COMP.SGN.100 Introduction to Signal Processing,
Exercise 8, 29.9.-1.10.2021

Pen & paper task solutions should be submitted to Moodle at least one hour before your exercise session. Matlab tasks are done during the exercise session.

Task 1. *(Pen & paper)* Assume that the input $x(n)$ and the output $y(n)$ of a causal LTI system satisfy the following difference equation:

$$y(n) = x(n) - 2x(n-1) + x(n-2) + 3y(n-1) - \frac{37}{16}y(n-2).$$

(a) Determine the transfer function $H(z)$ of the system.

(b) Draw the pole-zero plot of the system.

(c) Is the system stable?

Task 2. *(Pen & paper)* The signal $x(n) = u(n)\sin(\frac{1}{5} \cdot 2\pi n)$ is filtered by a system having the impulse response

$$h(n) = \begin{cases} \frac{1}{2}, & \text{when } n = 0 \text{ or } n = 2, \\ 1, & \text{when } n = 1, \\ 0, & \text{otherwise.} \end{cases}$$

The output is of the form $y(n) = Au(n)\sin(\frac{1}{5} \cdot 2\pi n + \phi)$. Determine the values of the real numbers A and ϕ.

Task 3. *(Matlab)* The transfer function of an LTI system is

$$H(z) = \frac{0.0122 + 0.0226z^{-1} + 0.0298z^{-2} + 0.0204z^{-3} + 0.0099z^{-4}}{1 - 0.9170z^{-1} + 0.0540z^{-2} - 0.2410z^{-3} + 0.1990z^{-4}}.$$

Assign the coefficients to the vectors `a` and `b` and plot the pole-zero plot (`help zplane`), the amplitude and phase responses (`help freqz`) and the impulse response (`help impz`) of the system. Compare these to the corresponding ones of the inverse system

$$H^{-1}(z) = \frac{1 - 0.9170z^{-1} + 0.0540z^{-2} - 0.2410z^{-3} + 0.1990z^{-4}}{0.0122 + 0.0226z^{-1} + 0.0298z^{-2} + 0.0204z^{-3} + 0.0099z^{-4}}.$$

Note that you can utilize the original system when studying the inverse system. So do not rewrite the coefficients.

Load Matlab's test signal `handel` to variable `y` with the command `load handel`. Filter it with filter $H(z)$ and plot the spectrogram. Filter the obtained result with the inverse filter $H^{-1}(z)$ and plot the spectrogram of the result. This spectrogram should be similar to the spectrogram of the original signal. Is it?

Task 4. *(Matlab)* Create a signal `x` with steadily increasing frequency with the commands

```
t=0:1/8192:4;
x=chirp(t,0,1,1000);
```

Listen to the result (if possible) with the command `soundsc(x)`. Alternatively, you can study the signal with the command `spectrogram`. Filter the signal with the LTI system having the transfer function

$$H(z) = \frac{0.0675 + 0.1349z^{-1} + 0.0675z^{-2}}{1 - 1.143z^{-1} + 0.4128z^{-2}}.$$

Listen to the result and/or study its spectrogram. Compare the result to the amplitude response of the filter.

Task 5. *(Matlab)* Download the file `number.mat` from the course Moodle (`Ex_8.zip`). The file contains dialing tone (seven digits) for the push-button phone. Your task is to find out what phone number it is.

Load the signal to Matlab using the command `load number.mat`. The signal is then in the vector called `secret`. You can listen to it with the command `sound(secret)`.

Dialing tones consist of the sum of two components having different frequency (Table 1). Identify these components using Matlab command `spectrogram(secret,256,'yaxis')`. If the identification is difficult, you can zoom in the image by using the magnifying glass tool of Matlab.

Table 1: Dual tone multiple frequencies (DTMF) for push-button phone when the sampling rate is 8192 Hz. For example, the signal corresponding to the button '5' is $x(n) = \sin(0.1880\pi n) + \sin(0.3262\pi n)$.

	0.2952	0.3262	0.3606
0.1702	1	2	3
0.1880	4	5	6
0.2080	7	8	9
0.2297	*	0	#