

DEEP LEARNING

ASSIGNMENT 1

Note; The assignment should be submitted in handwritten format on 4th march 2022.

**** INDIVIDUAL ASSIGNMENT AND NOT A GROUP WORK**

QUESTION 1;

- a) Consider the matrix X and the vectors y and z below:

$$X = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix} \quad y = \begin{bmatrix} 1 \\ 3 \end{bmatrix} \quad z = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

What is the inner product of the vectors y and z ? (This is also sometimes called the dot product, and is sometimes written $y^T z$)

- b) What is the product Xy ?
c) Is X invertible? If so, give the inverse, and if no, explain why not?
d) What is the rank of X ? Explain your answer

QUESTION 2:

- a) If $y = x^3 + x - 5$ then what is the derivative of y with respect to x ?

QUESTION 3:

Consider a sample of data $S = \{1, 1, 0, 1, 0\}$ created by flipping a coin x five times, where 0 denotes that the coin turned up heads and 1 denotes that it turned up tails.

- a) What is the sample mean for this data?
b) What is the sample variance for this data?
c) What is the probability of observing this data, assuming it was generated by flipping a coin with an equal probability of heads and tails (i.e. the probability distribution is $p(x = 1) = 0.5, p(x = 0) = 0.5$).

QUESTION 4

Consider the following joint probability table over variables y and z , where y takes a value from the set $\{a, b, c\}$, and z takes a value from the set $\{T, F\}$:

	y		
	a	b	c
z			
T	0.2	0.1	0.2
F	0.05	0.15	0.3

- a) What is $p(z = T \text{ AND } y = b)$?
b) What is $p(z = T | y = b)$?

QUESTION 5

Below is a diagram of a single artificial neuron (unit):

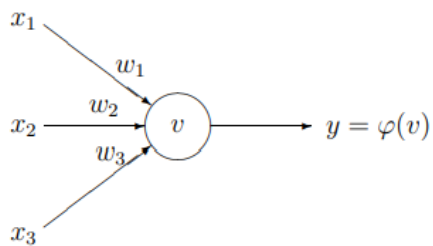


Figure 1: Single unit with three inputs.

Consider the unit shown on Figure 1. Suppose that the weights corresponding to the three inputs have the following values:

w_1	=	2
w_2	=	-4
w_3	=	1

and the activation of the unit is given by the step-function:

$$\varphi(v) = \begin{cases} 1 & \text{if } v \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

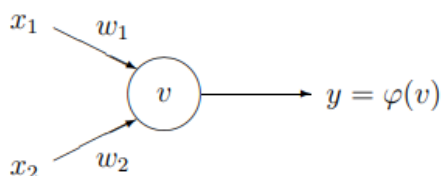
Calculate what will be the output value y of the unit for each of the following input patterns:

Pattern	P_1	P_2	P_3	P_4
x_1	1	0	1	1
x_2	0	1	0	1
x_3	0	1	1	1

QUESTION 6:

Logical operators (i.e. NOT, AND, OR, XOR, etc) are the building blocks of any computational device. Logical functions return only two possible values, true or false, based on the truth or false values of their arguments. For example, operator AND returns true only when all its arguments are true, otherwise (if any of the arguments is false) it returns false. If we denote truth by 1 and false by 0, then logical function AND can be represented by the following table:

$x_1 :$	0	1	0	1
$x_2 :$	0	0	1	1
$x_1 \text{ AND } x_2 :$	0	0	0	1



if the weights are $w_1 = 1$ and $w_2 = 1$ and the activation function is:

$$\varphi(v) = \begin{cases} 1 & \text{if } v \geq 2 \\ 0 & \text{otherwise} \end{cases}$$

Note that the threshold level is 2 ($v \geq 2$).

- Test how the neural AND function works.
- Suggest how to change either the weights or the threshold level of this single-unit in order to implement the logical OR function (true when at least one of the arguments is true):

$x_1 :$	0	1	0	1
$x_2 :$	0	0	1	1
$x_1 \text{ OR } x_2 :$	0	1	1	1

- The XOR function (exclusive or) returns true only when one of the arguments is true and another is false. Otherwise, it returns always false. This can be represented by the following table:

$x_1 :$	0	1	0	1
$x_2 :$	0	0	1	1
$x_1 \text{ XOR } x_2 :$	0	1	1	0

Do you think it is possible to implement this function using a single unit? A network of several units?