

### SECANT METHOD (derivation)

The secant method can also be derived from geometry, as shown in Figure 1. Taking two initial guesses,  $x_{i-1}$  and  $x_i$ , one draws a straight line between  $f(x_i)$  and  $f(x_{i-1})$  passing through the  $x$ -axis at  $x_{i+1}$ .  $ABE$  and  $DCE$  are similar triangles.

Hence

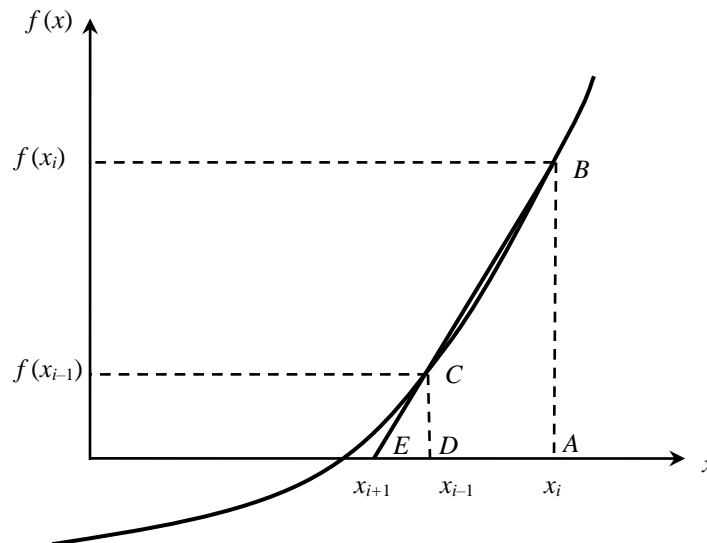
$$\frac{AB}{AE} = \frac{DC}{DE}$$

$$\frac{f(x_i)}{x_i - x_{i+1}} = \frac{f(x_{i-1})}{x_{i-1} - x_{i+1}}$$

On rearranging, the secant method is given as

$$x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$

$$X_r = x_u - f(x_u)(x_u - x_l) / f(x_u) - f(x_l)$$



**Figure 1** Geometrical representation of the secant method.

Qa). Find out the root of the function  $f(x) = x^4 - x - 10$  using Secant method with  $[1, 2]$  initial bounds.

b) stop the iterative procedure when the following conditions get satisfied:

- i.  $|f(x_r)| < E_s$
- ii.  $E_{abs} < E_s$

iii.  $E_r < E_s$  , where  $E_s = 0.001$

c) **comment on the efficiency** of the following conditions to approximate the root of the given function.

			$x^4 - x - 10$		secant method				
s.no	$x_0$	$f(x_0)$	$x_1$	$f(x_1)$	$x_2$	$f(x_2)$	error < 0.001	Eabs < 0.001	$E_r < 0.001$
1	1	-10	2	4	1.71428571	-3.07788	FALSE		
2	1.714286	-3.07788	2	4	1.83853125	-0.4128	FALSE	FALSE	FALSE
3	1.838531	-0.4128	2	4	1.85363596	-0.04777	FALSE	FALSE	FALSE
4	1.853636	-0.04777	2	4	1.85536335	-0.00543	FALSE	FALSE	TRUE
5	1.855363	-0.00543	2	4	1.85555944	-0.00062	TRUE	TRUE	TRUE

## Open methods

Characteristics:

- 1) Its works with at least one initial bounds
- 2) The root will always be located beyond the bound(s) or in other words the root will not be bracketed between the bounds.

Secant method:

1. belongs to open method category
2. it takes two initial bounds to start with