COMP.SGN.100 Introduction to Signal Processing
Exercise 7 - Task 1, 2, 3

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Task 1

TASK 1

a)

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)\, z^{-n}$$

$$X(z) = x(-1)z + x(0) + x(2)z^{-2} + x(3)z^{-3}$$

$$X(z) = 2z + 1 + 2z^{-2} + (-3)z^{-3}$$

$$\boxed{X(z) = 2z + 1 + 2z^{-2} - 3z^{-3}}$$

b)

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)\, z^{-n}$$

As $x(n) = 0$ for $n < 0$, so

$$X(z) = \sum_{n=0}^{\infty} 0.8^{n} z^{-n}$$

$$X(z) = \sum_{n=0}^{\infty} (0.8z^{-1})^{n}$$

$$X(z) = \frac{1}{1 - 0.8z^{-1}}$$ Using geometric series as $(0.8 < 1)$

$$\boxed{X(z) = \frac{z}{z - 0.8}}$$ By multiplying and dividing by z.

c) To calculate DTFT of the given signals, we simply replace z with $e^{i\omega}$ in the z-transform expression of above signals.

(a) $$X(e^{i\omega}) = 2e^{i\omega} + 1 + 2e^{-i\omega 2} - 3z^{-3i\omega}$$

(b) $$X(e^{i\omega}) = \frac{e^{i\omega}}{e^{i\omega} - 0.8}$$

Task 2

TASK 2

$$H(z) = \frac{1 + 2z^{-1} + z^{-2}}{1 + z^{-1} + z^{-2}}$$

Expand the negative powers away by multiplying numerator and denominator by z^2.

$$H(z) = \frac{z^2 + 2z + 1}{z^2 + z + 1}$$

Roots of numerator i.e zeroes are solved by applying quadratic formula.

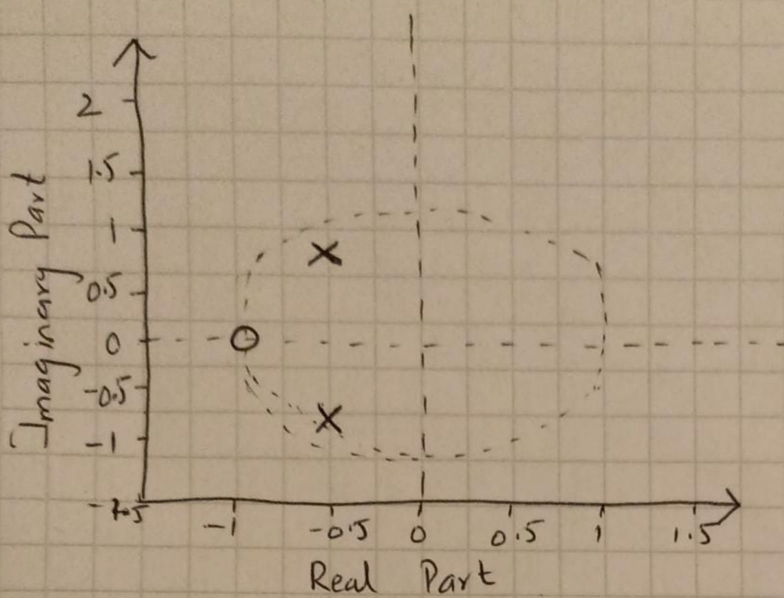
$$z_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{4 - 4}}{2}$$

$$z_{1,2} = \frac{-2}{2} = -1$$

Similarly, for poles.

$$P_{1,2} = \frac{-1 \pm \sqrt{1 - 4}}{2}$$

$$P_{1,2} = \frac{-1 \pm \sqrt{-3}}{2} = \frac{-1 \pm \sqrt{3}\,i}{2} = \frac{-1}{2} \pm \frac{\sqrt{3}\,i}{2}$$



This filter is stable as the poles lie inside the unit circle.

Task 3

TASK 3

$$y(n) = \frac{1}{4}x(n) - \frac{1}{2}x(n-1) + \frac{1}{4}x(n-2)$$

$$y(n) = 0.25\,x(n) - 0.5\,x(n-1) + 0.25\,x(n-2)$$

To find Tranfer function, $H(z)$

$$H(z) = \frac{Y(z)}{X(z)}$$

$$Y(z) = 0.25\,X(z) - 0.5\,X(z)z^{-1} + 0.25\,X(z)z^{-2}$$

$$H(z) = \frac{Y(z)}{X(z)} = 0.25 - 0.5z^{-1} + 0.25z^{-2}$$

$$H(e^{i\omega}) = 0.25 - 0.5e^{-i\omega} + 0.25e^{-2i\omega} \rightarrow (1)$$

Given : $F_s = 16000\,Hz$; $f = 4000\,Hz$

$$\omega = \frac{f}{F_s}2\pi = \frac{2\pi(4000)}{16000} = 0.25 \cdot 2\pi$$

$$H(e^{0.25\cdot 2\pi i}) = 0.25 - 0.5e^{-0.25\cdot 2\pi i} + 0.25e^{-2\cdot 0.25\cdot 2\pi i}$$

$$H(e^{0.25\cdot 2\pi i}) = 0.25 - 0.5\left(\cos(-0.25\cdot 2\pi) + i\sin(-0.25\cdot 2\pi)\right)$$

$$+ 0.25\left(\cos(-2\cdot 0.25\cdot 2\pi) + i\sin(-2\cdot 0.25\cdot 2\pi)\right) \longrightarrow \text{By Applying Eulers formula.}$$

$$H(e^{0.25\cdot 2\pi i}) = 0.25 - 0.5(0 - 1i) + 0.25(-1 + 0i)$$

$$H(e^{0.25\cdot 2\pi i}) = \cancel{0.25} + 0.5i - \cancel{0.25} = 0.5i$$

$$\text{Amplitude Response} = |0.5i| = \sqrt{(0.5)^2} = 0.5 \text{ Ans.}$$