# Local search algorithms

Chapter 4, Sections 3–4

## Outline

♢ Hill-climbing

♢ Simulated annealing

♢ Genetic algorithms (briefly)

♢ Local search in continuous spaces (very briefly)

## Iterative improvement algorithms

In many optimization problems, **path** is irrelevant;
the goal state itself is the solution

Then state space = set of "complete" configurations;
find **optimal** configuration, e.g., TSP
or, find configuration satisfying constraints, e.g., timetable

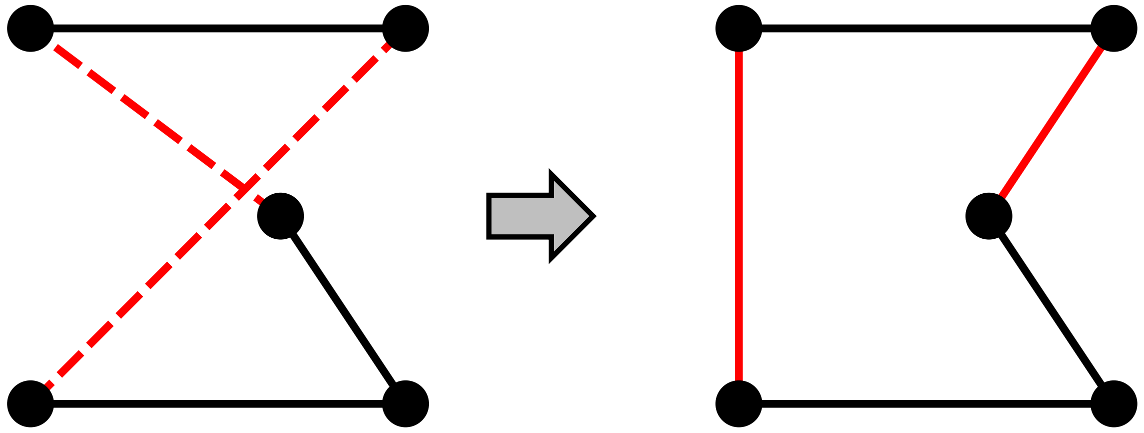
In such cases, can use iterative improvement algorithms;
keep a single "current" state, try to improve it

Constant space, suitable for online as well as offline search

TSP

## Example: Travelling Salesperson Problem

Start with any complete tour, perform pairwise exchanges

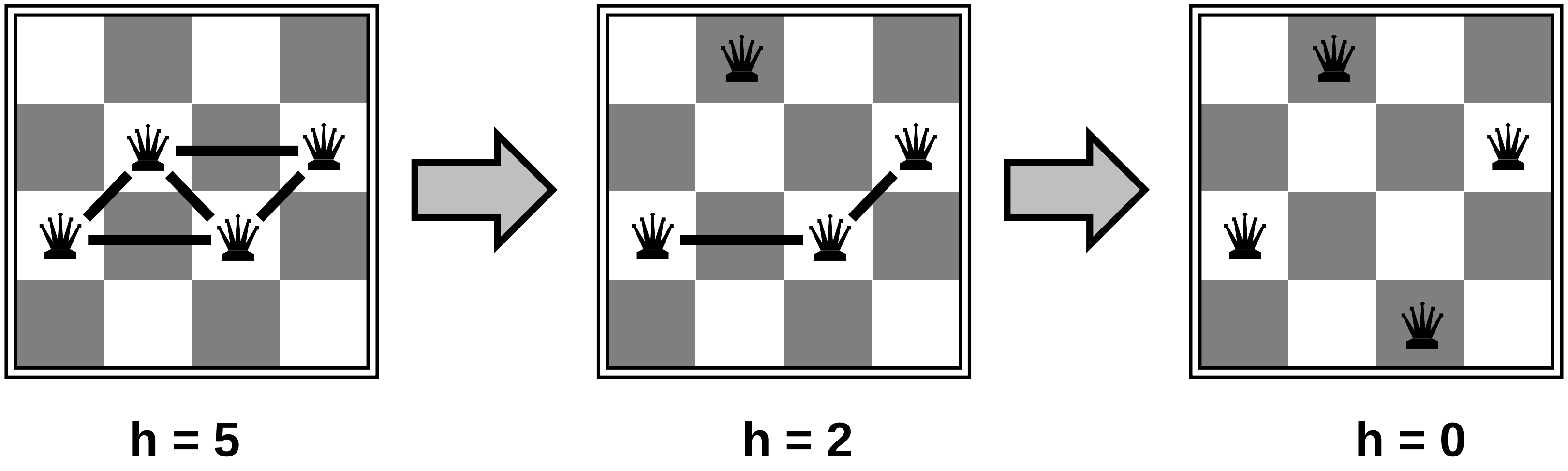


Variants of this approach get within 1% of optimal very quickly with thousands of cities

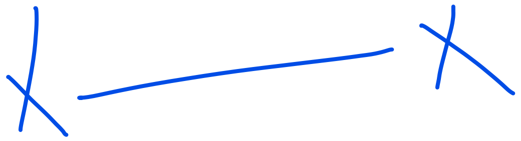
## Example: $n$-queens

Put $n$ queens on an $n \times n$ board with no two queens on the same row, column, or diagonal

Move a queen to reduce number of conflicts



Almost always solves $n$-queens problems almost instantaneously for very large $n$, e.g., $n = 1 million$



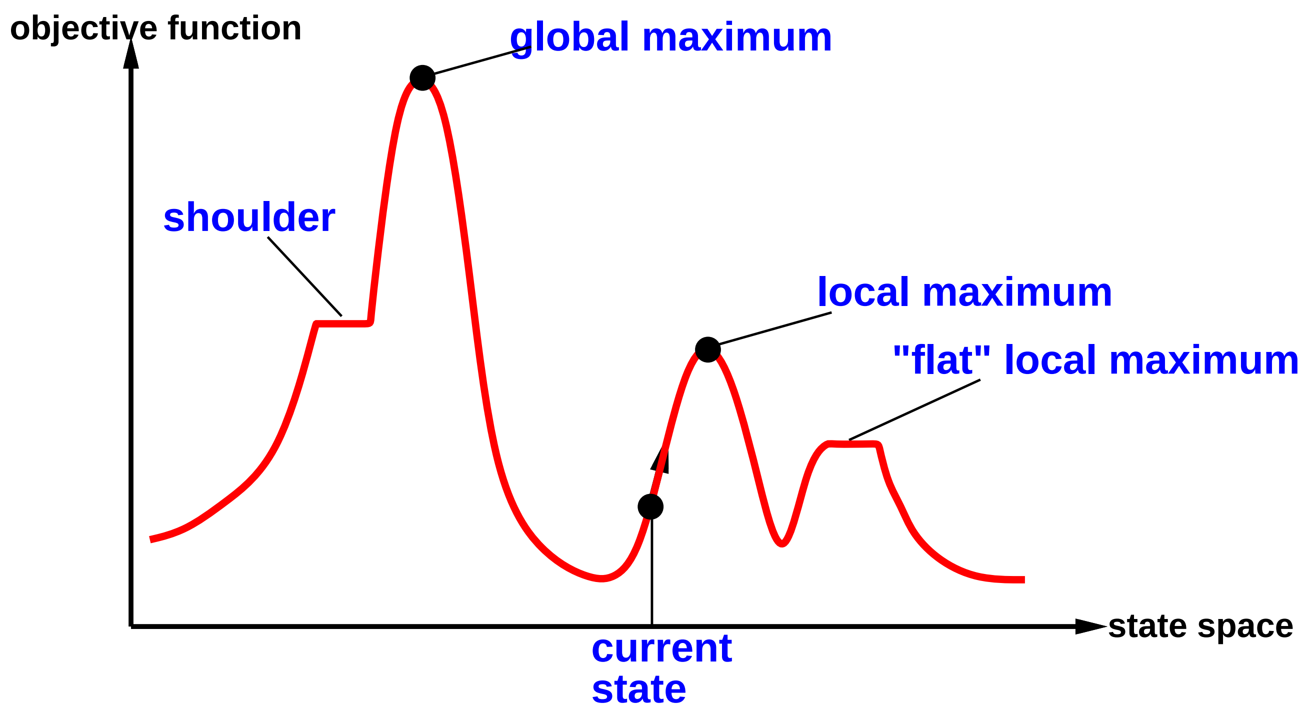
## Hill-climbing (or gradient ascent/descent)

"Like climbing Everest in thick fog with amnesia"

```
function Hill-Climbing(problem) returns a state that is a local maximum
    inputs: problem, a problem
    local variables: current, a node
                     neighbor, a node
    current ← Make-Node(Initial-State[problem])
    loop do
        neighbor ← a highest-valued successor of current
        if Value[neighbor] ≤ Value[current] then return State[current]
        current ← neighbor
    end
```

# Hill-climbing contd.

Useful to consider state space landscape



Random-restart hill climbing overcomes local maxima—trivially complete

Random sideways moves 😊 escape from shoulders ☹ loop on flat maxima

## Simulated annealing

Idea: escape local maxima by allowing some "bad" moves
**but gradually decrease their size and frequency**

```
function SIMULATED-ANNEALING(problem, schedule) returns a solution state
    inputs: problem, a problem
            schedule, a mapping from time to "temperature"
    local variables: current, a node
                     next, a node
                     T, a "temperature" controlling prob. of downward steps

    current ← MAKE-NODE(INITIAL-STATE[problem])
    for t ← 1 to ∞ do
        T ← schedule[t]
        if T = 0 then return current
        next ← a randomly selected successor of current
        ΔE ← VALUE[next] – VALUE[current]
        if ΔE > 0 then current ← next
        else current ← next only with probability $e^{\Delta E/T}$
```

## Properties of simulated annealing

At fixed "temperature" $T$, state occupation probability reaches Boltzman distribution

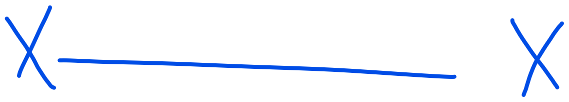
$$p(x) = \alpha e^{\frac{E(x)}{kT}}$$

$T$ decreased slowly enough $\Longrightarrow$ always reach best state $x^*$
because $e^{\frac{E(x^*)}{kT}} / e^{\frac{E(x)}{kT}} = e^{\frac{E(x^*)-E(x)}{kT}} \gg 1$ for small $T$

Is this necessarily an interesting guarantee??

Devised by Metropolis et al., 1953, for physical process modelling

Widely used in VLSI layout, airline scheduling, etc.



## Local beam search

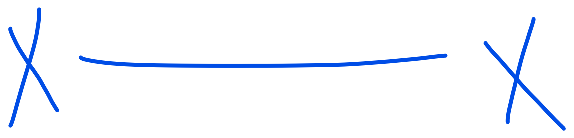
Idea: keep $k$ states instead of 1; choose top $k$ of all their successors

Not the same as $k$ searches run in parallel!
Searches that find good states recruit other searches to join them

Problem: quite often, all $k$ states end up on same local hill

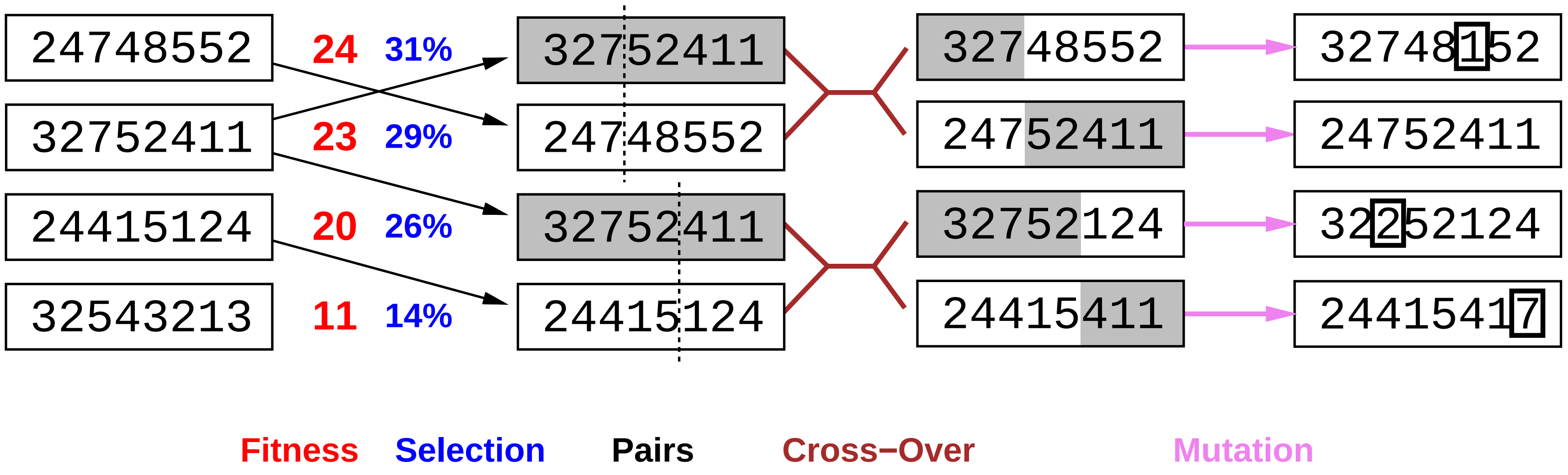
Idea: choose $k$ successors randomly, biased towards good ones

Observe the close analogy to natural selection!



## Genetic algorithms

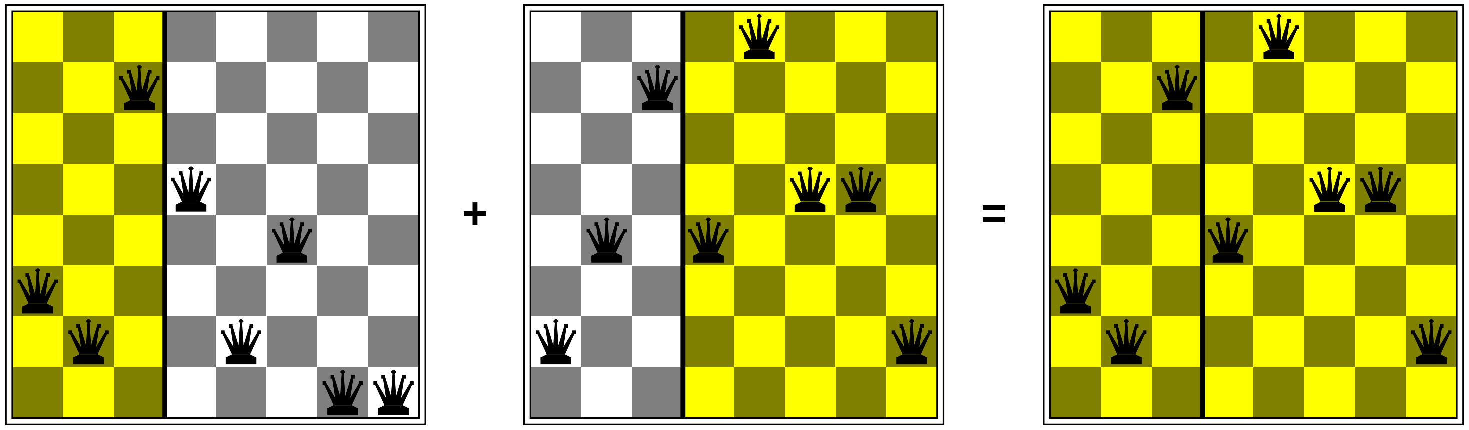
= stochastic local beam search + generate successors from **pairs** of states



## Genetic algorithms contd.

GAs require states encoded as strings (GPs use programs)

Crossover helps **iff substrings are meaningful components**



GAs $\neq$ evolution: e.g., real genes encode replication machinery!

## Continuous state spaces

Suppose we want to site three airports in Romania:
- 6-D state space defined by $(x_1, y_2)$, $(x_2, y_2)$, $(x_3, y_3)$
- objective function $f(x_1, y_2, x_2, y_2, x_3, y_3) =$
  sum of squared distances from each city to nearest airport

Discretization methods turn continuous space into discrete space, e.g., empirical gradient considers $\pm\delta$ change in each coordinate

Gradient methods compute

$$\nabla f = \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial y_1}, \frac{\partial f}{\partial x_2}, \frac{\partial f}{\partial y_2}, \frac{\partial f}{\partial x_3}, \frac{\partial f}{\partial y_3}\right)$$

to increase/reduce $f$, e.g., by $\mathbf{x} \leftarrow \mathbf{x} + \alpha \nabla f(\mathbf{x})$

Sometimes can solve for $\nabla f(\mathbf{x}) = 0$ exactly (e.g., with one city).
Newton–Raphson (1664, 1690) iterates $\mathbf{x} \leftarrow \mathbf{x} - \mathbf{H}_f^{-1}(\mathbf{x}) \nabla f(\mathbf{x})$
to solve $\nabla f(\mathbf{x}) = 0$, where $\mathbf{H}_{ij} = \partial^2 f / \partial x_i \partial x_j$