COMP.SGN.100 Introduction to Signal Processing
Exercise 12: Task 1,2

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Task 1

EXERCISE 12

TASK 1.

$$\mu_0 = \begin{pmatrix} -1 \\ 1 \end{pmatrix} ; \mu_1 = \begin{pmatrix} -5 \\ 2 \end{pmatrix}$$

$$C_0 = \begin{pmatrix} 5 & 4 \\ 4 & 5 \end{pmatrix} ; C_1 = \begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix}$$

$$\vec{w} = (C_0 + C_1)^{-1} \cdot (\mu_0 - \mu_1)$$

$$C_0 + C_1 = \begin{pmatrix} 5 & 4 \\ 4 & 5 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix} = \begin{pmatrix} 6 & 4 \\ 4 & 9 \end{pmatrix}$$

$$(C_0 + C_1)^{-1} = \begin{pmatrix} 6 & 4 \\ 4 & 9 \end{pmatrix}^{-1}$$

$$A^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$(C_0 + C_1)^{-1} = \frac{1}{54 - 16} \begin{pmatrix} 9 & -4 \\ -4 & 6 \end{pmatrix}$$

$$= \frac{1}{38} \begin{pmatrix} 9 & -4 \\ -4 & 6 \end{pmatrix} = \begin{pmatrix} 0.2368 & -0.1052 \\ -0.1052 & 0.1578 \end{pmatrix}$$

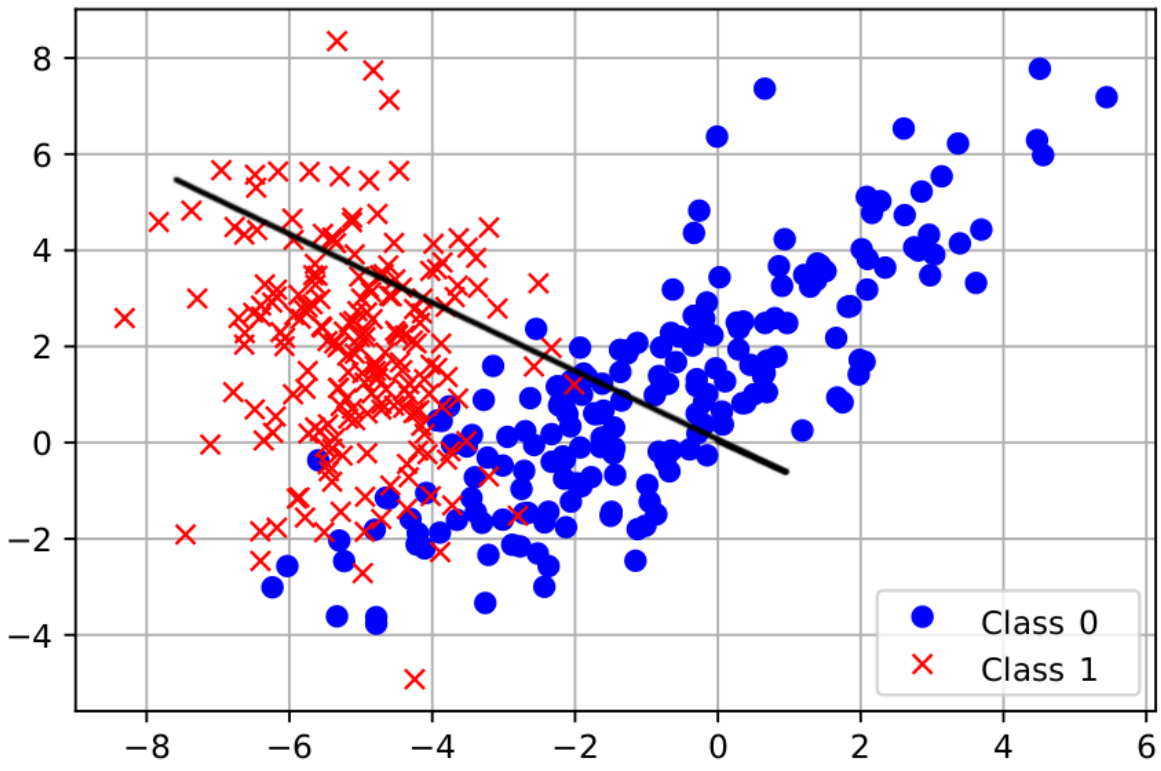
$$\mu_0 - \mu_1 = \begin{pmatrix} -1 \\ 1 \end{pmatrix} - \begin{pmatrix} -5 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$$

$$\vec{w} = (C_0 + C_1)^{-1} \cdot (\mu_0 - \mu_1)$$

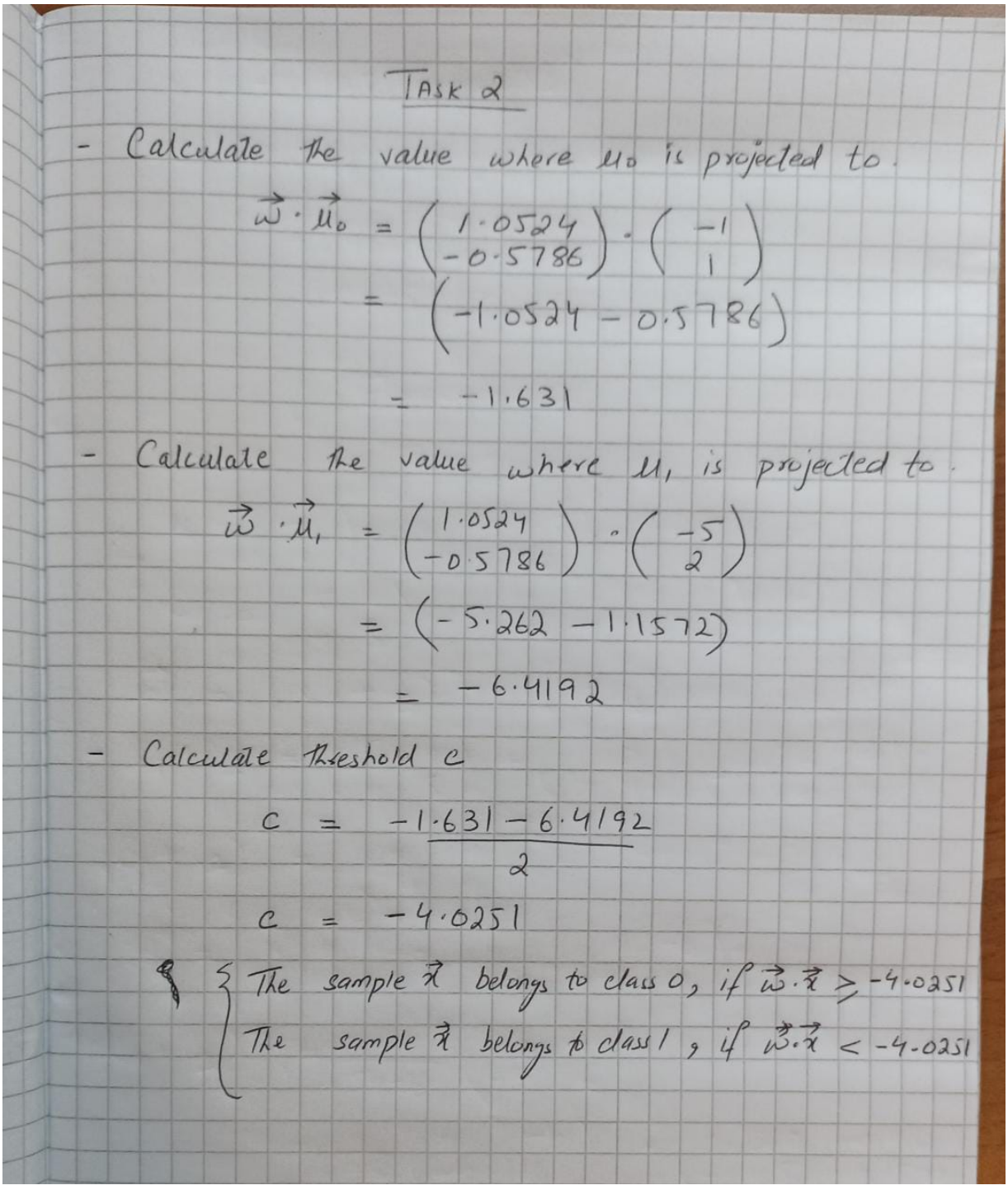
$$= \begin{pmatrix} 0.2368 & -0.1052 \\ -0.1052 & 0.1578 \end{pmatrix} \cdot \begin{pmatrix} 4 \\ -1 \end{pmatrix}$$

$$= \begin{pmatrix} 0.9472 + 0.1052 \\ -0.4208 + (-0.1578) \end{pmatrix} = \begin{pmatrix} 1.0524 \\ -0.5786 \end{pmatrix}$$

$$\vec{w} = \begin{pmatrix} 1.0524 \\ -0.5786 \end{pmatrix}$$



Task 2



TASK 2

- Calculate the value where μ_0 is projected to.

$$\vec{w} \cdot \vec{\mu_0} = \begin{pmatrix} 1.0524 \\ -0.5786 \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} -1.0524 - 0.5786 \end{pmatrix}$$

$$= -1.631$$

- Calculate the value where μ_1 is projected to.

$$\vec{w} \cdot \vec{\mu_1} = \begin{pmatrix} 1.0524 \\ -0.5786 \end{pmatrix} \cdot \begin{pmatrix} -5 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} -5.262 - 1.1572 \end{pmatrix}$$

$$= -6.4192$$

- Calculate threshold c

$$c = \frac{-1.631 - 6.4192}{2}$$

$$c = -4.0251$$

$$\begin{cases} \text{The sample } \vec{x} \text{ belongs to class 0, if } \vec{w} \cdot \vec{x} \geq -4.0251 \\ \text{The sample } \vec{x} \text{ belongs to class 1, if } \vec{w} \cdot \vec{x} < -4.0251 \end{cases}$$