Course: Numerical Analysis
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What is the bisection method and what is it based on?

One of the first numerical methods developed to find the root of a nonlinear equation $f(x)=0$ was the bisection method (also called *binary-search* method). The method is based on the following theorem.

Theorem

An equation $f(x)=0$, where $f(x)$ is a real continuous function, has at least one root between x_ℓ and x_u if $f(x_\ell)f(x_u)<0$ (See Figure 1).

Note that if $f(x_\ell)f(x_u)>0$, there may or may not be any root between x_ℓ and x_u.If $f(x_\ell)f(x_u)<0$, then there may be more than one root between x_ℓ and x_u (Figure 4). So the theorem only guarantees one root between x_ℓ and x_u.

Bisection method

Since the method is based on finding the root between two points, the method falls under the category of bracketing methods.

Since the root is bracketed between two points, x_ℓ and x_u, one can find the mid-point, x_m between x_ℓ and x_u. This gives us two new intervals

1. x_ℓ and x_m, and
2. x_m and x_u.

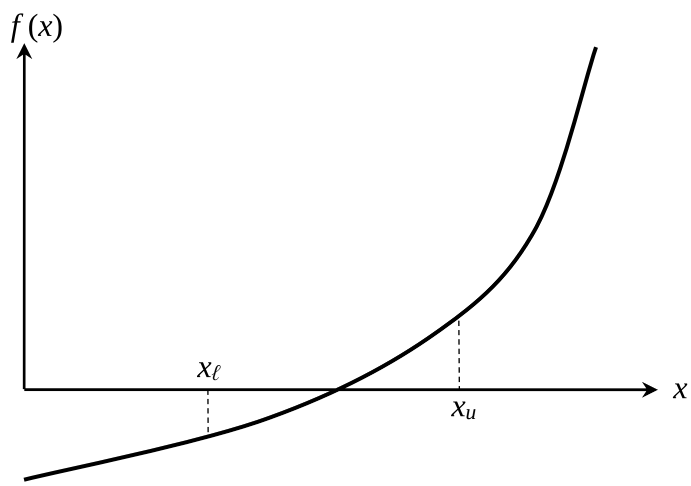

Figure 1 At least one root exists between the two points if the function is real, continuous, and changes sign.

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Algorithm for the bisection method

The steps to apply the bisection method to find the root of the equation $f(x)=0$ are

1. Choose x_ℓ and x_u as two guesses for the root such that $f(x_\ell)f(x_u)<0$, or in other words, $f(x)$ changes sign between x_ℓ and x_u.
2. Estimate the root, x_m, of the equation $f(x)=0$ as the mid-point between x_ℓ and x_u as

$$x_m = \frac{x_\ell + x_u}{2}$$

3. Now check the following
   a) If $f(x_\ell)f(x_m)<0$, then the root lies between x_ℓ and x_m; then $x_\ell = x_\ell$ and $x_u = x_m$.
   b) If $f(x_\ell)f(x_m)>0$, then the root lies between x_m and x_u; then $x_\ell = x_m$ and $x_u = x_u$.
   c) If $f(x_\ell)f(x_m)=0$; then the root is x_m. Stop the algorithm if this is true.
4. Find the new estimate of the root

$$x_m = \frac{x_\ell + x_u}{2}$$

   Find the absolute relative approximate error as

$$|\in_a| = \left|\frac{x_m^{\text{new}} - x_m^{\text{old}}}{x_m^{\text{new}}}\right| \times 100$$

   where

   x_m^{new} = estimated root from present iteration

   x_m^{old} = estimated root from previous iteration

5. Compare the absolute relative approximate error $|\in_a|$ with the pre-specified relative error tolerance $\in_s$. If $|\in_a| > \in_s$, then go to Step 3, else stop the algorithm. Note one should also check whether the number of iterations is more than the maximum number of iterations allowed. If so, one needs to terminate the algorithm and notify the user about it.

ACCURACY: REFERES TO HOW CLOSELY MEASURED OR CALCULATED VALUE AGRESS WITH THE TRUE SOLUTION/ VALUE

PRECISION: REFERS TO HOW CLOSELY MEASURED OR CALCULATED VALUES AGREE WITH EACH OTHER WHEN THEY ARE REPEATED.