Report

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Logistic Regression

1. Main Objective

Logistic regression is a kind of parametric classification model. It uses a linear combination of features to come out with a probability to assign two values 0 and 1 fail or true and false to the response variable.
The middle value of probabilities is considered as threshold to establish what belong to the class 1 and to the class 0. In a general note, if the probability is greater that 0.5, then the observation belongs to class 1, otherwise it is belongs to class 0.

2. Logit Function

Logistic regression is expressed as:

$$log(\frac{p(X)}{1-p(X)}) = \beta_0 + \beta_1 X$$

Where the left-hand side is called the *log-odds* and $\frac{p(X)}{1-p(X)}$ is called *odds*, and it tells about the probability of success to probability of failure.

When taking the inverse of the logit function we will get:

$$p(X) = (\frac{e^{\beta_0+\beta_1 X}}{1+e^{\beta_0+\beta_1 X}})$$

This function is the **Sigmoid** function and it creates the *S-shaped* curve, and returns a probability value between 0 and 1.

3. Maximum Likelihood Estimation

Since **Maximum Likelihood** is the more general method of non-linear least squares and it has a better statistical properties, it is used to fit the logistic regression model.
The maximum likelihood estimation defines the coefficients for which the the probability of getting the observed data is maximized.

The *likelihood* function formalizing the stated intuition is:

$$L(\beta, y) = \prod_{i=1}^{N} (\frac{\pi_i}{1-\pi_i})^y (1-\pi_i)$$

for $y_i = [0, 1]$

π_i is the probability of success if y_i belongs to class 1

In order, to determine the parameters' values, we apply $log()$ to the likelihood function, since it does not change its initiatl propoerties. Then we apply **iterative** optimisation techniques such as **Gradient Descent**.

4. Pros & Cons

Pros	Cons
- Logistic Regression is easy to interpret and very efficient to train	- Logistic Regression may lead to overfitting when the number of features is greater than the number of observations
- In addition to providing coefficients' sizes, it tells the direction of association	- It assumes linearity between the dependent variable and the target
- It doesn't need hyperparameter tuning	- It is not considered as a very powerful algorithm and can be easily outperformed by other algorithms

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Linear Discriminant Analysis

1. Main Objective

The aim of LDA is to maximize the **between-class** variance and **minimize the within-class** variance through a linear discriminant function. It assumes that all classes are linearly separable, and the data in each class is described by a **Gaussian** probability density function which means that it has a bell-curve shape when plotted.

2. Linear Descriminante function

Since the LDA uses the Bayes' Theorem to make predictions based on the probability that the observation x belongs to each class. The class having the highest probability is designated as the output class, and then prediction is made by the LDA.
The *Bayes' theorem states that:

$$Pr(Y = k|X = x) = \frac{\pi_k f_k(x)}{\sum_{l=1}^{k} \pi_l f_l(x)}$$

Where x= input

k= output class

π_k = prior probability that an observation belongs to class k

$f_k(x)$ = estimated probability of x belonging to class k

Supposing that $f_k(x)$ follows a Gaussian Distribution, it takes the form:

$$f_k(x) = \frac{1}{\sqrt{2\pi\sigma_k^2}} \exp(-\frac{1}{2\sigma_k^2}(x - \mu_k)^2)$$

After plugging this into the probability equation and taking the log of it, we get the following discriminant function:

$$\delta_k(x) = x.\frac{\mu_k}{\sigma^2} - \frac{\mu_k^2}{2\sigma^2} + \log(\pi_k)$$

3. LDA assumptions

As already mentioned, LDA makes two important assumptions about the data:
- Each data variable is bell curve shaped
- The values of each variable vary around the mean by the same amount on the average.

Based on that, LDA method approximates the Bayes classifier by plugging estimates for π_k, μ_k and σ^2 into the linear discriminant function.
The following estimates ares used:

$$\hat{\mu_k} = \frac{1}{n_k} \sum_{i:y=k} x_i$$

$$\hat{\sigma}^2 = \frac{1}{n-k} \sum_{k=1}^{k} \sum_{i:y=k} (x_i - \hat{\mu_k})^2$$

$$\hat{\pi}_k = n_k / n$$

Where $\hat{\pi}_k$ = variance across all inputs x

n = number of instances

k = number of classes

$\hat{\mu_k}$ = mean for input x

4. Pros & Cons

Pros	Cons
- LDA is simple to implement and the classification is robust - It uses information from both the features to create a new axis which in turn minimizes the variance and maximizes the class distance of the two variables	- LDA's linear decision boundaries may not adequately separate the classes - It requires normal distribution assumption on features - It has a large time complexity