Assignment : 1

Question : 1

a) Consider the Matrix X & the vectors y & z

$$X = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix} \quad y = \begin{bmatrix} 1 \\ 3 \end{bmatrix} \quad z = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

What is the inner product of the vectors y & z?
(This is also ~~called~~ sometimes called Dot product & is sometimes written $y \cdot z$)

Dot product of vectors y & z

$$y = \begin{bmatrix} 1 \\ 3 \end{bmatrix} \quad \& \quad z = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$\text{Dot product} = y \cdot z$$

$$= \begin{bmatrix} 1 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 9 \end{bmatrix}$$

Dot product of y & z are $= \begin{bmatrix} 2 \\ 9 \end{bmatrix}$ $\boxed{(y,z) = 11}$

{ line ar product of $y^t z$

b) What is the product Xy?
The product of Matrix & Vector.

$$X = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}_{2\times 2}, \quad y = \begin{bmatrix} 1 \\ 3 \end{bmatrix}_{2\times 1}$$

so, it can be multiplied.

$$\text{product of } Xy = \begin{bmatrix} (2\times1) + (4\times3) \\ (1\times1) + (3\times3) \end{bmatrix}$$

$$Xy = \begin{bmatrix} 2+12 \\ 1+9 \end{bmatrix} = \begin{bmatrix} 14 \\ 10 \end{bmatrix}$$

c) If X invertible? If so, give the inverse and if no, explain why not?

For Invertible

1) X should be a square matrix
2) Determinant $\neq 0$

$$X = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$$ it is a square matrix

Rows x Column

2×2

$$\text{Determinant} = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$$

$= 2 \times 3 - 4 \times 1 \Rightarrow 6 - 4 \Rightarrow \boxed{2}$

So, to prove invertible

$A \cdot A^{-1} = I$

For A^{-1}:

$$X = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$X^{-1} = \frac{1}{\text{Determinant}} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$X^{-1} = \frac{1}{2} \begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix}$$

$\boxed{X X^{-1} = I}$

$$\frac{1}{2} \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 3 & -4 \\ -1 & 2 \end{bmatrix}$$

$$\frac{1}{2} \begin{bmatrix} [illegible] & (2 \times -4) + (4 \times 2) \\ (1 \times 3) - (3 \times 1) & (1 \times -4) + (3 \times 2) \end{bmatrix}$$

$$\frac{1}{2}\begin{bmatrix} 6-4 & -8+8 \\ 3-3 & -4+6 \end{bmatrix}$$

$$\frac{1}{2}\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 2/2 & 0/2 \\ 0/2 & 2/2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$ It is an identity Matrix.

$X \cdot X^{-1} = I$ is proved

X is Invertible.

d) What is the rank of X ? Explain your Answer ?

$$X = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$$

$$X = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix} = 6-4 \quad \boxed{=2}$$

Rank of $X = 2$

The column Rank is 2, Since $\begin{bmatrix} 4 \\ 3 \end{bmatrix} \neq c\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

for all $c \in \mathbb{R}$

Question:2

If $y = x^3 + x - 5$, what is the derivative of y with respect to x?

$y = x^3 + x - 5$

derivative y w.r.t x

$y' = 3x^2 + 1 - 0$

$\boxed{y' = 3x^2 + 1}$

Question:3

Consider sample of data $S = \{1, 1, 0, 1, 0\}$ created by flipping a coin x five time, where the 0 denotes that the coin turned up heads & 1 denotes that the turned up tails.

a) What is the sample mean for this data?

$X = \frac{\sum x_i}{n}$

$X = \frac{1+1+0+1+0}{5}$

$X = \frac{3}{5}$

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b) What is the sample variance for this data?

$$s^2 = \frac{\Sigma (x_i - \bar{x})^2}{n-1}$$

$$s^2 = \frac{(1-0.6)^2 + (1-0.6)^2 + (-0.6)^2 + (1-0.6)^2 + (-0.6)^2}{5-1}$$

$$s^2 = \frac{0.16 + 0.16 + 0.36 + 0.16 + 0.36}{4}$$

$$s^2 = \frac{1.2}{4} \boxed{= 0.3}$$

c) What is the probability of observing this data, assuming it was generated by flipping a coin with an equal probability of heads & tails (i.e the distribution is $p(x=1) = 0.5$, & $p(x=0) = 0.5$)

Probability of Sample data $= (0.5)^5$

Probability of Seeing data $= \frac{1}{32}$

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Question: 4

Consider the following joint probability table over variables y & z, where y takes a value from the set {a, b, c} & z takes value from set ϕ {T, F}

		y		
		a	b	c
z	T	0.2	0.1	0.2
	F	0.05	0.15	0.3

a) What is $p(z = T \text{ AND } y = b)$?

$P(z) = T \text{ AND } y = b) = 0.1$

b) What is $p(z = T \mid y = b)$?

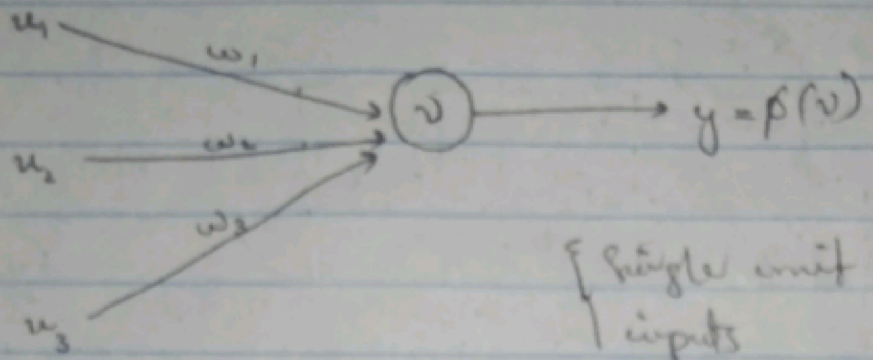
Using the definition of conditional probability

$$P(z = T / y = b) = \frac{P(z = T \text{ AND } y = b)}{P(y = b)}$$

$$P(z = T \mid y = b) = \frac{0.1}{0.1 + 0.15} = 0.4$$

Question : 5

Below is diagram of single artificial neural network (unit):



(1)

{ Single unit with 3 inputs (2)

Consider the unit shown in figure. 1 Suppose that weight corresponding to the three inputs have following values.

$w_1 =$	2
$w_2 =$	-4
$w_3 =$	1

& the activation of unit is given by step-function:

$$\phi(v) = \begin{cases} 1 & \text{if } v \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

Calculate what will be the output value y of unit for each of following input pattern.

Pattern	P_1	P_2	P_3	P_4
x_1	1	0	1	1
x_2	0	1	0	1
x_3	0	1	1	1

neuron

itte 3

suppose

[illegible]

ue/ y

pattern.

function: $\delta(v) = \begin{cases} 1 & \text{if } v \geq 0 \\ 0 & \text{otherwise} \end{cases}$

To find the output value of y for each pattern we have to:

① Calculate the weight sum

$v = \Sigma_i w_i x_i = w_1 x_1 + w_2 x_2 + \cdots + w_n x_n$

② Apply activation function to v

Calculation x_i of each pattern:

$P_1 = 2(1) - 4(0) + 1(0) = 2, (2>0), y = \delta(2) = 1$

$P_2 = 2(0) - 4(1) + 1(1) = -3, (-3 \; 0), y = \delta(-3) = 0$

$P_3 = 2(1) - 4(0) + 1(1) = 3, (3 \; 0), y = \delta(3) = 1$

$P_4 = 2(1) - 4(1) + 1(1) = -1, (-1<0), y = \delta(-1) = 0$

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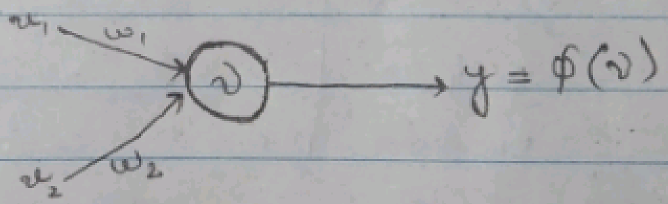
Question: 6

Logical operators (i.e. NOT, AND, OR, XOR etc) are the buildings blocks of any computational device. Logical funtions return only two possible values, true or false, based on the truth or false values of their arguments. For example, operator AND return true only when all its arguments are true, otherwise (if any of the arguments is false) it returns falses.

If we denote truth by 1 & false by 0, then logical funtion AND can be represented by the following table.

x_1:	0	1	0	1
x_2:	0	0	1	1
x_1 & x_2:	0	0	0	1



If the weights are $w_1 = 1$ & $w_2 = 1$ & activation funtion is:

$$\phi(v) = \begin{cases} 1 & \text{if } v \geq \phi\, 2 \\ 0 & \text{otherwise} \end{cases}$$

Note that the threshold level is 2 ($v \geq 2$).

7) Test how the neural AND function works.

b)

Weights:

$w_1 = 1$

$w_2 = 1$

Activation function: $f(v) = \begin{cases} 1 & \text{if } v \geq 2 \\ 0 & \text{otherwise} \end{cases}$

* And Table

$x_1 =$	0	1	0	1
$x_2 =$	0	0	1	1
x_1 & x_2	0	0	0	1
Neuron	0	0	0	1

Input $(0,0) = x_1w_1 + xw = 0(1) + 0(1) = 0 \ngtr 2$ $\boxed{y=0}$

Input $(1,0) = x_1w_1 + xw = 1(1) + 0(1) = 1 \ngtr 2$ $\boxed{y=0}$

Input $(0,1) = x_1w + xw = 0(1) + 1(1) = 1 \ngtr 2$ $\boxed{y=0}$

Input $(1,1) = xw + xw = 1(1) + 1(1) = 1 \ngtr 2$ $\boxed{y=1}$

So the neurons outputs verify the AND table outputs for different value if x_1 & x_2.

b) Suggest how to change either the weights or the threshold level of this single-unit in order to implement the logical OR function (true when atleast one of the arguments is true):

x_1:	0	1	0	1
x_2:	0	0	1	1
x_1 OR x_2:	0	1	1	1
Neurons	0	1	1	1

$w_1 = 1$

$w_2 = 1$

: The

$$f(v) = \begin{cases} 1 & \text{if } v > 0 \\ 0 & \text{otherwise.} \end{cases}$$

Processing:

$g(0,0) = 0(1) + 0(1) = 0 \geq 0$

$g(0,1) = 0(1) + 1(1) = 1 > 0$

$g(1,0) = 1(1) + 0(1) = 1 > 0$

$g(1,1) = 1(1) + 1(1) = 2 > 0$

2 $y=0$

2 $y=0$

2 $y=0$

2 $y=1$

c) The XOR function (exclusive or) returns true if only when one of the arguments is true & another is false. Otherwise, it returns always false. This can be represented by following table:

x_1 =	0	1	0	1
x_2 =	0	0	1	1
x_1 XOR x_2 =	0	1	1	0

Do you think it is possible to implement this function using a single unit? A network of several units?

It is impossible to implement the XOR function neither by a single unit nor by a single layer feed forward network (Single layer perceptron). This is also known as XOR problem which can be resolved by using a feed forward network with hidden layer.

The XOR network uses 2 hidden nodes & 1 output nodes.