

COMP.SGN.100 Introduction to Signal Processing
Exercise 11: Task 1,2,3

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Task 1

EXERCISE 11

TASK 1

Original Sampling Rate = 12 KHz

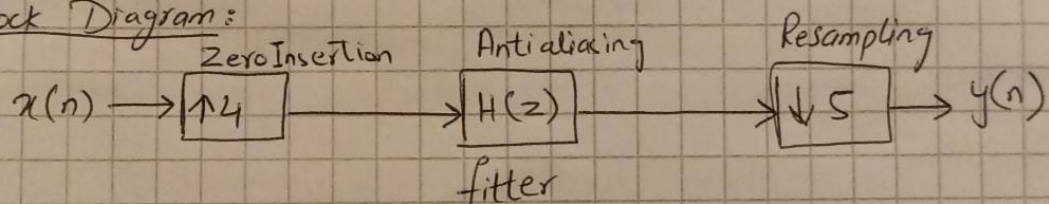
Converted Sampling Rate = 15 KHz

To find $\frac{L}{M}$ values, we take ratio of these two.

$$\frac{L}{M} = \frac{12 \text{ KHz}}{15 \text{ KHz}} = \frac{4}{5}$$

So $L = 4$ and $M = 5$

Block Diagram:

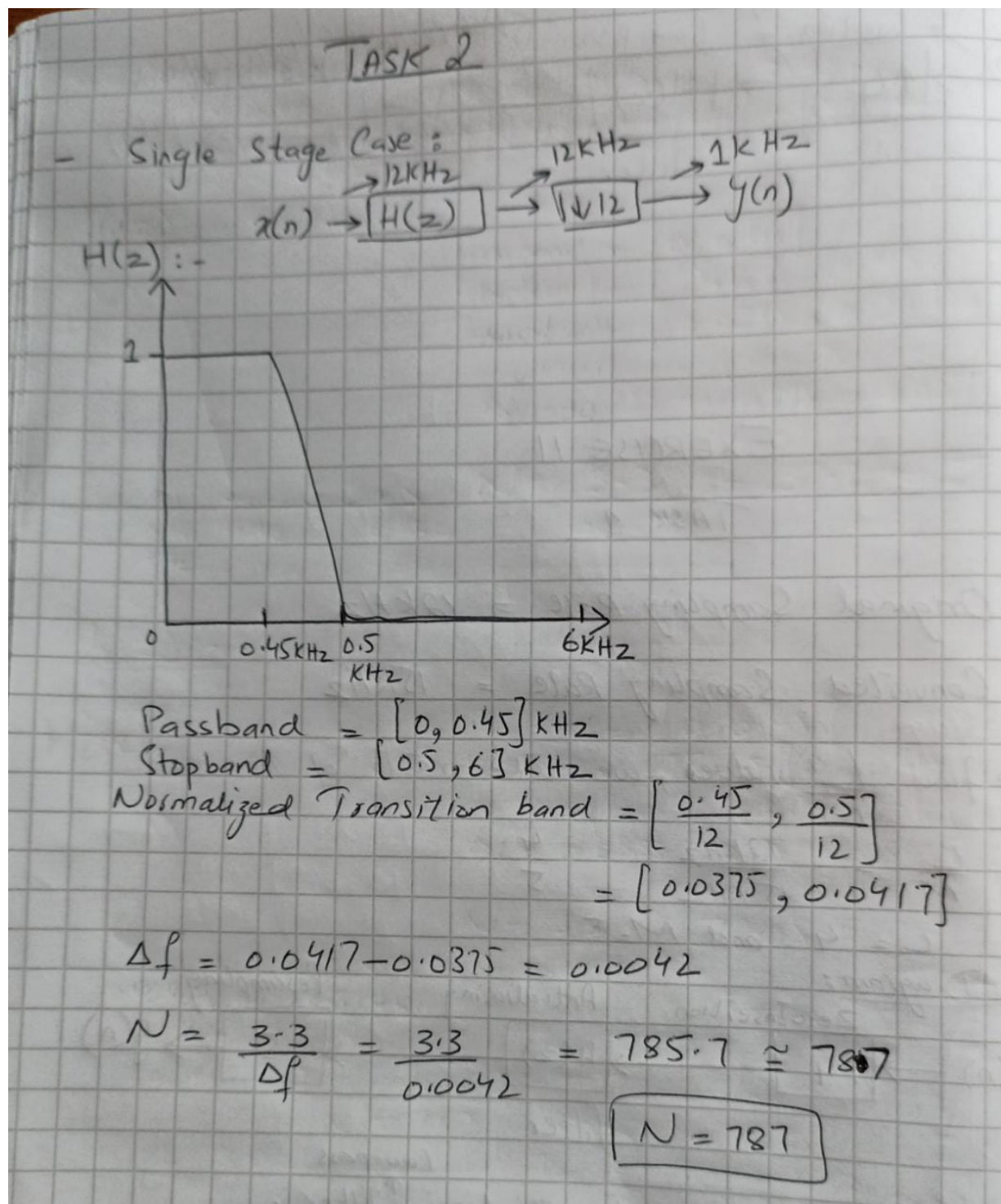


As $M > L$, so we look antialiasing ^{Low-pass} filter.

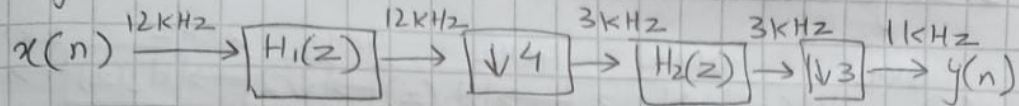
Passband of $H(z) = [0, 5 \cdot 5]$

Stopband of $H(z) = [\frac{1}{2M}, \frac{1}{2}] = [\frac{1}{10}, \frac{1}{2}]$

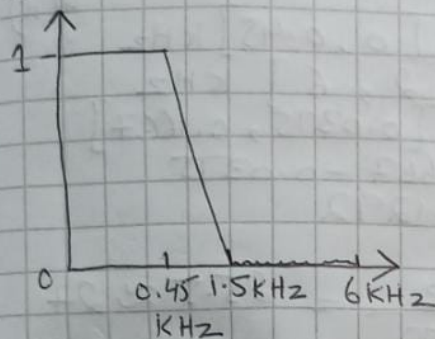
Task 2



- 2-Stage Case :-



$H_1(z)$:-

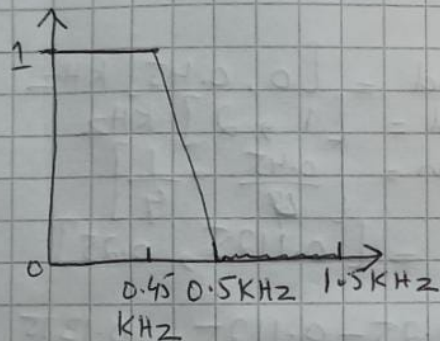


$$\begin{aligned}\text{Passband} &= [0, 0.45] \text{ KHz} \\ \text{Stopband} &= [1.5, 6] \text{ KHz} \\ \text{Transition} &= [0.0375, 0.125]\end{aligned}$$

$$\Delta f = 0.125 - 0.0375 = 0.0875$$

$$N = \frac{3.3}{0.0875} = 37.7 \approx 39$$

$H_2(z)$:-



$$\begin{aligned}\text{Passband} &= [0, 0.45] \text{ KHz} \\ \text{Stopband} &= [0.5, 1.5] \text{ KHz} \\ \text{Transition} &= \left[\frac{0.45}{3}, \frac{0.5}{3} \right] \\ &= [0.15, 0.166]\end{aligned}$$

$$\Delta f = 0.166 - 0.15 = 0.0167$$

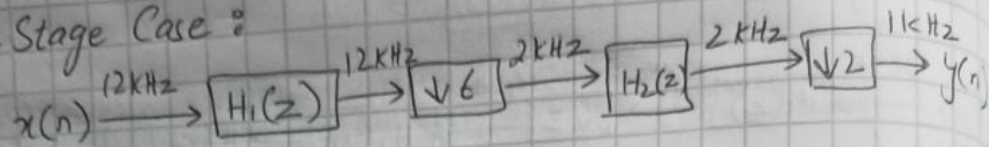
$$N = \frac{3.3}{0.0167} = 206.25 \approx 207$$

$$N = 197.6 \approx 199$$

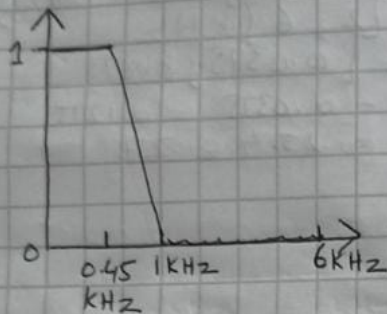
$$\text{Total} = 39 + 199 = 238$$

$$N = 238$$

- 2-Stage Case :



$H_1(z)$:-

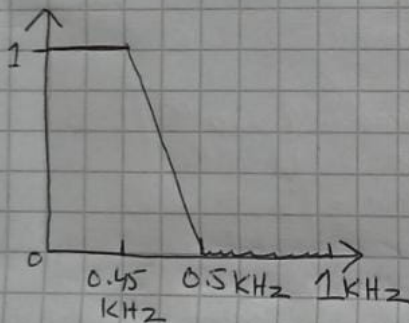


$$\begin{aligned} \text{Passband} &= [0, 0.45] \text{ kHz} \\ \text{Stopband} &= [1, 6] \text{ kHz} \\ \text{Transition} &= [0.0375, 0.0833] \\ \Delta f &= 0.0833 - 0.0375 \\ &= 0.0458 \end{aligned}$$

$$N = \frac{3.3}{\Delta f} = \frac{3.3}{0.0458} = 72.05$$

$$N = 73$$

$H_2(z)$:-



$$\begin{aligned} \text{Passband} &= [0, 0.45] \text{ kHz} \\ \text{Stopband} &= [0.5, 1] \text{ kHz} \\ \text{Transition} &= \left[\frac{0.45}{2}, \frac{0.5}{2} \right] \\ &= [0.225, 0.25] \end{aligned}$$

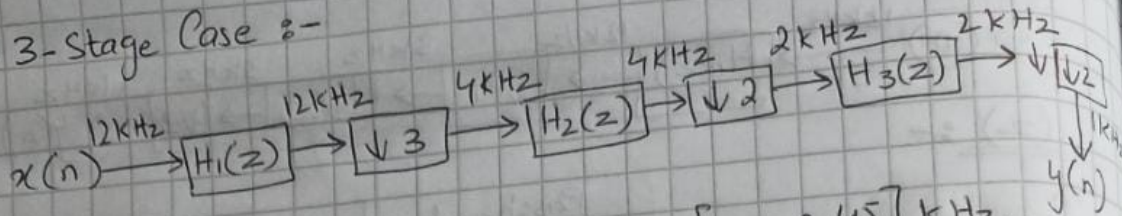
$$\Delta f = 0.25 - 0.225 = 0.025$$

$$N = \frac{3.3}{0.025} = 132 \approx 133$$

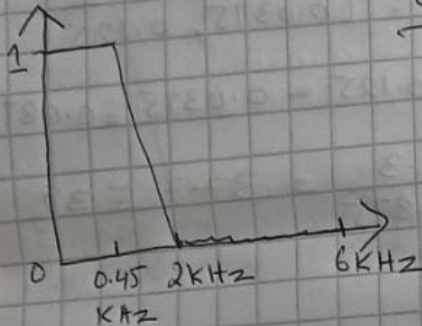
$$\text{Total No. of coefficients} = 73 + 133 = 206$$

$$N = 206$$

- 3-Stage Case :-



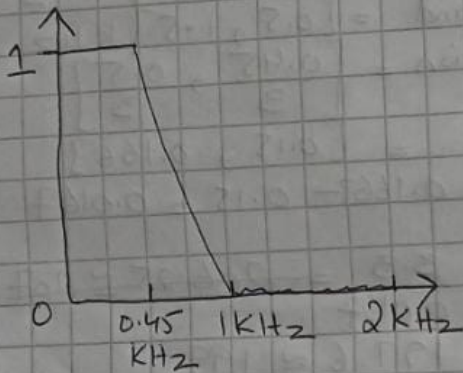
$H_1(z)$:-



$$\begin{aligned} \text{Passband} &= [0, 0.45] \text{ kHz} \\ \text{Stopband} &= [2, 6] \text{ kHz} \\ \text{Transition} &= [0.0375, 0.1667] \\ \Delta f &= 0.1667 - 0.0375 \\ &= 0.1292 \end{aligned}$$

$$N = \frac{3.3}{0.1292} = 25.5 \approx 27$$

$H_2(z)$:-

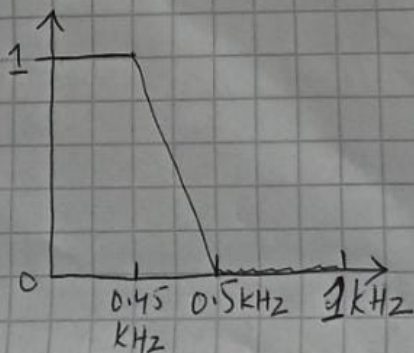


$$\begin{aligned} \text{Passband} &= [0, 0.45] \text{ kHz} \\ \text{Stopband} &= [1, 2] \text{ kHz} \\ \text{Transition} &= \left[\frac{0.45}{4}, \frac{1}{4} \right] \\ &= [0.1125, 0.25] \end{aligned}$$

$$\Delta f = 0.25 - 0.1125 = 0.1375$$

$$N = \frac{3.3}{0.1375} = 24 \approx 25$$

$H_3(z)$:-



$$\begin{aligned} \text{Passband} &= [0, 0.45] \text{ kHz} \\ \text{Stopband} &= [0.5, 1] \text{ kHz} \end{aligned}$$

$$\begin{aligned} \text{Transition} &= \left[\frac{0.45}{2}, \frac{0.5}{2} \right] \\ &= [0.225, 0.25] \end{aligned}$$

$$\Delta f = 0.25 - 0.225 = 0.025$$

$$N = \frac{3.3}{0.025} = 132 \approx 133$$

$$\text{Total } N = 27 + 25 + 133 = 185$$

$$\boxed{N = 185}$$

Task 3

TASK 3

MPS for 1-Stage Case :- $x(n) \xrightarrow{12\text{kHz}} [H(z)] \xrightarrow{12\text{kHz}} [\downarrow 12] \xrightarrow{1\text{kHz}} y(n)$

$$\text{MPS} = N_1 \times F_1$$

$$\text{MPS} = 787 \times 12 = 9444 \times 1000 = 9444000$$

MPS for 2-Stage Case :-

$x(n) \xrightarrow{12\text{kHz}} [H_1(z)] \xrightarrow{12\text{kHz}} [\downarrow 4] \xrightarrow{3\text{kHz}} [H_2(z)] \xrightarrow{3\text{kHz}} [\downarrow 3] \xrightarrow{1\text{kHz}} y(n)$

$$\begin{aligned} \text{MPS} &= (N_1 \times F_1) + (N_2 \times F_2) \\ &= (39 \times 12) + (199 \times 3) \\ &= 468 + 597 \\ &= 1065 \times 1000 = 1065000 \end{aligned}$$

MPS for 2-Stage Case :-

$x(n) \xrightarrow{12\text{kHz}} [H_1(z)] \xrightarrow{12\text{kHz}} [\downarrow 6] \xrightarrow{2\text{kHz}} [H_2(z)] \xrightarrow{2\text{kHz}} [\downarrow 2] \xrightarrow{1\text{kHz}} y(n)$

$$\begin{aligned} \text{MPS} &= (N_1 \times F_1) + (N_2 \times F_2) \\ &= (73 \times 12) + (133 \times 2) = 876 + 266 = 1142 \times 1000 \\ &= 1142000 \end{aligned}$$

MPS for 3-Stage Case :-

$x(n) \xrightarrow{12\text{kHz}} [H_1(z)] \xrightarrow{12\text{kHz}} [\downarrow 3] \xrightarrow{4\text{kHz}} [H_2(z)] \xrightarrow{4\text{kHz}} [\downarrow 2] \xrightarrow{2\text{kHz}} [H_3(z)] \xrightarrow{2\text{kHz}} [\downarrow 2] \xrightarrow{1\text{kHz}} y(n)$

$$\begin{aligned} \text{MPS} &= (N_1 \times F_1) + (N_2 \times F_2) + (N_3 \times F_3) \\ &= (27 \times 12) + (25 \times 4) + (133 \times 2) \\ &= 324 + 100 + 266 \\ &= 690 \times 1000 \\ &= 690000 \end{aligned}$$