COMP.SGN.100 Introduction to Signal Processing
Exercise 10: Task 1

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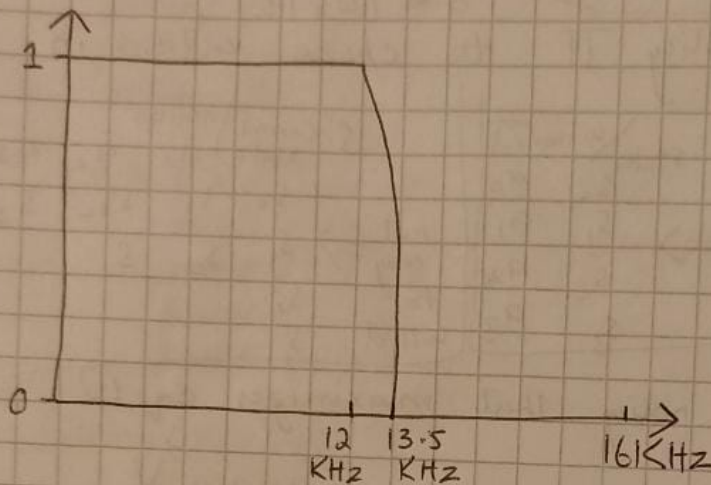
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Task 1

EXERCISE 10

TASK 1

Let's first draw the filter requirements.

→ Transition bandwidth $= \Delta f = \dfrac{13.5 - 12}{32} = \dfrac{1.5}{32}$

→ $f_c = \dfrac{(12 + 13.5)}{2 \times 32} = \dfrac{25.5}{64}$

→ According to the requirements, Hamming and Blackman are ok.
But we choose Hamming since it uses less coefficients

$$W(n) = \begin{cases} 0.54 + 0.46 \cos\left(\dfrac{2\pi n}{N}\right), & |n| \leq \dfrac{N-1}{2} \\ 0, & \text{otherwise} \end{cases}$$

→ $\Delta f = \dfrac{3.3}{N} \implies N = \dfrac{3.3}{\Delta f} = \dfrac{32(3.3)}{1.5} = 70.4$

$N = 71 \implies$ rounded up[1] to nearest odd integer.

→ Ideal Impulse Response (Low-pass)

$$h(n) = \begin{cases} 2f_c \, \text{sinc} \, (n \cdot 2\pi f_c), & n \neq 0 \\ 2f_c, & n = 0 \end{cases}$$

$$h(n) = \begin{cases} \dfrac{25.5}{32} \, \text{sinc}\left(\dfrac{25.5}{32} \cdot \pi n\right) & , \ n \neq 0 \\[3mm] \dfrac{25.5}{32} & , \ n = 0 \end{cases}$$

Impulse Response of the designed filter

$$N = 71 \implies \frac{N-1}{2} = \frac{70}{2} = 35$$

$$h_t(n) = w(n) \, h(n)$$

$$h_t(n) = \begin{cases} \left(0.54 + 0.46 \cos\left(\dfrac{2\pi n}{71}\right)\right)\left(\dfrac{25.5}{32} \, \text{sinc}\left(\dfrac{25.5}{32}\pi n\right)\right) & , \ 0 < |n| \leq 35 \\[3mm] \dfrac{25.5}{32} & , \ n = 0 \\[3mm] 0 & , \ \text{otherwise} \end{cases}$$