Stochastic Model
Markov process and Markov chain

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Markov Process

Question: Consider a survey that is made on two brands of cotton fabric, let's say Gul Ahmed and Alkaram. Every time a customer (he/she) buys the same brand or switch to another brand. The transition probabilities are given below:

Transition probability matrix

Future

2x2

Present

From To	Gul Ahmed	Alkaram
Gul Ahmed	0.7	0.3
Alkaram	0.6	0.4

Consider first row , we can say if a customer purchased gulahmed brand in the last visit , the probability of switching from gul ahmed to Alkaram brand is 30% in his next visit.

Cola example (COKE VS PEPSI)

- Given that a person last cola purchase was coke, there is a 90% chance that is next cola purchase will also be coke. If a person last cola purchase was Pepsi, there is a 80% chance that is next cola purchase will also be Pepsi. Construct the TPM.

- Note: since there are two states coke and pepsi , so the TPM is gonna be of 2*2 matrix

	coke	Pepsi
Coke	0.9	0.1
Pepsi	0.2	0.8

Question: Three players Kamran, Ali and Salman, are throwing a plastic ring to each other. Kamran always throws the ring to Ali and Ali always throws the ring to Salman, but Salman is as likely to throw the ball to Ali and Kamran. Find the transition matrix.

Since there are three states: so the transition matrix will be of 3*3 matrix

From To	Kamran	Ali	Salman
Kamran			
Ali			
Salman			

Kamran Always throws to Ali

From To	Kamran	Ali	Salman
Kamran	0	1	0
Ali			
Salman			

Ali Always throws to Salman

From To	Kamran	Ali	Salman
Kamran	0	1	0
Ali	0	0	1
Salman			

Salman is as likely to throw the ring to Kamran and Ali

From To	Kamran	Ali	Salman
Kamran	0	1	0
Ali	0	0	1
Salman	0.5	0.5	0

khaadi = 40%
saphie = J.

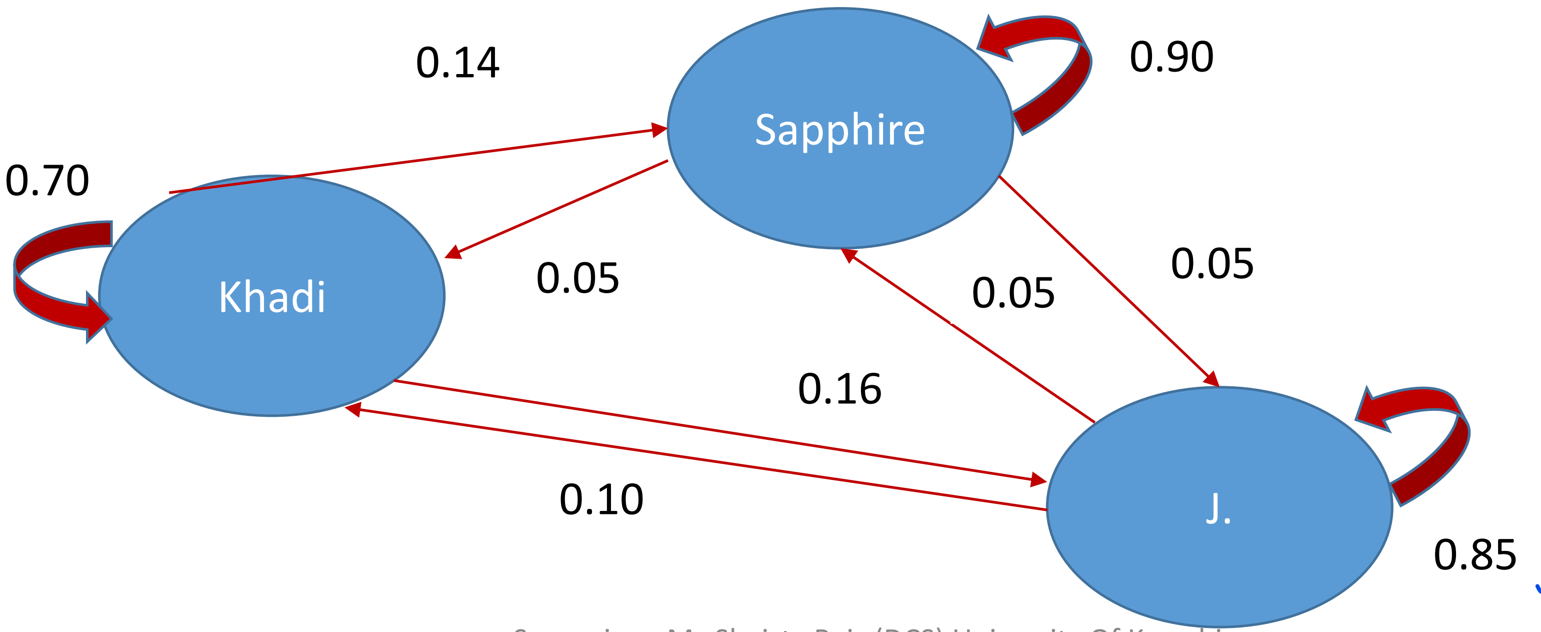
Question: Brand Khadi has 40% market share in the local markets for its clothing product, while the other two branded companies, Sapphire and J. have equal share each on January 1st, 2022. A study by the market research company has disclosed the following data for every year.

- Brand khadi retains 70% of its customers and gains 5% from Saphhire brand and 10% from J. brand.
  khaadi -> 70% C
  5% --> saphire, 10% --> J.
- Sapphire retains 90% of its customers and gains 14% from Khadi and 5% from J.
  sapphire -> 90% C
  14% --> khaadi. 5%--> J.
- J. retains 85% of its customers and gains 16% from Khadi and 5% from Sapphire.
  j. -> 85% C
  16%--> khaadi, 5%--> sapphire
- Construct the TPM.

From To	Khadi	Sapphire	J.
Khadi	0.70	0.14	0.16
Sapphire	0.05	0.90	0.05
J.	0.10	0.05	0.85

Make a transition diagram from the TPM

From To	Khadi	Sapphire	J.
Khadi	0.70	0.14	0.16
Sapphire	0.05	0.90	0.05
J.	0.10	0.05	0.85



MARKOV CHAIN

- How to calculate the probability of the states.

Example: $P(X_3 = 4);$ $P(X_2 = 5)$ etc

- How to calculate the probability after n-steps.

Example: $p_{ij}{}^{(2)},\ p_{23}{}^{(3)}, P(X_3 = 4|X_1 = 2)$ etc

- How to find the probability of the chain.

Example: $P(X_3 = 4, X_1 = 2),$

$P(X_3 = 2, X_2 = 3, X_1 = 1, X_0 = 4)$

- 1- step transition probability:

$$pij = P(X_{n+1} = j \,|X_n = i)$$

- n-step transition probability:

$$p_{ij}{}^{(n)} = P(X_{n+1} = j|X_1 = i)$$

Remember (n) = time , i = from, j = to

Transition probabilities

Probabilities FROM state i TO state j after 1 step time period, denoted by p_{ij} is defined as,

$$\boxed{p_{ij} = P\{X_{n+1} = j | X_n = i\}}$$

Examples:

$$P(X_2 = 3 | X_1 = 2) = p_{23}^{(1)}$$

$$P(X_2 = 2 | X_1 = 0) = p_{02}$$

...

1-day

n-step probabilities

Probabilities FROM state i TO state j after n- step time period, denoted by $p_{ij}(n)$ or $p_{ij}^{(n)}$ is defined as,

$$\boxed{p_{ij}^{(n)} = P\{X_{n+1} = j | X_1 = i\}}$$

Example:

$$P(X_2 = 3 | X_0 = 2) = p_{23}^{(2)}$$

$$P(X_3 = 2 | X_1 = 1) = p_{12}^{(2)}$$

Notations:

q_0	Initial probability of the states.
q_1	Probabilities of the states after the 1 time period.
q_2	Probabilities of the states after the 2 time period.
...	
q_n	Probabilities of the states after the n-time period.
P	TPM after 1 time period.
P^2	TPM after 2 time period.
...	
P^n	TPM after n- time period.

Calculating probability of the states

$$X_n = i$$

Where n = time n i= state

$$P(X_n = a) = \ q_n\ (a)$$

We have formula for calculating probability of states:

$$q_n = \ q_o P^n$$

OR

$$q_{n+1} = q_n P$$

Ali uses his pointer or , ball point or marker to write notes well each day. The tpm of the markov chain with these three states 1(pointer) , 2(ball point) , 3(marker) is

- P =

	P	B	M
P	0.1	0.5	0.4
B	0.6	0.2	0.2
M	0.3	0.4	0.3

And the initial probability is q0 = (0.7,0.2,0.1). Calculate P(X_2 =3)

P(X2=3) = $q_2 = 3 \, , it \ means \ m \ trying \ find \ the \ probability \ after \ 2 \ time \ period$,
$q2 = q_0 P^2$

0.1	0.5	0.4
0.6	0.2	0.2
0.3	0.4	0.3

*

0.1	0.5	0.4
0.6	0.2	0.2
0.3	0.4	0.3

P^2=

0.43	0.31	0.26
0.24	0.42	0.34
0.3	0.35	0.29

(0.7,0.2,0.1).*P^2 = [0.385 0.336 0.279]

P$(X_2 = 3) = 0.279$ (ans)