"I have done this assignment myself. I have not copied it from anywhere and no other student have copied from my work."

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DISCRETE MATHEMATICS

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Ch: Relations

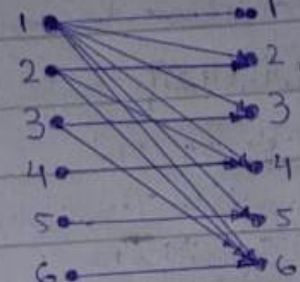
Exercise : 9.1

Q2: a) List all the ordered pairs in relation $R = \{(a,b) / a \text{ divisible } b\}$ on the set $\{1,2,3,4,5,6\}$.

$\Rightarrow$ Because (a,b) in R if and only if a & b are positive integers.

$R = \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,2),(2,4),(2,6),(3,3),(3,6),$
$(4,4),(5,5),(6,6)\}$

b) Display this relation graphically.



c) Display relation in tabular form:

R	1	2	3	4	5	6
1	×	×	×	×	×	×
2		×		×		×
3			×			×
4				×		
5					×	
6						×

Q3: For each of these relation on set $\{1,2,3,4\}$, decide whether its reflexive, symmetric, antisymmetric, transitive.

a) $\{(2,2),(2,3),(2,4),(3,2),(3,3),(3,4)\}$

- Reflexive: not include $(1,1)$, $(4,4)$
- Symmetric: not, $(2,4)$ but not $(4,2)$
- Antisymmetric: include $(2,3)$ & $(3,2)$, not
- Transitive: Yes it is, (a,b) & $(b,c) \rightarrow (a,c)$. $\underset{a\ b}{(2,3)}, \underset{b\ c}{(3,2)} \Rightarrow \underset{a\ c}{(2,2)}$

a $\rightarrow$ must be 2 or 3 $(2,c)$
$(3,c)$

b) $\{(1,1),(1,2),(2,1),(2,2),(3,3),(4,4)\}$

- Reflexive: Yes, b/c $(1,1),(2,2),(3,3),(4,4)$ present.
- Symmetric: Yes b/c being both $(1,2),(2,1)$
- Antisymmetric: No
- Transitive: Yes $\Rightarrow (1,2),(2,1) \Rightarrow (1,1)$

c) $\{(2,4),(4,2)\}$

- Reflexive: No
- Symmetric: Yes, $(2,4),(4,2)$
- Antisymmetric: No
- Transitive: No $(2,2)$ not

d) $\{(1,2),(2,3),(3,4)\}$

- Reflexive: No
- Symmetric: No
- Antisymmetric: Yes
- Transitive: No, since $(1,2)$ & $(2,3)$ in relation but $(1,3)$ is not.

e) $\{(1,1),(2,2),(3,3),(4,4)\}$

- Reflexive: Yes
- Symmetric: Yes
- Antisymmetric: Yes $\{$trivially$\}$
- Transitive: Yes

f) $\{(1,3),(1,4),(2,3),(2,4),(3,1),(3,4)\}$

- Reflexive: No
- Symmetric: No
- Antisymmetric: No
- Transitive: No

Q6: Determine whether relation R on set of all real no. is reflexive, symmetric, antisymmetric, &/or transitive, where $(x, y) \in R$ if & only if:

a) $x+y=0$

- Reflexive: Not, b/c $x+x=0$ is only true when x is 0, but not for all.
- Symmetric: Yes, if $x+y=0$ then $y+x=0$; $-3+3=0 \Rightarrow 3-3=0$
- Anti Symmetric: Not, $2+(-2)=0$ & $(-2)+2=0$; $2 \neq -2$ $a \neq b$
- Transitive: Not, $1+(-1)=0$ & $(-1)+1=0$ while $1+1 \neq 0$.

b) $x = \pm y$ $R = \{(x,y) | x = \pm y\}$

- Reflexive: Yes, $x=x$ is always true.
- Symmetric: Yes, if $x=\pm y$ then $y=\pm x$
- Antisymmetric: Not, $5=-(-5)$ & $(5)=-5$ while $-5 \neq 5$
- Transitive: Yes, $x=\pm y$ & $y=\pm z$ then $x=\pm y=\pm(\pm z)$ $\boxed{=\pm z}$

c) $x-y$ is rational number. $R=\{(x,y) | (x-y) \text{ is RN}\}$

- Reflexive: Yes, $x-x=0$ & 0 is always rational.
- Symmetric: Yes, if $x-y$ is Rational then $y-x$ is also.
- Anti Symmetric: Not, b/c $5-6$ & $6-5$ both ration
- Transitive: Yes, $x-y$ & $y-z$ is RN & $x-z$ is also.

d) $x=2y$ $R=\{(x,y) | x=2y\}$

- Reflexive: Not, $x=2y$ is only when $x=0$, not for all.
- Symmetric: Not, $2=2(1)$ while $1 \neq 2(2)$
- Anti Symmetric: Yes, $x=y=0$ $[x=2y=2(2x)=4x]$ can only true when $x=0$
- Transitive: Not, if $x=2y$ & $y=2z$ then $x \neq 2z$

e) $xy \geq 0$ $R = \{(x,y) \mid xy \geq 0\}$

- **Reflexive**: Yes, $x.x \geq 0$, always greater than 0.
- **Symmetric**: Yes, $x.y \geq 0$ & $y.x \geq 0$ [commulative]
- **Anti symmetric**: Not, $[-2][-1] \geq 0$ & $[-1][-2] \geq 0$ while $-2 \neq -1$
- **Transitive**: Not, $x=-1$, $y=0$ & $z=1$ then $zx < 0$ $-1 \neq 1$

f) $xy = 0$ $R = \{(x,y) \mid xy = 0\}$

- **Reflexive**: Not, $xy \neq xx \neq 0$, Only when $x=0$ not for all.
- **Symmetric**: Yes. $xy = 0$ then $yx = 0$
- **Antisymmetric**: Not, $(0)(1) = 0$ & $(1)(0) = 0$ where $0 \neq 1$
- **Transitive**: Not, $x=-1$, $y=0$, $z=1$ then $xy=0$, $y=0$
  $xz \neq 0$.

g) $x = 1$ $R = \{(x,y) \mid x=1\}$

- **Reflexive**: Not, $x.x \neq 1$
- **Symmetric**: Not, $x=1$ & $y=0$ then $(x,y) \in R$ & $(y,x) \notin R$
- **Antisymmetric**:
- **Transitive**:

h) $x=1$ or $y=1$

- **Reflexive**: Not, $(0,0)$ is not in R (Since $0 \neq 1$)
- **Symmetric**: Yes, if $x=1$ & $y=0$ then $(x,y) \in R$ & $(y,x) \in R$
- **Anti Symmetric**: Not, $x=1$ & $y=0$ " $x \neq y$
- **Transitive**: Not, $x=0$, $y=1$, $z=2$ then $(x,y) \in R$ & $(y,z) \in R$ while $(x,z) \notin R$

Q7. Determine whether relation R on set of all integers is reflexive, symmetric, antisymmetric &/or transitive, where $(x,y) \in R$ if and only if:

a) $x \neq y$

- Reflexive: Not, since it not the case $1 \neq 1$
- Symmetric: Yes, if $x \neq y$ then ofcourse $y \neq x$
- Antisymmetric: Not, $1 \neq 2$ & also $2 \neq 1$
- Transitive: Not, $1 \neq 2$ & $2 \neq 1$ but it in not the case $1 \neq 1$

b) $x = \pm y$ $xy > 1$

- Reflexive: Not, b/c $(0,0)$ not including.
- Symmetric: Yes, commutative property of multiplication, $xy = yx$. of of term greater or equal to 1
- Antisymmetric: Not, $(2,3)$ & $(3,2)$ both include.
- Transitive: x & y both +ve or -ve $(a,b) \in R$ & $(b,c) \in R$ Yes $\rightarrow (a,c) \in R$

c) ~~$x-y$ is rational number~~ $x = y \pm 1$

- Reflexive: Not, since $(1,1)$ not include.
- Symmetric: Yes, $x = y-1$ equivalent to $y = x+1$, y relate to x, so x & y reversed role
- Antisymmetric: Not, $(1,2)$ & $(2,1)$ in relation.
- Transitive: Not. " " in relation but $(1,1)$ not.

d) $x \equiv y \pmod 7$ $x-y$ multiple of 7 $\therefore x-y = 7t$ t = some integer

- Reflexive: Yes, $x - x = 7.0$ for all x.
- Symmetric: Yes $x-y = 7t$, $y-x = 7(-t)$ $\therefore y \equiv x \pmod 7$
- Antisymmetric: Not, $2 \equiv 9$ & $9 \equiv 2 \pmod 7$.
- Transitive: Yes, $x \equiv y$ & $y \equiv z$. $\therefore x-y = 7s$ & $y-z = 7t$

$$x - z = 7s + 7t$$
$$x - z = 7(s+t)$$
$$x - z = (7 \pmod)$$

e) x is multiple of y.

- Reflexive: Yes
- Symmetric: Not, 6 multiple of 2 but 2 not multiple of 6.
- Antisymmetric: Not, 2 is multiple of -2 & vice versa, but $2 \neq -2$
- Transitive: Yes, x multiple of y $(x = ty)$ & y multiple of z $(y = sz)$ then, $x = t(sz) = (ts)z$.

f) x & y both negative or both nonnegative.

- Reflexive: Yes, since a & a either both $-$ve or non-$-$ve.
- Symmetric: Yes
- Antisymmetric: No, since 5 related to 6 & 6 related to 5 but $5 \neq 6$
- Transitive: Yes, a related to b & b related to c then all three must be $-$ve or non-$-$ve.

g) $x = y^2$

- Reflexive: Not, $17 \neq 17^2$
- Symmetric: Not, $289 = 17^2$ but not the case $17 = 289^2$
- Antisymmetric: Yes, (x,y) & (y,x), then $x = y^2$ & $y = x^2 \Rightarrow x - x^4 = 0$
  $x(1-x^3) = x(1-x)(1+x+x^2) = 0/1$ y also $0/1$
  $x = y$ This case right
- Transitive: Not, $16 = 4^2$ & $4 = 2^2$ but $16 \neq 2^2$

h) $x \geq y^2$

- Reflexive: Not, $17 \not\geq 17^2$
- Symmetric: Not $289 \geq 17^2$ but not $17 \geq 289^2$
- Antisymmetric: (x,y) & (y,x). Then $x \geq y^2$ & $y \geq x^2$. $y^2 \geq x^4$. $\Rightarrow x \geq x^4$
  $x - x^4 \geq 0$, Yes
- Transitive: Yes . $x \geq y^2$ & $y \geq z^2 \Rightarrow y^2 \geq z^4 \Rightarrow x \geq z^4 \rightarrow z^4 \geq z^2$
  $x \geq z^2$

$t-2$

$(=Sz)$

$\neq 6$

Q35: Let $R_1 = \{(1,2),(2,3),(3,4)\}$

$R_2 = \{(1,1),(1,2),(2,1),(2,2),(2,3),(3,1),(3,2),(3,3),(3,4)\}$

$\{1,2,3\}$ to $\{1,2,3,4\}$

a) $R_1 \cup R_2$

$\Rightarrow \{(1,1),(1,2),(2,1)(2,2),(2,3),(3,1),(3,2),(3,3),(3,4)\}$

b) $R_1 \cap R_2$

$\Rightarrow \{(1,2),(2,3),(3,4)\}$

c) $R_1 - R_2$

$\Rightarrow \{\}$

d) $R_2 - R_1$

$\Rightarrow \{(1,1),(2,1),(2,2),(3,1),(3,2),(3,3)\}$

Q36: Find:

$R_1 = \{(a,b) \in R^2 \mid a > b\}$

$R_2 = \{(a,b) \in R^2 \mid a \geq b\}$

$R_3 = \{(a,b) \in R^2 \mid a < b\}$

$R_4 = \{(a,b) \in R^2 \mid a \leq b\}$

$R_5 = \{(a,b) \in R^2 \mid a = b\}$

$R_6 = \{(a,b) \in R^2 \mid a \neq b\}$

a) $R_1 \circ R_1$

$$\left.\begin{array}{l} \bullet\ (a,b) \in R_1 \longrightarrow a > b \\ \bullet\ (b,c) \in R_1 \longrightarrow b > c \end{array}\right\} \quad a > c$$

$\boxed{R_1 \circ R_1 = R_1}$ Transitive.

b) $R_1 \circ R_2$

$$\left.\begin{array}{l} \bullet\ (a,b) \in R_2 \longrightarrow a \geq b \\ \bullet\ (b,c) \in R_1 \longrightarrow b > c \end{array}\right\} \quad a > c$$

$\boxed{R_1 \circ R_2 = R_1}$

c) $R_1 \circ R_3$

$$\left.\begin{array}{l} \bullet\ (a,b) \in R_3 \longrightarrow a < b \\ \bullet\ (b,c) \in R_1 \longrightarrow b > c \end{array}\right\} \quad a \geq c$$

$\boxed{R_1 \circ R_3 = R_2}$

d) $R_1 \circ R_4$

$\left.\begin{array}{l} \bullet\ (a,b) \in R_4 \longrightarrow a \leq b \\ \bullet\ (b,c) \in R_1 \longrightarrow b > c \end{array}\right\}$ $a > c$

$\boxed{R_1 \circ R_4 = R_1}$

e) $R_1 \circ R_5$

$\left.\begin{array}{l} \bullet\ (a,b) \in R_5 \longrightarrow a = b \\ \bullet\ (b,c) \in R_1 \longrightarrow b > c \end{array}\right\}$ $a > c$

$\boxed{R_1 \circ R_5 = R_1}$

f) $R_1 \circ R_6$

$\left.\begin{array}{l} \bullet\ (a,b) \in R_6 \longrightarrow a \neq b \\ \bullet\ (b,c) \in R_1 \longrightarrow b > c \end{array}\right\}$ $a \geq c$

$\boxed{R_1 \circ R_6 = R_2}$

g) $R_2 \circ R_3$

$\left.\begin{array}{l} \bullet\ (a,b) \in R_3 \longrightarrow a < b \\ \bullet\ (b,c) \in R_2 \longrightarrow b \geq c \end{array}\right\}$ $a < c$

$\boxed{R_2 \circ R_3 = R_3}$

h) $R_3 \circ R_3$

$\left.\begin{array}{l} \bullet\ (a,b) \in R_3 \longrightarrow a < b \\ \bullet\ (b,c) \in R_3 \longrightarrow b < c \end{array}\right\}$ $a < c$

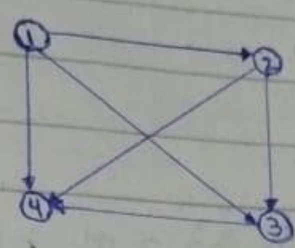
$\boxed{R_3 \circ R_3 = R_3}$ Transitive

Exercise : 9.3

Q2: Represent each of these relations on $\{1,2,3,4\}$ with matrix (with elements of this set listed in increasing order).

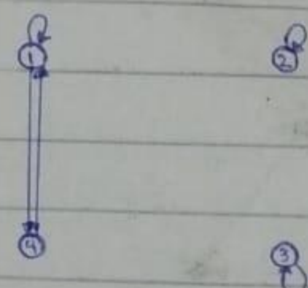
a) $\{(1,2),(1,3),(1,4),(2,3),(2,4),(3,4)\}$

$$\begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$



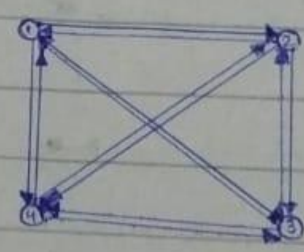
b) $\{(1,1),(1,4),(2,2),(3,3),(4,1)\}$

$$\begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$



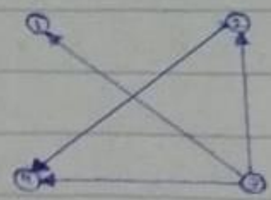
c) $\{(1,2),(1,3),(1,4),(2,1),(2,3),(2,4),(3,1),(3,2),(3,4),(4,1)$
$(4,2),(4,3)\}$

$$\begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$



d) $\{(2,4), (3,1), (3,2), (3,4)\}$

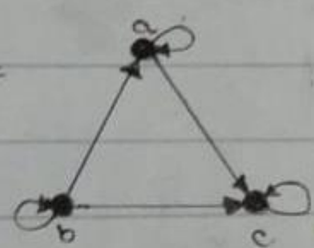
$$\begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & & & & \\ 2 & & \left[\begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{matrix}\right] & & \end{array}$$



Q19: Draw directed graph representing each of the relations from Exe-2.

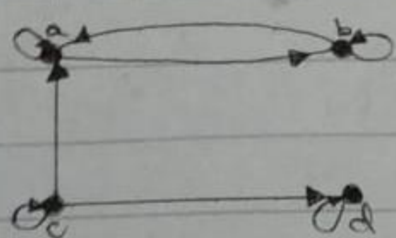
2) ⇒ With G_1, Draw.

Q24:



$R = \{(a,a), (a,c), (c,c), (b,b), (b,a), (b,c)\}$

Q26:



$R = \{(a,a), (b,b), (c,c), (d,d), (a,b), (b,a), (c,a), (c,d)\}$

Exercise: 9.4

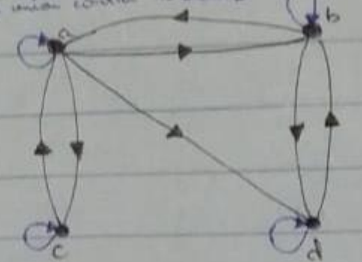
Q2: Let R be relation $\{(a,b) \mid a \neq b\}$ on set of integers. What is reflexive closure of R? $R = \{(a,b) \mid a \neq b\}$; A = set of integers = Z

$\Rightarrow$ to form reflexive closure of R with all the pairs of (a,b). For this we need to add $(a,a) \in R$ or aRa in Relation.
The Reflexive closure of R is $R \cup \Delta = R \cup \{(a,a)\}$

$R \cup \Delta = \{R \cup \{(a,a) \mid a \in a\}$
$= \{(a,b) \mid a \neq b\} \cup \{(a,b) \mid a = b\}$
The union contain all ordered pair in either relation

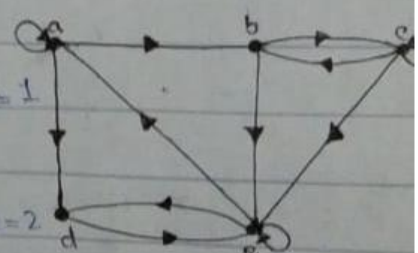
$= \{(a,b) \mid a \neq b \text{ or } a = b\}$
The statement "$a \neq b$ or $a = b$" true for all pair of integers.
$= \{(a,b) \mid a,b \in Z\} \Rightarrow \boxed{Z \times Z}$

Q7:



$\Rightarrow$ We form reflexive closure by appending loop.

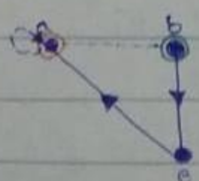
Q18: Determine whether there is path in digraph in Ex. 16 beginning at first vertex given & ending at second vertex given.



a) a, b

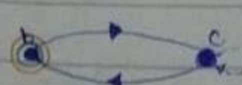
path of length = 1

b) b, a



path of length = 2

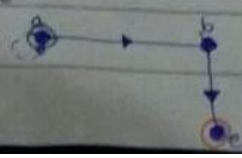
c) b, b



path of length = 2

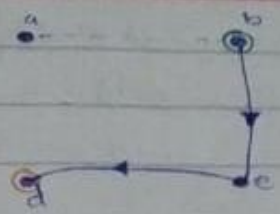
• → Initial state
• → Terminal state

d) a, e



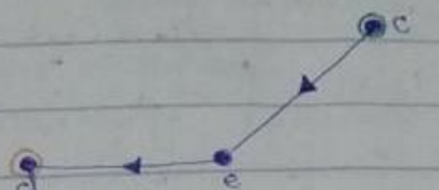
path of length = 2

e) b, d



path of length = 2

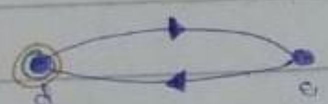
f) c, d



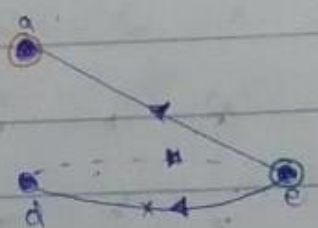
path of length = 2

g) d, d

path of length = 2

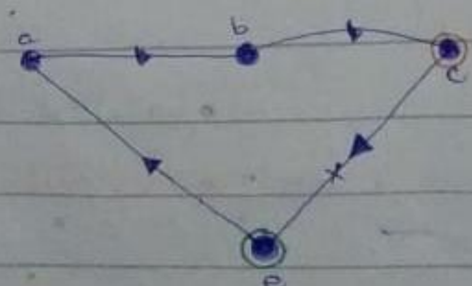


h) e, a



path of length = 1

i) e, e



path of length = 3

Q19: Let R be relation on set {1, 2, 3, 4, 5} containing order pairs (1,3), (2,4), (3,1), (3,5), (4,3), (5,1), (5,2) & (5,4). Find:

a) R^2

$$M_R = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 0 & 1 & 0 & 0 \\ 2 & 0 & 0 & 0 & 1 & 0 \\ 3 & 1 & 0 & 0 & 0 & 1 \\ 4 & 0 & 0 & 1 & 0 & 0 \\ 5 & 1 & 1 & 0 & 1 & 0 \end{array}$$

$\boxed{A \times B}$ $R_1 = \{(1,3), (2,4), (3,1), (3,5), (4,3), (5,1), (5,2), (5,4)\}$

$\boxed{B \times C}$ $R_2 = \{(1,3), (2,4), (3,1), (3,5), (4,3), (5,1), (5,2), (5,4)\}$

$R^2 = \{(1,1), (4,1), (1,5), (4,5), (3,3), (5,3), (2,3), (5,3), (3,1), (3,2), (3,4), (5,4)\}$

$\boxed{A \times C}$

$$R^2 = M_{R^2} = M_R^{[2]} = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 1 & 0 & 0 & 0 & 1 \\ 2 & 0 & 0 & 1 & 0 & 0 \\ 3 & 1 & 1 & 1 & 1 & 0 \\ 4 & 1 & 0 & 0 & 0 & 1 \\ 5 & 0 & 0 & 1 & 1 & 0 \end{array}$$

Answer!

b) R^3

$\boxed{A \times B}$ $R_1 = \{(1,1), (4,1), (1,5), (4,5), (3,3), (5,3), (2,3), (5,3), (3,1), (3,2), (3,4), (5,$

$\boxed{B \times C}$ $R_2 = \{(1,3), (2,4), (3,1), (3,5), (4,3), (5,1), (5,2), (5,4)\}$

$\boxed{A \times C}$ $R_3 = \{(1,3), (4,3), (3,3), (3,4), (3,1), (5,1), (2,1), (5,1), (3,5), (5,5), (2,5), (5,$
$(3,3), (5,5), (1,1), (1,2), (1,4), (4,1), (4,2), (4,4)\}$

$$R^3 = M_{R^3} = M_R^{[3]} = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 1 & 1 & 1 & 1 & 0 \\ 2 & 1 & 0 & 0 & 0 & 1 \\ 3 & 1 & 0 & 1 & 1 & 1 \\ 4 & 1 & 1 & 1 & 1 & 0 \\ 5 & 1 & 0 & 1 & 0 & 1 \end{array}$$

Answer!

c) R^4

$\boxed{a \vee b}$

$R_1 = \{(1,3), (4,3), (3,3), (3,4), (3,1), (5,1)\ (2,1)\ (5,1), (3,5)\ (5,5)\ (2,5)\ (5,5)$
$(3,3), (5,3), (1,1), (1,2), (1,4), (4,1), (4,2), (4,4)\}$

$\boxed{b \vee c}$

$R_2 = \{(1,3), (2,4), (3,1), (3,5), (4,3), (5,1), (5,2), (5,4)\}$

$\boxed{a \vee c}$

$R_4 = \{(1,1), (4,1)\ (3,1)\ (5,1), (1,5)\ (4,5)\ (3,5), (5,5), (3,3), (1,3), (4,3)$
$\cancel{(3,3)}\ (5,3)\ (2,3)\ \cancel{(5,3)}\ \cancel{(1,3)}\ \cancel{(4,3)}, (1,4), (4,4), \cancel{(2,4)}\ (2,2), (3,2)$
$(5,2)\ (3,4), (5,4), (2,4)\}$

$$R^4 = M_{R^4} = M_R^{[4]} = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 1 & 0 & 1 & 1 & 1 \\ 2 & 1 & 1 & 1 & 1 & 0 \\ 3 & 1 & 1 & 1 & 1 & 1 \\ 4 & 1 & 0 & 1 & 1 & 1 \\ 5 & 1 & 1 & 1 & 1 & 1 \end{array}$$

Ans.

d) R_5

$\boxed{a \vee b}$

$R_1 = \{\cancel{(1,3), (4,3)\ (3,3)\ (3,4)}$
$\{(1,1)\ (4,1)\ (3,1)\ (5,1), (2,1), (1,5)\ (4,5), (3,5)\ (5,5), (3,3)\ (1,3)\ (4,3), (5,3)\ (2,$
$(1,4)\ (4,4), (3,4)\ (5,4)\ (2,4), (2,2)\ (3,2), (5,2)\}$

$\boxed{b \vee c}$

$R_2 = \{(1,3), (2,4)\ (3,1), (3,5), (4,3)\ (5,1), (5,2)\ (5,4)\}$

$\boxed{a \vee c}$

$R_5 = \{(1,3), (4,3), (3,3), (5,3), (2,3), (2,4), (3,4), (5,4), (3,1), (1,1), (4,1), (5,1), (2,$
$(3,5), (1,5)\ (4,5)\ (5,5), (2,5), (1,2), (4,2), (3,2), (5,2), (1,4)\ (4,4)\}$

$$R^5 = M_R^5 = M_R^{[5]} = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 1 & 1 & 1 & 1 & 1 \\ 2 & 1 & 0 & 1 & 1 & 1 \\ 3 & 1 & 1 & 1 & 1 & 1 \\ 4 & 1 & 1 & 1 & 1 & 1 \\ 5 & 1 & 1 & 1 & 1 & 1 \end{array}$$

Ans.

e) R^6

$R_1 = \{(3,1)(1,1),(4,1),(5,1),(2,1),(1,2),(4,2)(3,2)(5,2),(1,3),(4,3),(2,3),(5,3)$
$(2,4),(3,4),(5,4)(1,4)(4,4),(3,5),(1,5),(4,5)(5,5)(2,5)\}$ $(2,3)$

$R_5 = \{(1,3),(2,4),(3,1),(3,5),(4,3),(5,1),(5,2),(5,4)\}$

$R_6 = \{$ —

(1) $\Rightarrow \{(3,1),(1,1),(4,1),(5,1),(2,1)\}$ $\Rightarrow \{(3,3),(1,3),(4,3),(5,3),(2,3)\}$
$\Rightarrow \{(\cancel{1},3)\}$

(2) $\Rightarrow \{(1,2)(4,2)(3,2),(5,2)\}$ $\Rightarrow \{(1,4)(4,4)(3,4)(5,4)\}$
$\Rightarrow \{(\cancel{2},4)\}$

(3) $\Rightarrow \{(1,3),(4,3),(3,3)(5,3),(2,3)\}$ $\Rightarrow \{(1,1)(4,1)(3,1)(5,1)(2,1),(1,5),(4,5),(3,5),$
$(5,5),(2,5)\}$
$\Rightarrow \{(\cancel{3},1)(\cancel{3},5)\}$

(4) $\Rightarrow \{(2,4),(3,4)(5,4)(1,4)(4,4)\}$ $\Rightarrow \{(2,3)(3,3)(5,3)(1,3)(4,3)\}$
$\Rightarrow \{(\cancel{4},3)\}$

(5) $\Rightarrow \{(3,5)(1,5),(4,5)(5,5)(2,5)\}$ $\Rightarrow \{(3,1)(1,1)(4,1)(5,1)(2,1),(3,2)(1,2)$
$(4,2)(5,2)(2,2),(3,4)(1,4),(5,4)$
$(2,4)\}$
$\Rightarrow \{(\cancel{5},1)(\cancel{5},2)(\cancel{5},4)\}$

$R_6 = \{(3,3),(1,3),(4,3),(5,3),(2,3),(1,4),(4,4),(3,4),$
$(5,4),(1,1),(4,1)(3,1)(5,1)(2,1),(1,5)(4,5),$
$(3,5),(5,5),(2,5),(3,2),(1,2),(4,2)(5,2),$
$(2,2),(2,4)\}$

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}\begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

Ans

f) R^*

$\Rightarrow$ The matrix of R^* is join of first matrix displayed above and answers to part (a) through (d) namely:

$$M_{R^*} = M_R \vee M_R^{[2]} \vee M_R^{[3]} \vee M_R^{[4]} \vee M_R^{[5]} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Ans

Exercise: 9.5

Q1: Which of these Relations on {0,1,2,3} are equivalence Relatio[ns]. Determine properties of equivalence relation that other lacks.

b) {(0,0), (0,2), (2,0), (2,2), (2,3), (3,2), (3,3)}

- Reflexive: Not, (1,1) not present
- Symmetric: Yes
- Transitive: Not, {(0,0), (2,2), (3,3)} are Transitive, ✓

(0,2), (2,0) $\rightarrow$ (0,0) ✓

(2,3), (3,2) $\rightarrow$ (2,2) ✓

(0,2), (2,3) $\rightarrow$ $\underline{(0,3)}$ ✗ Not present

This relation is not equivalence.

d) $\{(0,0), (1,1), (1,3), (2,2), (2,3), (3,1), (3,2), (3,3)\}$

- Reflexive: Yes
- Symmetric: Yes
- Transitive: Not, $\{(0,0), (1,1), (2,2), (3,3)\}$ are transitive.

$(1,3)\ (3,1) \longrightarrow (1,1)$

$(2,3), (3,2) \longrightarrow (2,2)$

$(1,3), (3,2) \longrightarrow (1,2)$ × (1,2) not present.

∴ This relation is not equivalence

Q24: Determine whether relation represented by these zero-one matrix are equivalence relation.

b)
$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

⇒ Reflexive b/c diagonal = 1

⇒ Symmetric same members.

⇒ Transitive:

In order to check / determine whether relation are equivalence relation, we need to check reflexively, symmetric & transitive

This relation is true reflexive, symmetric, and transitive. So, this relation is equivalence relation.

Q2: which of these collections of subsets are partition of $\{-3, -2, -1, 0, 1, 2, 3\}$?

if ① $\forall X \in S \quad X \neq \emptyset$ set not empty $\quad \cap \to \emptyset$
② $X \cap Y = \emptyset \quad \cup \to A$
③ $\bigcup_{X \in S} X = A$

a) $\{-3, -1, 1, 3\}, \{-2, 0, 2\}$

- 1) Set is not empty. ✓
- 2) disjoint of these 2 sets: $\{-3, -1, 1, 3\} \cap \{-2, 0, 2\} = \emptyset$ ✓
- 3) Union gives set A: $\{-3, -1, 1, 3\} \cup \{-2, 0, 2\} = \{-3, -2, -1, 0, 1, 2, 3\}$ ✓

$\Rightarrow$ These sets are partitions.

b) $\{-3, -2, -1, 0\}, \{0, 1, 2, 3\}$

- Sets not empty. ✓
- disjoint gives: $\{-3, -2, -1, 0\} \cap \{0, 1, 2, 3\} = \{0\}$ ✗
- Union gives: $\{-3, -2, -1, 0\} \cup \{0, 1, 2, 3\} = \{-3, -2, -1, 0, 1, 2, 3\}$ ✓

$\Rightarrow$ These subsets are not Partition

c) $\{-3, 3\}, \{-2, 2\}, \{-1, 1\}, \{0\}$

- Sets not empty ✓
- disjoint: $\{-3, 3\} \cap \{-2, 2\} \cap \{-1, 1\} \cap \{0\} = \emptyset$ ✓
- Union: $\{-3, 3\} \cup \{-2, 2\} \cup \{-1, 1\} \cup \{0\} = \{-3, -2, -1, 0, 1, 2, 3\}$ ✓

$\Rightarrow$ These subsets are partition

d) $\{-3, -2, 2, 3\}, \{-1, 1\}$

- Sets not empty ✓
- disjoint: $\{-3, -2, 2, 3\} \cap \{-1, 1\} = \emptyset$ ✓
- Union: $\{-3, -2, 2, 3\} \cup \{-1, 1\} = \{-3, -2, -1, 1, 2, 3\}$ ✗

$\Rightarrow$ These sets are not partition.

Exercise: 9-6

Q2: Which of these relation on $\{0,1,2,3\}$ are partial orderings? Determine properties of partial ordering, that others lack.

a) $\{(0,0),(2,2),(3,3)\}$ Reflexive, Anti-Symmetric, Transitive

- Reflexive: No $R = A \times A - X$
- Anti-Symmetric: Yes
- Transitive: Yes

$\Rightarrow$ Not, partial ordering.

(b) $\{(0,0),(1,1),(1,2),(2,2),(3,1),(3,3)\}$

- Reflexive: Yes 12
- Anti-Symmetric: Yes 22 12
- Transitive: Yes (No)

No
$\Rightarrow$ Yes, partial ordering.

c) $\{(0,0),(0,1),(0,2),(0,3),(1,0),(1,1),(1,2),(1,3),(2,0),(2,2),(3,3)\}$

- Reflexive: Yes
- Anti-Symmetric: No (0 1) (0,2) (0,3) (1 2) (1 3)
  (1 0) (2,0)
- Transitive: Yes
  (1 2)
  1 3

$\Rightarrow$ Not partial ordering.

Que Is (S, R) a poset if S is set of all people in world & (a, b) ∈ R where a & b are people. If:

a) a is no shorter than b?

- Reflexive: Yes, can be.
- Anti-Symmetric: Not, a & b can have same height
- Transitive: Not sure,

⇒ No shorter means a is taller or have equal height as b. Since two people can have same height. Not, poset.

b) a weights more than b?

- Reflexive: Not, a weights more than b.
- Anti-Symmetric:
- Transitive:

⇒ Not post.

c) a = b or a is descendant of b?

- Reflexive:
- Anti-Symmetric:
- Transitive:

d) a and b do not have common friend?

- Reflexive:
- Anti-Symmetric:
- Transitive:

Q: Determine whether relation represented by these zero-one matrices are partial orders.

a) $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

→ Reflexive : Yes
→ Anti-Symmetric : Yes
→ Transitive : No, (2,1) & (1,3) → (2,3)

No!

b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

→ Reflexive : Yes
→ Anti-Symmetric : Yes
→ Transitive :

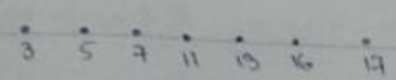
c) $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$

→ Reflexive : Yes
→ Anti-Symmetric : Yes
→ Transitive : Yes, (1,3), (3,4) → (1,4)
(2,3) (3,4) → (2,4)

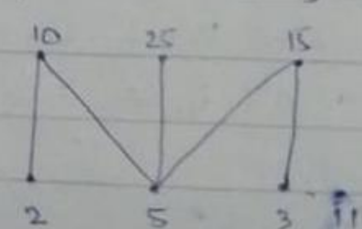
Yes!

Q22: Draw Hasse diagram for divisibility on the set.

b) {3, 5, 7, 11, 13, 16, 17}



c) {2, 3, 5, 10, 11, 15, 25}



Ch: Graph

Exercise: 10-1

10.1: 2 | 10.2: 24, 33, 58 | 10.3: 36, 40

Q: What kind of graph (from table) can be used to model a highway system b/w major cities where.

Types	Edges	Multiple Edges Allowed?	Loops allowed?
Simple Graph	Undirected	No	No
Multi-Graph	Undirected	Yes	No
Pseudograph	"	Yes	Yes
Simple digraph	Direct	No	No
Direct multigraph	"	Yes	Yes
Mixed Graph	Direct + Undirect	Yes	Yes

a) ~~What kind of graph~~ There is an edge b/w vertices representing cities if there is an interstate highway b/w them?

⇒ Simple Graph

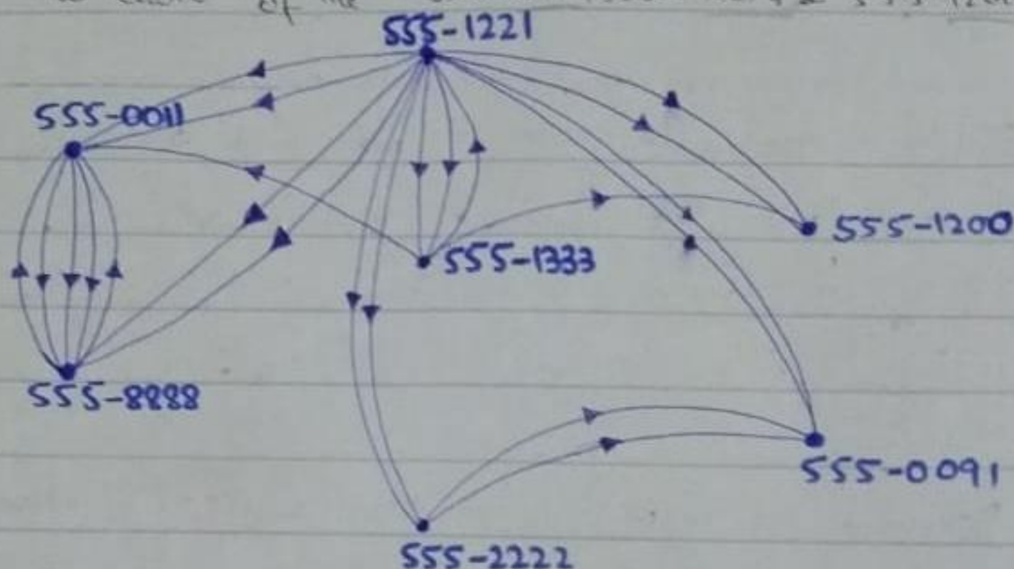
b) There is an edge b/w vertices representing cities for each interstate highway b/w them?

⇒ Multigraph, because it also present of having atmost two edges connecting the same vertices.

c) There is an edge b/w vertices representing cities for each interstate highway b/w them & there is a loop at vertex representing a city if there is an interstate highway that circles this city?

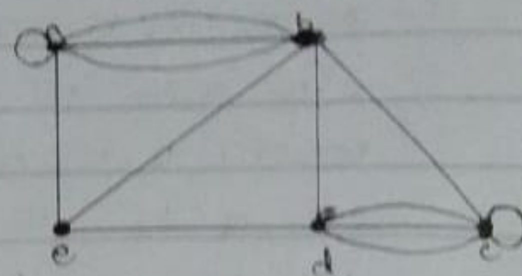
⇒ Pseudograph, b/c loops are also count.

Q22: Construct call graphs for set of seven telephone no's 555-0011, 555-1221, 555,1333, 555-8888, 555-2222, 555-00
& 555-1200. If there were 3 calls from 555-0011 to 555-
& two calls from 555-8888 to 555-0011, two calls from 555-2222 to 555-0091, two calls from 555-1221 to each of the other numbers, & one call from 555-1333 to each of the 555-0011, 555-1221, & 555-1200.



Exercise: 10.2

Q2. Find no. of vertices, the no. of edges & degree of each vertex in given undirected graph.

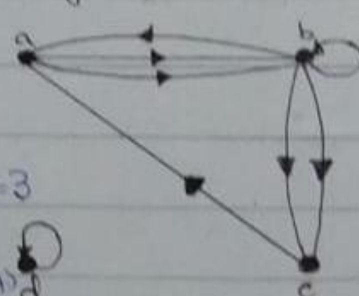


- Vertices: 5
- Edges: 13
- Degrees: $deg(a)=6$, $deg(b)=6$, $deg(c)=6$, $deg(d)=5$, $deg(e)=3$

Q4. Find sum of degrees of vertices of above graph and verify that it equals twice the number of edges in graph.

⇒ The sum of edges degrees is: $6+6+6+5+3=26$, which is twice the number of edges (13).

Q8. Determine no. of vertices and edges and find in-degree & out-degree of each vertex for the given directed multigraph.



- Vertices: 4
- Edges: 8
- In-degree: $deg^-(a)=2$, $deg^-(b)=3$
  $deg^-(c)=2$, $deg^-(d)=1$ (pendant) d

- Out-degree: $deg^+(a)=2$, $deg^+(b)=\cancel{3}4$
  $deg^+(c)=1$ (pendant), $deg^+(d)=1$ (pendant)

Q10: For previous graph, determine sum of in-degrees & out-degrees of vertices. Show that that they are both equal to no. of edges in graph.

⇒ The sum of degrees (in-degrees + out-degrees) are:
$2+4+1+1=8$, which is equal to the number of edges in the digraph.

Q14: What does degree of vertex in Hollywood graph represent? What does neighbourhood of vertex represent? What do isolated and pendant vertices represent?

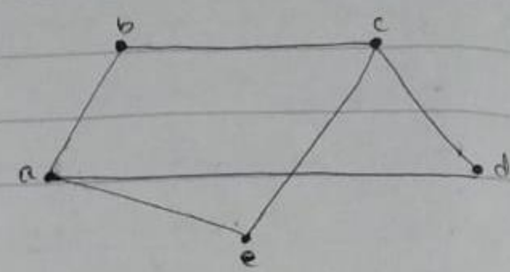
⇒ In Hollywood graph,
[V = Set of all Actors, E = has worked together]

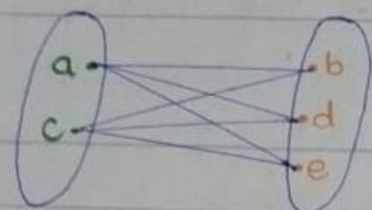
- We do not use loop b/c person is not work with himself/herself.
- Degree of vertex is no. of edges that connect to vertex.
- The neighbourhood of vertex u are all vertex connected to u.
- Neighbourhood of vertex u = All people that u worked with on movies or tv-shows.
- A vertex is isolated if vertex has 0 degree.
- Isolated vertex = All actors who did not work with any other actor on movies/tv-show.
- A vertex is pendant if vertex has degree 1
- Pendant vertex = All actors who did work with exactly one actor

Q23. Determine whether graph is bipartite. (Theorem 4) & answer the question by determining whether it possible to assign either red or blue to each vertex so that no ~~two~~ adjacent assign same colour

$V_1 = \{a, c\}$

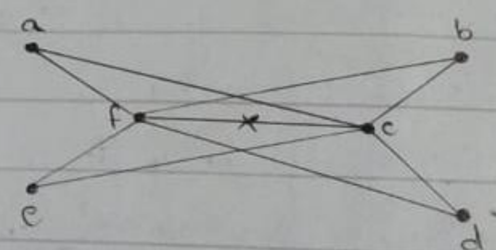
$V_2 = \{b, d, e\}$





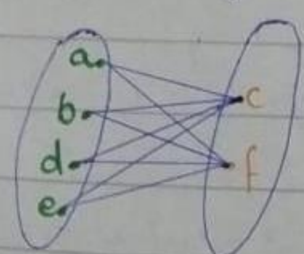
⇒ Yes. This is Complete Bipartite Graph. Every edge joins vertice in d/f parts. Take $\{a, c\}$ to be one & $\{b, d, e\}$ to be other.

Q24c



$V_1 = \{a, b, d, e\}$

$V_2 = \{c, f\}$

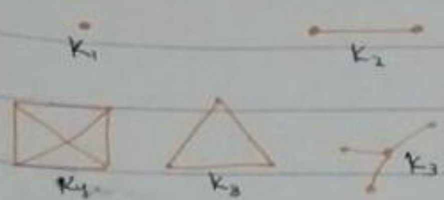


⇒ Yes, This is Complete Bipartite Graph. Every edge joins vertices in d/f parts. Take $\{a, b, d, e\}$ to be one & $\{c, f\}$ to be other.

Q26: For which values of n are these graphs bipartite?

a) K_n

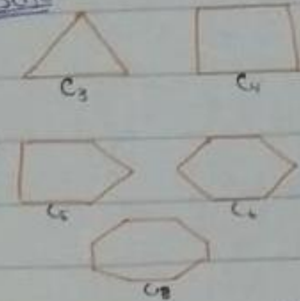
Sol:



So, here K_1 & K_2 are bipartite.

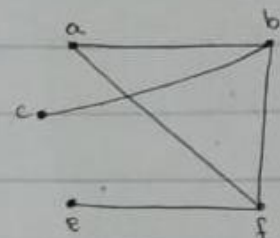
b) C_n

Sol:

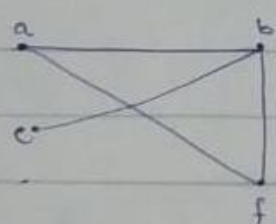


⇒ when 'n' is even then it is bipartite.

Q39: For the graph G in Exe 1 find:

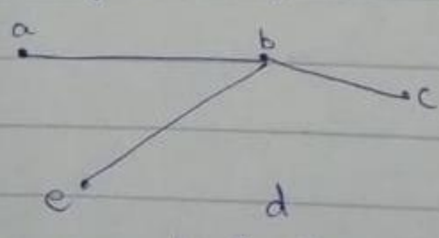


a)



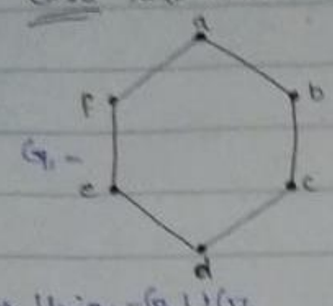
Subgraph ⇒ {a, b, c, f}

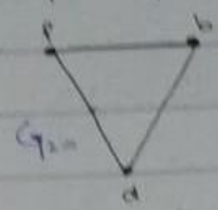
b)



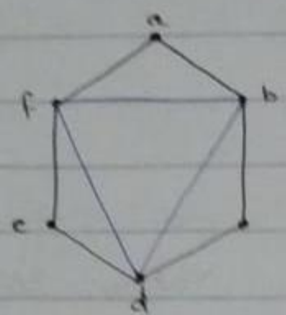
edge are b & f.

Q56: Find Union of given pair of simple graphs.

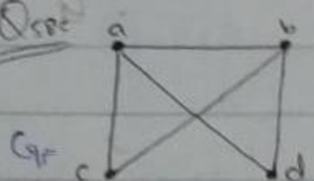


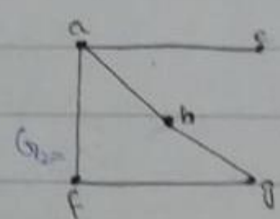


⇒ Union = $G_1 \cup G_2$

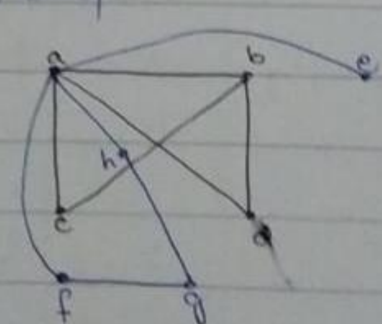


Q58:

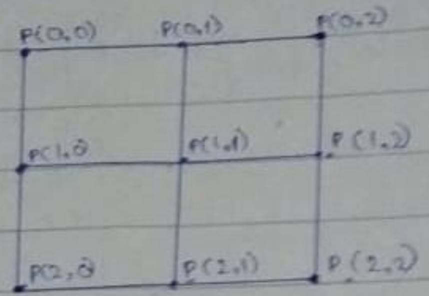




⇒ Union = $G_1 \cup G_2$



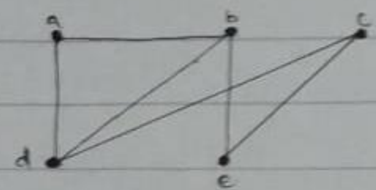
Q2: Draw mesh network for interconnecting nine parallel processors.



Exercise: 10.3

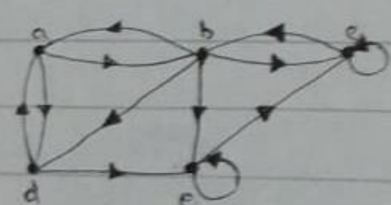
• Use adjacency list to represent the given graph.

Q3:



Vertex	Adjacent Vertex
a	b, d
b	a, d, e, c
c	d, e
d	a, b, c
e	b, c

Q4:

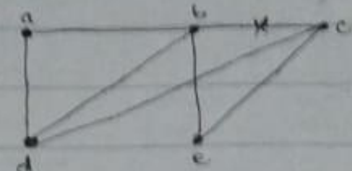


Vertex	Adjacent Vertex
a	b, d
b	a, d, e, c
c	c, b.
d	a, e
e	e, c

Q. Represent graph Ex-2 with adjacency matrix.

→ We order vertices a, b, c, d, e. The matrix representing graph

$$
\begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array}
\begin{array}{c}
\begin{array}{ccccc} a & b & c & d & e \end{array} \\
\begin{bmatrix}
0 & 1 & 0 & 1 & 0 \\
1 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 1 & 1 \\
1 & 1 & 1 & 0 & 0 \\
0 & 1 & 1 & 0 & 0
\end{bmatrix}
\end{array}
$$



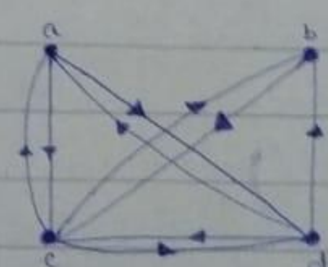
Q. Represent Ex-4 with adjacency matrix.

→ We order vertices a, b, c, d, e. The matrix representation graph,

$$
\begin{array}{c} \\ a \\ b \\ c \\ d \\ e \end{array}
\begin{array}{c}
\begin{array}{ccccc} a & b & c & d & e \end{array} \\
\begin{bmatrix}
0 & 1 & 0 & 1 & 0 \\
1 & 0 & 1 & 1 & 1 \\
0 & 1 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 & 1
\end{bmatrix}
\end{array}
$$

Q. Draw graph with given adjacency matrix.

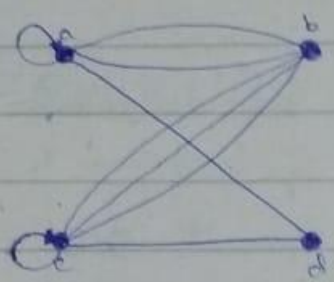
$$
\begin{array}{c} \\ a \\ b \\ c \\ d \end{array}
\begin{array}{c}
\begin{array}{cccc} a & b & c & d \end{array} \\
\begin{bmatrix}
0 & 0 & 1 & 1 \\
0 & 0 & 1 & 0 \\
1 & 1 & 0 & 1 \\
1 & 1 & 1 & 0
\end{bmatrix}
\end{array}
$$



Q14: Represent given graph using adjacent matrix.

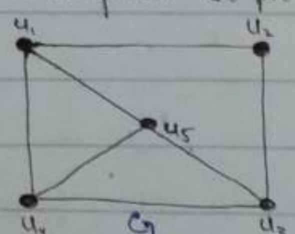
$$\begin{array}{c|cccc} & a & b & c & d \\ \hline a & 0 & 3 & 0 & 1 \\ b & 3 & 0 & 1 & 0 \\ c & 0 & 1 & 0 & 3 \\ d & 1 & 0 & 3 & 0 \end{array}$$

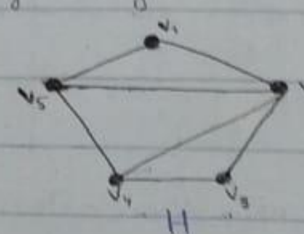
Q17: Draw undirected graph represented by given adjacency matrix.



$$\begin{array}{c|cccc} & a & b & c & d \\ \hline a & 1 & 2 & 0 & 1 \\ b & 2 & 0 & 3 & 0 \\ c & 0 & 3 & 1 & 1 \\ d & 1 & 0 & 1 & 0 \end{array}$$

Q36: Determine whether given pair of graphs is isomorphic. Exhibit an isomorphism or provide a rigorous argument that none exist



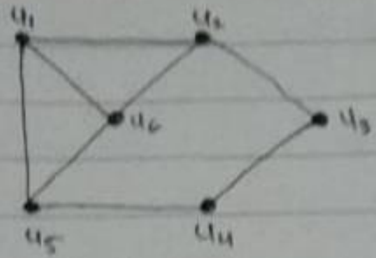


Degree change

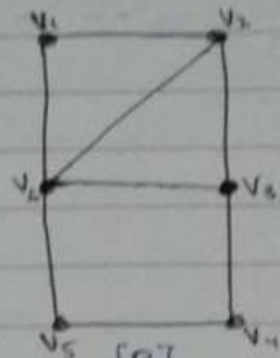
Vertex ⇒ $G = H = 5$

Edges ⇒ $G = H = 7$

Ans



[A]



[B]

⇒ There is not actually isomorphic because of v_6 has degree of 4 ($v_6 = 4$), but no other vertices has degree of 4 in other graph.

X ——— X

Ch: The Foundations: Logic & Proofs

1.1

Exercise : 1.1

Q2. Which of these propositions? What are truth values of those that are propositions

a) Do not pass go.
⇒ Not proposition

b) What time is it?
⇒ Not proposition

c) There are no black flies in Maine
⇒ Proposition (False)

d) $4 + x = 5$
⇒ Not proposition (var. contains)

e) The moon is made of green cheese
⇒ Proposition (False)

f) $2^n \geq 100$
⇒ Not proposition

Q4. What is negation of each of these proposition.

a) Jennifer & Teja are friends.
⇒ Jennifer & Teja are <u>not</u> friends.

b) There are 13 items in a baker's dozen.
⇒ There are <u>not</u> 13 items in a baker's dozen.

c) Abby sent more than 100 text messages every day.
→ This is not the case that Abby sent more than 100 text msgs everyday.

d) 121 is a perfect square.
→ 121 is not a perfect square.

Q: Let p and q be propositions. as an English sentence.

p: I bought a lottery ticket this week.
q: I won the million dollar jackpot.

Express each of these propositions as an English sentence.

a) $\neg p$; I didn't bought a lottery ticket this week.

b) $p \vee q$; I bought a lottery ticket this week, or I won the million dollar jackpot.

c) $p \to q$; If I bought a lottery ticket this week, then I won the million dollar jackpot.

d) $p \wedge q$; I bought a lottery ticket this week, and I won the million dollar jackpot.

e) $p \leftrightarrow q$; I bought a lottery ticket this week if and only if I won the million dollar jackpot.

f) $\neg p \to \neg q$; If I didn't ~~won~~ bought a lottery ticket this week then I didn't won the million dollar jackpot.

g) $\neg p \wedge \neg q$; I didn't bought a lottery ticket this week and I didn't won the million dollar jackpot.

h) $\neg p \vee (p \wedge q)$; I didn't bought a lottery ticket this week or either I bought a lottery ticket this week and I won the million dollar jackport.

Q12: Let p, q & r be propositions.

- p: You have the flu
- q: You miss the final examination
- r: You pass the course.

Express as English sentence.

a) $p \rightarrow q$; If you have the flu, then you miss the final examination.

b) $\neg q \leftrightarrow r$; This is not the case that you miss the final examination if and only if you pass the course.

c) $q \rightarrow \neg r$; If you miss the final examination, then you didnt pass the course.

d) $p \vee q \vee r$; You have the flu or you miss the final examination or you pass the course.

e) $(p \rightarrow \neg r) \vee (q \rightarrow \neg r)$; If you have the flu, then you didnt pass the course or either if you miss the final examination than you didnt pass the course.

f) $(p \wedge q) \vee (\neg q \wedge r)$; You have the flu and you miss the final examination or either you don't miss the final examination and you pass the course.

Que Let p, q and r be propositions.

- p: You get an A on the final exam.
- q: You do every exercise in this book.
- r: You get an A in this class.

Write these propositions using these and logical connective (including negation).

a) You get an A in this class, but you do not do every exercise in this book.

$\Rightarrow$ $(r \land \neg q)$

b) You get an A on the final, you do every exercise in this book, and you get an A in this class.

$\Rightarrow$ $(p \land q \land r)$

c) To get an A in this class, it is necessary for you to get an A on the final.

$\Rightarrow$ $(r \longleftrightarrow p)$

d) You get an A on the final, ~~&~~ but you don't do every exercise in this book; nevertheless, you get an A in this class.

$\Rightarrow$ $(p \land \neg q \land r)$

e) Getting an A on the final and doing every exercise in this book is sufficient for getting an A in this class.

$\Rightarrow$ $(p \land q) \longleftrightarrow r$

f) You will get an A in this class if and only if you either do every exercise in this book or you get an A on the final

$\Rightarrow$ $r \longleftrightarrow (q \lor p)$

T F → False

Q8: Determine whether each of these conditional statements is true or false.

a) If $1+1=3$, then unicorns exist
⇒ True

b) If $1+1=3$, then dogs can fly.
⇒ True

c) If $1+1=2$, then dogs can fly.
⇒ False

d) If $2+2=4$, then $1+2=3$
⇒ True

Q9: Write each of these statements in the form "if p, then q" in English.

a) I will remember to send you address only if you send me an email message.
⇒ If I will remember to send you address then you can send me an email msg.

b) To be a citizen of this country, it is sufficient that you were born in the United States.
⇒ If to be a citizen of this country then you have to born in the United States.

c) If you keep your textbook, it will be a useful reference in your future course.
⇒ If you keep your textbook then it will be a useful reference in your future course.

d) The Red Wings will win the Stanley Cup if their goalie plays well.
⇒ If ~~the Red wings~~ goalie plays well, then Red wings will win the stanley cup.

e) That you get the job implies that you had the best credentials.

⇒ If you get the job than you had the best credentials

f) The beach erodes whenever there is a storm.

⇒ If there is a storm, then beach erodes.

g) It is necessary to have a valid password to log onto the server

⇒ If you have to log on to server than you have an valid password.

h) You will reach the summit unless, you begin your climb too late.

⇒ If you begin your climb too late/early than you will reach the summit.

Q28. State converse, contrapositive, and inverse of each of these condition statements. $q \to p$, $\neg q \to \neg p$, $\neg p \to \neg q$

a) If it ~~know~~ snows tonight, then I will stay at home.

① If I will stay at home, than it snows tonight.

② If I will not stay at home, that it not snows tonight.

③ If it not snows tonight than I will not stay at home.

If it is sunny summer day than I goto beach

b) I go to the beach whenever it is a sunny summer day

① If I goto beach than it is sunny summerday.

② If I don't goto beach than it is not sunny sumerday.

③ If I donot go to beach than it is not sunny summerday.

c) When I stay up late it is necessary that I sleep untill noon.

① When I sleep until noon, it is necessary that I stay up late.

② When I didn't stay up late it is necessary that I will'nt sleep until noon.

③ When I did'nt sleep until noon it is necessary that I will'nt stay up late.

Q32: Construct truth table.

b) $p \leftrightarrow \neg p$

P	$\neg P$	$P \rightarrow \neg P$	$P \leftrightarrow \neg P$
T	F	F	F
F	T	T	F

d) $(p \wedge q) \rightarrow (p \vee q)$

P	q	$P \wedge q$	$P \vee q$	$(P \wedge q) \rightarrow (P \vee q)$
T	T	T	T	T
T	F	F	T	F
F	T	F	T	F
F	F	F	F	T

e) $(q \rightarrow \neg p) \leftrightarrow (p \leftrightarrow q)$

p	q	$\neg p$	$q \rightarrow \neg p$	$p \leftrightarrow q$	$(q \rightarrow \neg p) \leftrightarrow (p \leftrightarrow q)$
T	T	F	F	T	F
T	F	F	T	F	F
F	T	T	T	F	F
F	F	T	T	T	T

Exer: 1.2

Q: 20 Relate to inhabitants of island of knights & knaves created by Sumallyan, where knight always tell the truth & knaves always lie. You encounter two people, A & B. Determine if possible, what A & B are if they address you in the ways described. If you cannot determine what these two people are, can you draw any conclusion.

Q20: A says "The two of us are both knights" ~~"One of us is knaves"~~ & B says ~~nothing~~. "A is knave."

⇒ If we suppose A is knight so in this case A & B both be knights. Which is contradiction. So, we consider B as knight & say "A is knave". So this can be true.

A → knave

B → knight

Q22: Both A & B say "I am a knight".
A → Either knave or knight
B → Either knave or knight

Q24: ~~A says "C is knave~~ In Island there are ~~8~~ inhabitants & three people;
Knight: always tell truth
Knave: always tell lie
Spies: Either lie or truth
You encounter 3 people; A, B & C. Every one have above roles each. Each of person know the type of person each of other two is. If possible Determine for unique solution & determine their person type.

Q25: A says "C is knave", B says "A is knight", C say "I am spy"
A → Knight
B → Spy
C → Knave

Q26: A says "I am knave", B says "I am knave" & C says "I am knave"
This arise contradiction. May be all these 3 are spies.

Q28: A says "I am the knight", B says "A is not the knave"
C says "B is not the knave".
A → knight / Knave
B → Spy
C → Knave / Knight

Q30: A says "I am not the spy" B says 'I am not the spy'
C say "A is spy".

A ⟶ knight

B ⟶ Spy

C ⟶ knave

Ex: 1·3

Q12: Show that each conditional statement in Ex. 10 is a tautology without using truth table.

$\cancel{\neg[\neg p \land (p \lor q}$ $[\neg p \land (p \lor q)] \to r$

a) $\neg[\neg P \land (P \lor r)] \lor r$ — conditional law

$[\neg(\neg P) \lor \neg(P \lor r)] \lor r$ — Demorgan's law

$[P \lor (\neg P \lor \neg q)] \lor r$ — double negation

$(P \lor \neg P) \land (\neg r \lor r)$ — Negation law

$T \land T$

T

b) $[(p \to q) \land (q \to r)] \to (p \to r)$

(17)

c) $[p \wedge (p \rightarrow q)] \rightarrow q$

$[p \wedge (\neg p \vee q)] \rightarrow q$ Conditional law

$[(p \wedge \neg p) \vee (p \wedge q)] \rightarrow q$ distributive "

$[F \vee (p \wedge q)] \rightarrow q$ Negation law

$(p \wedge q) \rightarrow q$ Identity law

$\neg(p \wedge q) \vee q$ conditional "

$\neg p \vee \neg q \vee q$ demorgans "

$\cancel{F} \neg p \vee \neg q \vee q$

$\neg p \vee T$ Negation "

$\underline{T}$ Domination

Proved

d) $[(p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow r$

Q13: Use truth tables to verify the absorption laws.

a) $p \vee (p \wedge q) \equiv p$

p	q	$p \wedge q$	$p \vee (p \wedge q)$
T	T	T	T
T	F	F	T
F	T	F	F
F	F	F	F

b) $p \wedge (p \vee q) \equiv p$

p	q	$p \vee q$	$p \wedge (p \vee q)$
T	T	T	T
T	F	T	T
F	T	T	F
F	F	F	F

Q14: Determine whether $(\neg p \wedge (p \to q)) \to \neg q$ is tautology.

Take R.H.S

$\Rightarrow p$

Q22. Show that $(p \to q) \wedge (p \to r)$ and $p \to (q \wedge r)$ are logically equivalent.

Take R.H.S :-

$p \to (q \wedge r) \equiv \neg p \vee (q \wedge r)$ conditional

$\equiv (\neg p \vee q) \wedge (\neg p \vee r)$ distributive

$\equiv (p \to q) \wedge (p \to r)$ conditional

hence R.H.S = L.H.S

Q26. Show that $\neg p \to (q \to r)$ and $q \to (p \vee r)$ are logically equivalent

$\neg p \longrightarrow (\neg q \vee r)$ conditional

$\neg(\neg p) \vee (\neg q \vee r)$ "

$p \vee (\neg q \vee r)$ double negation

$\neg q \vee (p \vee r)$

$q \longrightarrow (p \vee r)$ conditional.

hence R.H.S = L.H.S

Ex: 1.4

Q2: Let $P(x)$ be statement "the world x contains letter a". What truth values.

a) P(orange)
↳ true

b) P(lemon)
↳ false

c) P(true)
↳ false

d) P(False)
↳ true

Q5: Let $P(x)$ be statement "x spend more than five hours every weekday in class". When domain for x consist all students. Express each quantification in English.

a) $\exists x\, P(x)$

⇒ There Exist a student, who spends more than five hrs in every weekday in class.

b) $\forall x\, P(x)$

⇒ All the students have spend more than five hours every weekday in class.

c) $\exists x \neg P(x)$

⇒ There Exist a student who did not spend more than five hours every weekday in class.

d) $\forall_x \neg P(x)$

⇒ All the students did not spend more than five hours every weekday in class.

Q. Translate statements into English, where $C(x)$ is "x is a comedian" and $F(x)$ is "x is funny" & domain contains all people.

a) $\forall x\,[C(x) \rightarrow F(x)]$

$\Rightarrow$ All the comedians are funny.

b) $\forall x\,[C(x) \wedge F(x)]$

$\rightarrow$ All the people are comedian and funny too.

c) $\exists x\,[C(x) \rightarrow F(x)]$

$\Rightarrow$ There Exist some people if they're funny then also comedyian.

d) $\exists x\,[C(x) \wedge F(x)]$

$\Rightarrow$ There Exist some people who are funny and are comedian too.

Q. Let $P(x)$ be statement "$x = x^2$". If domain is integers what are these truth values?

a) $P(0) \longrightarrow$ True

b) $P(1) \longrightarrow$ True

c) $P(2) \longrightarrow$ False

d) $P(-1) \longrightarrow$ False

e) $\exists x\, P(x) \longrightarrow$ True

f) $\forall x\, P(x) \longrightarrow$ False

Q15: Determine truth value of each statement if domain of all variable consist of all integers.

a) $\forall n\ (n^2 \geq 0) \longrightarrow$ True

b) $\exists n\ (n^2 = 2) \longrightarrow$ False

c) $\forall n\ (n^2 \geq n) \longrightarrow$ True

d) $\exists n\ (n^2 < 0) \longrightarrow$ False

Q20: Suppose each domain of propositional function P(x) consist of -5, -3, -1, 1, ~~2~~, 3, 5. Express statements without using quantifier, instead use negation, disjunction & conjunction.

a) $\exists x\ P(x)$

$\Rightarrow P(-5) \vee P(-3) \vee P(-1) \vee P(1) \vee P(3) \vee P(5)$

b) $\forall x\ P(x)$

$\Rightarrow P(-5) \wedge P(-3) \wedge P(-1) \wedge P(1) \wedge P(3) \wedge P(5)$

c) $\forall x [(x \neq 1) \rightarrow P(x)]$

$\Rightarrow P(-5) \wedge P(-3) \wedge P(-1) \wedge P(3) \wedge P(5)$

d) $\exists x\ [(x \geq 0) \wedge P(x)]$

$\Rightarrow P(1) \vee P(3) \vee P(5)$

e) $\exists x\ (\neg P(x)] \wedge \forall x[(x < 0) \rightarrow P(x)]$

$\Rightarrow [\neg P(-5) \vee \neg P(-3) \vee \neg P(-1) \vee \neg P(1) \vee \neg P(3) \vee \neg P(5)]$

$[P(-5) \wedge P(-3) \wedge P(-1)]$

* User reqt Soft. req. system

Q21: For each of these statements find domain for which statement is true & domain for which statement is false.

a) Everyone is studying discrete mathematics.

⇒ True Domain: Student of class

⇒ False Domain: Student of University.

b) Everyone is older than 21 years.

⇒ True Domain: 19 less than 2000 years.

⇒ False Domain: All

c) Every two people have the same mother.

⇒ True Domain: Siblings (Brother/Sister)

⇒ False Domain: Stranger / Friend

d) No two d/f people have the same grandmother.

⇒ True Domain: Friends / Stranger

⇒ False Domain: family

Q23: Translate in two ways each of these statements into logical expressions using predicators, quantifiers & logical connectives. First let domain consist of student in your class & second, let it consist of all people.

a) Someone in your class can speak Hindi

⇒ $A(x)$ = Hindi speaker $B(x)$ = x is your class

* 1st Domain → $\exists x\, A(x)$
* 2nd Domain → $\exists x\, [B(x) \wedge A(x)]$

b) Everyone in your class is friendly.

* 1^{st} Domain $\longrightarrow$
* 2^{nd} Domain $\longrightarrow$

c) There is a person in your class who was not born in California

$A(x)$ = born in California $C(x)$ = x person in class

* 1^{st} Domain $\longrightarrow \exists(x)\ [\neg A(x)]$
* 2^{nd} Domain $\longrightarrow \exists(x)\ [C(x) \longrightarrow \neg A(x)]$

d) A student in your class has been in a movie.

$A(x)$ = been in movie $B(x)$ = x student in class

* 1^{st} Domain $\longrightarrow \exists(x)\ [A(x)]$
* 2^{nd} Domain $\longrightarrow \exists(x)\ [B(x) \land A(x)]$