COMP.SGN.100 Introduction to Signal Processing
Exercise 5 - Task 1, 2

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Task 1

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TASK 1

$$x(t) = \begin{cases} 1, & \text{when } -\pi/4 \le t \le \pi/4, \\ 0, & \text{in other points of interval } [-\pi, \pi] \end{cases}$$

$x(t) = x(t+2\pi)$ i.e, $x(t)$ is periodic cont. signal with period 2π So,

1- When $x(t) = 1$ for period $-\pi/4 \le t \le \pi/4$ and $n \neq 0$

$$X(n) = \frac{1}{2\pi} \int_{-\pi/4}^{\pi/4} x(t)\, e^{-int}\, dt$$

$$X(n) = \frac{1}{2\pi} \int_{-\pi/4}^{\pi/4} 1 \cdot e^{-int}\, dt \Rightarrow X(n) = \frac{1}{2\pi} \int_{-\pi/4}^{\pi/4} e^{-int}\, dt$$

$$X(n) = -\frac{1}{2\pi} \cdot \frac{1}{in} \left| e^{-int} \right|_{-\pi/4}^{\pi/4}$$

$$X(n) = \frac{1 \cdot 1}{2\pi i n} \left(e^{+in\pi/4} - e^{-in\pi/4} \right)$$

$$X(n) = \frac{1}{n\pi} \cdot \frac{1}{2i} \left(e^{in\pi/4} - e^{-in\pi/4} \right)$$

$$X(n) = \frac{1}{n\pi} \left(\sin\left(n \cdot \pi/4\right) \right)$$

Using Euler's Formula.

$$X(n) = \frac{1}{4} \frac{\sin(n \cdot \pi/4)}{\frac{n\pi}{4}}$$

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2. When $n = 0$:

$$x(n) = \frac{1}{2\pi}\int_{-\pi/4}^{\pi/4} 1 \cdot e^{-i\cdot 0 \cdot t}\, dt \Rightarrow \frac{1}{2\pi}\int_{-\pi/4}^{\pi/4} 1 \cdot 1$$

$$x(n) = \frac{1}{2\pi}\, t \Big|_{-\pi/4}^{\pi/4}$$

$$x(n) = \frac{1}{2\pi}\left(\frac{\pi}{4} - \left(-\frac{\pi}{4}\right)\right) \Rightarrow \frac{\cancel{2}\cancel{\pi}}{\cancel{2}\cdot 4\cancel{\pi}}$$

$$x(n) = \frac{1}{4}$$

$$x(n) = \begin{cases} \frac{1}{4}\dfrac{\sin\left(\frac{n\pi}{4}\right)}{\frac{n\pi}{4}}, & \text{when } n \neq 0 \\ 1/4, & \text{when } n = 0 \end{cases}$$

Task 2

TASK 2

As it is shown from infinitely long signal:

at $n = 0$, $x(n) = 1\,u(n)$; $n = 1$, $x(n) = (-0.9)^1 u(n)$

$n = 2$, $x(n) = (-0.9)^2 u(n) \Rightarrow x(n) = 0.81\, u(n)$

and so on. So,

$$x(n) = (-0.9)^n u(n)$$

$$X(e^{i\omega}) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-i\omega n}$$

$$= \sum_{n=-\infty}^{\infty} -0.9^n e^{-i\omega n} u(n)$$

$$= \sum_{n=0}^{\infty} -0.9^n e^{-i\omega n}$$

$$= \sum_{n=0}^{\infty} \left(-0.9 e^{-i\omega}\right)^n \Rightarrow \sum_{n=0}^{\infty} (-1)\left(0.9 e^{-i\omega}\right)^n$$

$$X(e^{i\omega}) = \frac{-1}{1 - 0.9 e^{-i\omega}}$$

Applying gemetric series.