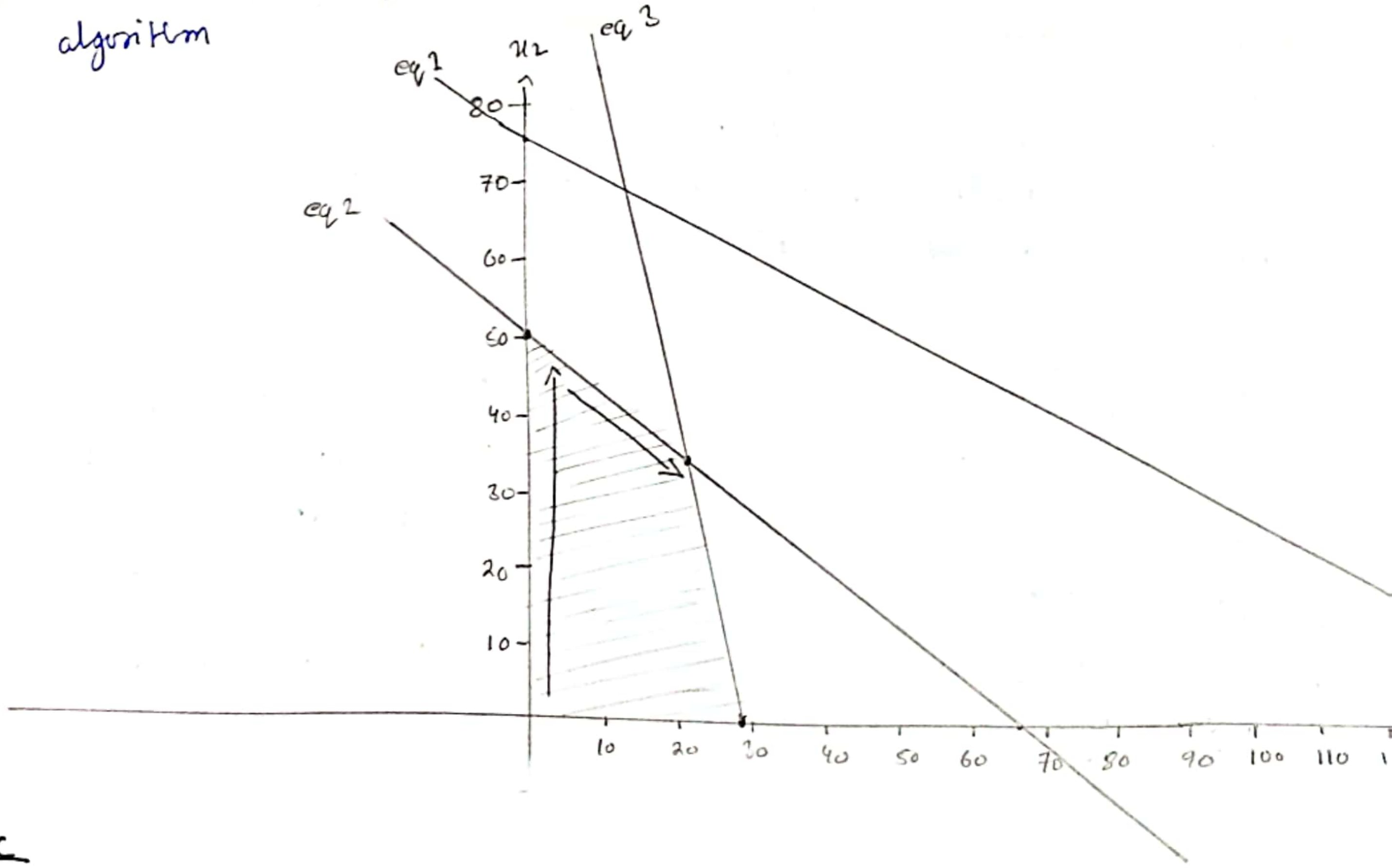
- 0   3/4   0   0   -1/28   1/7 $\frac{125}{7}$

0   0   1   0   2/7   -1/7 $\frac{225}{7}$

since no more negative values in our z-row so we stop

$\boxed{\therefore \text{ optimum value of } z = \frac{1775}{3} \text{ at } s_1 = \frac{1300}{21},\ x_1 = \frac{500}{21} \text{ and } x_2 = \frac{225}{7}}$

now lets plot a graph of the problem and see the path of the simplex algorithm



<u>part c</u>

$$\text{Max } z = 2x_1 + 3x_2$$

Subject to

$$6x_1 + 9x_2 \le 100$$
$$2x_1 + x_2 \le 20$$
$$x_1, x_2 \ge 0$$

<u>converting to standard form for algebraic solution</u>

$$\text{Max } z = 2x_1 + 3x_2 + 0s_1 + 0s_2$$

Subject to:

$$6x_1 + 9x_2 + s_1 = 100$$
$$2x_1 + x_2 + s_2 = 20$$
$$x_1, x_2, s_1, s_2 \ge 0$$

$m = 2 \quad n = 4 \quad n - m = 2$

Non-basic variables	Basic variables	Basic solution	Feasible	Objective Value, z
(x_1, x_2)	(S_1, S_2)	$(100, 20)$	YES	0
(x_1, S_1)	(x_2, S_2)	$(100/9, 80/9)$	YES	100/3
(x_1, S_2)	(x_2, S_1)	$(20, -80)$	NO	—
(x_2, S_1)	(x_1, S_2)	$(50/3, -40/3)$	NO	—
(x_2, S_2)	(x_1, S_1)	$(10, 40)$	YES	20
(S_1, S_2)	(x_1, x_2)	$(\frac{20}{3}, \frac{20}{3})$	YES	100/3

optimum Solutions

② $S_2 = 20 - \frac{100}{9} = \frac{80}{9}$

③ $S_1 = 100 - 9(20) = -80$

④ $S_2 = 20 - 2(\frac{50}{3})$

⑤ $S_1 = 100 - 60 = 40$

⑥
$$\begin{aligned} \cancel{6x_1} + 9x_2 &= 100 \\ -\cancel{6x_1} - 3x_2 &= -60 \\ \hline 6x_2 &= 40 \end{aligned}$$

$x_2 = 40/6 = \frac{20}{3}$

$x_2 = \frac{20}{3}$, $x_1 = \frac{100 - 9(20/3)}{6} = \frac{20}{3}$

Now lets find the value of z at these Basic Solutions

① $z = 0$

② $z = \cancel{3(20)}\ 3(\frac{100}{9}) + 0(\frac{80}{9}) = \frac{100}{3}$

③ $z = 2(10) + 0 + 0(40) + 0 = 20$

④ $z = 2(\frac{20}{3}) + 3(\frac{20}{3}) + 0(0) + 0(0) = \frac{100}{3}$

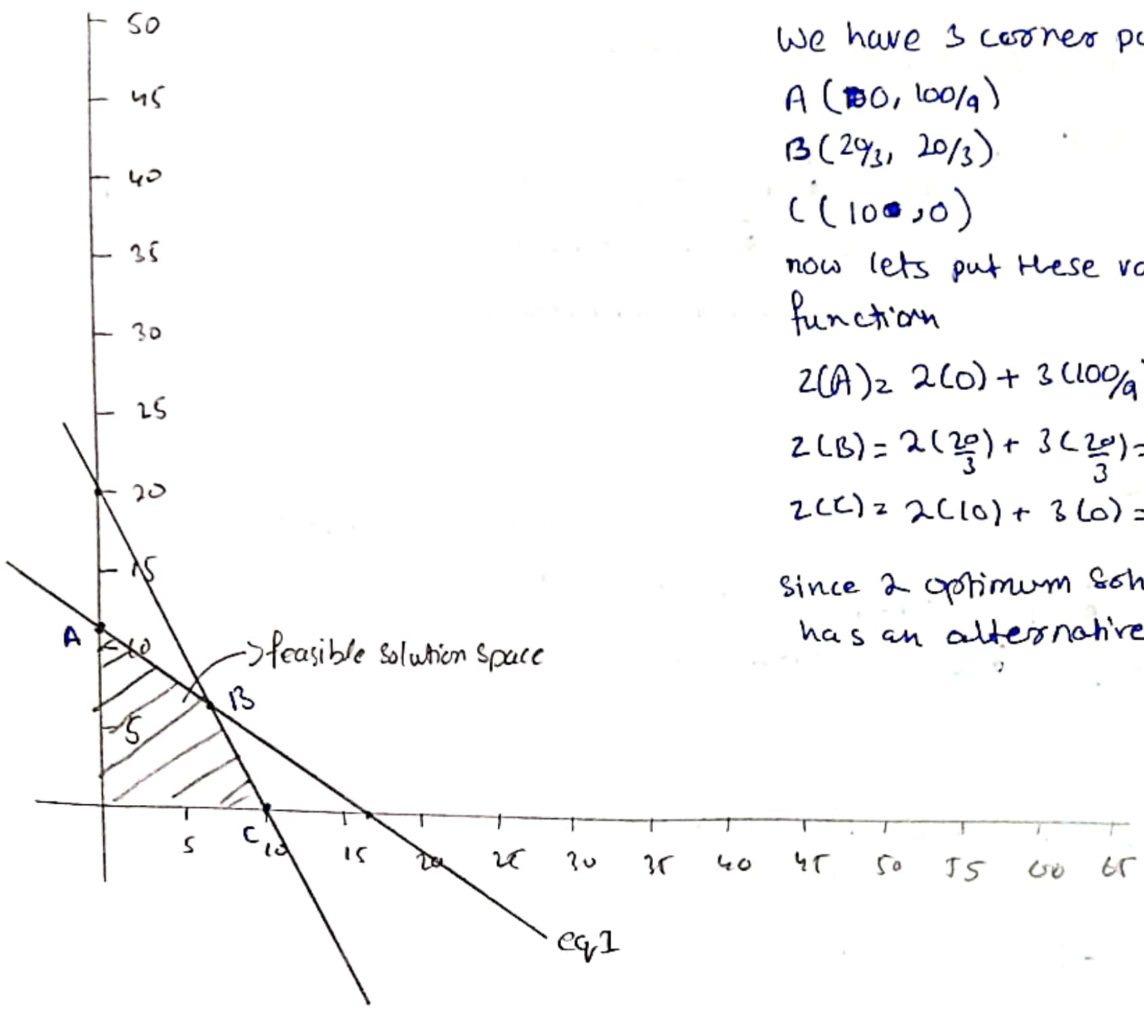
∴ as we can see that there are two optimal solutions so we can say that our LLP has alternate optimum solution

① $z = 100/3$ at $x_2 = \frac{100}{9}, S_2 = 80/9$

&

② $z = \frac{100}{3}$ at $x_1 = \frac{20}{3}, x_2 = \frac{20}{3}$

now lets represent it graphically



We have 3 corner point

A $(0, 100/9)$

B $(20/3, 20/3)$

C $(10, 0)$

now lets put these values in our objective function

$$Z(A) = 2(0) + 3(100/9) = 100/3$$

$$Z(B) = 2\left(\frac{20}{3}\right) + 3\left(\frac{20}{3}\right) = 100/3$$

$$\left.\begin{matrix} Z(A) = 100/3 \\ Z(B) = 100/3 \end{matrix}\right\} \text{optimum}$$

$$Z(C) = 2(10) + 3(0) = 20$$

since 2 optimum solutions $\therefore$ our LLP has an alternative optimal solution.

Question 5

part a using Two phase method

$$\text{Min } z = 2x_1 + x_2$$

Subject to

$$x_1 \geq 5$$

$$x_2 \geq 1$$

$$x_1, x_2 \geq 0$$

~~converting to standard form~~

$$\left.\begin{matrix} \cancel{x_1 + S_1 \quad = 5} \\ \cancel{x_2 \quad + S_2 = 1} \end{matrix}\right\} \text{do not slack so we add artificial variable } (R_1 \,\&\, R_2)$$

$$\Downarrow$$

$$x_1 + S_1 \quad + R_1$$

converting to standard form...

$$\text{Min } Z = 2x_1 + x_2 + R_1 + R_2$$

Subject to:

$$x_1 - S_1 + R_1 = 5$$
$$x_2 - S_2 + R_2 = 1$$
$$x_1, x_2, S_1, S_2, R_1, R_2 \geq 0$$

Phase-I

$$\text{Min } r = R_1 + R_2$$

Subject to:

$$x_1 - S_1 + R_1 = 5$$
$$x_2 - S_2 + R_2 = 1$$

$m = 2$ $n-m = 4$
$n = 6$

Basic	x_1	x_2	S_1	S_2	R_1	R_2	Sol
r	0	0	0	0	-1	-1	0
R_1	1	0	-1	0	1	0	5
R_2	0	1	0	-1	0	1	1

Since our r row has non-zero values of cofficient of R_1 & R_2 so we make them zero in the following way

new-r-row = Old r-row + (1×R_1 row + 1×R_2-row)

=	0	0	0	0	-1	-1	0
+	1	1	-1	-1	1	1	6
	1	1	-1	-1	0	0	6

Basic	x_1	x_2	S_1	S_2	R_1	R_2	Sol	
r	1	1	-1	-1	0	0	6	
R_1	1	0	-1	0	1	0	5	(5:1) = 5 minimum
R_2	0	1	0	-1	0	1	1	(1:0)

Since our problem is minimization so we take most positive value

entering = x_1, leaving = R_1 pivot element = 1 pivot row = current ÷ pivot element

Basic	x_1	x_2	s_1	s_2	R_1	R_2	Sol	
r	0	1	0	-1	-1	0	1	
x_1	1	0	-1	0	1	0	5	(5:0)
R_2	0	1	0	-1	0	1	1	(1:1) => minimum

new r-row = 1 1 -1 -1 0 0 6

− 1 0 -1 0 1 0 5

0 1 0 -1 -1 0 1

new R_2-row = 0 1 0 -1 0 1 1

− 0 0 0 0 0 0 0

0 1 0 -1 0 1 1

entering = x_2 leaving = R_2 pivot element = 1

new pivot row = 0 1 0 -1 0 1 1

Basic	x_1	x_2	s_1	s_2	R_1	R_2	Sol
r	0	0	0	0	-1	-1	0
x_1	1	0	-1	0	1	0	5
x_2	0	1	0	-1	0	1	1

no more positive values so we stop here. Since r = 0 we have the basic feasible solution as $x_1 = 5$ and $x_2 = 1$

new r-row = 0 1 0 -1 -1 0 1

− 0 1 0 -1 0 1 1

0 0 0 0 -1 -1 0

new x_1-row = 1 0 -1 0 1 0 5

− 0 0 0 0 0 0 0

1 0 -1 0 1 0 5

Phase II

now we delete the columns of the artificial variables R_1 & R_2

our system becomes

$$\text{minimize} \quad z = 2x_1 + x_2$$

Subject to:

$\cancel{x_1 - s_1}$

$$x_1 \quad - s_1 \quad = 5$$

$$x_2 \quad - s_2 = 1$$

our tablue is as follows

Basic	x_1	x_2	s_1	s_2	sol
z	-2	-1	0	0	0
x_1	1	0	-1	0	5
x_2	0	1	0	-1	1

=> ~~make basic variables~~ zero

Since our basic variable x_1 and x_2 have non-zero coefficients in ~~row~~ z-row so we make them zero using the below method.

new z-row =

	-2	-1	0	0	0
+	2	0	-2	0	10
	0	1	0	-1	1
	0	0	-2	-1	11

Basic	x_1	x_2	s_1	s_2	sol
z	0	0	-2	-1	11
x_1	1	0	-1	0	5
x_2	0	1	0	-1	1

Since no more positive values in z-row so we stop and

$\boxed{\text{optimum (min) } z = 11 \text{ at } x_1 = 5 \text{ and } x_2 = 1}$

part b using M-method

$$\text{Min } z = 2x_1 + x_2$$

Subject to:

$$x_1 \geq 5$$
$$x_2 \geq 1$$
$$x_1, x_2 \geq 0$$

Converting to Standard form

$$\text{Min } z = 2x_1 + x_2 + MR_1 + MR_2$$

$$x_1 - s_1 + R_1 = 5$$
$$x_2 - s_2 + R_2 = 1$$
$$x_1, x_2, s_1, s_2, R_1, R_2 \geq 0$$

taking $M = 100$ $\quad n-m=2 \quad m=2 \quad n=6$

Basic	x_1	x_2	s_1	s_2	R_1	R_2	sol
z	-2	-1	0	0	-100	-100	0
R_1	1	0	-1	0	1	0	5
R_2	0	1	0	-1	0	1	1

making coefficients of R_1 & $R_2 = 0$

new z-row =	-2	-1	0	0	-100	-100	0
+	100	0	-100	0	100	0	500
	0	100	0	-100	0	100	100
	98	99	-100	-100	0	0	600

Basic	x_1	x_2	s_1	s_2	R_1	R_2	sol	
z	98	99	-100	-100	0	0	600	
R_1	1	0	-1	0	1	0	5	(5:0) -
R_2	0	1	0	-1	0	1	1	(1:1) => minimum

entering variable = x_2 leaving variable = R_2 pivot element = 1 new pivot row = same as old one

Basic	x_1	x_2	s_1	s_2	R_1	R_2	Sol	
z	98	0	-100	-1	0	-99	501	
R_1	1	0	-1	0	1	0	5	(5:1) → minimum
x_2	0	1	0	-1	0	1	1	(1:0) –

new z-row =

	98	99	-100	-100	0	0	600
-	0	99	0	-99	0	99	99
	98	0	-100	-1	0	-99	501

new R1-row =

	1	0	-1	0	1	0	5
-	0	0	0	0	0	0	0
	1	0	-1	0	1	0	5

entering variable = x_1 leaving variable = x_1 pivot element = 1

new pivot row = same as old one

Basic	x_1	x_2	s_1	s_2	R_1	R_2	Sol
z	0	0	-2	-1	-98	-99	11
x_1	1	0	-1	0	1	0	5
x_2	0	1	0	-1	0	1	1

⇒ since no more positive values so we stop

$\boxed{\text{optimum (min) } z = 11 \text{ at } x_1 = 5 \text{ and } x_2 = 1}$

new z-row =

	98	0	-100	-1	0	-99	501
-	98	0	-98	0	98	0	490
	0	0	-2	-1	-98	-99	11

new x_2-row =

	0	1	0	-1	0	1	1
-	0	0	0	0	0	0	0
	0	1	0	-1	0	1	1