Trapezoidal Rule of Integration

Prepared By Shaista Rais

What is Integration

Integration:

The process of measuring the area under a function plotted on a graph.

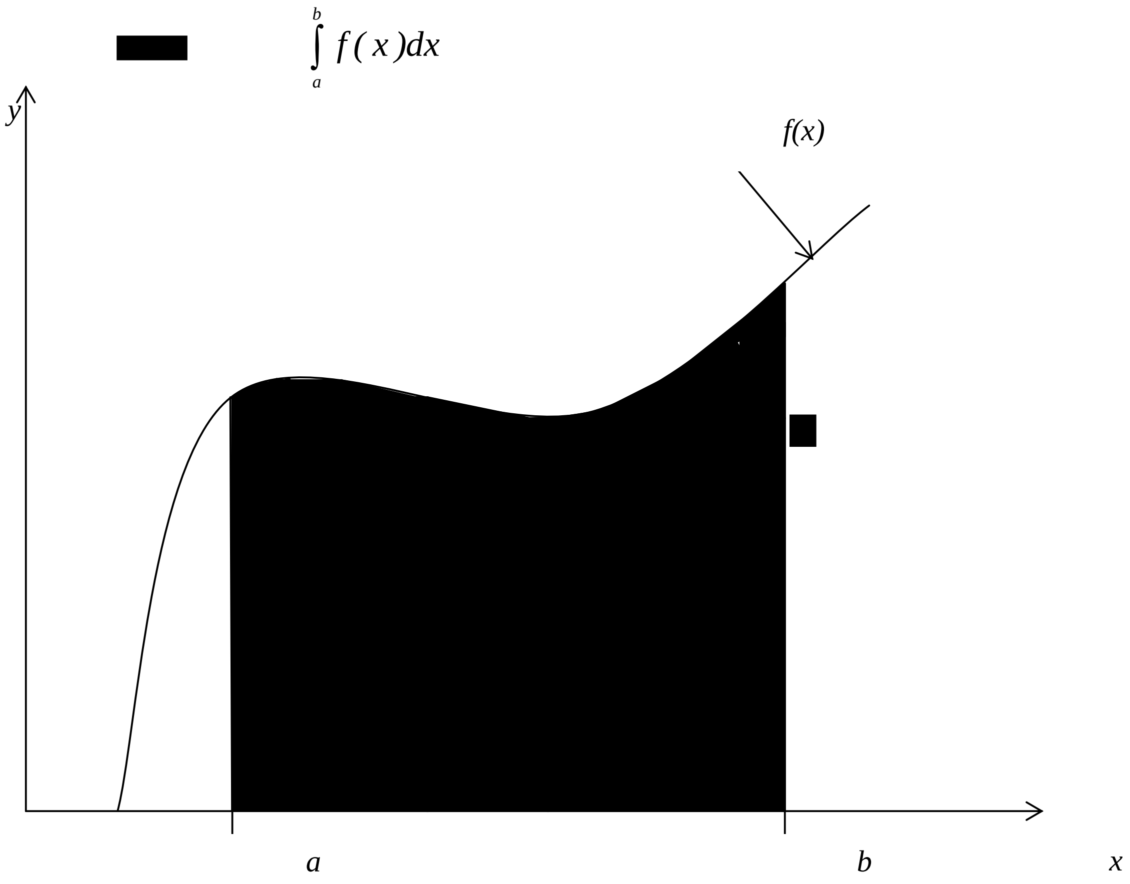
$$\boxed{I = \int_a^b f(x)dx}$$

Where:

f(x) is the integrand

a= lower limit of integration

b= upper limit of integration



Basis of Trapezoidal Rule

Trapezoidal Rule is based on the Newton-Cotes Formula that states if one can approximate the integrand as an n^{th} order polynomial…

$$I = \int_a^b f(x)\,dx \qquad \text{where} \qquad f(x) \approx f_n(x)$$

and

$$f_n(x) = a_0 + a_1 x + \ldots + a_{n-1} x^{n-1} + a_n x^n$$

Basis of Trapezoidal Rule

Then the integral of that function is approximated by the integral of that n^{th} order polynomial.

$$\int_a^b f(x) \approx \int_a^b f_n(x)$$

Trapezoidal Rule assumes n=1, that is, the area under the linear polynomial,

$$\int_a^b f(x)dx = (b-a)\left[\frac{f(a)+f(b)}{2}\right]$$

Derivation of the Trapezoidal Rule

Method Derived From Geometry

The area under the curve is a trapezoid. The integral

$$\int_a^b f(x)dx \approx Area\ of\ trapezoid$$

$$= \frac{1}{2}(Sum\ of\ parallel\ sides)(height)$$

$$= \frac{1}{2}\left(f(b) + f(a)\right)(b-a)$$

$$= (b-a)\left[\frac{f(a)+f(b)}{2}\right]$$

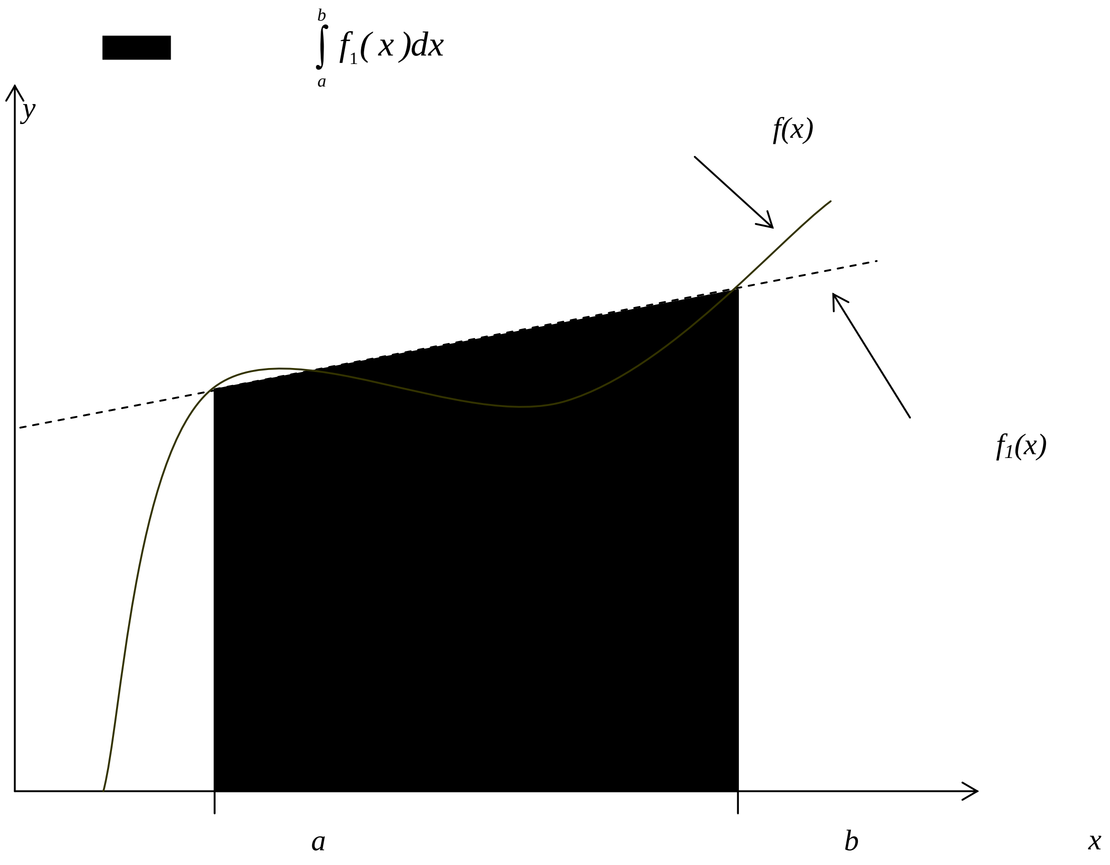


Figure 2: Geometric Representation

Example 1

The vertical distance covered by a rocket from t=8 to t=30 seconds is given by:

$$x = \int_{8}^{30} \left(2000\, ln\left[\frac{140000}{140000 - 2100t} \right] - 9.8t \right) dt$$

a) Use single segment Trapezoidal rule to find the distance covered.
b) Find the true error, E_t for part (a).
c) Find the absolute relative true error, $|\in_a|$ for part (a).

Solution

a) $$I \approx (b-a)\left[\frac{f(a)+f(b)}{2}\right]$$

$$a = 8 \qquad b = 30$$

$$f(t) = 2000\,ln\left[\frac{140000}{140000-2100t}\right] - 9.8t$$

$$f(8) = 2000\,ln\left[\frac{140000}{140000-2100(8)}\right] - 9.8(8) \qquad = 177.27\ m/s$$

$$f(30) = 2000\,ln\left[\frac{140000}{140000-2100(30)}\right] - 9.8(30) \quad = 901.67\ m/s$$

Solution (cont)

a) $$I = (30 - 8)\left[\frac{177.27 + 901.67}{2}\right]$$

$$= 11868\ m$$

b) The exact value of the above integral is

$$x = \int_{8}^{30}\left(2000\ ln\left[\frac{140000}{140000 - 2100t}\right] - 9.8t\right)dt \quad = 11061\ m$$

Solution (cont)

b) $$E_t = True\ Value - Approximate\ Value$$

$$= 11061 - 11868$$

$$= -807\ m$$

c) The absolute relative true error, $|\in_t|$, would be

$$|\in_t| = \left|\frac{11061 - 11868}{11061}\right| \times 100 = 7.2959\%$$