Assignment: Statistics and Probability

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Class: CSSE-417 (Morning)

Question: 1

a: Define statistics by given examples.

Statistics:

A branch of mathematics dealing with collection, analysis, interpretation & presentations of masses of commercial data.

[OR]

Statistics are defined as numerical data & is the field of math that deals with the collection, tabulation & interpretation of numerical data.

examples: some of the examples where statistics plays a major role are below:

i) Weather forecasts
ii) Banking / Business statistics
iii) Consumer goods
iv) A report of number saying how many followers are of each religion in a country.

b: In the given data, we measure lengths of 50 neem leaves.

i) Construct a stem-and leaf plot & find value of median from it.

Stem	Leaf
4	9
5	1,2,3,4,4 555555 666 777 88 9999
6	000000 111 2222 333 44 6 77 8 99
7	70

Key : 4/9 means 4.9

Median: $\frac{6.0 + 6.0}{2}$

$= \frac{12.0}{2}$

$\boxed{= 6.0 \text{ units}}$

ii) Comment on the shape of distribution.
stem & leaf plot shows that the distribution is symmetric.

Question: 2

The number of road accident reported by Police per day for last two months.

a) Construct a frequency distribution.

Upper boundary = 57
lower boundary = 0
$n = 60$
No. of Classes $= \sqrt{n}$
$= \sqrt{60}$
≈ 7 or 8.

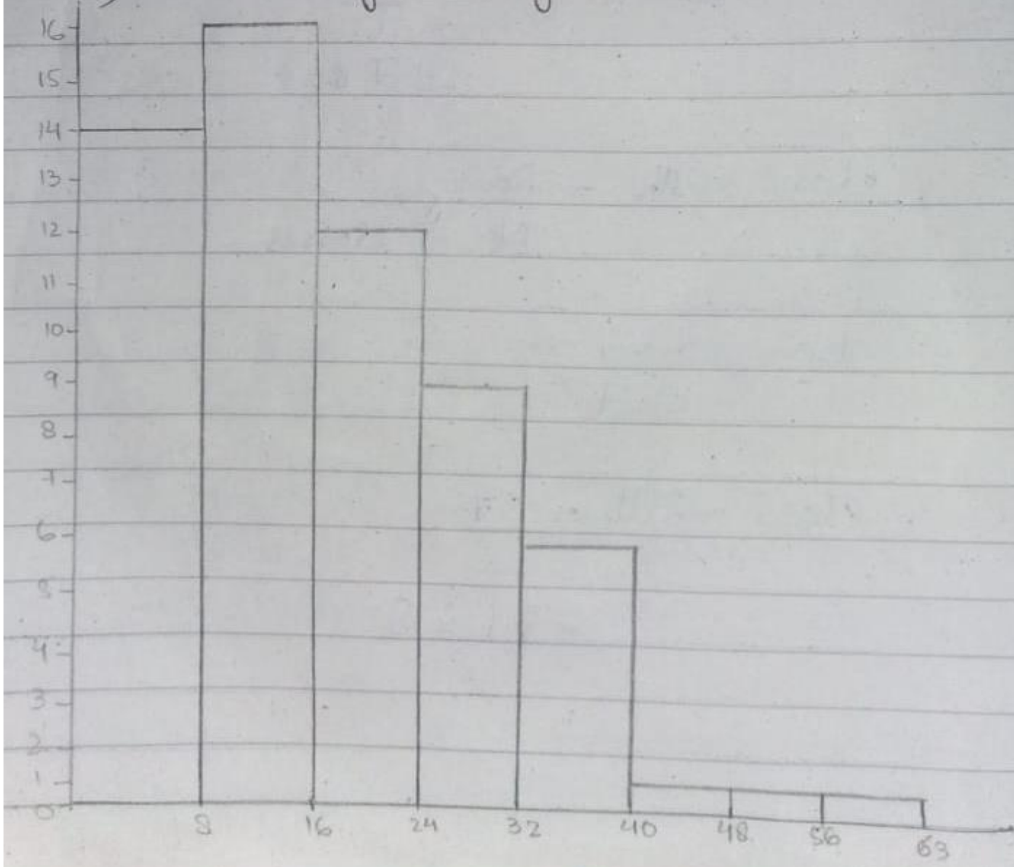
$$\text{class width} = \frac{\text{Range}}{\text{No. of classes}}$$

Range $= 57 - 0$
$= 57$

$$\text{Class width} = \frac{57}{8}$$
$$= 7.1 = 8$$

Class limits	Tally	Frequency
0 – 7	卌 卌 ||||	14
8 – 15	卌 卌 卌 |	16
16 – 23	卌 卌 ||	12
24 – 31	卌 ||||	9
32 – 39	卌 |	6
40 – 47	|	1
48 – 55	|	1
56 – 63	|	1

b) Constructing a histogram



c) Comments: on the shape of distribution.

The histogram is skewed to the right

d) As insurance person, what you understand for your motor department.

As an insurance person fewer accidents mean fewer claims. As the graph shows decrease in the number of accidents then as an insurance person. I could recieve outsized profits so it is in the favour of my motor department.

Question: 3

Consider the frequency distribution of the lengths of snakes measured in cms.

Class-In	35-39	40-44	45-49	50-54	55-59	60-64	65-69	70-74	75-79	80-84
frequen	7	8	12	26	34	42	40	15	17	9

a) Find the relation b/w Mean, Median and Mode.

Class Interval	frequency	Mid point	frequency x Mid point
35-39	7	37	259
40-44	8	42	336
45-49	12	41	564
50-54	26	52	1352
55-59	34	57	1930
60-64	42	62	2604
65-69	40	67	2680
70-74	15	72	1080
75-79	17	77	1309
80-84	9	82	738
	$\Sigma f = 210$		$= 121860$

Mean:

$$\text{Mean} = \frac{\Sigma (f.m)}{\Sigma f}$$

$$= \frac{121860}{210}$$

$\boxed{\text{Mean} = 61.2}$

Median:

class interval

$$\text{Median} = L + \frac{h}{f}\left[\frac{\Sigma f}{2} - C\right]$$

$$= 59.5 + \frac{5}{42}\left[\frac{210}{2} - 87\right]$$

$$\boxed{\text{Median} = 61.64}$$

Mode:

$$\text{Mode} = L + \left[\frac{f_m - f_1}{2f_m - f_2 - f_2}\right] \times h$$

$$= 59.5 + \left[\frac{42 - 34}{2(42) - 34 - 40}\right] \times 5$$

$$\boxed{\text{Mode} = 63.5}$$

⇒ Relatively b/w mean, median & mode i negative skewed.

$$\boxed{\text{Mean} < \text{Median} < \text{Mode}}$$

b) Write comments on the nature of distrib

Nature of distribution is left skewed. (nega

c) Calculate the skewness & kurtosis & write comments on nature of distribution.

C.I	m.p	f	$x-\bar{x}$	$f(x-\bar{x})^2$	$f(x-\bar{x})^3$	$f(x-\bar{x})^4$
35-39	37	7	-24.24	4113.04	-99700.17	2416732.00
40-44	42	8	-19.24	2961.42	-56977.74	1096251.64
45-49	47	12	-14.24	2433.33	-34650.64	493425.06
50-55	52	20	-9.24	2219.82	-20511.12	189522.70
56-~~60~~59	57	34	-4.8 24	611.24	-2591.65	10988.60
60-64	62	42	0.8	24.26	18.44	14.012
65-69	69	46	5.8	1327.104	7644.12	44030.13
70-75	72	15	10.8	1736.66	18686.50	201066.60
76-79	77	17	15.8	4222.42	66545.33	10648754.35
80-84	82	9	20.8	3818.80	80523.85	671675.22
		=210	-17	=23528.09	=-41013.08	=7172460.52

$\mu_1 = 0$ (Us always be zero)

$$\mu_2 = \frac{\Sigma f(x-\bar{x})^2}{\Sigma f} = \frac{(23528.09)^2}{210} = 112.04$$

$$\mu_3 = \frac{\Sigma f(x-\bar{x})^3}{\Sigma f} = \frac{(-41013.024)^3}{210} = -195.30$$

$$\mu_4 = \frac{\Sigma f(x-\bar{x})^4}{\Sigma f} = \frac{7172460.52}{210} = 34154.57$$

Skewness:-

$$\beta_1 = \frac{\mu_3^2}{\mu_2^2} = \frac{(-195.30)^2}{(112.04)^2} = 0.027$$

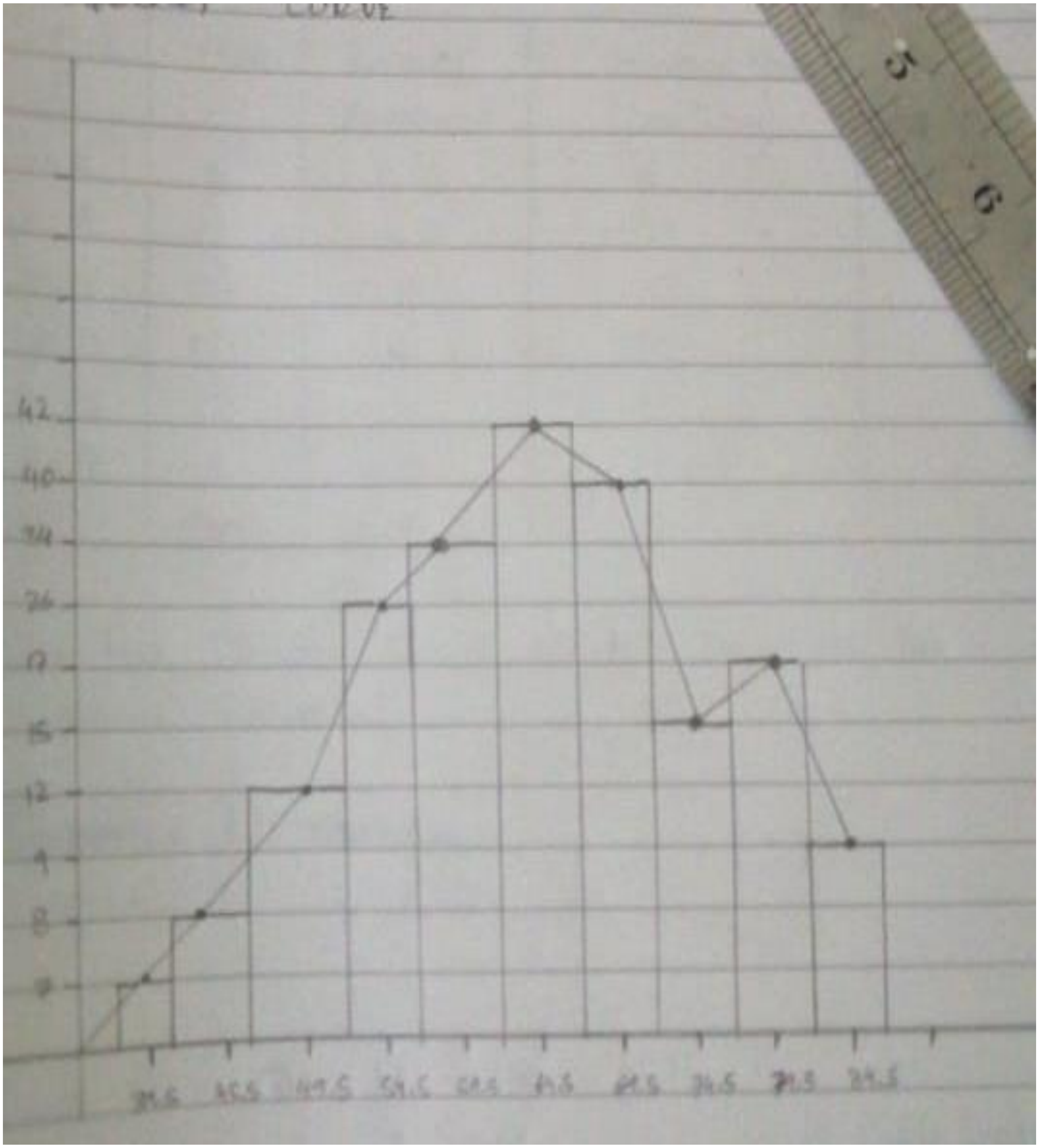
Kurtosis:-

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{34154.57}{(112.04)^2} = 2.72$$

Comments:-

Graph is leptokurtic.

d) Calculate a histogram & superimpose on it a frequency curve.


CURVE
42
40
12

e) Discuss the shape of the distribution with the results obtained in part (a) to (d)

- In part (b) the shape of the distribution is negative because according to part(a) means < median < mode

- In part (d) there is a large accureness in right side so the shape of distribution of histogram is left skewed (negative)

Question:4

The following data incomplete

C.I	freq.	Relative frequency	Commulative freq(less)
6–8	6	0.06	6
8–10	23	0.23	29
10–12	34	0.34	63
12–14	17	0.17	80
14–16	12	0.12	92
16–18	8	0.08	100
Total	100	1.00	

Calculate Quartile deviation.

Quartile deviation:

Formula: $QD = \dfrac{Q_3 - Q_1}{2}$

Find Q1: $\dfrac{N}{4} = \dfrac{100}{4} = 25^{th}$

For Exact value of Q_1

$$Q_1 = \text{Lower limit}_{(l)} + \frac{\frac{N}{4} - CF}{F} \times i \,(\text{Class Interval})$$

$$= 8 + \frac{25 - 6}{23} \times 2$$

$\boxed{Q_1 = 9.65}$

∵ Class Interval

$C.I = \text{Range} / \text{No of Cl.}$
$= 18 - 6 / 6 \;\boxed{= 2}$

Find Q_3:

$$Q_3 = \frac{3N}{4} = \frac{3(100)}{4} = 75^{th}$$

for exact value of Q_3

$$\therefore Q_3 = \text{lower limit}_{(L)} + \frac{\frac{3N}{4} - CF}{F} \times i(C \cdot L)$$

$$= 12 + \frac{75 - 63}{17} \times 2$$

$$\boxed{Q_3 = 13.41}$$

Now,

$$Q \cdot D = \frac{13.41 - 9.65}{2}$$

$$\boxed{Q \cdot D = 1.88}$$

Ansal

Question: 5

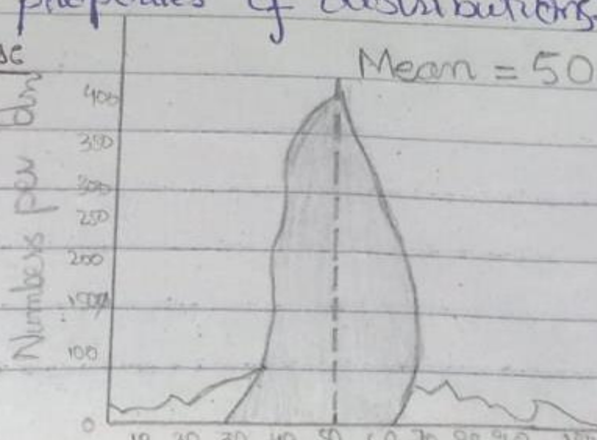
1) Explain with suitable examples the term 'dispersion'. State the relative & absolute measures of dispersion & describe the situations for the using these measures.

* **Dispersion:-**

In statistics, dispersion (also called variability, or scatter, or spread) is the extent to which a distribution is stretched or squeezed common examples of measures of statical dispersion are the variance, standard deviation and inter quartile range.

Dispersion is contrasted with location or central tendency, and together they are the most used properties of distributions.

• Examples:



Examples of samples from two populations with the same mean but different dispersion. The non-shaded population is more dispersed than the shaded population.

* Measures Of Absolute dispersions-

The measure which express the scattering of observation in terms of distances i.e. range, quartile deviation.

* Measures of Relative Dispersion:-

The relative measure of dispersion are used for comparing distribution of two or more data set and for unit free comparision. They are the coefficient of range, the coefficient of mean deviation, The coefficient of standard deviation.

* Range:

A range is the most common and easily understandable measure of dispersion. It is the difference b/w two extreme observations of the data set. If X_{max} and X_{min} are the two extreme observations then

$$\boxed{Range = X_{max} - X_{min}}$$

* Quartile Deviation:

The quartile divide a data set into quarters. The first quartile, (Q_1) is the middle number between the smallest number and the median of the data. The second quartile, (Q_2), is the median of the data set. The third quartile, (Q_3) is the middle number b/w the median and the largest number.

∴ Quartile deviation or semi-inter quartile deviation is

$$\boxed{Q = 1/2 \times (Q_3 - Q_1)}$$

* Mean Deviation:-

Mean deviation is the arithmetic mean of the absolute deviations of the observations from a measure of central central tendency. If $x_1, x_2, ---, x_n$ are the set of observations, then the mean deviation of x about the average, A (Mean, Median, Mode) is mean deviation from average

$$\boxed{A = \frac{1}{n}\left[\Sigma_i |x_i - A|\right]}$$

For a grouped frequency, it is calculated as Mean deviation from average

$$\boxed{A = \frac{1}{N}\left[\Sigma_i f_i \, |x_i - A|\right], \; N = \Sigma f_i}$$

Here, x_i and f_i are respectively mid value and the frequency of the i^{th} class interval.

* Standard Deviation:

A standard deviation is the positive square root of the arithematic mean of the square of the deviations of the given values from their arithematic mean. It is denoted by greek letter stigma sigma σ. It is also reffered to as root mean square deviation. The standard deviation is given as

$$\boxed{\sigma = [(\Sigma_i (y_i - \bar{y})/n]^{1/2} = [(\Sigma_i y_i^2/n) - \bar{y}^2]^{1/2}}$$

For a grouped frequency distribution is,

$$\boxed{\sigma = [(\Sigma_i f_i (y_i - \bar{y})/N]^{1/2} = [(\Sigma_i f_i y_i^2/n) - \bar{y}^2]^{1/2}}$$

Variance:

The square root of the standard deviation is the variance. It is also a measure of dispersion.

~~$\sigma^2 = [$~~

$$\boxed{\sigma^2 = [(\Sigma_i (y_i - \bar{y}) / n]^{1/2} = [(\Sigma_i y_i^2 / n) - \bar{y}^2]}$$

For a grouped frequency distribution; ti

$$\boxed{\sigma^2 = [(\Sigma_i f_i (y_i - \bar{y}) / N]^{1/2} = [(\Sigma_i f_i x_i^2 / n) \bar{y}^2]}$$

Suitable situation to use measure of dispersion:-

- Standard deviation is used as a measure of dispersion when mean is used as measure of central tendency (i.e. for symmetric numerical data)

- For ordinal data or skewed numerical data, median and interquartile range are used.

b) Mean of the 6 numbers 6, 9, 3, 2, x, y is 6 and variance is 10. find value of 'x' and 'y'.

Given:

6, 9, 3, 2, x, y

Mean of 6 numbers $= 6 = \bar{x}$

variance $= 10 = \sigma^2$

Formula:

$$\bar{x} = \frac{(x_1 + x_2 + x_3 + \cdots + x_n)}{n}$$

$$\sigma^2 = \frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \cdots + (x_n - \bar{x})}{n}$$

Solution:-

* Mean:

$$\bar{x} = 6 = \frac{6 + 9 + 3 + 2 + y + x}{6}$$

$$36 = 20 + x + y$$

$$x + y = 16$$

$$y = 16 - x \quad \text{— (1)}$$

* Variance

$$\sigma^2 = 10 = \frac{(6-6)^2+(9-6)^2+(3-6)^2+(2-6)^2+(x-6)^2+(y-6)^2}{6}$$

$$10 = \frac{(0)^2+(3)^2+(-3)^2+(-4)^2+x^2-12x+36+(16-x-6)^2}{6}$$

$\longrightarrow$ using equati (i)

$$60 = 0+9+9+16+x^2-12x+36+(10-x^2)$$

$$60 = x^2-12x+70+100-20x+x^2$$

$$60 = 2x^2-32x+170$$

$$2x^2-32x+110=0$$

$$2(x^2-16+55)=0$$

$$x^2-16+55=0$$

$\boxed{x=11}$ $\boxed{x=5}$

$\Rightarrow$ Put $x=11$ in eq (1)

$y=16-11$

$\boxed{y=5}$

$(11,5)$

$\Rightarrow$ Put $x=5$ in eq (1)

$y=16-5$

$\boxed{y=11}$

$(5,11)$

Answer

Question: 6

a) What is conditional probility? Explain with the help of an example.

Conditional Probability:

Conditional probability is the probability of one event occuring with some relationship to one or more other relationships.

Example:

- Event 'a' is that it is raining outside it has 0.3% (30%) chance raining today.
- Event 'b' is that you will need to go outside, and that has a probability of (0.5%) 50%.

A conditional probability would like at these two events in relationship with one another. Such as the probability that is both raining and you will need to go outside.

b) Let A and B be the two possible outcomes of an experiment and suppose $P(A) = 0.4$, $P(A \cup B) = 0.7$, and $P(B) = p$.

i) For what value of p, are A & B mutually exclusive?

Sol:

$$P(A \cup B) = P(A) + P(B)$$

$$0.7 = 0.4 + P$$

$$P = 0.7 - 0.4$$

$$\boxed{P = 0.3}$$

ii) For what value of p, are A & B independently?

Sol:

$$P(A \cap B) = P(A) \cdot P(B)$$

$$P(B) = \frac{P(A \cap B)}{P(A)}$$

$$P(A \cap B) = P(A) \cdot P(B)$$

$$P(A \cap B) = (0.4)\, P$$

$$P(B) = p = \frac{0}{0.4} = 0$$

$$\boxed{p = 0}$$

iii) If A & B are independently events then prove followings:

c) A' and B are independent.

Sol:

We have to proved $A' \cap B$ independly

$P(A)' = 1 - P(A)$

$P(B) = P(B)$

$P(A \cap B) = (1 - P(A))\, P(B)$

$= P(B) - P(A)\, P(B)$

$$\boxed{P(A' \cap B) = P(B) - P(A \cap B)}$$

Proved!

d) A' & B' are independent.

Sol:

We have to prove;

$= (A' \cap B')$

$\therefore P(A \cap B) = P(A) \cdot P(B)$

$P(A)' = 1 - P(A)$

$P(B)' = 1 - P(B)$

$P(A' \cap B') = P(A)' \cdot P(B)'$

$P(A' \cap B') = (1 - P(A)) \cdot (1 - P(B))$

$P(A' \cap B') = 1 - P(B) - P(A) + P(A)\, P(B)$

$P(A' \cap B') = 1 - P(B) - P(A) + P(A \cap B)$

$$\boxed{P(A' \cap B') = 1 - P(B) - P(A) + P(A \cap B)}$$

Proved!

Question : 7

The following function of the random variable "x", is given by:

$$f(x) = \begin{cases} c/x^5 & ; x \geq 1 \\ 0 & ; x < 1 \end{cases}$$

a) What is the value of c ?

Sol:

We have to find the value of c,

$f(x) = \frac{c}{5}$ $\quad x \geq 1$

$x\cancel{0}$ $\quad x < 1$

$$f(x) = \int_1^\infty \frac{c}{x^5}\, dx$$

$$= \int_1^\infty c x^{-5}\, dx$$

$$= c \left. \frac{x^{-5+1}}{-5+1} \right|_1^\infty$$

Area under curve = 1

$$1 = c \frac{x^{-4}}{-4}$$

$$1 = c \left| \frac{1}{-4x^4} \right|_1^\infty$$

$$1 = c\left[\frac{1}{-4_{\infty}} + \frac{1}{4}\right]$$

$$1 = c\left(0 + \frac{1}{4}\right)$$

$$\boxed{c = 4}$$

b) Plot pdf and cdf.

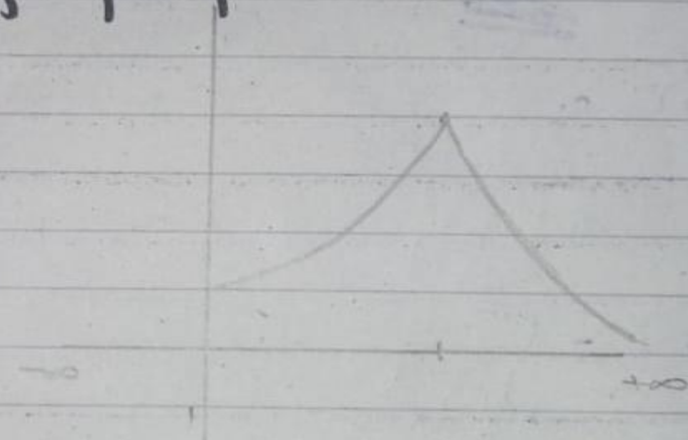
Graph of pdf:-

$\frac{4}{x^5}$ $x \geq 1$

$f(1) = \frac{4}{1} = 4$

$f(1/2) = \frac{4}{(1/2)^5} = 128$

$f(\infty = \frac{4}{\infty} = 0$

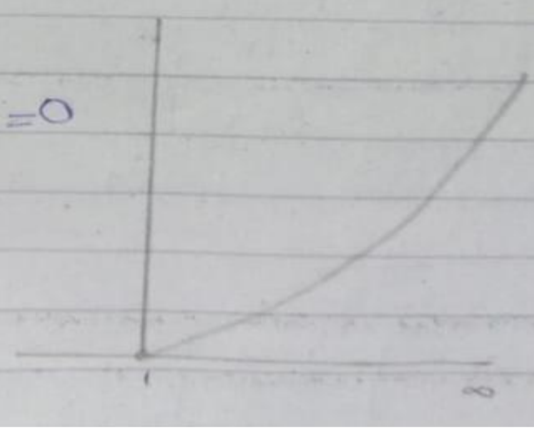


Graph of cdf:-

$-\left[\frac{1}{x^4} + 1\right]$

$f(-\infty) = -\left[\frac{1}{1} + 1\right] = 0$

$f(+\infty) = -\left[\frac{1}{\infty} + 1\right]$



c) Find $E(X)$

Sol:

$$E(x) = \int_1^\infty x f(x)$$

$$= \int_1^\infty x \frac{4}{x^5}\, dx$$

$$E(x) = \int_1^\infty 4x^{-4}\, dx$$

$$E(x) = \frac{4x^{-4+1}}{-4+1}\Big|_1^\infty$$

$$E(x) = \frac{4(x)^{-3}}{-3}$$

$$= \left| \frac{-4}{3} \frac{1}{x^3} \right|_1^\infty$$

$$= \frac{-4}{3}\left| \frac{1}{-\infty} - \frac{1}{1} \right|$$

$$= \frac{-4}{3}\left[0-1\right]$$

$$\boxed{E(X) = \frac{4}{3}}$$

Answer.

e) find the value of 'M' (median)?

Sol:

Median $= \frac{1}{2}$

$$4\int_1^m \frac{1}{x^5}\,dx = \frac{1}{2}$$

$$4\left|\frac{x^{-5+1}}{-5+1}\right|_1^m = \frac{1}{2}$$

$$4\left|\frac{x^{-4}}{-4}\right|_1^m = \frac{1}{2}$$

$$\frac{\cancel{4}}{\cancel{4}}\left|\frac{1}{-x^4}\right|^m = \frac{1}{2}$$

$$\frac{1}{-m^4} + 1 = \frac{1}{2}$$

$$\frac{\cancel{+}1}{-m^4} = \frac{1}{2} - 1$$

$$\cancel{-}\frac{1}{m^4} = \cancel{-}\frac{1}{2}$$

$$2 = m^4$$

$$\boxed{m = (2)^{4}}$$

Answer

d) What is cdf of X

Sol:

$$F(x) = \int_1^x \frac{4}{x^5}\, dx$$

$$= 4\int_1^x x^{-5}\, dx$$

$$= 4 \left| \frac{x^{-5+1}}{-5+1} \right|_1^x$$

$$= 4 \left| \frac{x^{-4}}{-4} \right|_1^x$$

$$= \frac{\cancel{4}}{\cancel{4}} \left| \frac{x^{-4}}{-1} \right|_1^x$$

$$= \left| \frac{-1}{x^4} \right|_1^x$$

$$= \left[\frac{-1}{x^4} - \frac{1}{1} \right]$$

$$\boxed{F(x) = -\left[\frac{1}{x^4} + 1 \right]}$$ Ans.

Question: 8

The random variable X representing the no. of errors per 100 lines of software code has the following pdf.

a) Find mean & variance of X

Sol:

x	$f(x)$	$xf(x)$	$x^2 f(x)$
2	0.01	0.02	0.04
3	0.25	0.75	2.25
4	0.4	1.6	6.4
5	0.3	1.5	7.5
6	0.04	0.24	1.44

$$\text{Mean of } x = \frac{\sum x f(x)}{n}$$

$$= \frac{4.11}{100}$$

$$\boxed{E(x) = 0.0411}$$

$$\text{Variance of } x = V(x) = \frac{\sum x^2}{n} - \left[\frac{\sum x}{n}\right]^2$$

$$= \frac{17.63}{100} - \left[\frac{4.11}{100}\right]^2$$

$$\boxed{V(x) = 0.1352}$$

b) Find the mean and variance of $Z = 3X - 2$

Sol:

x	$Z = 3X - 2$	$f(x)$	$z f(x)$	$z^2 f(x)$
2	4	0.01	0.04	0.16
3	7	0.25	1.75	12.25
4	10	0.4	4	40
5	13	0.3	3.9	50.7
6	16	0.04	0.64	10.24

- Mean of $Z = \frac{\sum z f(x)}{n}$

$$= \frac{10.33}{100}$$

$$\boxed{E(z) = 0.1033}$$

- Variance of $Z = \sum z^2 f(x) - \left[\frac{\sum z f(x)}{n}\right]^2$

$$= \frac{113.35}{100} - \left[\frac{10.33}{100}\right]^2$$

$$\boxed{= 1.122}$$

Question: 9

d) Determine value of C so that the given function is pdf.

$$f(x) = C(x^2+4) \quad ; \quad x = 0, 1, 2, 3$$

Sol:

$f(x) = C(x^2+4)$ $\quad x = 0, 1, 2, 3$

$0 \le x \le 3$

we have to find the value of C.

$$f(x) = c\int_0^3 (x^2+4)\,dx$$

$$= c\left|\frac{x^3}{3} + 4x\right|_0^3$$

$$= c\left[\frac{(3)^3}{3} - \frac{0}{3} + 4(3) - 4(0)\right]$$

Area under the curve = 1

$1 = 21c$

$$\boxed{c = \frac{1}{21}}$$ Ans.

b)

Sol: a) What is pmf of X?

$$f(x) = c(x^2+4)$$

$$f(x) = \frac{1}{21}\int_{-\infty}^{0} (x^2+4)$$

$$= \frac{1}{21}\left|\frac{x^3}{3} + 4x\right|_{-\infty}^{0}$$

$$f(x)_0 = 0$$

$$f(x_0) = \int_0^1 (x^2+4)dx$$

$$= \frac{1}{21}\left|\frac{x^3}{3} + 4x\right|_0^1$$

$$= \frac{1}{21}\left[\frac{(1)^3}{3} - \frac{(0)^3}{3} + 4(1) - 4(0)\right]$$

$$\boxed{f(x_1) = 0.206}$$

$$f(x)_2 = \frac{1}{21}\int_1^2 (x^2+4)dx$$

$$= \frac{1}{21}\left|\frac{x^3}{3} + 4x\right|_1^2$$

$$= \frac{1}{21}\left[\frac{(2)^3}{3} - \frac{1}{3} + 4(2) - 4(1)\right]$$

$$\boxed{f(x) = 0.301}$$

$$f(x)_3 = \frac{1}{21}\int_2^3 (x^2+4)\,dx$$

$$= \frac{1}{21}\left|\frac{x^3}{3} + 4x\right|_2^3$$

$$= \frac{1}{21}\left[\frac{(3)^3}{3} - \frac{(2)^3}{3} + 4(3) - 4(2)\right]$$

$$\boxed{f(x) = 0.492}$$

pmf of X is

x	0	1	2	3
$d(x)$	0	0.206	0.301	0.492

b) Find $P(4 < x < 7)$

Sol:

$$f(x) = \begin{cases} 0 & x < 1 \\ 0.4 & 1 \le x < 3 \\ 0.6 & 3 \le x < 5 \\ 0.8 & 5 \le x < 7 \\ 1 & x \ge 7 \end{cases}$$

$$f(x) = P(7) - P(4)$$

$$= 1 - 0.8$$

$$\boxed{f(x) = 0.2}$$ Answer.

Question: 10

a) Ten percent of population is left handed. use normal approx. to Binomial distribution to find probability that there are atleast 60 left handed students in a school of 400 students.

Total no. of events = $n = 400$
no. of success = $(P) = 0.10$
no. of failure$(q) = p-1 = 0.90$

mean $= p * n$
$\mu = 0.10 \times 400 = 40$

$SD = \sqrt{n \times p \times q}$

$6 = \sqrt{400 \times 0.10 \times 0.90} = 6$

for normal approximation $B(n=1000, p=0.10)$
with $N(\mu = 100, \sigma = 6)$

$P(X >= 60)$

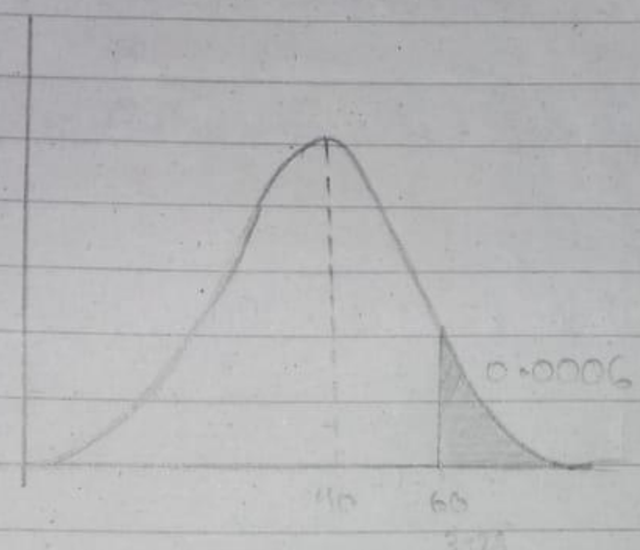
$\therefore P(x >= 39.5)$

$$Z = \frac{59.5 - 40}{6} = 3.25$$

$P(Z >= 3.25)$

$\therefore$ $1 - 0.9994$

$\boxed{P = 0.0006}$

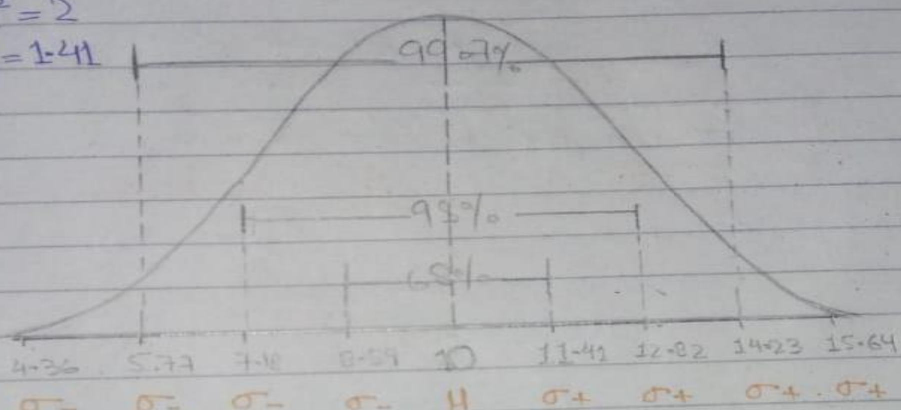


b) Plot the following Normal curves on the same graph paper and give comments on shape of curve for d/f values of parameter.

i) $N(\mu = 10, \sigma^2 = 2)$

$\sigma^2 = 2$

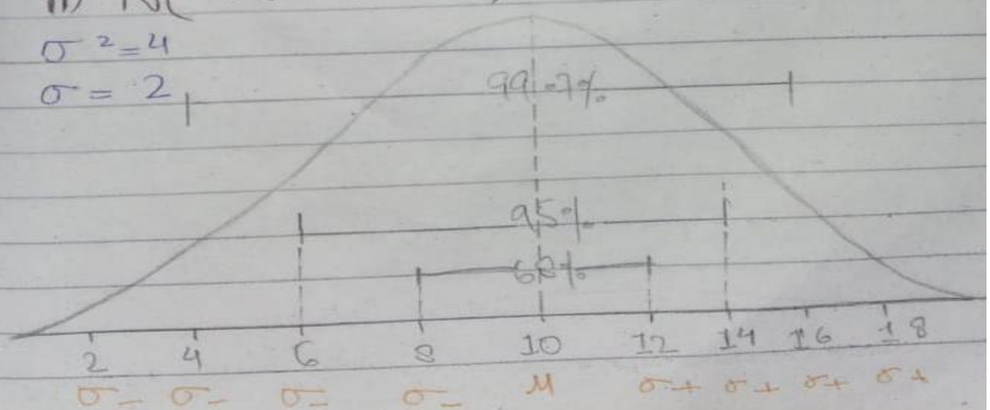
$\sigma = 1.41$

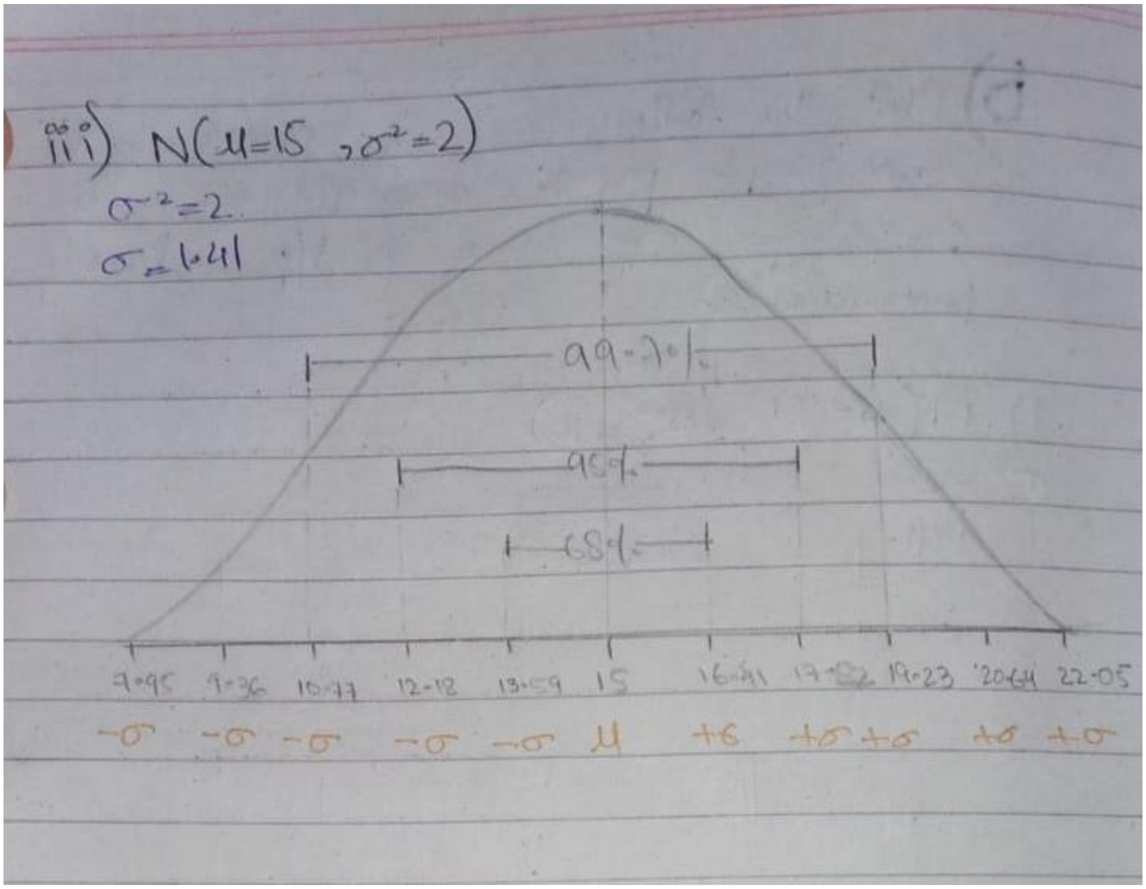


ii) $N(\mu = 10, \sigma^2 = 4)$

$\sigma^2 = 4$

$\sigma = 2$





iii) N(μ=15 , σ²=2)
σ²=2
σ=1.41
99.7%
95%
68%
7.95 9.36 10.77 12.18 13.59 15 16.41 17.82 19.23 20.64 22.05
-σ -σ -σ -σ -σ μ +σ +σ +σ +σ +σ