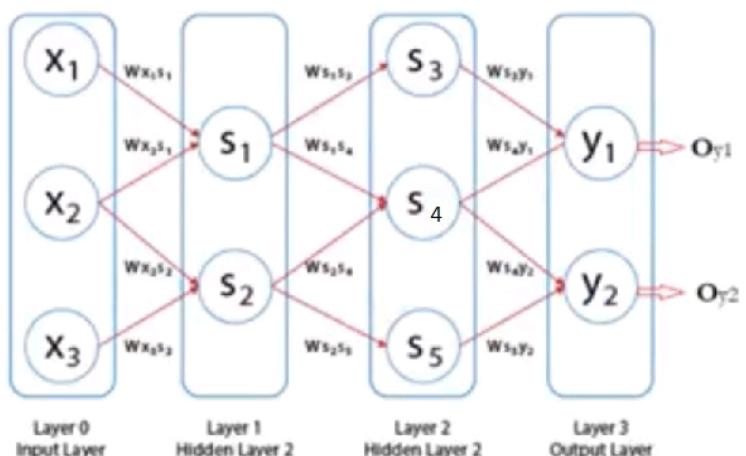


# Task

For the given Network, perform feed forward and back propagation and find all the appropriate values.

Term	Value	Term	Value	Term	Value
Input x1	0.7	Input x2	0.15	Input x3	0.61
Wx1s1	0.25	Wx2s5	0.85	θs3	0.5
Wx2s1	0.33	Ws2y1	0.11	θs4	0.19
Wx2s2	0.12	Ws4y1	0.51	θs5	0.33
Wx3s2	-0.7	Ws4y2	-0.35	θy1	0.8
Wx1s3	-0.36	Ws5y2	-0.42	θy2	0.09
Ws1s4	0.7	θs1	0.7	Ty1	1
Ws2s4	0.22	θs2	0.2	Ty2	0



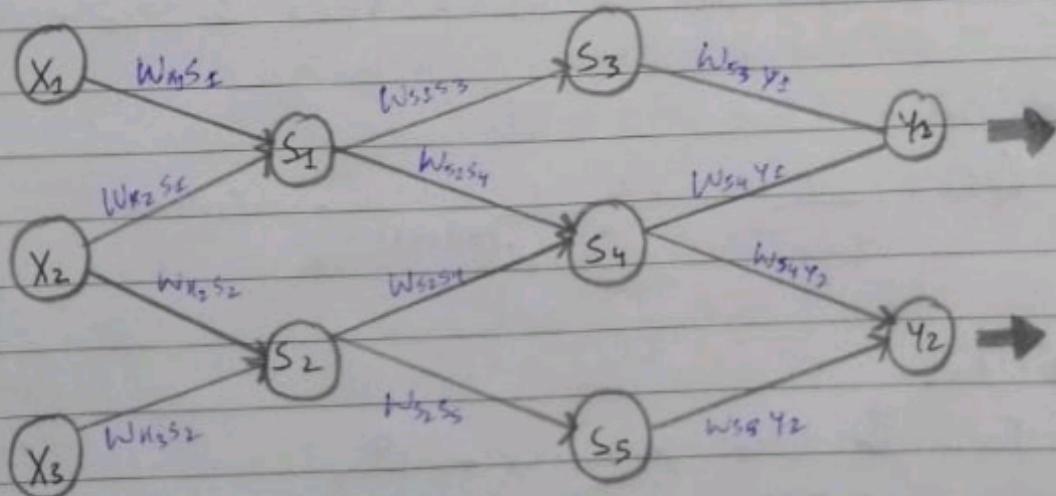
NAME: SHAIKH HARIS ALI

SEAT NO: B18158049

### TASK:

For the given Network, perform feed forward and back propagation and find all the appropriate values.

TERM	VALUE	TERM	VALUE	TERM	VALUE
Input $x_1$	0.7	Input $x_2$	0.15	Input $x_3$	0.61
$w_{x1s1}$	0.25	$w_{x2s5}$	0.85	$o_{s3}$	0.5
$w_{x2s1}$	0.33	$w_{x2y1}$	0.11	$o_{s4}$	0.19
$w_{x2s2}$	0.12	$w_{s4y1}$	0.51	$o_{s5}$	0.33
$w_{x3s2}$	-0.7	$w_{s4y2}$	-0.35	$o_{y1}$	0.8
$w_{x1s3}$	-0.36	$w_{s5y2}$	-0.42	$o_{y2}$	0.09
$w_{s1s4}$	0.7	$o_{s1}$	0.7	$T_{y1}$	?
$w_{s2s4}$	0.22	$o_{s2}$	0.2	$T_{y2}$	0



## OUTPUT LAYER ERROR:

$$\begin{aligned} \text{Err } y_1 &= O_{y_1} (1 - O_{y_1}) (T_{y_1} - O_{y_1}) \\ &= 0.771 (1 - 0.771) (1 - 0.771) \\ &\approx 0.04 \end{aligned}$$

$$\begin{aligned} \text{Err } y_2 &= O_{y_2} (1 - O_{y_2}) (T_{y_2} - O_{y_2}) \\ &= 0.394 (1 - 0.394) (0 - 0.394) \\ &= -0.094 \end{aligned}$$

## HIDDEN LAYER ERROR:

$$\begin{aligned} \text{Err } s_3 &= O_{s_3} (1 - O_{s_3}) [\text{Err } y_1 \times W_{31} y_1] \\ &\quad \cancel{\times O_{s_3}} (1 - 0.56) [0.04 \times 0.11] \\ &= 0.56 (1 - 0.56) [0.04 \times 0.11] \\ &\approx 0.001 \end{aligned}$$

$$\begin{aligned} \text{Err } s_4 &= O_{s_4} (1 - O_{s_4}) [\text{Err } y_2 \times W_{41} y_1] \\ &= O_{s_4} (1 - O_{s_4}) (\text{Err } y_2 W_{41} y_1 + \text{Err } y_1 W_{42} y_2) \\ &= 0.688 (1 - 0.688) (0.04 \times 0.51 + (-0.094) \times -0.35) \\ &\approx 0.01 \end{aligned}$$

$$\begin{aligned} \text{Err } s_5 &= O_{s_5} (1 - O_{s_5}) [\text{Err } y_2 \times W_{51} y_2] \\ &= 0.671 (1 - 0.671) (-0.094 \times -0.42) \\ &\approx 0.0087 \end{aligned}$$

$$\begin{aligned} \text{Err } s_6 &= O_{s_6} (1 - O_{s_6}) [\text{Err } s_3 \times W_{61} s_3 + \text{Err } s_4 \times W_{62} s_4] \\ &= 0.716 (1 - 0.716) (0.001 \times -0.36 + 0.01 \times 0.7) \\ &\approx 0.0013 \end{aligned}$$

FOR  $O_{S_3}$ :

$$O_{S_3} = \frac{1}{1+e^{-I_{S_3}}} = \frac{1}{1+e^{-(0.242)}} = 0.56$$

FOR  $I_{S_4}$ :

$$I_{S_4} = O_{S_1} \times W_{S_1 S_4} + O_{S_2} \times W_{S_2 S_4} + O_{S_3} \\ = 0.76 \times 0.7 + 0.448 \times 0.22 + 0.19 = 0.78976$$

FOR  $O_{S_4}$ :

$$O_{S_4} = \frac{1}{1+e^{-I_{S_4}}} = \frac{1}{1+e^{-(0.78976)}} = 0.688$$

FOR  $I_{S_5}$ :

$$I_{S_5} = O_{S_2} \times W_{S_2 S_5} + O_{S_3} = (0.2 \times 0.85) + 0.33 \\ = 0.711$$

FOR  $O_{S_5}$ :

$$O_{S_5} = \frac{1}{1+e^{-I_{S_5}}} = \frac{1}{1+e^{-(0.711)}} = 0.671$$

FOR  $I_{Y_2}$

$$I_{Y_2} = O_{S_3} \times W_{S_3 Y_2} + O_{S_4} \times W_{S_4 Y_2} + O_{Y_2} \\ = 0.56 \times 0.11 + 0.688 \times 0.51 + 0.8 = 1.212$$

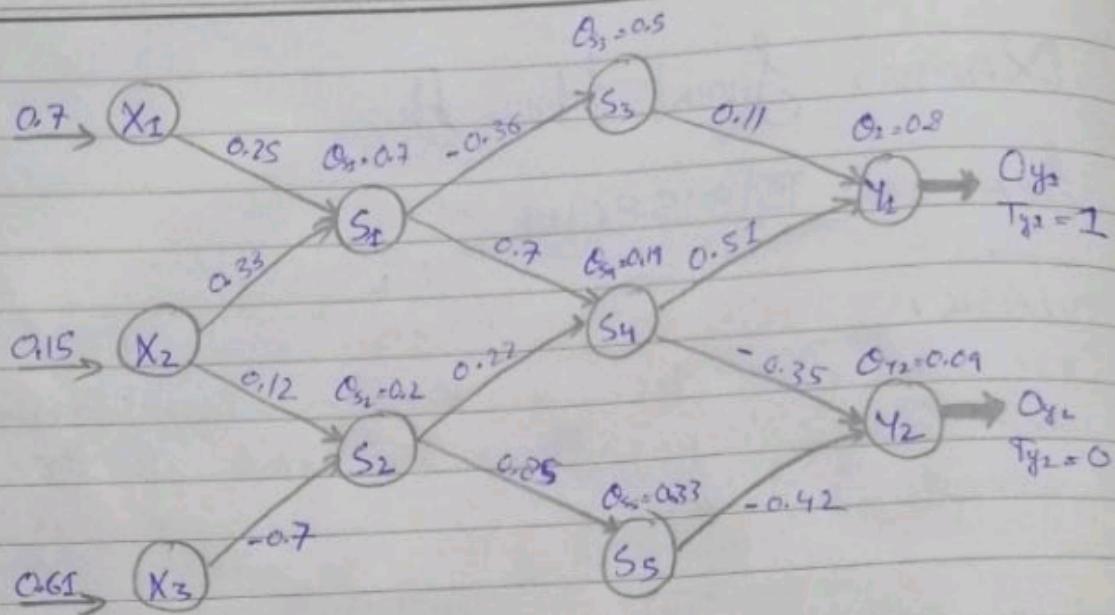
FOR  $O_{Y_2}$ :

$$O_{Y_2} = \frac{1}{1+e^{-I_{Y_2}}} = \frac{1}{1+e^{-(1.212)}} = 0.394$$

FOR  ~~$I_{Y_2}$~~   $I_{Y_2} : O_{S_4} \times W_{S_4 Y_2} + O_{S_5} \times W_{S_5 Y_2} + O_{Y_2}$

$$I_{Y_2} = 0.688 \times (-0.35) + 0.671 \times (-0.42) + 0.09 = -0.43202$$

FOR  $O_{Y_2} : O_{Y_2} = \frac{1}{1+e^{-I_{Y_2}}} = 0.394$



FOR  $I_{S_1} =$

$$I_{S_1} = T_{X_1} \times W_{X_1 S_1} + T_{X_2} \times W_{X_2 S_1} + O_{S_1}$$

$$= (0.7 \times 0.25) + (0.15 \times 0.33) + 0.7 = 0.9245$$

FOR  $O_{S_1} :$

$$O_{S_1} = \frac{1}{1+e^{-I_{S_1}}} = \frac{1}{1+e^{-(0.9245)}} = 0.716$$

FOR  $I_{S_2} :$

$$I_{S_2} = T_{X_2} \times W_{X_2 S_2} + T_{X_3} \times W_{X_3 S_2} + O_{S_2}$$

$$= 0.15 \times 0.12 + 0.61 \times (-0.7) + 0.2 = -0.209$$

FOR  $O_{S_2} :$

$$O_{S_2} = \frac{1}{1+e^{-I_{S_2}}} = \frac{1}{1+e^{-(0.209)}} = 0.448$$

FOR  $I_3 :$

$$I_3 = O_{S_1} \times W_{S_1 S_3} + O_{S_3}$$

$$I_3 = 0.716 \times (-0.36) + 0.5 = 0.242$$

## FEED FORWARD

$$\begin{aligned}
 I_{S1} &= I_{x_1} \times W_{x_1 S1} + I_{x_2} \times W_{x_2 S1} + \theta_{S1} \\
 &= 0.7 \times 0.25 + 0.15 \times 0.33 + 0.7 \\
 &= 0.9245
 \end{aligned}$$

$$O_{S1} = \frac{1}{1 + e^{-I_{S1}}} = 0.716$$

$$\begin{aligned}
 I_{S2} &= I_{x_2} \times W_{x_2 S2} + I_{x_3} \times W_{x_3 S2} + \theta_{S2} \\
 &= 0.15 \times 0.12 + 0.61 \times -0.7 + 0.2 \\
 &= -0.209
 \end{aligned}$$

$$O_{S2} = 0.448$$

$$\begin{aligned}
 I_{S3} &= O_{S1} \times W_{S1 S3} + \theta_{S3} \\
 &= 0.716 \times -0.36 + 0.5 \\
 &= 0.242
 \end{aligned}$$

$$O_{S3} = \frac{1}{1 + e^{-I_{S3}}} = 0.56$$

$$\begin{aligned}
 I_{S4} &= O_{S1} \times W_{S1 S4} + O_{S2} \times W_{S2 S4} + \theta_{S4} \\
 &= 0.716 \times 0.7 + 0.448 \times 0.22 + 0.19 \\
 &= 0.78976
 \end{aligned}$$

$$O_{S4} = \frac{1}{1 + e^{-I_{S4}}} = 0.688$$

$$\begin{aligned}
 I_{S5} &= O_{S2} \times W_{S2 S5} + \theta_{S5} \\
 &= 0.448 \times 0.85 + 0.33 \\
 &= 0.711
 \end{aligned}$$

$$O_{S5} = \frac{1}{1 + e^{-I_{S5}}} = 0.671$$

# ASSIGNMENT # 04

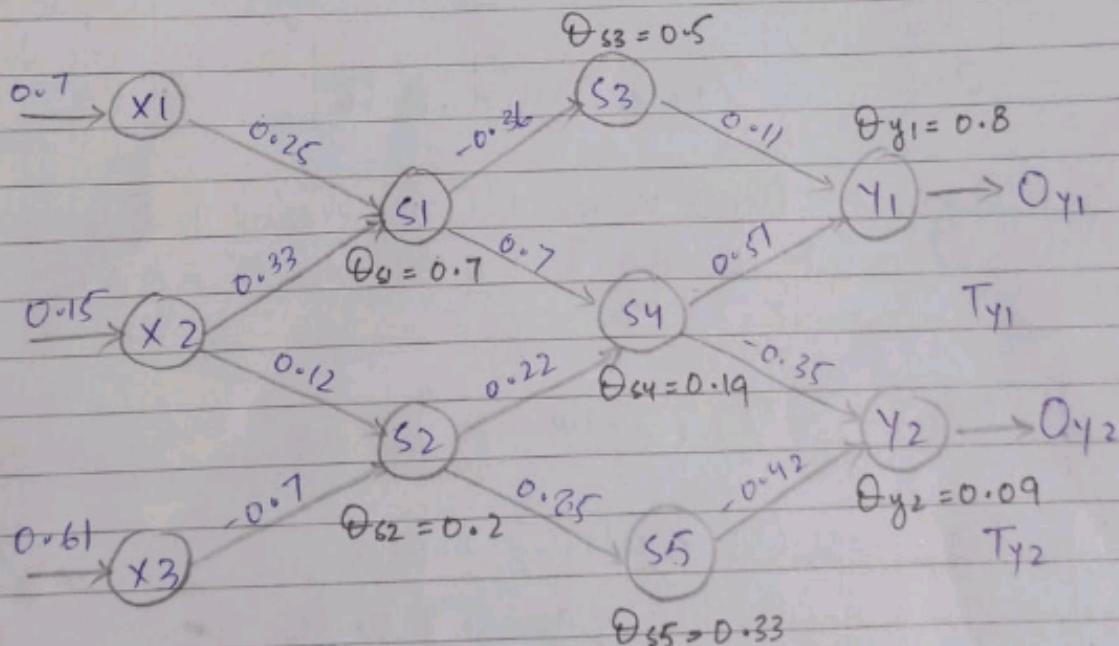
Date: 22-Nov-2021

y<sub>2</sub>

SUBMITTED By: BISMA KHAN (BL8150008)

QUESTION 1: For the given network, perform feed forward and back propagation and find all the appropriate values.

Term	Value	Term	Value	Term	Value
Input x <sub>1</sub>	0.7	Input x <sub>2</sub>	0.15	Input x <sub>3</sub>	0.61
w <sub>x1s1</sub>	0.25	w <sub>s2s5</sub>	0.85	θ <sub>s3</sub>	0.5
w <sub>x2s1</sub>	0.33	w <sub>s3y1</sub>	0.11	θ <sub>s4</sub>	0.19
w <sub>x2s2</sub>	0.12	w <sub>s4y1</sub>	0.51	θ <sub>s5</sub>	0.33
w <sub>x3s2</sub>	-0.7	w <sub>s4y2</sub>	-0.35	θ <sub>y1</sub>	0.8
w <sub>s1s3</sub>	-0.36	w <sub>s5y2</sub>	-0.42	θ <sub>y2</sub>	0.09
w <sub>s1s4</sub>	0.7	θ <sub>s1</sub>	0.7	T <sub>y1</sub>	1
w <sub>s2s4</sub>	0.22	θ <sub>s2</sub>	0.2	T <sub>y2</sub>	0



Layer 0 Input layer	Layer 1 Hidden Layer 2	Layer 2 Hidden Layer 2	Layer 3 Output layer
------------------------	------------------------------	------------------------------	-------------------------

$$\begin{aligned}
 E_{ws5} &= 0.55 (1 - 0.55) [E_{wz2} W_{c5z2}] \\
 &= 0.671 (1 - 0.671) (-0.094 \times -0.42) \\
 &= 0.0087
 \end{aligned}$$

$$\begin{aligned}
 E_{ws1} &= 0.51 (1 - 0.51) [E_{ws3} W_{c1s3} + E_{wz4} W_{c1s4}] \\
 &= 0.716 (1 - 0.716) (0.001 \times -0.36 + 0.01 \times 0.7) \\
 &= 0.00135
 \end{aligned}$$

$$\begin{aligned}
 E_{ws2} &= 0.52 (1 - 0.52) [E_{wz4} W_{c2s4} + E_{ws5} W_{c2s5}] \\
 &= 0.448 (1 - 0.448) (0.01 \times 0.22 + 0.0087 \times 0.85) \\
 &= 0.00237
 \end{aligned}$$

Here, the errors at output layer and hidden layer is very minimum, so we can assume that network follows vanishing gradient problem and weights and bias will not change, we can calculate update bias and weights to prove this problem.

### UPDATE BIAS

$$\begin{aligned}
 \text{Formula : } \theta_j &= \theta_j + \Delta \theta_j \\
 &= \theta_j + l E_{wj}
 \end{aligned}$$

We assume that learning rate = 0.1, because error is very minimum / small.

So,

$$l = 0.1$$

$$\begin{aligned}
 \theta_{s1} &= \theta_{s1} + l E_{ws1} \\
 &= 0.7 + 0.1 (0.00135) \\
 &= 0.7
 \end{aligned}$$

$$\begin{aligned}\theta_{s2} &= \theta_{s2} + l \epsilon_{ws2} \\ &= 0.2 + 0.1(0.00237) \\ &= 0.2\end{aligned}$$

$$\begin{aligned}\theta_{s3} &= \theta_{s3} + l \epsilon_{ws3} \\ &= 0.5 + 0.1(0.001) \\ &= 0.5\end{aligned}$$

$$\begin{aligned}\theta_{s4} &= \theta_{s4} + l \epsilon_{ws4} \\ &= 0.19 + 0.1(0.01) \\ &= 0.19\end{aligned}$$

$$\begin{aligned}\theta_{s5} &= \theta_{s5} + l \epsilon_{ws5} \\ &= 0.33 + 0.1(0.0087) \\ &= 0.33\end{aligned}$$

$$\begin{aligned}\theta_{y1} &= \theta_{y1} + l \epsilon_{wy1} \\ &= 0.8 + 0.1(0.04) \\ &= 0.8\end{aligned}$$

$$\begin{aligned}\theta_{y2} &= \theta_{y2} + l \epsilon_{wy2} \\ &= 0.09 + 0.1(-0.094) \\ &= 0.0806\end{aligned}$$

### UPDATE WEIGHTS

$$\text{Formula : } w_{ij} = w_{ij} + \Delta w_{ij} \\ = w_{ij} + l \epsilon_{wj} o_i$$

$$\begin{aligned}w_{x1s1} &= w_{x1s1} + l \epsilon_{ws1} o_{x1} \\ &= 0.25 + 0.1(0.00135)(0.7) \\ &= 0.25\end{aligned}$$

$$\begin{aligned}I_{y_1} &= O_{s3} \times W_{s3y_1} + O_{s4} \times W_{s4y_1} + O_{y_1} \\&= 0.56 \times 0.11 + 0.688 \times 0.51 + 0.8 \\&= 1.212.\end{aligned}$$

$$O_{y_1} = \frac{1}{1 + e^{-I_{y_1}}} = 0.771$$

$$\begin{aligned}I_{y_2} &= O_{s4} \times W_{s4y_2} + O_{s5} \times W_{s5y_2} + O_{y_2} \\&= 0.688 \times -0.35 + 0.671 \times -0.42 + 0.09 \\&= -0.43262.\end{aligned}$$

$$O_{y_2} = \frac{1}{1 + e^{-I_{y_2}}} = 0.394$$

## BACK PROPAGATION

### OUTPUT LAYER ERROR

$$\begin{aligned}Err_{y_1} &= O_{y_1} (1 - O_{y_1}) (T_{y_1} - O_{y_1}) \\&= 0.771 (1 - 0.771) (1 - 0.771) \\&= 0.04\end{aligned}$$

$$\begin{aligned}Err_{y_2} &= O_{y_2} (1 - O_{y_2}) (T_{y_2} - O_{y_2}) \\&= 0.394 (1 - 0.394) (0 - 0.394) \\&= -0.094\end{aligned}$$

### HIDDEN LAYER ERROR

$$\begin{aligned}Err_{s3} &= O_{s3} (1 - O_{s3}) [Err_{y_1} W_{s3y_1}] \\&= 0.56 (1 - 0.56) (0.04 \times 0.11) \\&= 0.001\end{aligned}$$

$$\begin{aligned}Err_{s4} &= O_{s4} (1 - O_{s4}) [Err_{y_1} W_{s4y_1} + Err_{y_2} W_{s4y_2}] \\&= 0.688 (1 - 0.688) (0.04 \times 0.51 + (-0.094 \times -0.35)) \\&= 0.01\end{aligned}$$

Date: \_\_\_\_\_

$$\begin{aligned}W_{x2s1} &= W_{x2s1} + l \cdot E_{ws1} \cdot O_{x2} \\&= 0.33 + 0.1 (0.00135) (0.15) \\&= 0.33\end{aligned}$$

$$\begin{aligned}W_{x2s2} &= W_{x2s2} + l \cdot E_{ws2} \cdot O_{x2} \\&= 0.12 + 0.1 (0.00237) (0.15) \\&= 0.12\end{aligned}$$

$$\begin{aligned}W_{x3s2} &= W_{x3s2} + l \cdot E_{ws2} \cdot O_{x3} \\&= -0.7 + 0.1 (0.00237) (0.61) \\&= -0.7\end{aligned}$$

$$\begin{aligned}W_{s1s3} &= W_{s1s3} + l \cdot E_{ws3} \cdot O_{s1} \\&= -0.36 + 0.1 (0.001) (0.716) \\&= -0.36\end{aligned}$$

$$\begin{aligned}W_{s1s4} &= W_{s1s4} + l \cdot E_{ws4} \cdot O_{s1} \\&= 0.7 + 0.1 (0.01) (0.716) \\&= 0.7\end{aligned}$$

$$\begin{aligned}W_{s2s4} &= W_{s2s4} + l \cdot E_{ws4} \cdot O_{s2} \\&= 0.22 + 0.1 (0.01) (0.448) \\&= 0.22\end{aligned}$$

$$\begin{aligned}W_{s2s5} &= W_{s2s5} + l \cdot E_{ws5} \cdot O_{s2} \\&= 0.85 + 0.1 (0.0087) (0.448) \\&= 0.85\end{aligned}$$

$$\begin{aligned}W_{s3s4} &= W_{s3s4} + l \cdot E_{ws4} \cdot O_{s3} \\&= 0.11 + 0.1 (0.04) (0.56) \\&= 0.11\end{aligned}$$

Date: \_\_\_\_\_

$$\begin{aligned}W_{s4y1} &= W_{suy1} + l \cdot Err_{y1} \cdot O_{s4} \\&= 0.51 + 0.1(0.04)(0.688) \\&= 0.51\end{aligned}$$

$$\begin{aligned}W_{s4y2} &= W_{suy2} + l \cdot Err_{y2} \cdot O_{s4} \\&= -0.35 + 0.1(-0.094)(0.688) \\&= -0.356\end{aligned}$$

$$\begin{aligned}W_{s5y2} &= W_{s5y2} + l \cdot Err_{y2} \cdot O_{s5} \\&= -0.42 + 0.1(-0.094)(0.671) \\&= -0.42\end{aligned}$$

### Final Result

As the weights and bias remain same and does not change and error is very small so we don't proceed further with successive iterations.

The final predictions (output) will be

$$O_{y1} = 0.771$$

$$O_{y2} = 0.394$$

$$\begin{aligned}
 Err_{S2} &= O_{S2} (1 - O_{S2}) (Err_{S3} \times W_{S2S3} + Err_{S4} \times W_{S2S4}) \\
 &= 0.448 (1 - 0.448) (0.01 \times 0.22 + 0.0027 \times 0.85) \\
 &= 0.00237
 \end{aligned}$$

## UPDATING WEIGHTS

$$\lambda = 0.1$$

$$\begin{aligned}
 W_{S2Y3} &= W_{S2Y2} + \Delta W_{S2Y3} \\
 &= 0.11 + \lambda Err_{Y3} O_{S2} \\
 &= 0.11 + (0.1)(0.04)(0.56)
 \end{aligned}$$

$$W_{S2Y2} = 0.11$$

$$\begin{aligned}
 W_{S4Y2} &= W_{S4Y1} + \Delta W_{S4Y2} \\
 &= 0.51 + (0.1)(0.04)(0.688) \\
 &= 0.5
 \end{aligned}$$

$$\begin{aligned}
 W_{S4Y2} &= W_{S4Y2} + \Delta W_{S4Y2} \\
 &= -0.35 + (0.1)(-0.35)(0.688)
 \end{aligned}$$

$$\begin{aligned}
 W_{S5Y2} &= W_{S5Y1} + \Delta W_{S5Y2} \\
 &= -0.42 + (0.1)(-0.094)(0.671) \\
 &= -0.42
 \end{aligned}$$

$$\therefore \Delta W_{S1S3} = \lambda Err_{S3} O_{S2}$$

$$\begin{aligned}
 W_{S1S3} &= W_{S1S2} + \Delta W_{S1S3} \\
 &= -0.36 + (0.1)(0.001)(0.716) \\
 &= -0.36
 \end{aligned}$$

$$\therefore \Delta W_{S2S4} = \lambda Err_{S4} O_{S2}$$

$$\begin{aligned}
 W_{S2S4} &= W_{S2S3} + \Delta W_{S2S4} \\
 &= 0.7 + (0.1)(0.05)(0.716) \\
 &= 0.7
 \end{aligned}$$

$$\begin{aligned}
 W_{S_2 S_4} &= W_{S_2 S_4} + \Delta W_{S_2 S_4} \\
 &= W_{S_2 S_4} + l \text{ Err}_{S_4} O_{S_2} \\
 &= 0.22 + (0.1) (0.01) (0.448) \\
 W_{S_2 S_4} &= 0.22
 \end{aligned}$$

$$\begin{aligned}
 W_{S_2 S_4} &= W_{S_2 S_2} + \Delta W_{S_2 S_2} \\
 &= 0.25 + (0.1) (0.0087) (0.448) \\
 &= 0.25
 \end{aligned}$$

$$\begin{aligned}
 W_{H_2 S_2} &= W_{H_2 S_2} + \Delta W_{H_2 S_2} \\
 &= 0.25 + \cancel{l} \text{ Err}_{S_2} I_{H_2} \\
 &= 0.25 + (0.1) (0.0013) (0.7) \\
 W_{H_2 S_2} &= 0.25
 \end{aligned}$$

$$\begin{aligned}
 W_{H_2 S_2} &= W_{H_2 S_2} + \Delta W_{H_2 S_2} \\
 &= 0.33 + \cancel{l} \text{ Err}_{S_2} I_{H_2} \\
 &= 0.33 + (0.1) (0.0013) (0.15) \\
 &= 0.33
 \end{aligned}$$

$$\begin{aligned}
 W_{H_2 S_2} &= W_{H_2 S_2} + \Delta W_{H_2 S_2} \\
 &= 0.12 + \cancel{l} \text{ Err}_{S_2} I_{H_2} \\
 &= 0.12 + (0.1) (0.00237) (0.15) \\
 &= 0.12
 \end{aligned}$$

$$\begin{aligned}
 W_{H_3 S_2} &= W_{H_3 S_2} + \Delta W_{H_3 S_2} \\
 &= -0.7 + \cancel{l} \text{ Err}_{S_2} I_{H_3} \\
 &= -0.7 + (0.1) (0.00237) (0.61) \\
 &= -0.7
 \end{aligned}$$

$$\hat{O}_{y_2} = O_{y_2} + l \text{ Err } y_2 \\ = 0.09 + (0.1)(-0.094)$$

$$\hat{O}_{y_2} = 0.0806$$

## CHANGE IN BASIS:

$$\Delta \theta_j = l \text{ Err}_j$$

$$\begin{aligned}\theta_{S1} &= \theta_{S1} + \Delta \theta_{S1} \\&= \theta_{S1} + l \text{ Err}_{S1} \\&= 0.7 + (0.1)(0.0013) \\&\leftarrow \theta_{S1} = 0.7\end{aligned}$$

$$\begin{aligned}\theta_{S2} &= \theta_{S2} + \Delta \theta_{S2} \\&= \theta_{S2} + l \text{ Err}_{S2} \\&= 0.2 + (0.1)(0.00237) \\&\leftarrow \theta_{S2} = 0.2\end{aligned}$$

$$\begin{aligned}\theta_{S3} &= \theta_{S3} + l \text{ Err}_{S3} \\&= 0.5 + (0.1)(0.001) \\&\leftarrow \theta_{S3} = 0.5\end{aligned}$$

$$\begin{aligned}\theta_{S4} &= \theta_{S4} + l \text{ Err}_{S4} \\&= 0.19 + (0.1)(0.001) \\&\leftarrow \theta_{S4} = 0.19\end{aligned}$$

$$\begin{aligned}\theta_{S5} &= \theta_{S5} + l \text{ Err}_{S5} \\&= 0.33 + (0.1)(0.0087)\end{aligned}$$

$$\leftarrow \theta_{S5} = 0.33$$

$$\begin{aligned}\theta_{Y1} &= \theta_{Y1} + l \text{ Err}_{Y1} \\&= 0.8 + (0.1)(0.04) \\&\leftarrow \theta_{Y1} = 0.8\end{aligned}$$