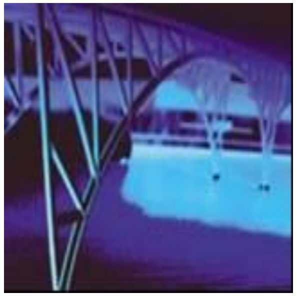
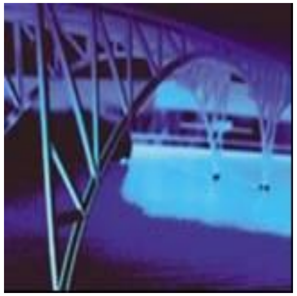
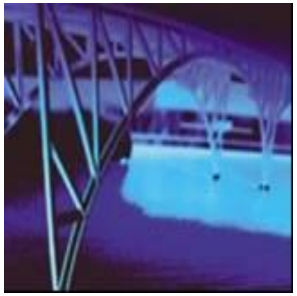


Time Value of Money



The Time Value of Money

- The Interest Rate
- Simple Interest
- Compound Interest
- Amortizing a Loan
- Compounding More Than Once per Year

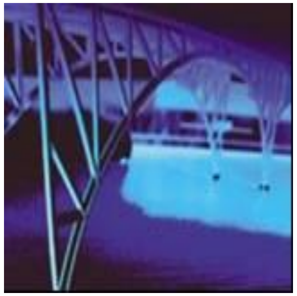


The Interest Rate

Which would you prefer -- $10,000 today or $10,000 in 5 years?

Obviously, $10,000 today.

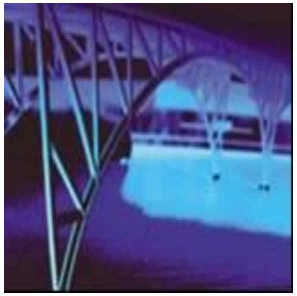
You already recognize that there is *TIME VALUE TO MONEY*!!



Why TIME?

Why is TIME such an important element in your decision?

TIME allows you the *opportunity* to postpone consumption and earn INTEREST.



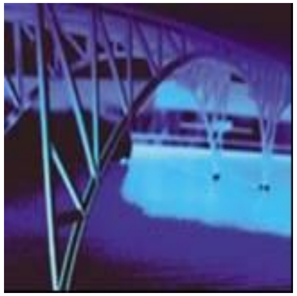
Types of Interest

- **Simple Interest**

  Interest paid (earned) on only the original amount, or principal, borrowed (lent).

- **Compound Interest**

  Interest paid (earned) on any previous interest earned, as well as on the principal borrowed (lent).



Simple Interest Formula

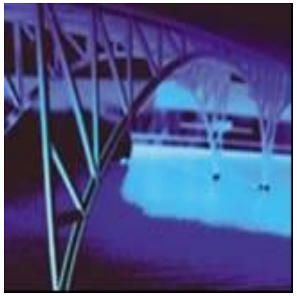
Formula $SI = P_0(i)(n)$

SI: Simple Interest

P_0: Deposit today (t=0)

i: Interest Rate per Period

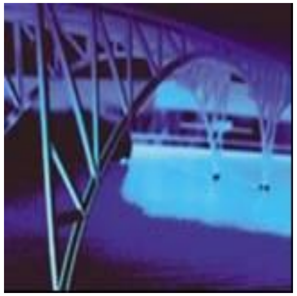
n: Number of Time Periods



Simple Interest Example

- Assume that you deposit $1,000 in an account earning 7% simple interest for 2 years. *What is the accumulated interest at the end of the 2nd year?*

- SI = $P_0(i)(n)$ = $1,000(.07)(2) = **$140**

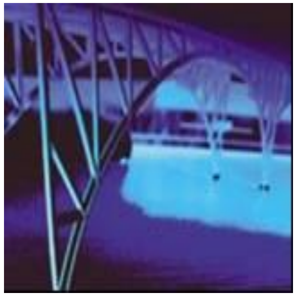


Simple Interest (FV)

- What is the **Future Value (FV)** of the deposit?

$$FV = P_0 + SI = \$1,000 + \$140 = \$1,140$$

- **Future Value** is the value at some future time of a present amount of money, or a series of payments, evaluated at a given interest rate.

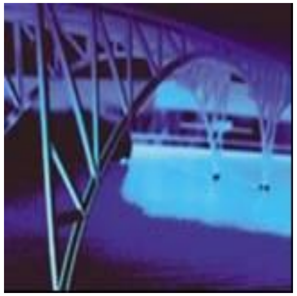


Simple Interest (PV)

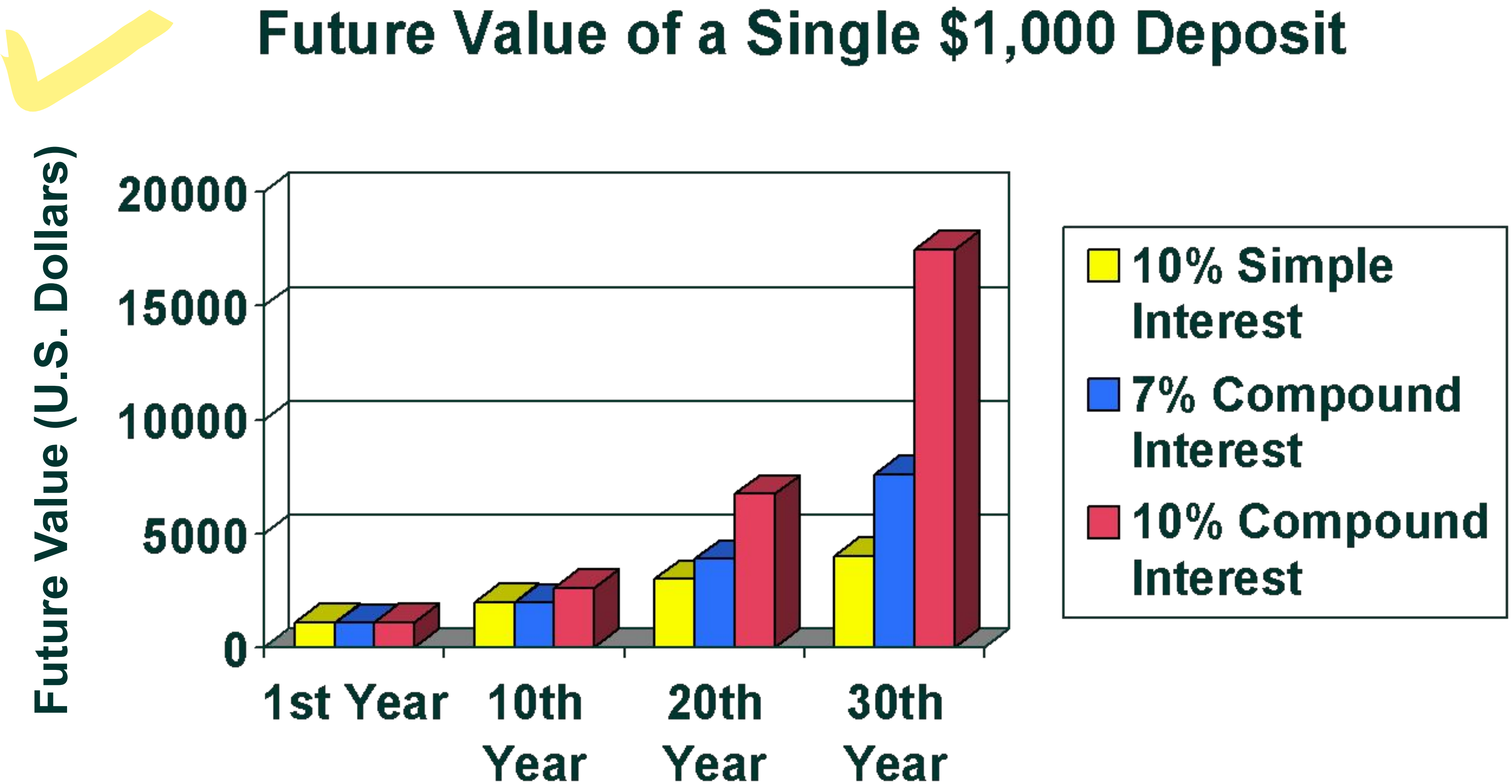
- What is the Present Value (PV) of the previous problem?

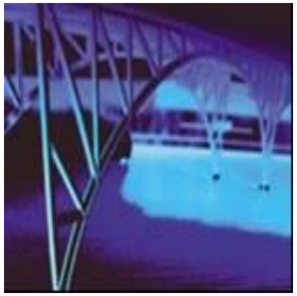
  The Present Value is simply the $1,000 you originally deposited. That is the value today!

- Present Value is the current value of a future amount of money, or a series of payments, evaluated at a given interest rate.



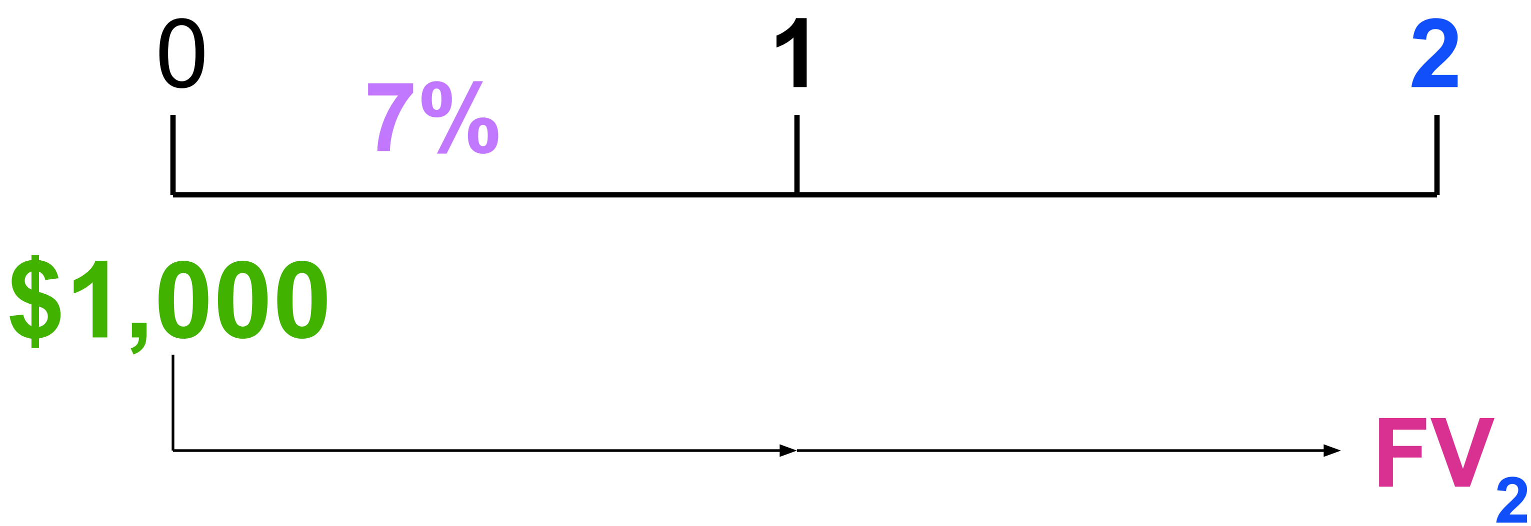
Why Compound Interest?

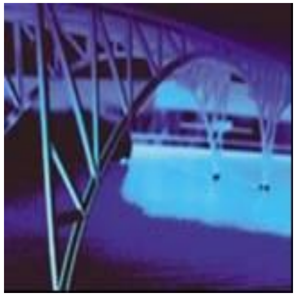




Future Value Single Deposit (Graphic)

Assume that you deposit $1,000 at a compound interest rate of 7% for 2 years.





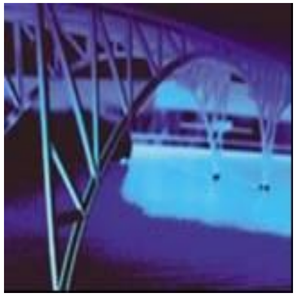
Future Value Single Deposit (Formula)

$FV_1 = P_0 (1+i)^1$ $= \$1,000 (1.07)$
$= \$1,070$

Compound Interest

You earned $70 interest on your $1,000 deposit over the first year.

This is the same amount of interest you would earn under simple interest.



Future Value Single Deposit (Formula)

$FV_1 = P_0 (1+i)^1 \qquad = \$1{,}000\ (1.07)$

$= \$1{,}070$

$FV_2 = FV_1\ (1+i)^1$

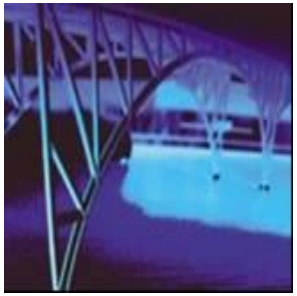
$= P_0 (1+i)(1+i) \quad =$

$\$1{,}000(1.07)(1.07) \qquad = P_0 (1+i)^2$

$= \$1{,}000(1.07)^2$

$= \$1{,}144.90$

You earned an *EXTRA $4.90* in Year 2 with compound over simple interest



General Future Value Formula

$$FV_1 = P_0(1+i)^1$$

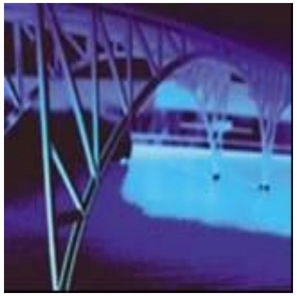
$$FV_2 = P_0(1+i)^2$$

etc.

General Future Value Formula:

$$FV_n = P_0 (1+i)^n$$

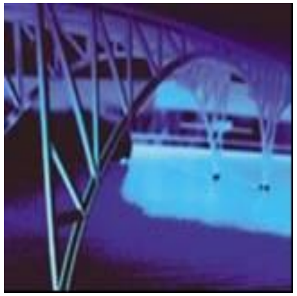
or $FV_n = P_0 (FVIF_{i,n})$ -- *See Table I*



Valuation Using Table I

$FVIF_{i,n}$ is found on Table I at the end of the book.

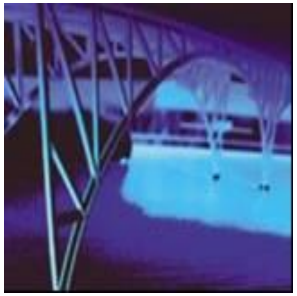
Period	6%	7%	8%
1	1.060	1.070	1.080
2	1.124	**1.145**	1.166
3	1.191	1.225	1.260
4	1.262	1.311	1.360
5	1.338	1.403	1.469



Using Future Value Tables

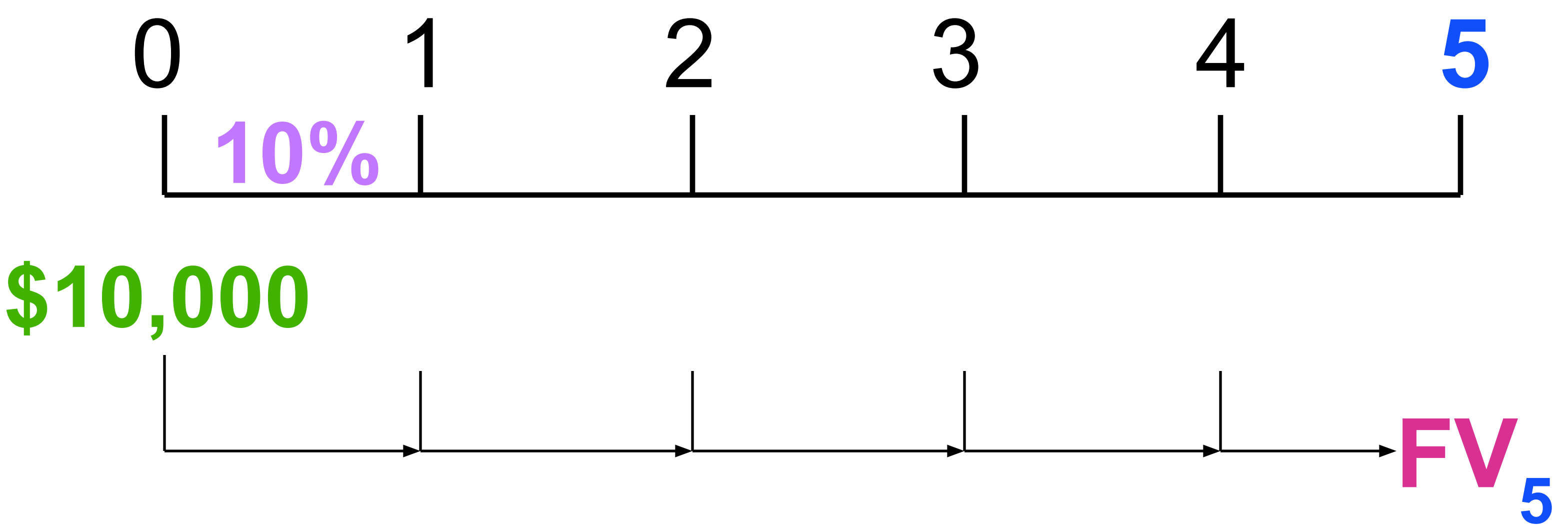
$FV_2 = \$1{,}000\ (FVIF_{7\%,2}) = \$1{,}000\ (1.145) = \$1{,}145$
[Due to Rounding]

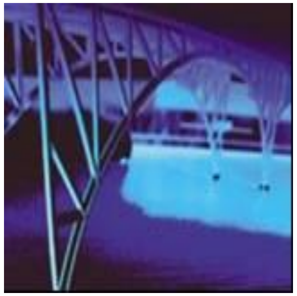
Period	6%	7%	8%
1	1.060	1.070	1.080
2	1.124	1.145	1.166
3	1.191	1.225	1.260
4	1.262	1.311	1.360
5	1.338	1.403	1.469



Story Problem Example

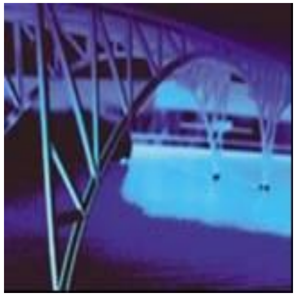
Julie Miller wants to know how large her deposit of $10,000 today will become at a compound annual interest rate of 10% for 5 years.





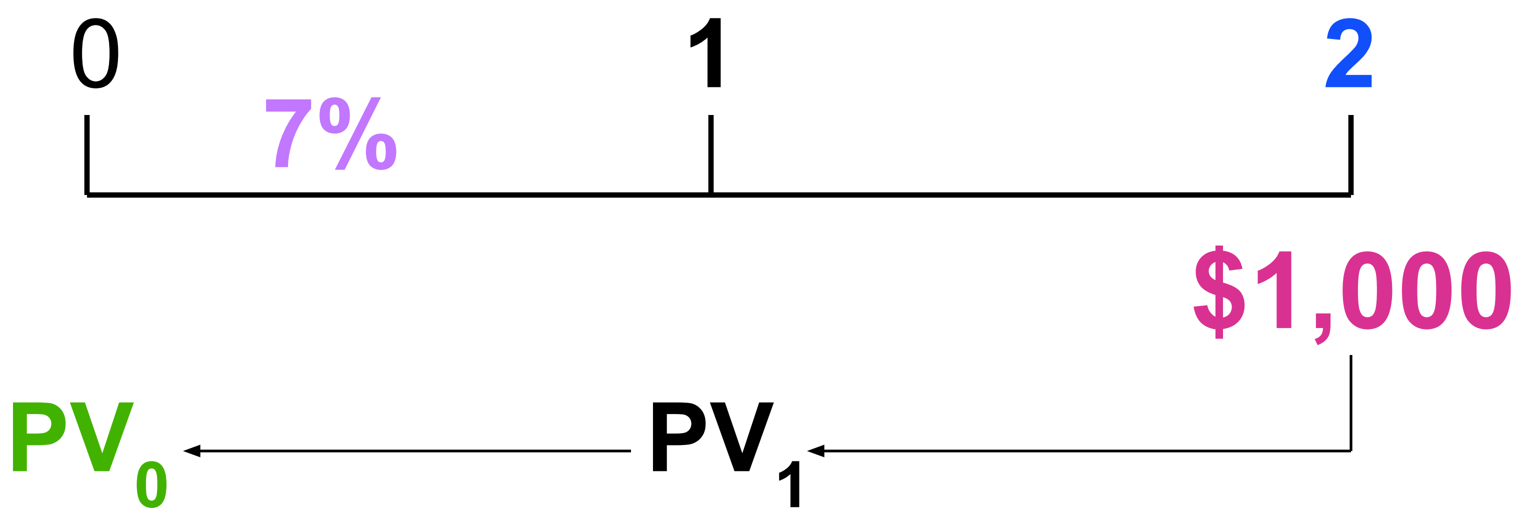
Story Problem Solution

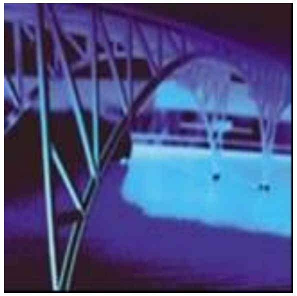
- Calculation based on general formula: $FV_n = P_0 (1+i)^n$ $FV_5 = \$10,000 (1+ 0.10)^5 = \$16,105.10$
- Calculation based on Table I: $FV_5 = \$10,000 \left(FVIF_{10\%,\, 5}\right) = \$10,000 (1.611) = \$16,110$ [*Due to Rounding*]



Present Value Single Deposit (Graphic)

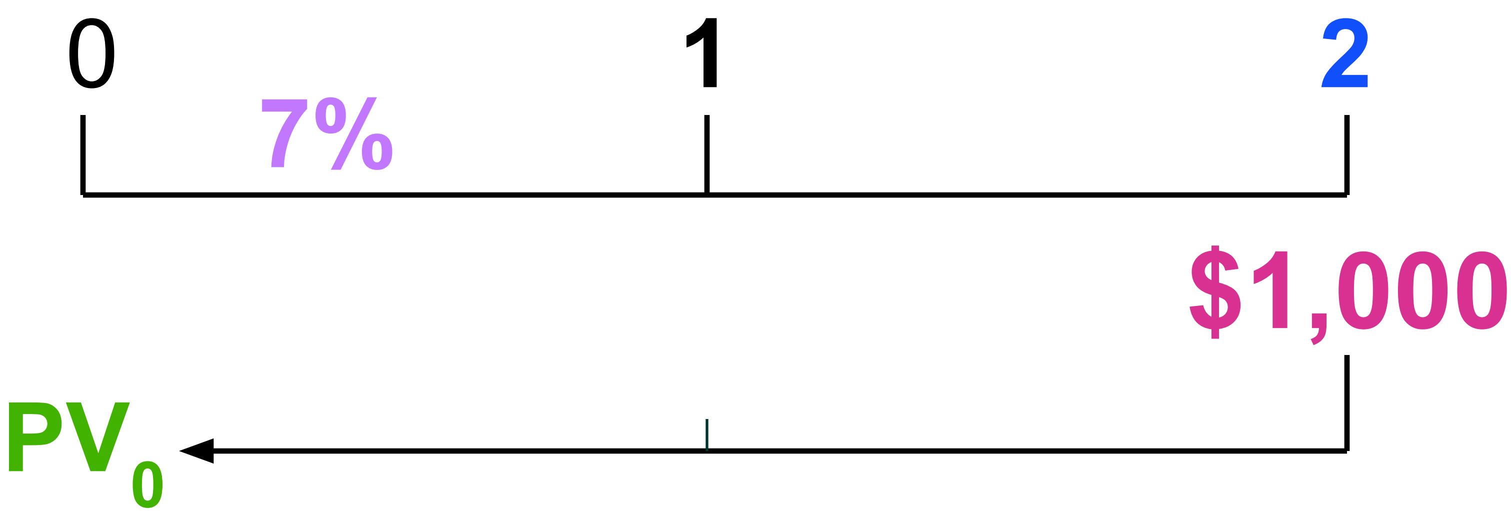
Assume that you need $1,000 in 2 years. Let's examine the process to determine how much you need to deposit today at a discount rate of 7% compounded annually.

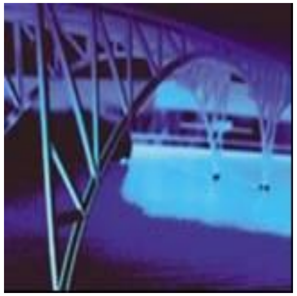




Present Value Single Deposit (Formula)

$PV_0 = FV_2 / (1+i)^2 = \$1,000 / (1.07)^2$
$= FV_2 / (1+i)^2 = \$873.44$





General Present Value Formula

$$PV_0 = FV_1 / (1+i)^1$$

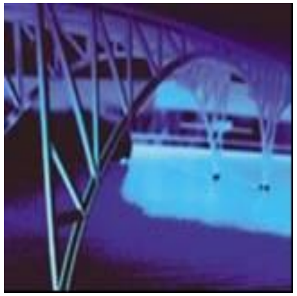
$$PV_0 = FV_2 / (1+i)^2$$

etc.

General Present Value Formula:

$$PV_0 = FV_n / (1+i)^n$$

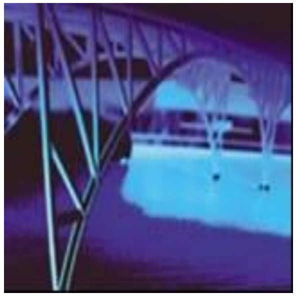
or $PV_0 = FV_n (PVIF_{i,n})$ -- *See Table II*



Valuation Using Table II

$PVIF_{i,n}$ is found on Table II at the end of the book.

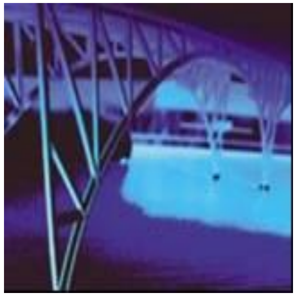
Period	6%	7%	8%
1	.943	.935	.926
2	.890	.873	.857
3	.840	.816	.794
4	.792	.763	.735
5	.747	.713	.681



Using Present Value Tables

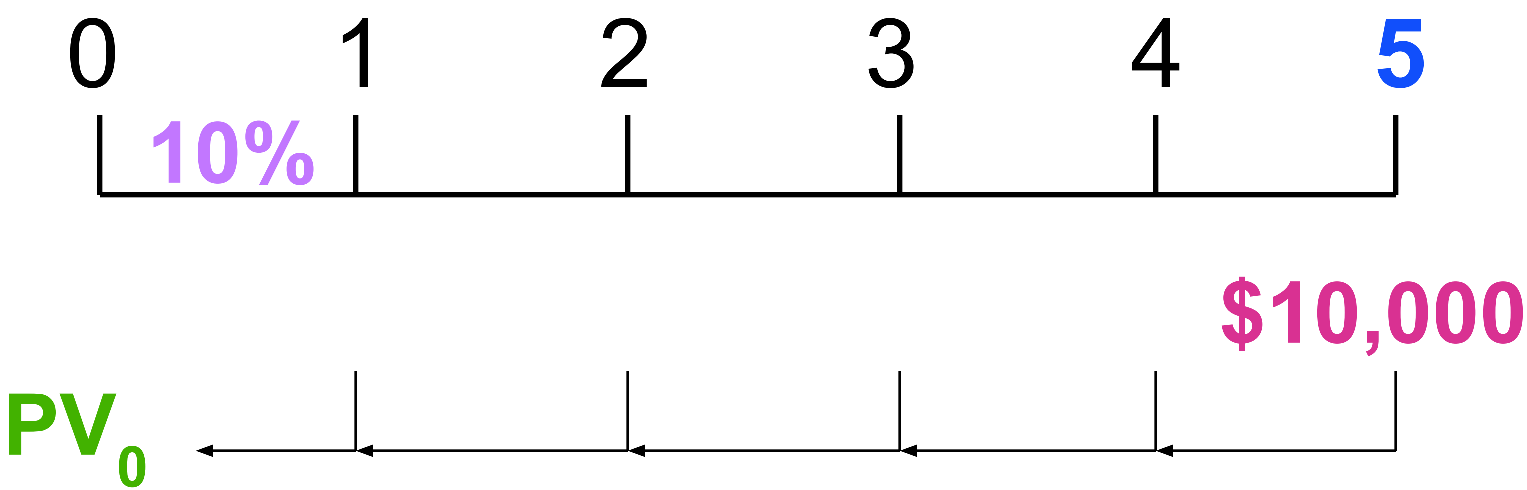
PV_2 = \$1,000 ($PVIF_{7\%,2}$) = \$1,000 (.873) = \$873 [Due to Rounding]

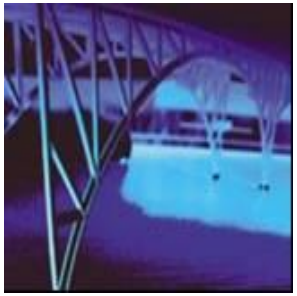
Period	6%	7%	8%
1	.943	.935	.926
2	.890	.873	.857
3	.840	.816	.794
4	.792	.763	.735
5	.747	.713	.681



Story Problem Example

Julie Miller wants to know how large of a deposit to make so that the money will grow to $10,000 in 5 years at a discount rate of 10%.

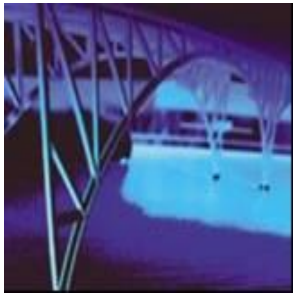




Story Problem Solution

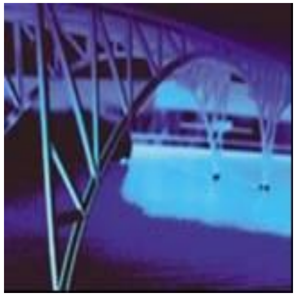
- **Calculation based on general formula:** $PV_0 = FV_n / (1+i)^n$ PV_0 = $\$10,000 / (1+0.10)^5$ = **$6,209.21**

- **Calculation based on Table I:** PV_0 = $\$10,000\ (PVIF_{10\%,\ 5})$ = $10,000 (.621) = **$6,210.00** ***[Due to Rounding]***



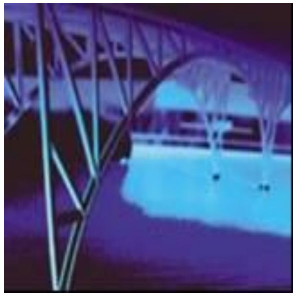
Types of Annuities

- *An Annuity* represents a series of equal payments (or receipts) occurring over a specified number of equidistant periods.
- Ordinary Annuity: Payments or receipts occur at the end of each period.
- Annuity Due: Payments or receipts occur at the beginning of each period.



Examples of Annuities

- Student Loan Payments
- Car Loan Payments
- Insurance Premiums
- Mortgage Payments
- Retirement Savings



Parts of an Annuity

(Ordinary Annuity)

End of Period 1 | End of Period 2 | End of Period 3

0 | 1 | 2 | 3

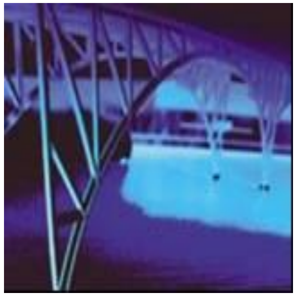
saal ka

$100 | $100

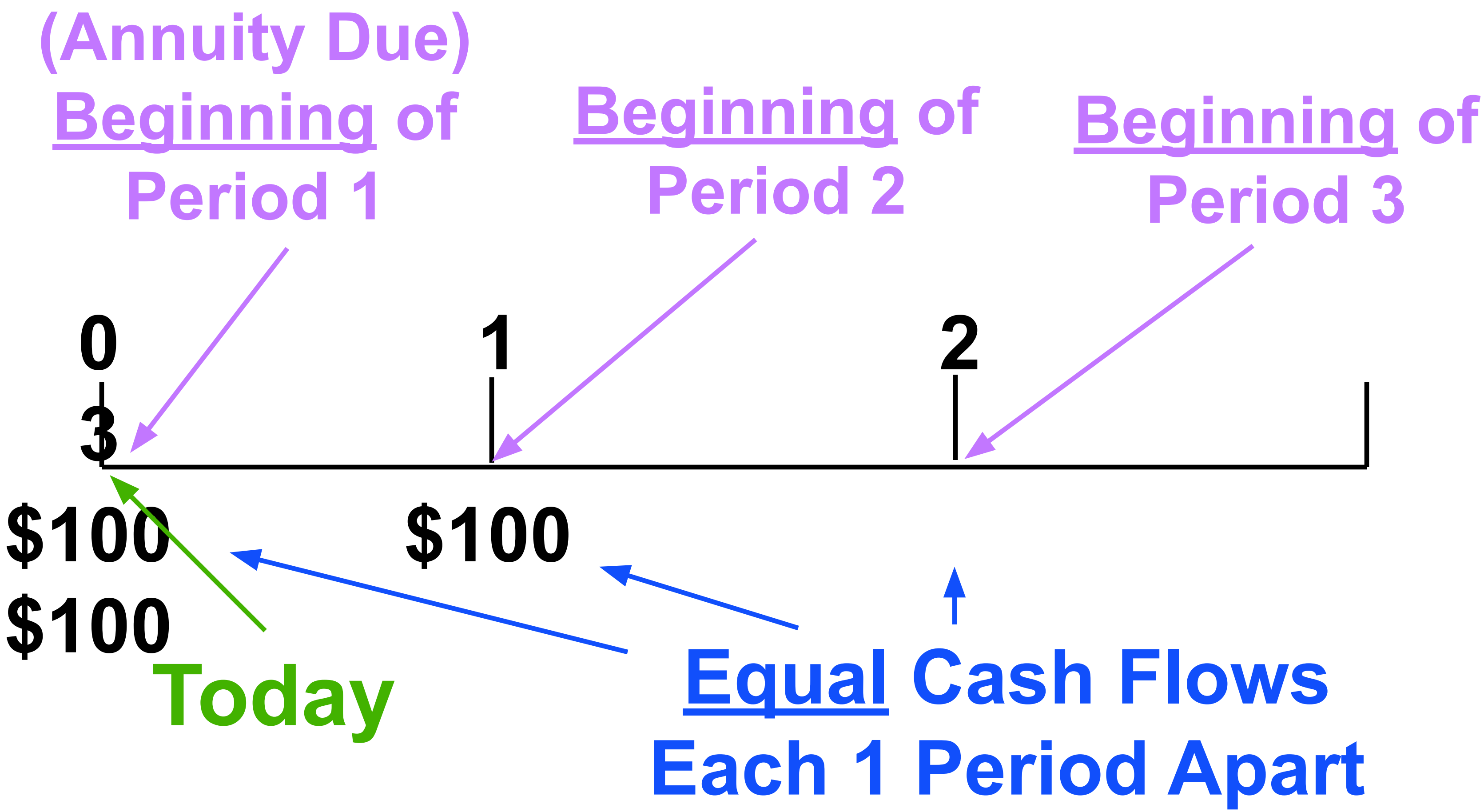
$100

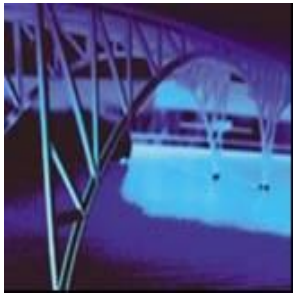
Today

Equal Cash Flows Each 1 Period Apart



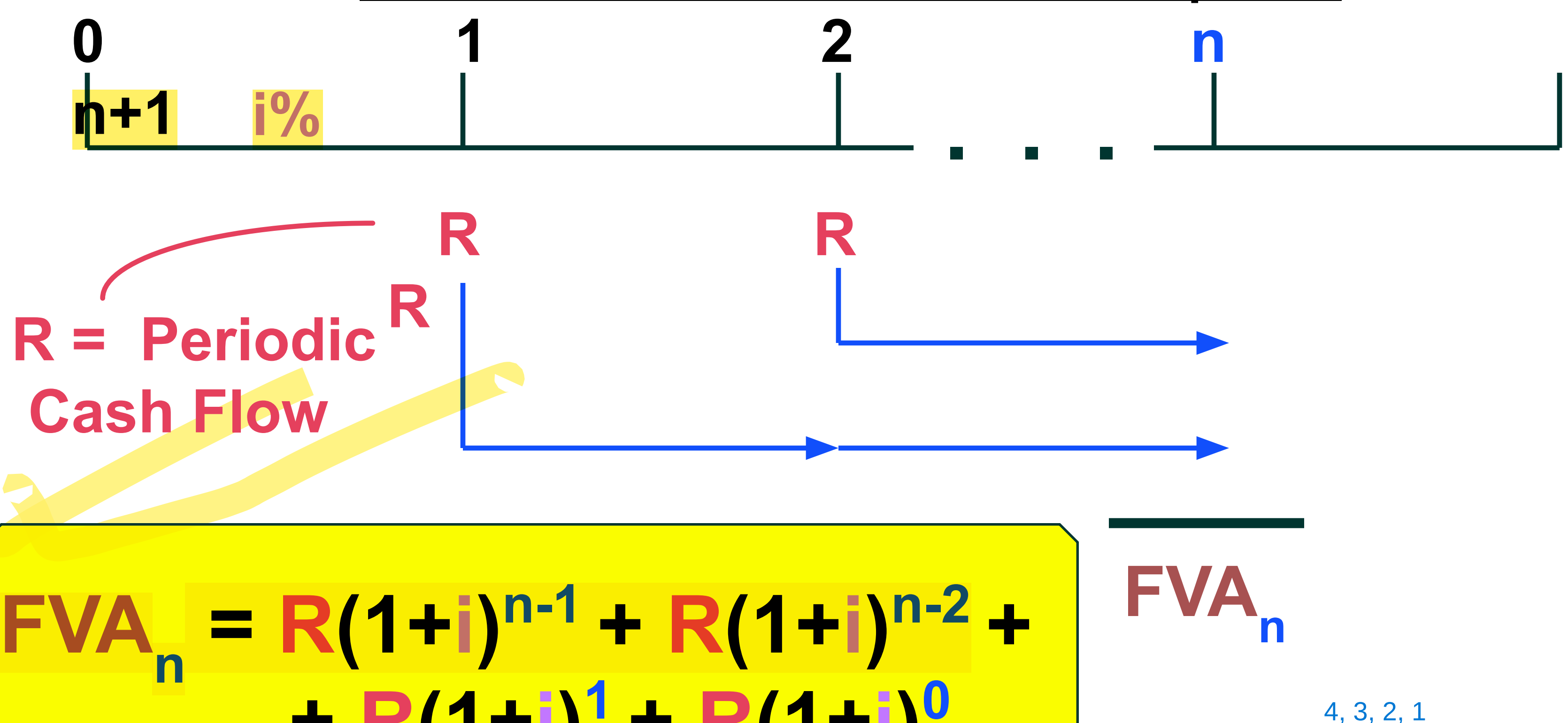
Parts of an Annuity



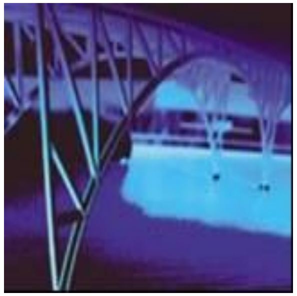


Overview of an Ordinary Annuity -- FVA

Cash flows occur at the end of the period

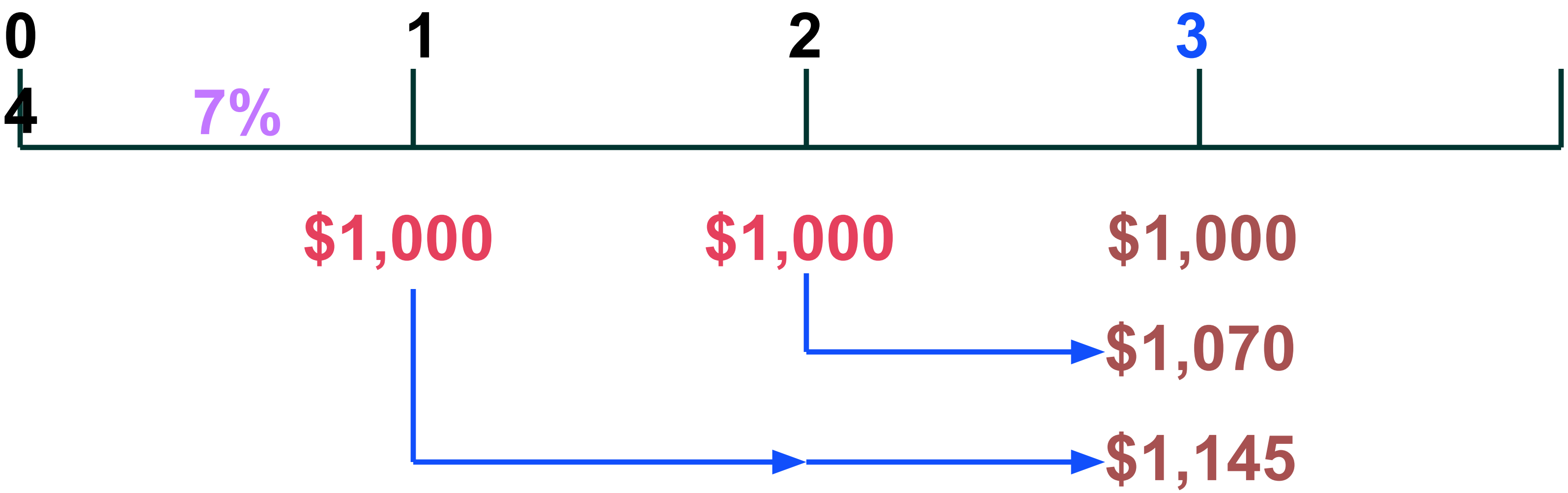


$$FVA_n = R(1+i)^{n-1} + R(1+i)^{n-2} + \ldots + R(1+i)^{1} + R(1+i)^{0}$$



Example of an Ordinary Annuity -- FVA

Cash flows occur at the end of the period

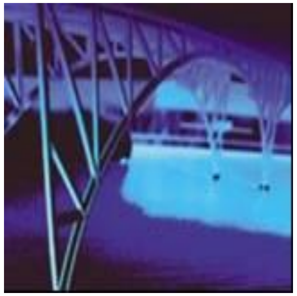


$\$3{,}215 = FVA_3$

$FVA_3 = \$1{,}000(1.07)^2 + \$1{,}000(1.07)^1 + \$1{,}000(1.07)^0$

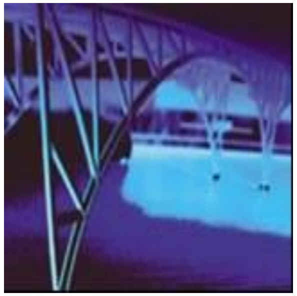
$= \$1{,}145 + \$1{,}070 + \$1{,}000$

$= \$3{,}215$



Hint on Annuity Valuation

The future value of an ordinary annuity can be viewed as occurring at the *end* of the last cash flow period, whereas the future value of an annuity due can be viewed as occurring at the *beginning* of the last cash flow period.



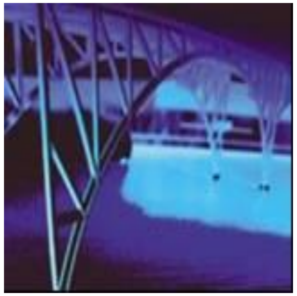
Valuation Using Table III

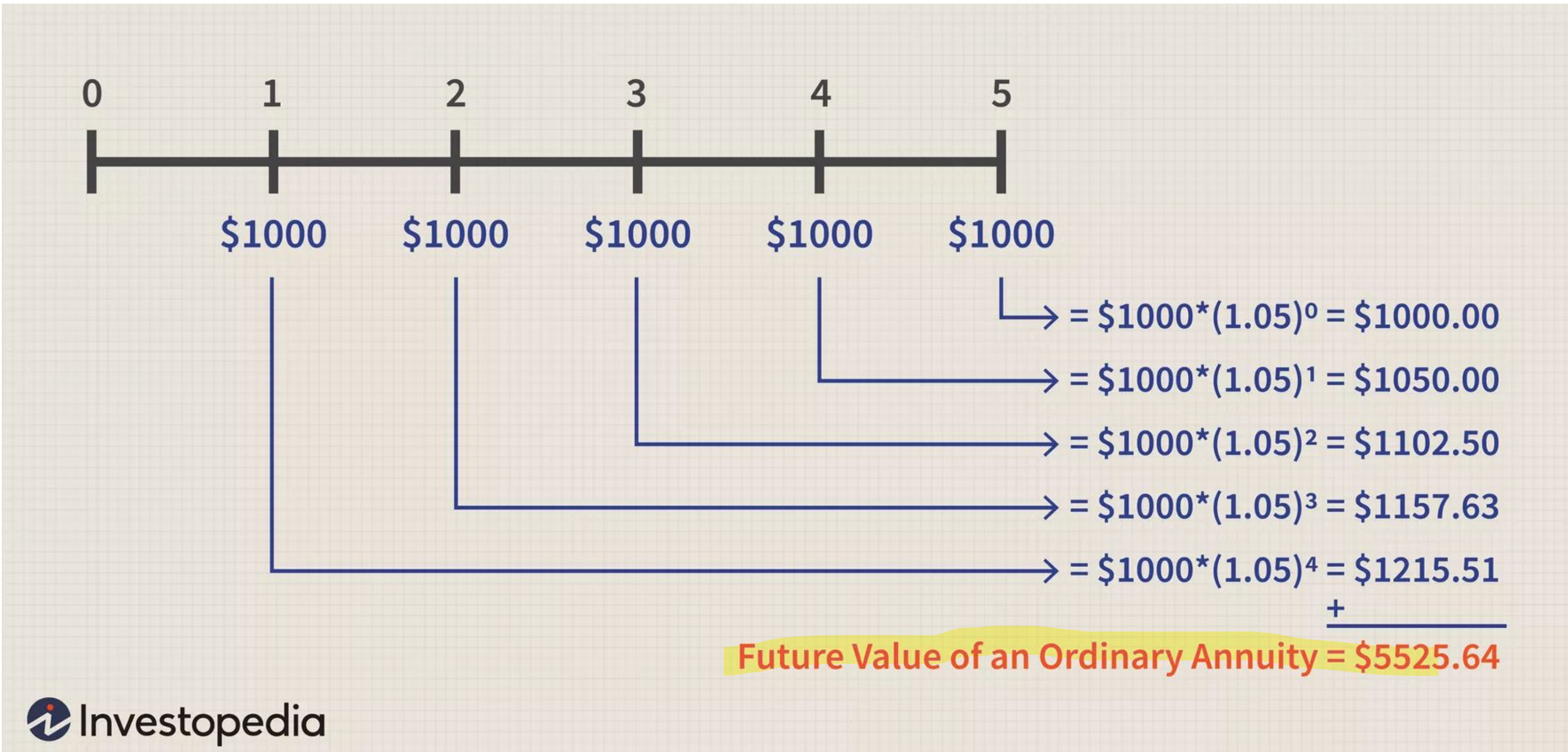
$FVA_n = R\,(FVIFA_{i\%,n})$

$FVA_3 = \$1,000\,(FVIFA_{7\%,3})$

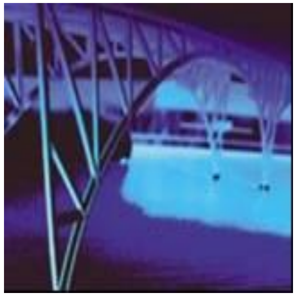
$= \$1,000\,(3.215) = \$3,215$

Period	6%	7%	8%
1	1.000	1.000	1.000
2	2.060	2.070	2.080
3	3.184	**3.215**	3.246
4	4.375	4.440	4.506
5	5.637	5.751	5.867





0
1
2
3
4
5
$1000
$1000
$1000
$1000
$1000
= $1000*(1.05)^0 = $1000.00
= $1000*(1.05)^1 = $1050.00
= $1000*(1.05)^2 = $1102.50
= $1000*(1.05)^3 = $1157.63
= $1000*(1.05)^4 = $1215.51
+
Future Value of an Ordinary Annuity = $5525.64
Investopedia



$$FV_{\text{Ordinary Annuity}} = \boxed{C \times \left[\frac{(1+i)^n - 1}{i}\right]}$$

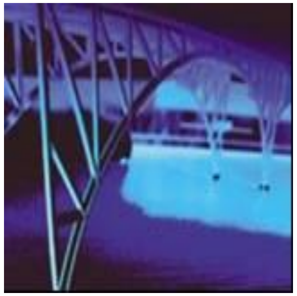
where:

C = cash flow per period

i = interest rate

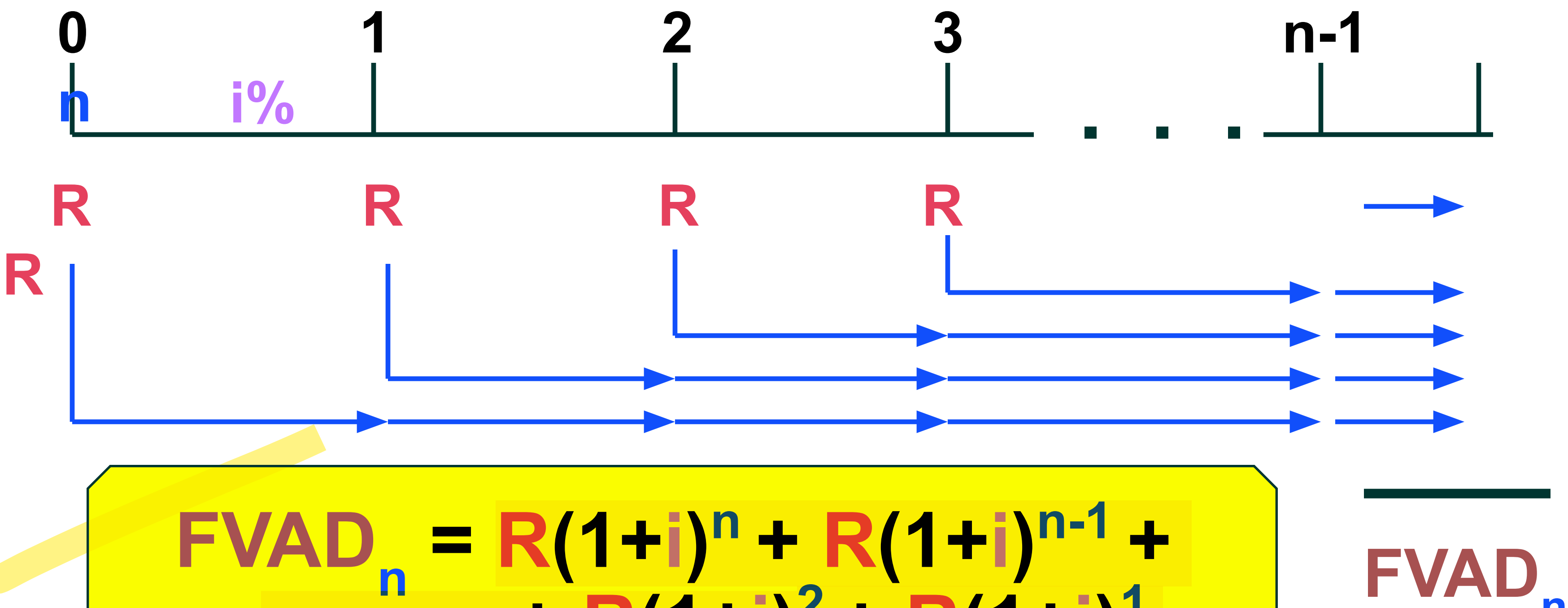
n = number of payments

$$FV_{\text{Annuity Due}} = C \times \left[\frac{(1+i)^n - 1}{i}\right] \times (1+i)$$

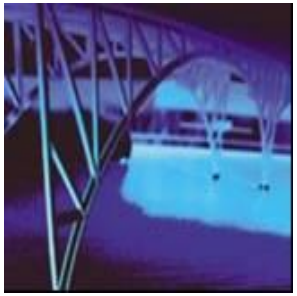


Overview View of an Annuity Due -- FVAD

Cash flows occur at the beginning of the period



$$FVAD_n = R(1+i)^n + R(1+i)^{n-1} + \ldots + R(1+i)^2 + R(1+i)^1$$
$$= FVA_n (1+i)$$



Example of an Annuity Due -- FVAD

Cash flows occur at the beginning of the period

0 1 2 3
4 7%

$1,000 $1,000 $1,000

$1,070

$1,145

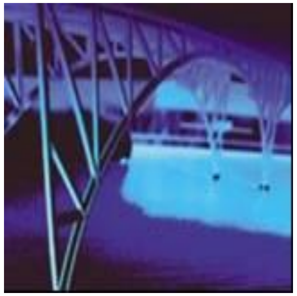
$1,225

$3,440 = $FVAD_3$

$FVAD_3$ = \$1,000$(1.07)^3$ + \$1,000$(1.07)^2$ + \$1,000$(1.07)^1$

= \$1,225 + \$1,145 + \$1,070

= \$3,440



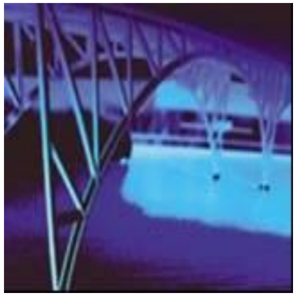
Valuation Using Table III

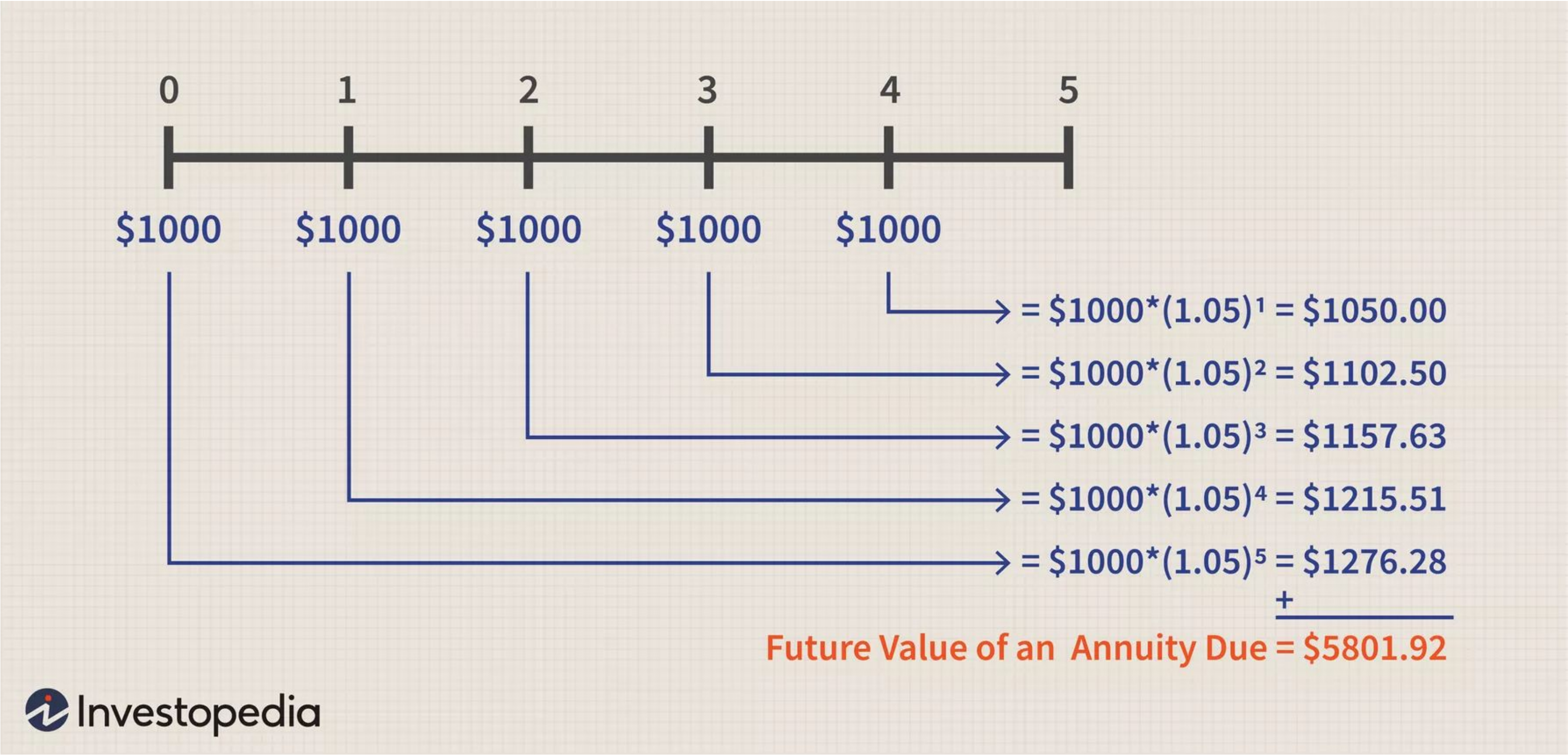
$FVAD_n = R\,(FVIFA_{i\%,n})(1+i)$

$FVAD_3 = \$1{,}000\,(FVIFA_{7\%,3})(1.07)$

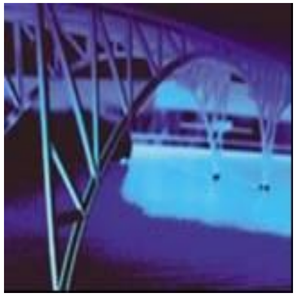
$= \$1{,}000\,(3.215)(1.07) = \$3{,}440$

Period	6%	7%	8%
1	1.000	1.000	1.000
2	2.060	2.070	2.080
3	3.184	**3.215**	3.246
4	4.375	4.440	4.506
5	5.637	5.751	5.867





0
1
2
3
4
5
$1000
$1000
$1000
$1000
$1000
= $1000*(1.05)¹ = $1050.00
= $1000*(1.05)² = $1102.50
= $1000*(1.05)³ = $1157.63
= $1000*(1.05)⁴ = $1215.51
= $1000*(1.05)⁵ = $1276.28
+
Future Value of an Annuity Due = $5801.92
Investopedia



$$\mathrm{FV}_{\text{Ordinary Annuity}} = \mathrm{C} \times \left[\frac{(1+i)^n - 1}{i}\right]$$

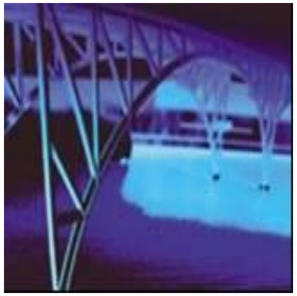
where:

C = cash flow per period

i = interest rate

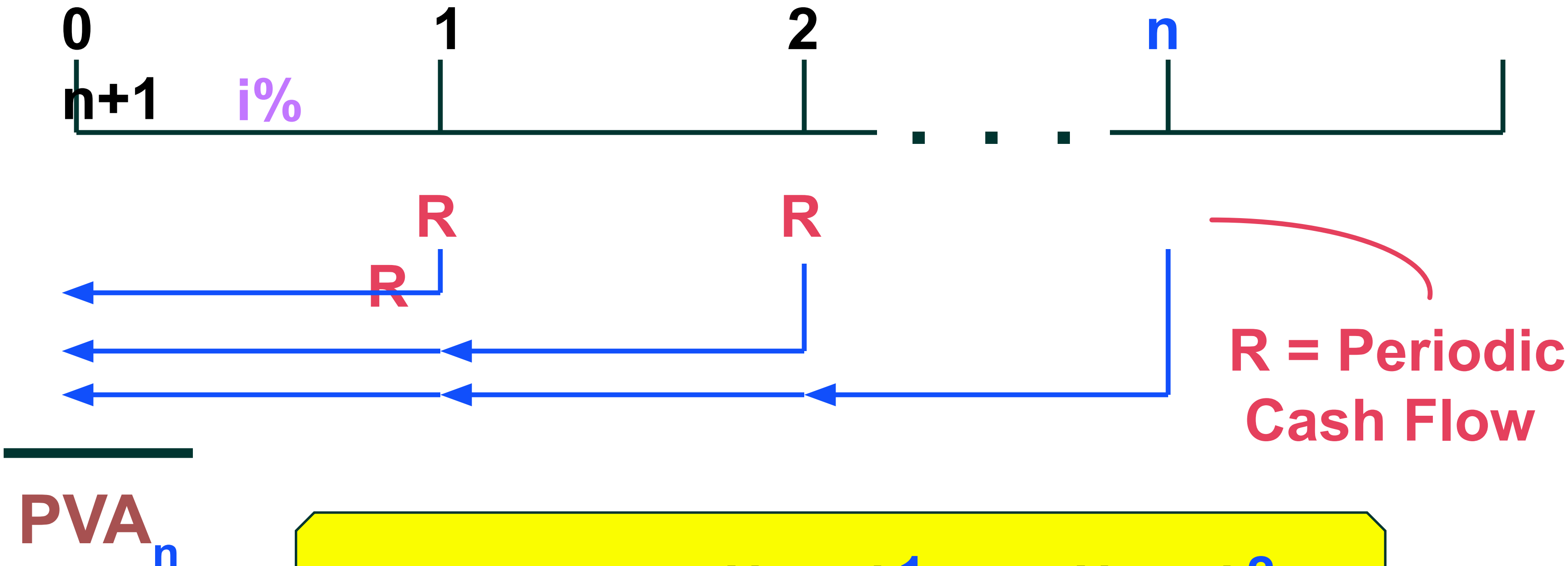
n = number of payments

$$\mathrm{FV}_{\text{Annuity Due}} = \mathrm{C} \times \left[\frac{(1+i)^n - 1}{i}\right] \times (1+i)$$

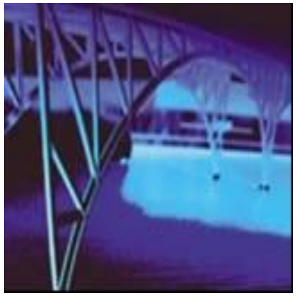


Overview of an Ordinary Annuity -- PVA

Cash flows occur at the end of the period



$$PVA_n = R/(1+i)^1 + R/(1+i)^2 + \ldots + R/(1+i)^n$$



Example of an Ordinary Annuity -- PVA

Cash flows occur at the end of the period

0 1 2 3 4

7%

$1,000 $1,000 $1,000

$934.58

$873.44

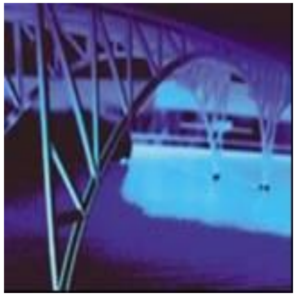
$816.30

$\$2,624.32 = PVA_3$

$PVA_3 = \$1,000/(1.07)^1 + \$1,000/(1.07)^2 + \$1,000/(1.07)^3$

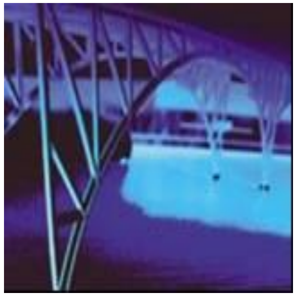
$= \$934.58 + \$873.44 + \$816.30$

$= \$2,624.32$



Hint on Annuity Valuation

The present value of an ordinary annuity can be viewed as occurring at the ***beginning*** of the first cash flow period, whereas the future value of an annuity due can be viewed as occurring at the ***end*** of the first cash flow period.



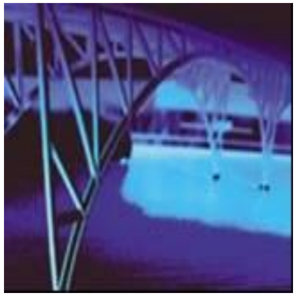
Valuation Using Table IV

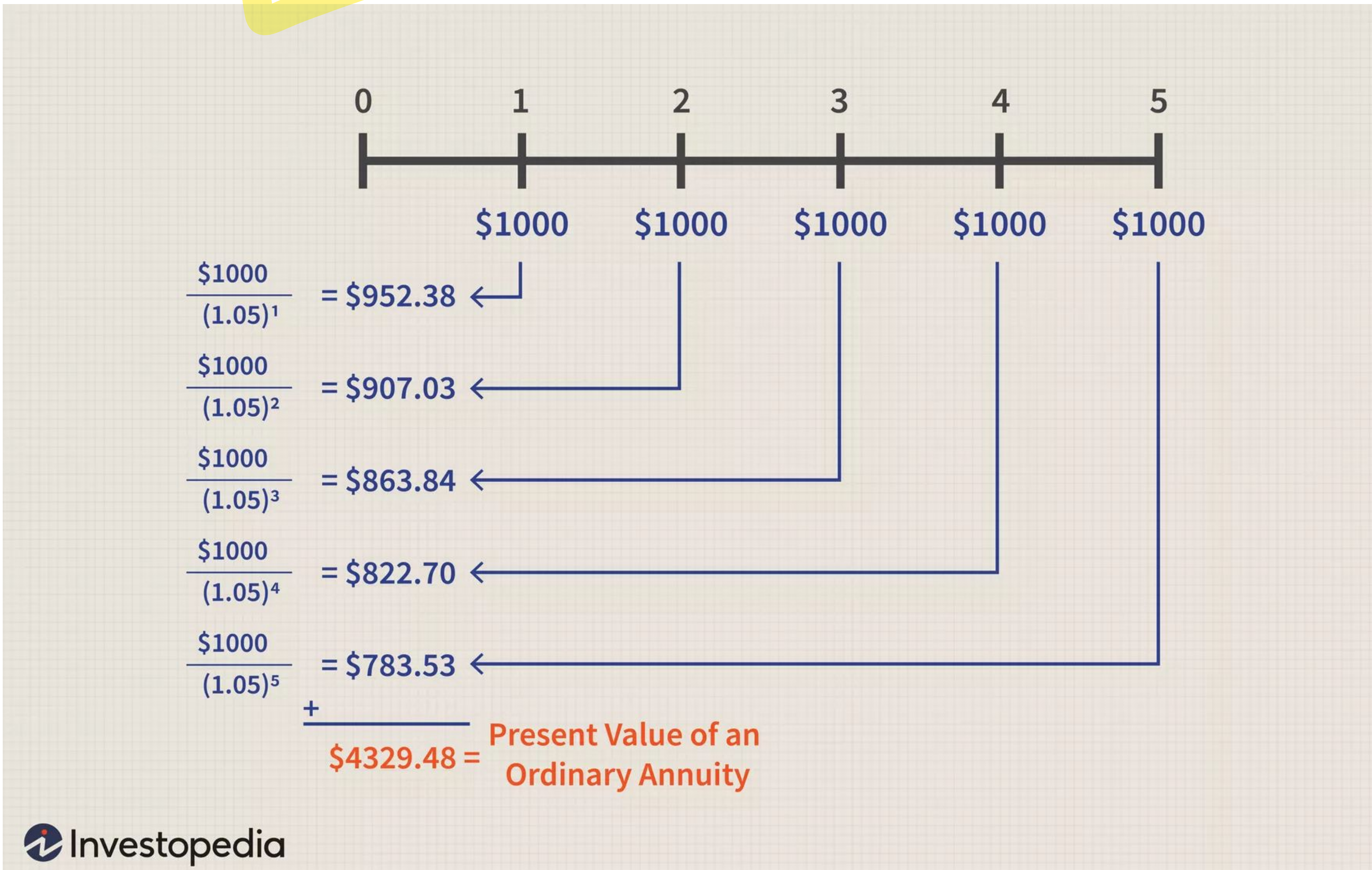
$PVA_n = R\,(PVIFA_{i\%,n})$

$PVA_3 = \$1{,}000\,(PVIFA_{7\%,3})$

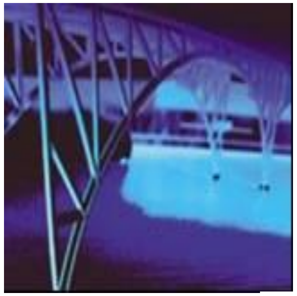
$= \$1{,}000\,(2.624) = \$2{,}624$

Period	6%	7%	8%
1	0.943	0.935	0.926
2	1.833	1.808	1.783
3	2.673	**2.624**	2.577
4	3.465	3.387	3.312
5	4.212	4.100	3.993



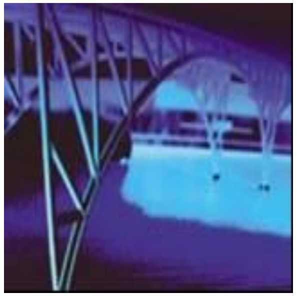


0
1
2
3
4
5
$1000
$1000
$1000
$1000
$1000
$1000 / (1.05)^1 = $952.38
$1000 / (1.05)^2 = $907.03
$1000 / (1.05)^3 = $863.84
$1000 / (1.05)^4 = $822.70
$1000 / (1.05)^5 = $783.53
+
$4329.48 = Present Value of an Ordinary Annuity
Investopedia



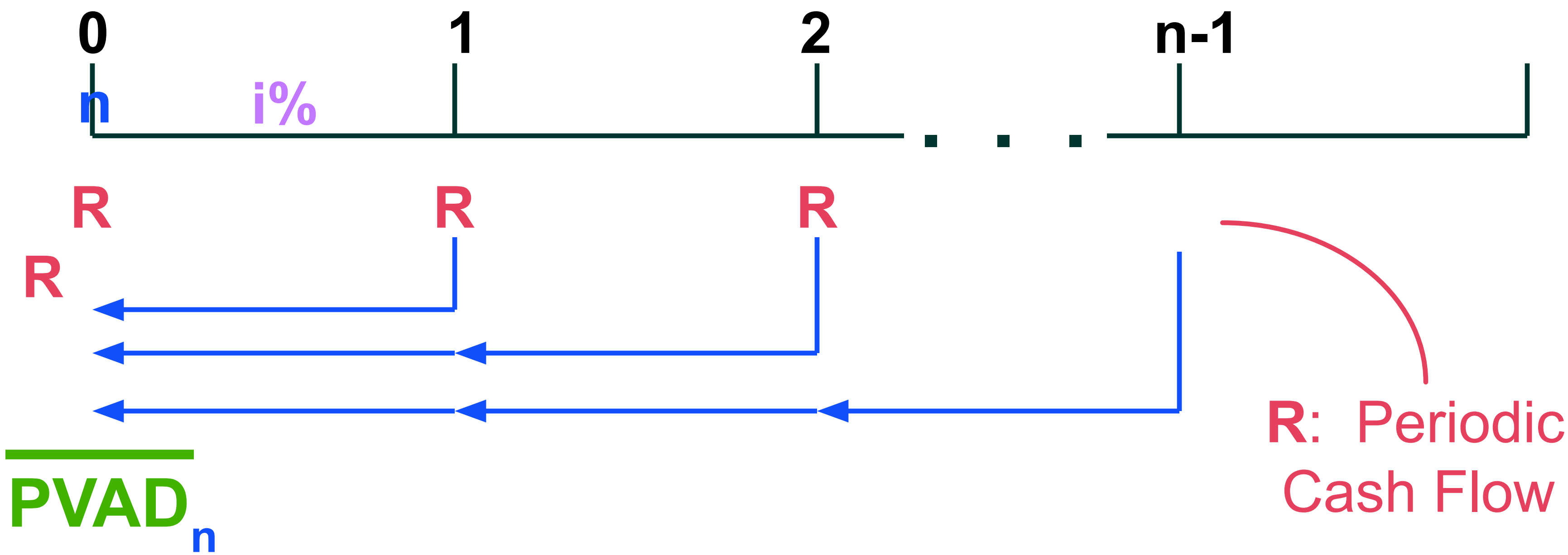
$$PV_{\text{Ordinary Annuity}} = C \times \left[\frac{1-(1+i)^{-n}}{i}\right]$$

$$PV_{\text{Annuity Due}} = C \times \left[\frac{1-(1+i)^{-n}}{i}\right] \times (1+i)$$

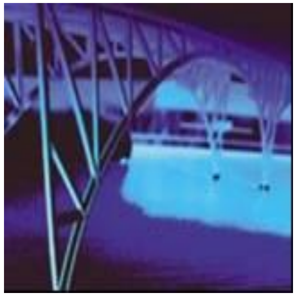


Overview of an Annuity Due -- PVAD

Cash flows occur at the beginning of the period



$$\mathrm{PVAD}_n = R/(1+i)^0 + R/(1+i)^1 + \ldots + R/(1+i)^{n-1}$$
$$= \mathrm{PVA}_n\,(1+i)$$



Example of an Annuity Due -- PVAD

Cash flows occur at the beginning of the period

0 1 2 3
4 7%

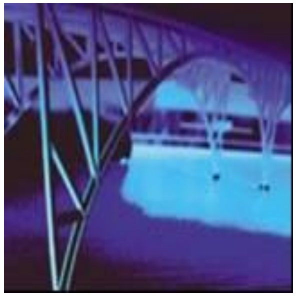
$1,000.00 $1,000 $1,000

$ 934.58

$ 873.44

$\$2{,}808.02 = PVAD_n$

$$PVAD_n = \$1{,}000/(1.07)^0 + \$1{,}000/(1.07)^1 + \$1{,}000/(1.07)^2 = \$2{,}808.02$$



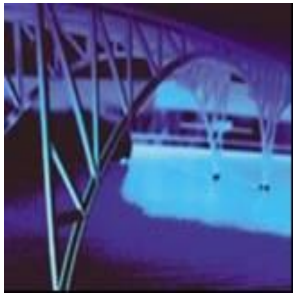
Valuation Using Table IV

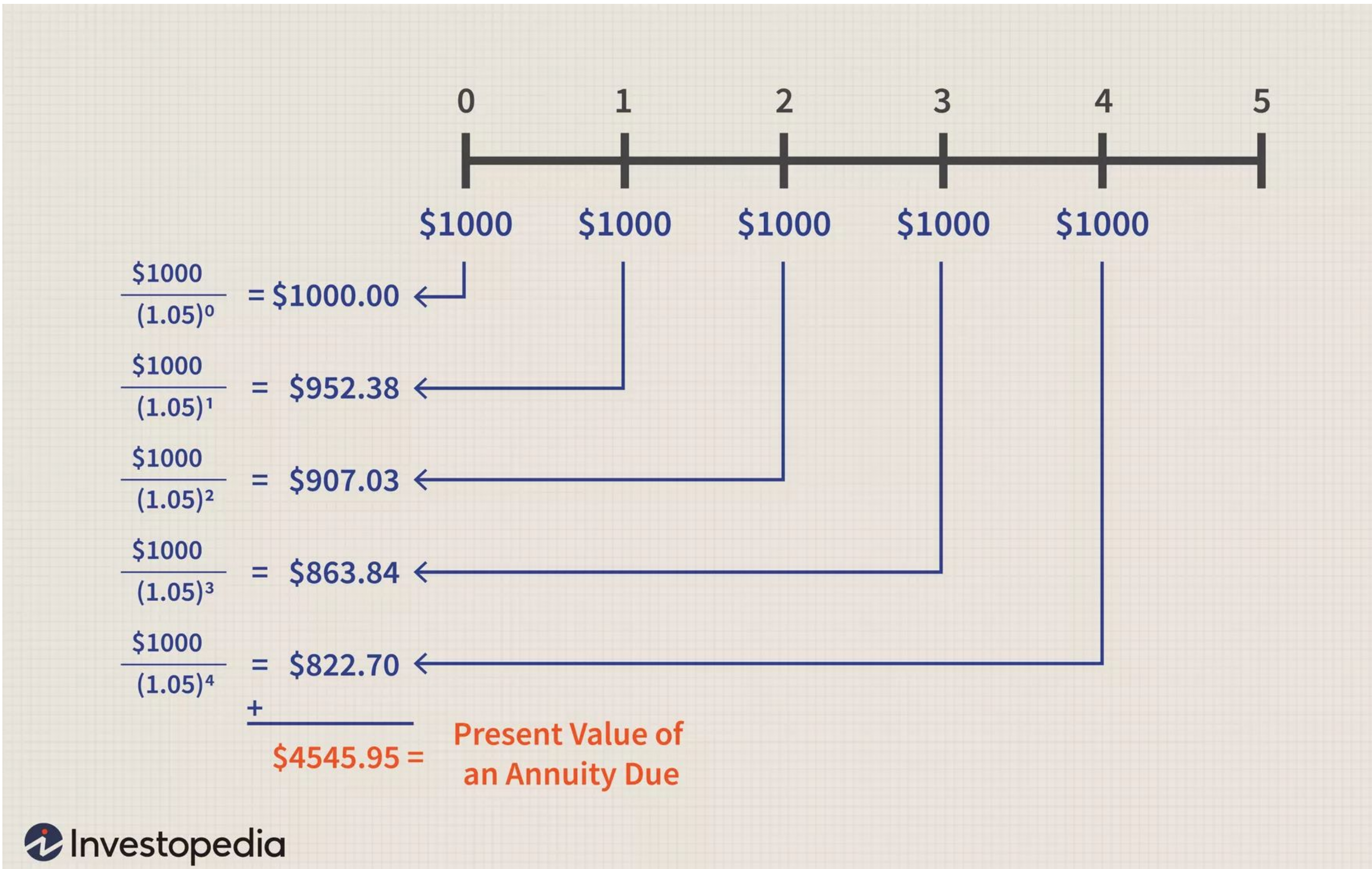
$$PVAD_n = R\,(PVIFA_{i\%,n})(1+i)$$

$$PVAD_3 = \$1,000\,(PVIFA_{7\%,3})(1.07)$$

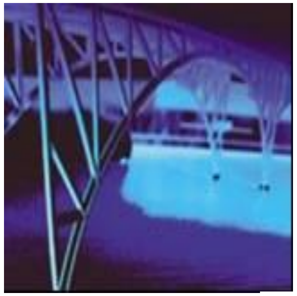
$$= \$1,000\,(2.624)(1.07) = \$2,808$$

Period	6%	7%	8%
1	0.943	0.935	0.926
2	1.833	1.808	1.783
3	2.673	**2.624**	2.577
4	3.465	3.387	3.312
5	4.212	4.100	3.993



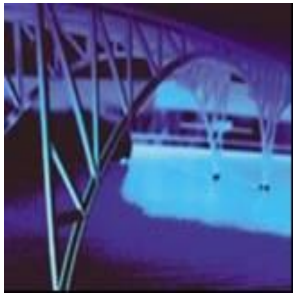


0
1
2
3
4
5
$1000
$1000
$1000
$1000
$1000
$1000 / (1.05)^0 = $1000.00
$1000 / (1.05)^1 = $952.38
$1000 / (1.05)^2 = $907.03
$1000 / (1.05)^3 = $863.84
$1000 / (1.05)^4 = $822.70
+
$4545.95 = Present Value of an Annuity Due
Investopedia



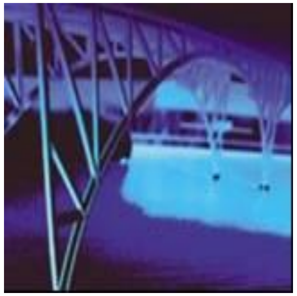
$$PV_{\text{Ordinary Annuity}} = C \times \left[\frac{1-(1+i)^{-n}}{i}\right]$$

$$PV_{\text{Annuity Due}} = C \times \left[\frac{1-(1+i)^{-n}}{i}\right] \times (1+i)$$



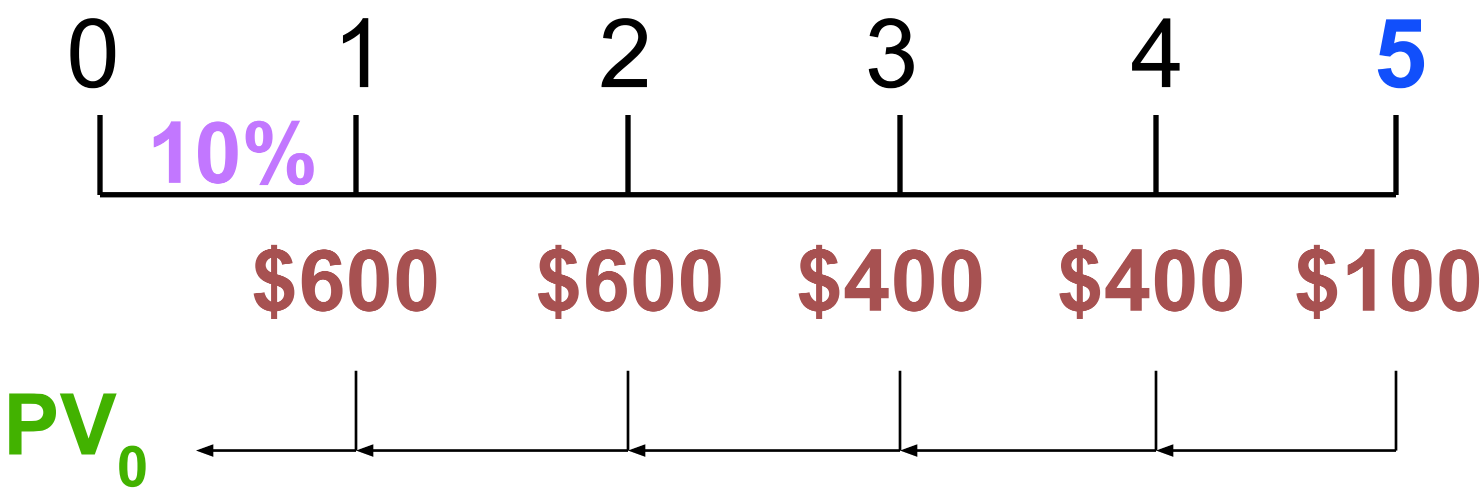
Steps to Solve Time Value of Money Problems

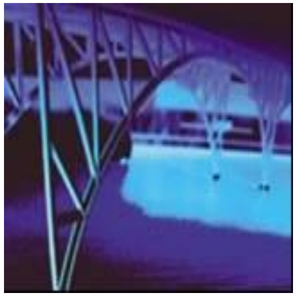
1. Read problem thoroughly
2. Create a time line
3. Put cash flows and arrows on time line
4. Determine if it is a PV or FV problem
5. Determine if solution involves a single CF, annuity stream(s), or mixed flow
6. Solve the problem
7. Check with financial calculator (optional)



Mixed Flows Example

Julie Miller will receive the set of cash flows below. What is the Present Value at a discount rate of 10%.

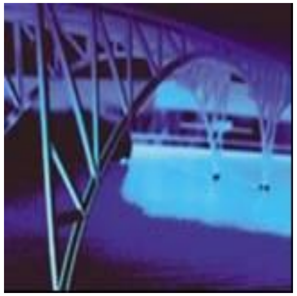




How to Solve?

1. Solve a "*piece-at-a-time*" by discounting each *piece* back to t=0.

2. Solve a "*group-at-a-time*" by first breaking problem into groups of annuity streams and any single cash flow groups. Then discount each *group* back to t=0.



"Piece-At-A-Time"

0 1 2 3 4 5

10%

$600 $600 $400 $400 $100

$545.45

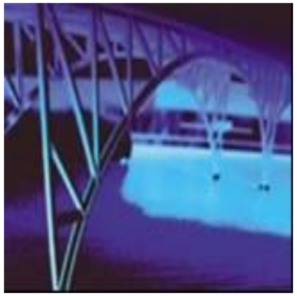
$495.87

$300.53

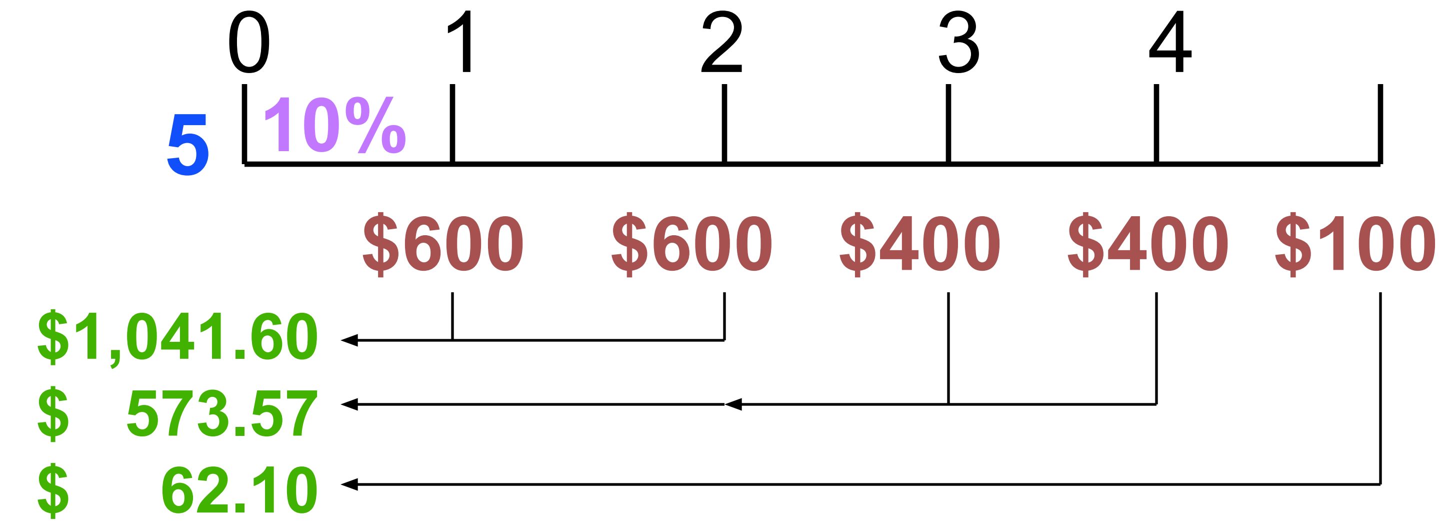
$273.21

$ 62.09

$1677.15 = PV_0 of the Mixed Flow



"Group-At-A-Time" (#1)

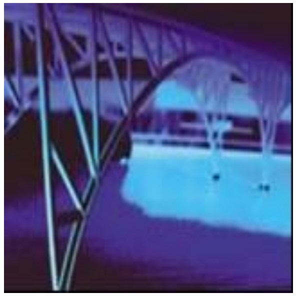


$1,677.27 = PV_0 **of Mixed Flow [Using Tables]**

$$\$600(PVIFA_{10\%,2}) = \$600(1.736) = \$1,041.60$$

$$\$400(PVIFA_{10\%,2})(PVIF_{10\%,2}) = \$400(1.736)(0.826) = \$573.57$$

$$\$100\,(PVIF_{10\%,5}) = \$100\,(0.621) = \$62.10$$



"Group-At-A-Time" (#2)

0 1 2 3 4

$400 $400 $400 $400

$1,268.00

0 1 2

Plus

$200 $200

$347.20

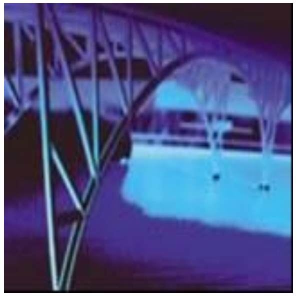
0 1 2 3 4 5

Plus

$100

$62.10

PV_0 equals $1677.30.



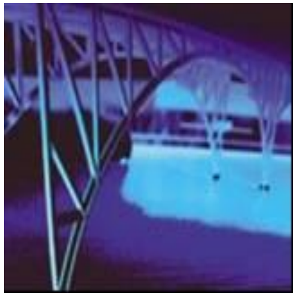
Frequency of Compounding

General Formula:

$$FV_n = PV_0(1 + [i/m])^{mn}$$

n: Number of Years m:
Compounding Periods per Year i:
Annual Interest Rate $FV_{n,m}$: FV at
the end of Year n

PV_0: PV of the Cash Flow today

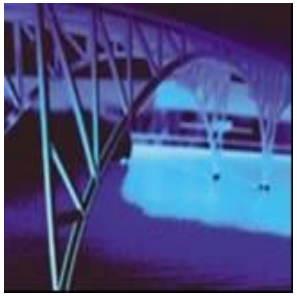


Impact of Frequency

Julie Miller has $1,000 to invest for 2 Years at an annual interest rate of 12%.

Annual $FV_2 = 1{,}000(1+[.12/1])^{(1)(2)} = 1{,}254.40$

Semi $FV_2 = 1{,}000(1+[.12/2])^{(2)(2)} = 1{,}262.48$



Impact of Frequency

Qrtly $FV_2 = 1{,}000(1+[.12/4])^{(4)(2)}$

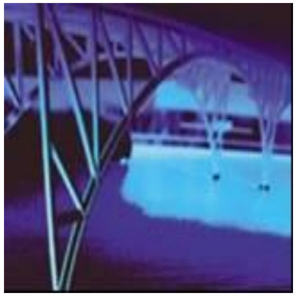
$= 1{,}266.77$

Monthly $FV_2 = 1{,}000(1+[.12/12])^{(12)(2)}$

$= 1{,}269.73$

Daily $FV_2 = 1{,}000(1+[.12/365])^{(365)(2)}$

$= 1{,}271.20$

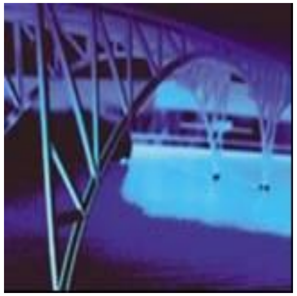


Effective Annual Interest Rate

Effective Annual Interest Rate

The actual rate of interest earned (paid) after adjusting the *nominal rate* for factors such as the number of compounding periods per year.

$$(1 + [\, i / m \,])^m - 1$$

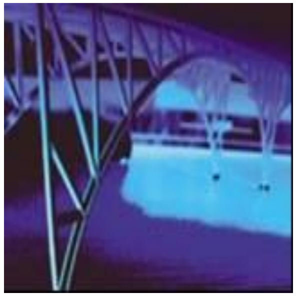


BWs Effective Annual Interest Rate

Basket Wonders (BW) has a $1,000 CD at the bank. The interest rate is 6% compounded quarterly for 1 year. What is the Effective Annual Interest Rate (EAR)?

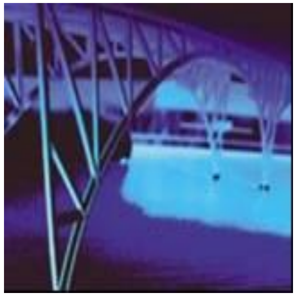
$$EAR = (1 + 6\% / 4)^4 - 1 =$$

$$1.0614 - 1 = .0614 \text{ or } 6.14\%!$$



Steps to Amortizing a Loan

1. Calculate the payment per period.
2. Determine the interest in Period t.
   (Loan Balance at t-1) x (i% / m)
3. Compute principal payment in Period t.
   (Payment - Interest from Step 2)
4. Determine ending balance in Period t.
   (Balance - principal payment from Step 3)
5. Start again at Step 2 and repeat.



Amortizing a Loan Example

Julie Miller is borrowing $10,000 at a compound annual interest rate of 12%. Amortize the loan if annual payments are made for 5 years.

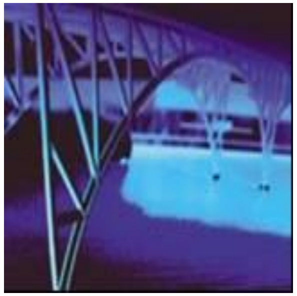
Step 1: Payment

$$PV_0 = R\ (PVIFA_{i\%,n})$$

$$\$10,000 = R\ (PVIFA_{12\%,5})$$

$$\$10,000 = R\ (3.605)$$

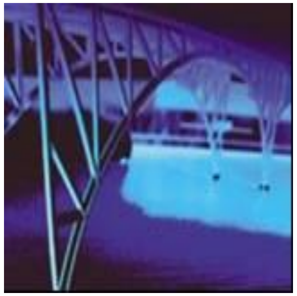
$$R = \$10,000 / 3.605 = \$2,774$$



Amortizing a Loan Example

End of Year	Payment	Interest	Principal	Ending Balance
0	---	---	---	$10,000
1	$2,774	10000 x (.12/1) + $1,200	2774-1200 $1,574	8,426 7226
2	2,774	1,011	1,763	6,663
3	2,774	800	1,974	4,689
4	2,774	563	2,211	2,478
5	2,775	297	2,478	0
	$13,871	$3,871	$10,000	

[Last Payment Slightly Higher Due to Rounding]



Usefulness of Amortization

1. Determine Interest Expense -- Interest expenses may reduce taxable income of the firm.
2. Calculate Debt Outstanding -- The quantity of outstanding debt may be used in financing the day-to-day activities of the firm.