What are iterative methods? Why are these methods more suitable for computer use ?(will do later)

What are bracketing methods? pg120 (chapra)

What are open methods? Pg 142 (chapra)

What do you mean by convergence and divergence? Pg 144 (chapra)

What are the advantages and disadvantages of newton Raphson?

Advantages:

- Converges fast (quadratic convergence), if it converges.
- Requires only one guess

Disadvantages:

- Must find the derivative
- Dependent on initial guess
    - May be too far from local root
    - May encounter a zero derivative
    - May loop indefinitely

What are Different types of equations:

different types of algebraic equations :
(1) Linear equation
(2) Quadratic equation
(3) Polynomial equation
(4) Trigonometric equation
(5) Radical equation
(6) Exponential equation

Solving equations means to find the value or set of values of unknown variable contained in it. Let us go ahead in this chapter and learn more about different types of differential equations and sample problems based on those.

Different Types of Equations

Numerical Analysis/Numerical Computing
Department of Computer science
University of Karachi
Supervisor: MS. Shaista Rais

An equation is an important part of calculus, which comes under mathematics. It helps us to solve many problem.

Types of Algebraic Equations

Lets discuss different types of algebraic equations:

1) Linear Equations:

A linear equation is an algebraic equation in which each term is either a constant or the product of a constant and a single variable. The graph of linear equation is a straight line if there are two variables.

General form of the linear equation with two variables:

$y = mx + c, m \neq 0.$

Where, m is known as slope and c at that point on which it cut y axis.

Linear equations with different variables:

a) The equation with one variable:

An equation who have only one variable.

Examples:

1. $8a - 8 = 0$
2. $9a = 72.$

b) The equation with two variables:

An equation who have only two types of variable in the equation.

Examples:

1. $7x + 7y = 12$
2. $8a - 8d = 74$
3. $9a + 6b - 82 = 0.$

c) The equation with three variables:

An equation who have only three types of variable in the equation.

Examples:

1. 5x + 7y - 6z = 12
2. 13a - 8b + 31c = 74
3. 6p + 14q -7r + 82 = 0.

2) Radical Equations:

An equation whose maximum exponent on the variable is 12 and have more than one term or we can say that a radical equation is an equation in which the variable is lying inside a radical symbol usually in a square root.

Examples:

1. x√+10=26
2. x2−5−−−−−√+x−1

3) Quadratic Equations:

Quadratic equation is the second degree equation in one variable contains the variable with an exponent of 2.

The general form:

$ax^2 + bx + c = 0, a \neq 0$

Examples of Quadratic Equations:

1. $8x^2 + 7x - 75 = 0$
2. $4y^2 + 14y - 8 = 0$
3. $5a^2 - 5a = 35$

4) Exponential Equations:

An equation who have variables in the place of exponents. Exponential equation can be solved using the property: ax = ay => x = y.

Examples:

1. $a^b = 0$ Here "a" is base and "b" is exponent.

2. $4^x = 0$
3. $8^x = 32$.

5) Rational Equations:

A rational equation is one that involves rational expressions.

Example:

x2 = x+24.

Factors and Roots of a Polynomial Equation

Here are three important theorems relating to the roots of a polynomial:

(a) A polynomial of n-th degree can be factored into n linear factors.

(b) A polynomial equation of degree n has exactly n roots.

(c) If $(x-r)$ is a factor of a polynomial, then $x=r$ is a root of the associated polynomial equation.

Let's look at some examples to see what this means.

Example 1

The cubic polynomial $f(x) = 4x^3 - 3x^2 - 25x - 6$ has degree 3 (since the highest power of x that appears is 3).

$4x^3 - 3x^2 - 25x - 6 = (x - 3)(4x + 1)(x + 2)$

So we see that a 3rd degree polynomial has 3 roots.

The associated polynomial equation is formed by setting the polynomial equal to zero:

$f(x) = 4x^3 - 3x^2 - 25x - 6 = 0$

In factored form, this is:

$(x-3)(4x+1)(x+2)=0$

We see from the expressions in brackets and using the 3rd theorem from above, that there are 3 roots, $x=3, -\frac{1}{4}, -2$.

In this example, all 3 roots of our polynomial equation of degree 3 are real.

Since $(x-3)$ is a factor, then $x=3$ is a root.

Since $(4x+1)$ is a factor, then $x=-1/4$ is a root.

Since $(x+2)$ is a factor, then $x=-2$ is a root.

Example 2

The equation $x^5 - 4x^4 - 7x^3 + 14x^2 - 44x + 120 = 0$ can be factored as:

$(x - 2)(x - 5)(x + 3)(x^2 + 4) = 0$

We see there are 3 real roots ,$x=2,5,-3$, and 2 complex roots $x=\pm 2j$, (where $j=\sqrt{-1}$).

So our 5th degree equation has 5 roots altogether.

Advantages and disadvantages of secant method:

Secant Method

The secant method is a recursive method used to find the solution to an equation like Newton's Method. The idea for it is to follow the secant line to its *x*-intercept and use that as an approximation for the root. This is like Newton's Method (which follows the tangent line) but it requires two initial guesses for the root.

The big advantage of the secant method over Newton's Method is that it does not require the given function *f(x)* to be a differential function or for the algorithm to have to compute a derivative. This can be a big deal in other languages since many derivatives can only be estimated.

Problems With the Secant Method

The number of iterations required can not be determined before the algorithm begins.

The algorithm will halt (program termination by division by zero if not checked for) if a horizontal secant line is encountered.

The secant method will sometimes find an extraneous root.

Advantages of the Bisection method

1. Bisection can be shown to be an "optimal" algorithm for functions that change sigh in [a,b] in that it produces the smallest interval of uncertainty in a given # of iterations
2. f(x) need not be continuous on [a,b]
3. convergence is guaranteed (linearly)

Disadvantages of the Bisection Method

4. Two initial guesses are required, with f(a) f(b) <0. This may prove difficult
5. If there are multiple zeros in the interval there is no guidance as to which will be chosen
6. Linear convergence may be slow compared to other methods