Newton Raphson Method

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Derivation

Derivation

The Newton-Raphson method is based on the principle that if the initial guess of the root of $f(x)=0$ is at x_i, then if one draws the tangent to the curve at $f(x_i)$, the point x_{i+1} where the tangent crosses the x-axis is an improved estimate of the root (Figure 1).

Using the definition of the slope of a function, at $x = x_i$

$$f'(x_i) = \tan\theta$$

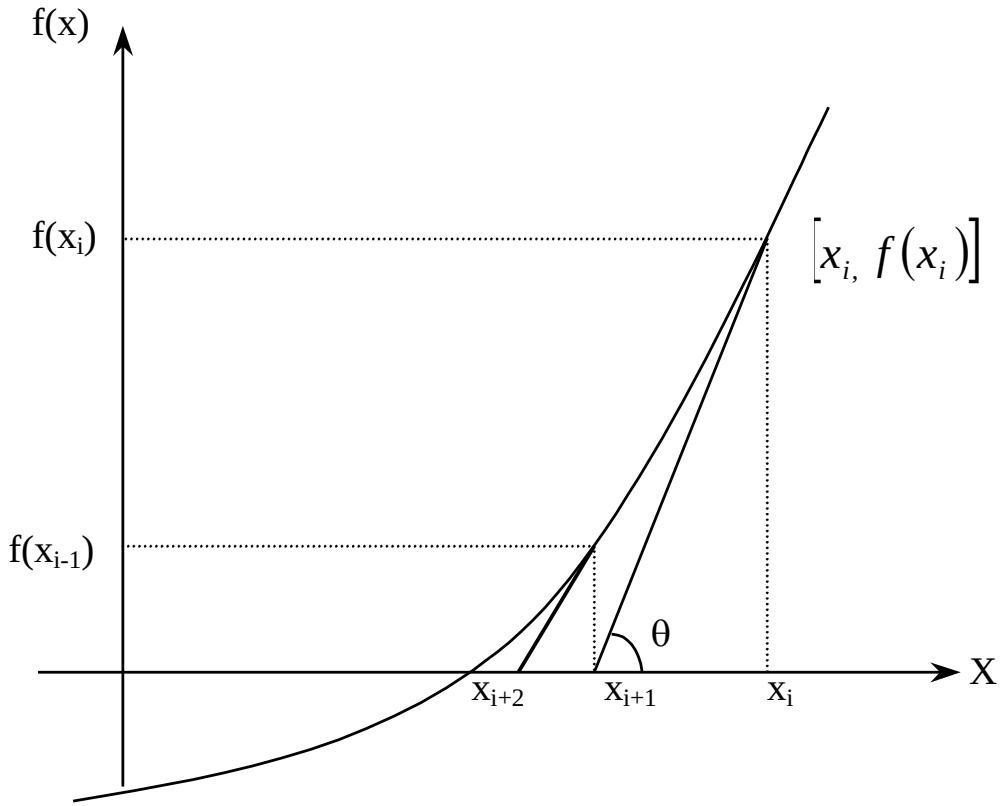
$$= \frac{f(x_i)-0}{x_i - x_{i+1}},$$

Which gives

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)} \qquad (1)$$

Equation (1) is called the Newton-Raphson formula for solving nonlinear equations of the form $f(x)=0$. So starting with an initial guess, x_i, one can find the next guess, x_{i+1}, by using Equation (1). One can repeat this process until one finds the root within a desirable tolerance.

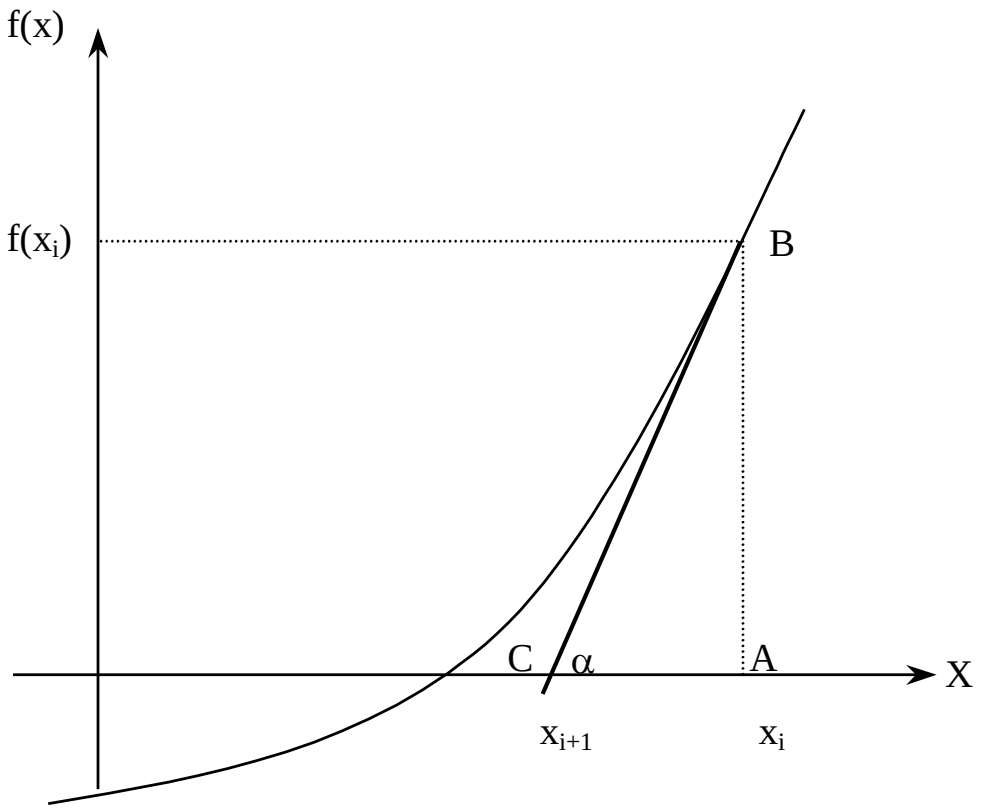
Newton-Raphson Method



$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

Figure 1 Geometrical illustration of the Newton-Raphson method.

Derivation



$$\tan(\alpha) = \frac{AB}{AC}$$

$$f'(x_i) = \frac{f(x_i)}{x_i - x_{i+1}}$$

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

Figure 2 Derivation of the Newton-Raphson method.

Algorithm for Newton-Raphson Method

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Step 1

Evaluate $f'(x)$ symbolically.

Step 2

Use an initial guess of the root, x_i , to estimate the new value of the root, x_{i+1} , as

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

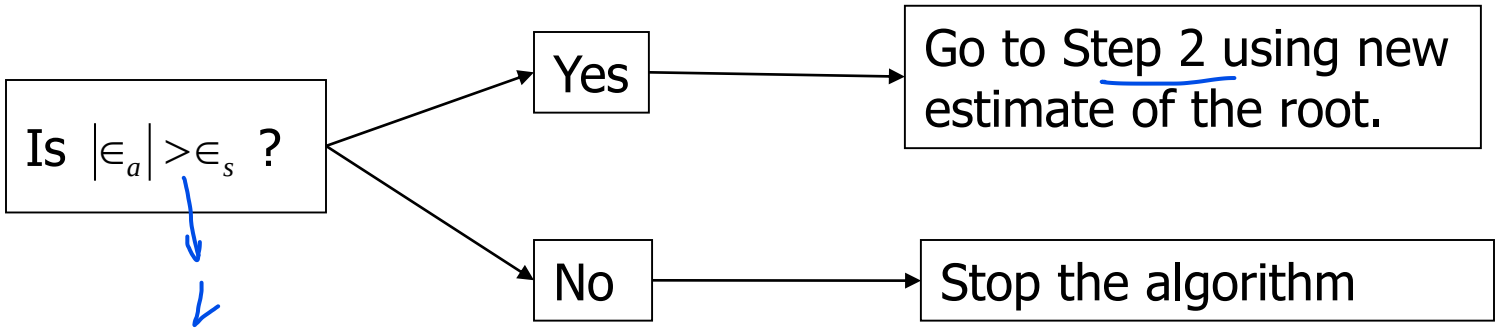
Step 3

Find the absolute relative approximate error $\left|\epsilon_a\right|$ as

$$\left|\epsilon_a\right| = \left|\frac{x_{i+1} - x_i}{x_{i+1}}\right| \times 100$$

Step 4

Compare the absolute relative approximate error with the pre-specified relative error tolerance $\in_s$.



Also, check if the number of iterations has exceeded the maximum number of iterations allowed. If so, one needs to terminate the algorithm and notify the user.

Question

	f(x) =	x^2-5	newton Raphson	
s.no	xi	f(xi)	f'(xi)	xi+1
1	1	-4	2	3
2	3	4	6	2.333333
3	2.333333	0.444444	4.666667	2.238095
4	2.238095	0.00907	4.47619	2.236069
5	2.236069	4.11E-06	4.472138	2.236068

- Q(a) Find out the approximation to the root of the function
- f(x)= x^2-5 using Newton Raphson method with xi=1.
- Your approximation must be significant to FIVE decimal places.

Newton Raphson (Characteristics)

- It’s a numerical root finding method
- Newton Raphson also belongs to category of open method.
- This method requires only 1 initial bound to start with
- Root will also be located beyond the Initial bound
- It is the fastest numerical root finding method

- **Open method :**
- Requires at least 1 initial bound
- Root will be located beyond the initial bound(s)