COMP.SGN.100 Introduction to Signal Processing
Exercise 8: Task 1, 2

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Task 1

EXERCISE 8

TASK 1

$$y(n) = x(n) - 2x(n-1) + x(n-2) + 3y(n-1) - \frac{37}{16}y(n-2)$$

(a) $$y(n) = x(n) - 2x(n-1) + x(n-2) + 3y(n-1) - 2.3125y(n-2)$$

$$H(z) = \frac{Y(z)}{X(z)}$$

First find z-transform of given equation.

$$Y(z) = X(z) - 2X(z)z^{-1} + X(z)z^{-2} + 3Y(z)z^{-1} - 2.3125Y(z)z^{-2}$$

$$Y(z) - 3Y(z)z^{-1} + 2.3125Y(z)z^{-2} = X(z) - 2X(z)z^{-1} + X(z)z^{-2}$$

$$Y(z)(1 - 3z^{-1} + 2.3125z^{-2}) = X(z)(1 - 2z^{-1} + z^{-2})$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 - 2z^{-1} + z^{-2}}{1 - 3z^{-1} + 2.3125z^{-2}}$$

Now, multiply and divide by z^2 to get rid of negative powers.

$$\boxed{H(z) = \frac{z^2 - 2z + 1}{z^2 - 3z + 2.3125}}$$

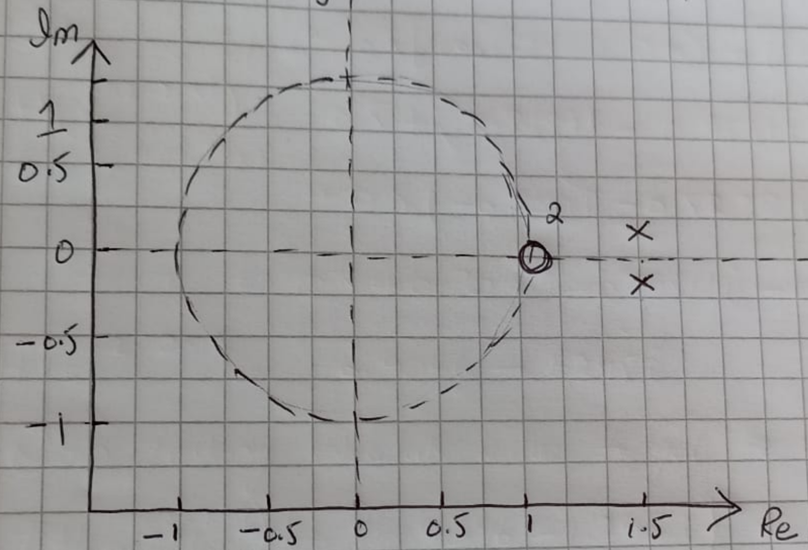
(b) Applying quadratic formula to numerator and denominator to get zeros and poles.

$$z_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{2 \pm \sqrt{4-4}}{2} = 1$$

$$P_{1,2} = \frac{3 \pm \sqrt{9-(9.25)}}{2} = \frac{3 \pm \sqrt{-0.25}}{2}$$

$$P_{1,2} = \frac{3}{2} \pm \frac{\sqrt{0.25}\, i}{2}$$

Now, drawing the pole-zero plot.



(c) Given system is unstable, as poles are outside the unit circle in pole-zero plot.

Task 2

TASK 2

$$x(n) = u(n) \sin\left(\frac{1}{5} \cdot 2\pi n\right)$$

$$h(n) = \begin{cases} 1/2, & \text{when } n = 0 \text{ or } n = 2, \\ 1, & \text{when } n = 1, \\ 0, & \text{otherwise} \end{cases}$$

$$y(n) = A\, u(n) \sin\left(1/5 \cdot 2\pi n + \phi\right)$$

$A = ?$ and $\phi = ?$

Let's find transfer function i.e, z-transform of given impulse response $h(n)$.

$$H(z) = \sum_{n=-\infty}^{+\infty} h(n) z^{-n} = 0.5 + z^{-1} + 0.5 z^{-2}$$

$$H(z) = 0.5 + z^{-1} + 0.5 z^{-2}$$

Replacing $z = e^{i\omega}$ to get frequency response.

$$H(e^{i\omega}) = 0.5 + e^{-i\omega} + 0.5 e^{-2i\omega}$$

$$H(e^{i\omega}) = 0.5 + e^{-i\omega} + 0.5e^{-2i\omega}$$

$$\omega = 0.2 \cdot 2\pi$$

$$H(e^{0.2\cdot 2\pi i}) = 0.5 + e^{-0.2\cdot 2\pi i} + 0.5\, e^{-2\cdot 0.2\cdot 2\pi i}$$

$$H(e^{0.2\cdot 2\pi i}) = 0.5 + [\cos(-0.2\cdot 2\pi) + i\sin(-0.2\cdot 2\pi)]$$
$$+ 0.5[\cos(-2\cdot 0.2\cdot 2\pi) + i\sin(-2\cdot 0.2\cdot 2\pi)]$$

$$H(e^{0.2\cdot 2\pi i}) = 0.5 + (0.3091 - 0.9511i)$$
$$+ 0.5(-0.8091 - 0.5878i)$$
$$= 0.5 + 0.3091 - 0.9511i - 0.4045 - 0.2939i$$

$$H(e^{0.2\cdot 2\pi i}) = 0.4046 - 1.245i$$

Now taking absolute value to get the amplitude response.

$$|H(e^{0.2\cdot 2\pi i})| = |0.4046 - 1.245i|$$
$$= \sqrt{(0.4046)^2 + (-1.245)^2}$$
$$= 1.3091$$

and phase response

$$\arg(0.4046 - 1.245i) = \arctan(-1.245/0.4046)$$
$$= -1.2565$$

Thus, amplitude is 1.3091 times the original and phase delayed by 1.2565 radians.
Thus, output would be,

$$y(n) = 1.3091\,u(n)\sin\left(\tfrac{1}{5}\cdot 2\pi n - 1.2565\right)$$

So, $A = 1.3091$, $\phi = -1.2565$