## COMP.SGN.100 Introduction to Signal Processing, Exercise 6, 22.-24.9.2021

Pen & paper task solutions should be submitted to Moodle at least one hour before your exercise session. Matlab tasks are done during the exercise session.

Task 1. *(Pen & paper)* Calculate manually the discrete Fourier transform of the vector $x(n) = (5, 1, -1, 0)^T$.

Task 2. *(Pen & paper)* Calculate the DFT of the sequence $x(n) = (-1, 3, 1, 0)$ using the FFT algorithm. You can skip part of the calculations by utilizing this information: the DFT of the sequence $(-1, 1)$ is $(0, -2)$ and the DFT of $(3, 0)$ is $(3, 3)$.

Task 3. *(Matlab)* Generate a one second long signal having frequency 2000 Hz with sampling rate 16000 Hz. Calculate the DFT of the signal using Matlab command `fft` and plot the graph of its absolute values. (`help fft, help plot, help abs`). The figure should have a clear spike in two positions on the horizontal axis (corresponding to the frequency 2000 Hz).

Task 4. *(Matlab)* We compare the computation times of DFT and FFT in this and the following task.

(a) Implement the function `dft(x)`, which works like the `fft(x)` but calculates the result directly by matrix multiplication. There are two steps in the implementation: (1) construct the DFT matrix and (2) left-multiply the input vector `x` by the matrix. The DFT matrix can be created with the command

```
F=exp(-2*pi*1i*(0:N-1)'*(0:N-1)/N);
```

The variable `N` is the number of elements in the vector `x`, which you can obtain with the `length` function.

(b) Check that the `dft` and `fft` give the same result e.g. for the vector $x = [1, 2, 3, 4]$.

Task 5. *(Matlab)* Now we are ready to compare the computation times.

(a) First, test both functions with a random vector of length 1024 `x=rand(1024,1)`. The execution time is obtained by using `tic/toc` pair as follows:

```
tic(); % Starts a stopwatch timer
X=dft(x); % DFT computation
elapsed_time=toc(); % Reads the elapsed time from the timer
```

(b) Put the calculation inside a `for`-loop and do the same calculation 100 times to get a more accurate estimate.

(c) Furthermore, put the 100 times calculation inside a second loop where you perform tests for lengths $N = 32, 64, 128, 256, 512, 1024$.

(d) Plot the time versus length graphs for FFT and DFT.