Python for Data Science Comprehensive Workshop

Part 01 – Python Basics & Numerical Computations Using Numpy

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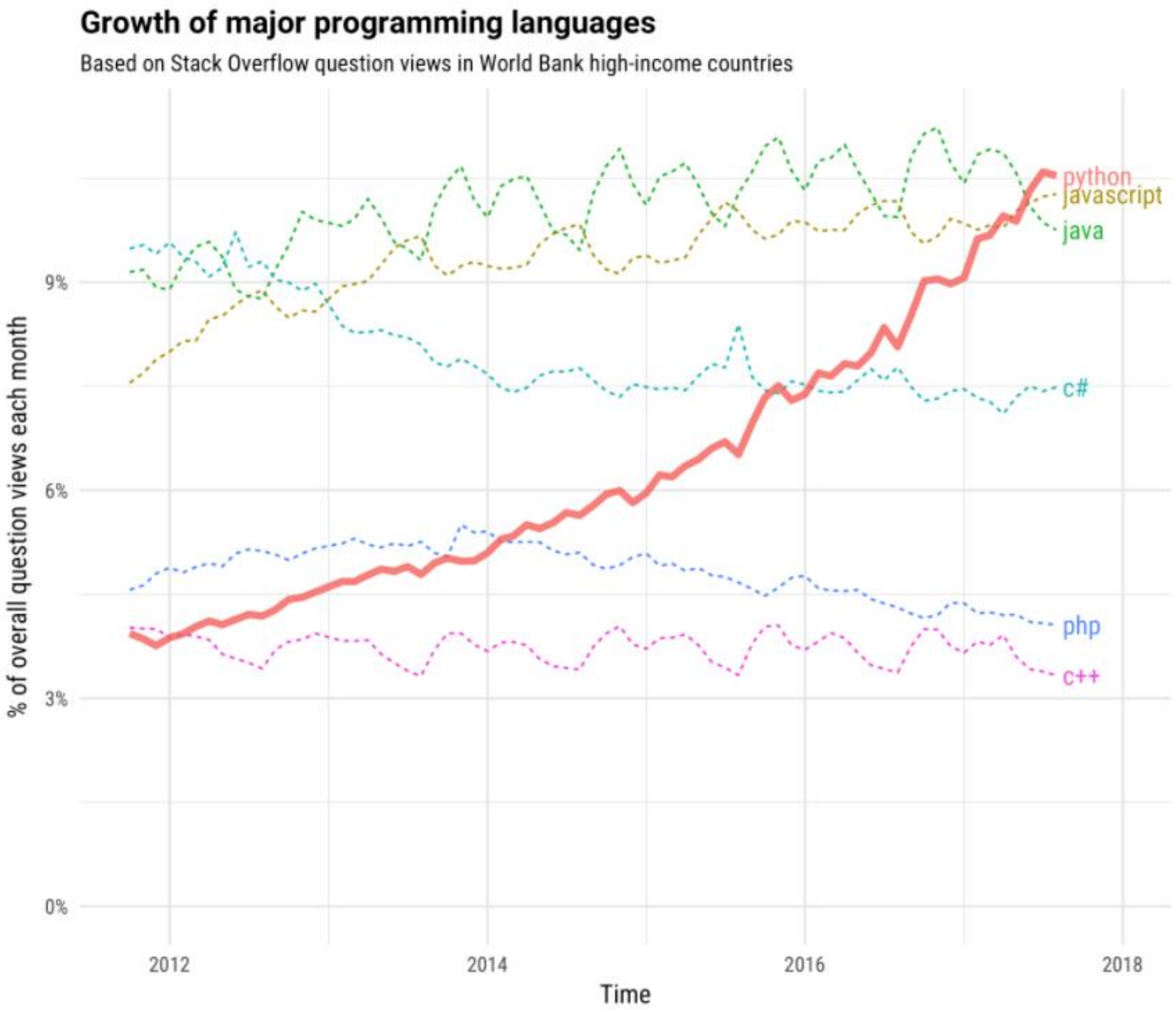
Why it is so popular?



Python

Python is an interpreted, high-level and general-purpose programming language. Python's design philosophy emphasizes code readability with its notable use of significant whitespace. Its language constructs and object-oriented approach aim to help programmers write clear, logical code for small and large-scale projects.

(Wikipedia)

Python IDEs

Mathematical Operations

Addition	+
Subtraction	-
Multiplication	*
Division	/
Integer Division	//
Power	**
Modulus Operator	%

Relational Operations

Greater than	>
Less than	<
Greater than or equal	>=
Less than or equal	<=
Equal	==
Not equal	!=

Logical Operations

And	and
Or	or
Not	not

Primitive Data Types

Python has 4 primitive data types:

- Integers
- Floats
- Booleans
- Strings

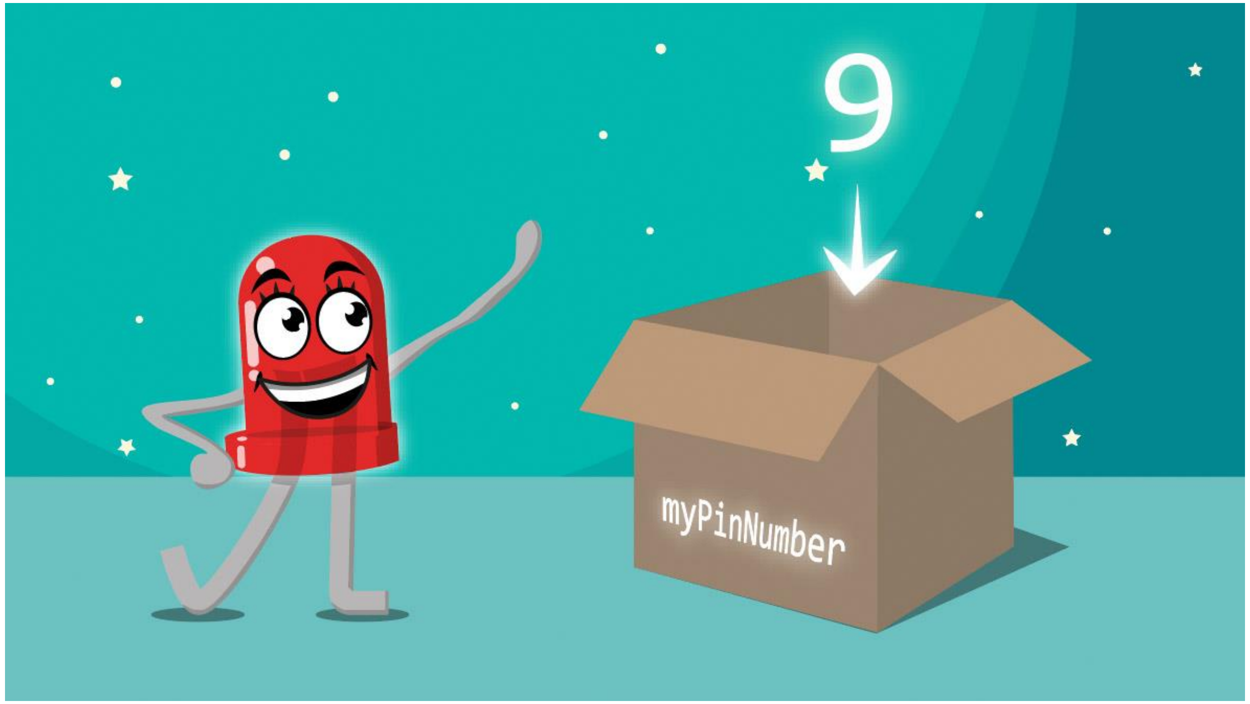
Compound Data Types

Python has 4 compound data types:

- Lists
- Tuples
- Sets
- Dictionaries

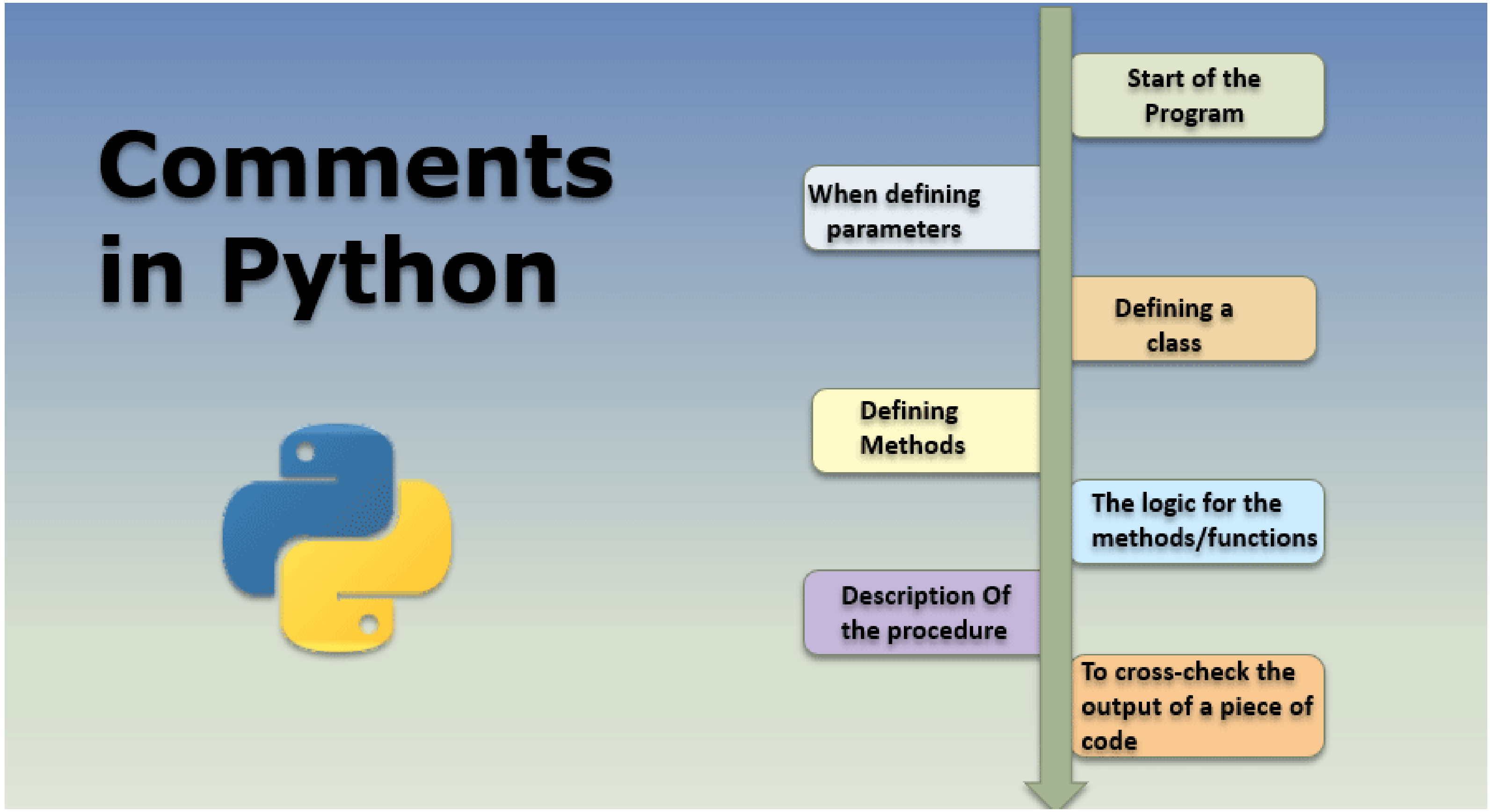
Variables

A Python variable is a reserved memory location to store values. In other words, a variable in a python program gives data to the computer for processing.



Comments

In computer programming, a comment is a programmer-readable explanation or annotation in the source code of a computer program. They are added with the purpose of making the source code easier for humans to understand, and are generally ignored by compilers and interpreters.



Compound Data Types

Lists

- Lists are used to store multiple items in a single variable.
- Square brackets are used.
- Lists are mutable.
- Lists can contain any type of data together.
- Indexing is starting from 0.
- Lists allow negative indexing.
- Several functions are available for operations in lists.

Compound Data Types

Tuples

- Tuples are used to store multiple items in a single variable.
- Curve brackets are used.
- Tuples are not mutable.
- Tuples can contain any type of data together.
- Indexing is starting from 0.
- Tuples allow negative indexing.
- Several functions are available for operations in tuples.

Compound Data Types

Sets

- Sets are used to store multiple items in a single variable.
- Curly brackets are used.
- Sets are not mutable.
- Sets can contain any type of data together.
- No indexing in sets.
- Several functions are available for operations in sets.

Compound Data Types

Dictionary

- Dictionaries have keys and values.
- Curly brackets are used.
- Dictionaries are mutable.
- Dictionaries can contain any type of data together.
- Can access the values through keys.
- Several functions are available for operations in dictionaries.

Conditions and Branching

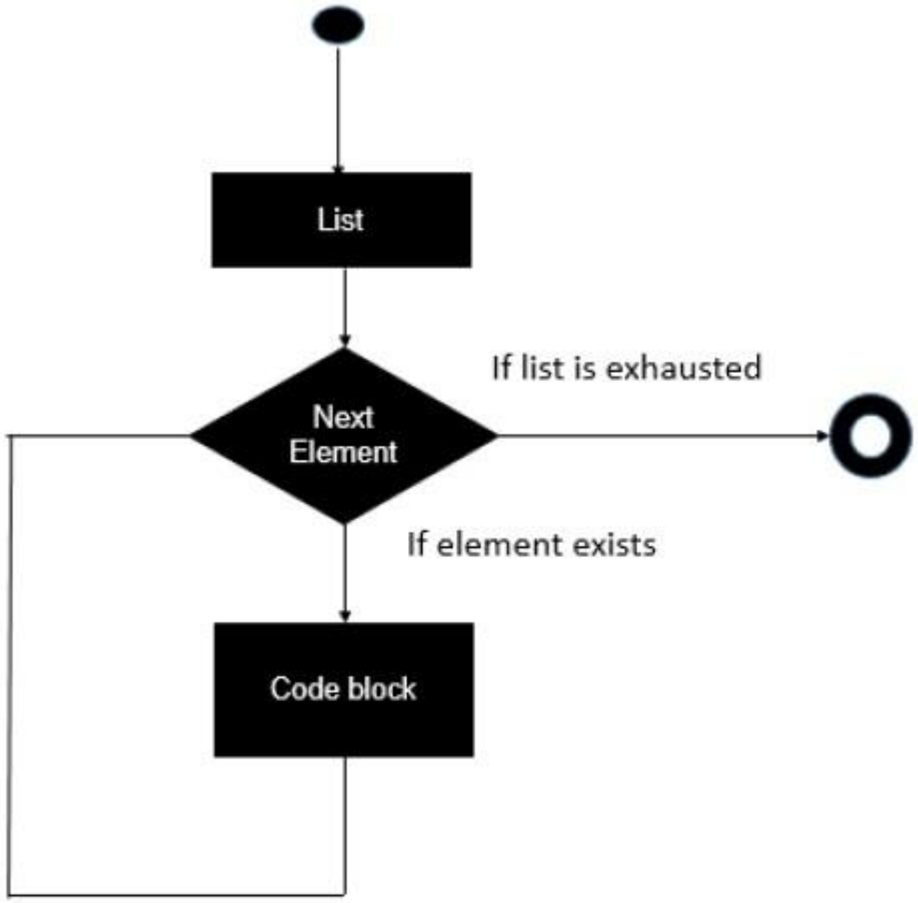
if **Criteria**:
    Do something

if **Criteria**:
    Do something
else:
    Do something

if **Criteria**:
    Do something
elif **Criteria**:
    Do something
elif **Criteria**:
    Do something
else:
    Do something

For Loops

for **iteration** in **Array**:
    Do something



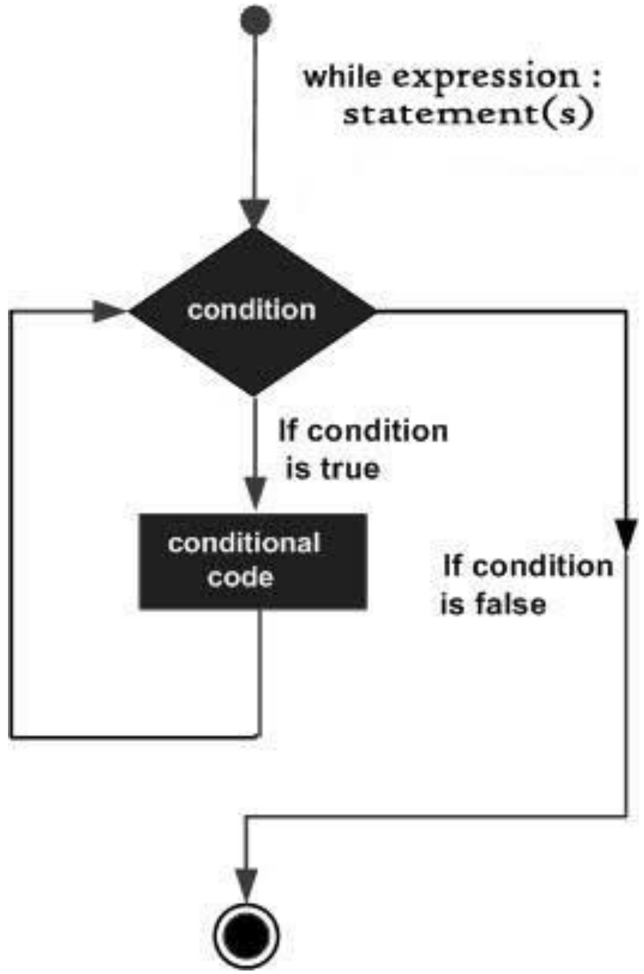
For Loops

- **Break** keyword breaks the entire loop when a condition is satisfied.
- **Continue** keyword skips the iteration when a condition is satisfied.

While Loops

Initiation

while **Criteria**:
    Do something
    Increment



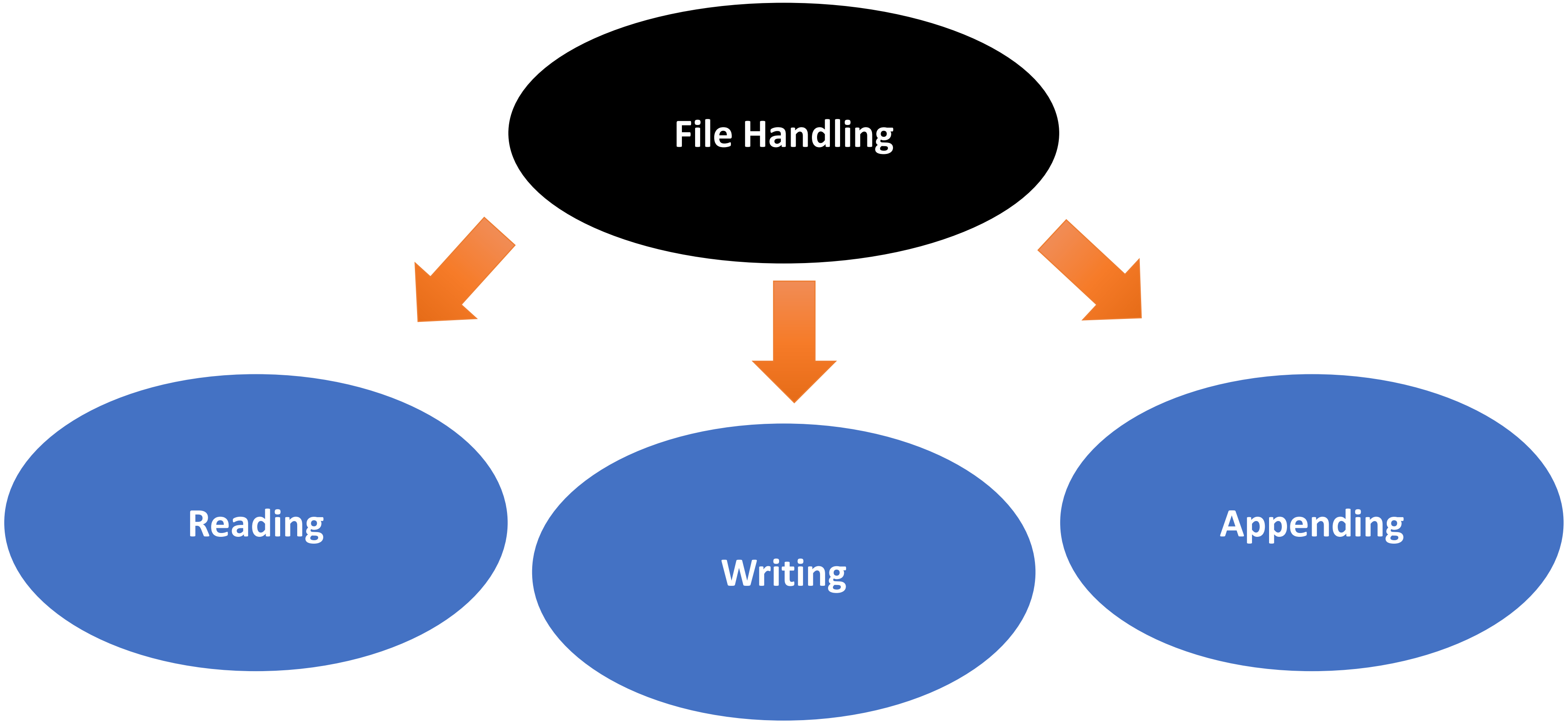
Python Functions

- Python functions are repeated blocks which will be used in several places in a program.
- Python functions can be created with arguments or without arguments.
- Python functions can be created with return values or without return values.

Python Strings

- A primitive data type of Python.
- Not mutable.
- Indexing is started from 0.
- Negative indexing is allowed.
- Several functions are available for doing string operations.

Python File Handling



Matrices

A matrix is a collection of numbers by rows and columns.

$$\boldsymbol{A} = \begin{bmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{m1} & \cdots & x_{mn} \end{bmatrix}_{m\times n}$$

Types of matrices

- A vector is a matrix which has only one row and only one column. If there is only one row it is called as a row vector and if there is only one column it is called as a column vector.

$$\boldsymbol{a} = \begin{bmatrix} 10 \\ 8 \\ 7 \end{bmatrix} \quad \boldsymbol{b} = \begin{bmatrix} -2 & 7 & 4 \end{bmatrix}$$

- A scalar matrix is a matrix with only one row and only one column.

$$\boldsymbol{a} = 5$$

- A square matrix is a matrix in which the number of rows equal to the number of columns.

$$\boldsymbol{A} = \begin{bmatrix} 1 & 6 \\ 3 & 2 \end{bmatrix}$$

Types of matrices

- A null matrix which is also called a zero matrix is any matrix in which all the elements are 0.

$$\boldsymbol{B} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

- A diagonal matrix is a square matrix where all the off diagonal elements are 0.

$$\boldsymbol{C} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 7 \end{bmatrix}$$

- An identity matrix is a diagonal matrix where all the diagonal elements are 1. It is generally denoted by I_n when the dimension is $n \times n$.

$$\boldsymbol{I}_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- A symmetric matrix is a square matrix in which $a_{ij} = a_{ji}$ for all i and j.

$$\boldsymbol{D} = \begin{bmatrix} 9 & 1 & 5 \\ 1 & 6 & 2 \\ 5 & 2 & 7 \end{bmatrix}$$

Matrix addition & subtraction

$$\boldsymbol{A}_{m\times n} \pm \boldsymbol{B}_{m\times n\times} = \boldsymbol{R}_{m\times n}$$

where $r_{ij} = a_{ij} + b_{ij}$

Ex:-

$$\begin{bmatrix} 2 & 4 & 6 \\ -1 & 3 & 2 \\ 1 & 6 & 2 \end{bmatrix} + \begin{bmatrix} 4 & 6 & 9 \\ 10 & -5 & 2 \\ 3 & 4 & 0 \end{bmatrix} =$$

Ex:-

$$\boldsymbol{A} = \begin{bmatrix} 2 & 3 \\ -2 & 6 \end{bmatrix} \; \boldsymbol{B} = \begin{bmatrix} 4 & 3 \\ 1 & 5 \end{bmatrix} \; \boldsymbol{C} = \begin{bmatrix} 6 & 2 \\ 1 & 3 \end{bmatrix}$$

Find **A**+**B-C** =

Answers

Ex:-

$$\begin{bmatrix} 2 & 4 & 6 \\ -1 & 3 & 2 \\ 1 & 6 & 2 \end{bmatrix} + \begin{bmatrix} 4 & 6 & 9 \\ 10 & -5 & 2 \\ 3 & 4 & 0 \end{bmatrix} = \begin{bmatrix} 6 & 10 & 15 \\ 9 & -2 & 4 \\ 4 & 10 & 2 \end{bmatrix}$$

Ex:-

$$\boldsymbol{A} = \begin{bmatrix} 2 & 3 \\ -2 & 6 \end{bmatrix} \ \boldsymbol{B} = \begin{bmatrix} 4 & 3 \\ 1 & 5 \end{bmatrix} \ \boldsymbol{C} = \begin{bmatrix} 6 & 2 \\ 1 & 3 \end{bmatrix}$$

Find **A**+**B-C** = $\begin{bmatrix} 0 & 4 \\ -2 & 8 \end{bmatrix}$

Matrix multiplication

- Scalar multiplication

$$k\boldsymbol{A} = \boldsymbol{B}$$

such that for all i, j $\quad b_{ij} = ka_{ij}$

Ex:-

$$2 \times \begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix} =$$

- Multiplying 2 matrices

$$\boldsymbol{C}_{m\times p} = \boldsymbol{A}_{m\times n} \times \boldsymbol{B}_{n\times p}$$

such that,

$$c_{ij} = \sum_{k=1}^{p} a_{ik} \times b_{kj}$$

Answers

Ex:-

$$2 \times \begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ -6 & 0 \end{bmatrix}$$

Ex:-

$$\begin{bmatrix} 2 & 8 & 1 \\ 3 & 6 & 4 \end{bmatrix} \times \begin{bmatrix} 1 & 7 \\ 9 & -2 \\ 6 & 3 \end{bmatrix} =$$

$$\boldsymbol{A} = \begin{bmatrix} 1 & 2 \\ 3 & 2 \\ -1 & 0 \end{bmatrix} \quad \boldsymbol{B} = \begin{bmatrix} 4 & 1 & 1 \\ 2 & 2 & 1 \end{bmatrix}$$

- Find **AB** and **BA**

Ex:-

$$\begin{bmatrix} 2 & 8 & 1 \\ 3 & 6 & 4 \end{bmatrix} \times \begin{bmatrix} 1 & 7 \\ 9 & -2 \\ 6 & 3 \end{bmatrix} = \begin{bmatrix} 80 & 1 \\ 81 & 21 \end{bmatrix}$$

$$\boldsymbol{A} = \begin{bmatrix} 1 & 2 \\ 3 & 2 \\ -1 & 0 \end{bmatrix} \quad \boldsymbol{B} = \begin{bmatrix} 4 & 1 & 1 \\ 2 & 2 & 1 \end{bmatrix}$$

- $\boldsymbol{AB} = \begin{bmatrix} 8 & 5 & 3 \\ 16 & 7 & 5 \\ -4 & -1 & -1 \end{bmatrix}$
- $\boldsymbol{BA} = \begin{bmatrix} 6 & 10 \\ 7 & 8 \end{bmatrix}$

Properties of matrix addition and subtraction

Let **A**, **B** and **C** are $m \times n$ matrices.

- $\boldsymbol{A} + \boldsymbol{B} = \boldsymbol{B} + \boldsymbol{A}$ (Commutative law)
- $\boldsymbol{A} + (\boldsymbol{B} + \boldsymbol{C}) = (\boldsymbol{A} + \boldsymbol{B}) + \boldsymbol{C}$ (Associative law)
- There is a unique $m \times n$ matrix 0 with $\boldsymbol{A} + \underline{0} = \boldsymbol{A}$ (Additive identity property)
- For any $\boldsymbol{A}_{m\times n}$ matrix there is a unique $\boldsymbol{B}_{m\times n}$ which is $-\boldsymbol{A}_{m\times n}$ matrix such that $\boldsymbol{A} + \boldsymbol{B} = \underline{0}$ (Additive inverse property)

Properties of matrix scalar multiplication

Let A and B are matrices with appropriate dimensions and r and s are real numbers.

- $r(s\boldsymbol{A}) = (rs)\boldsymbol{A}$
- $(r + s)\boldsymbol{A} = r\boldsymbol{A} + s\boldsymbol{A}$
- $r(\boldsymbol{A} + \boldsymbol{B}) = r\boldsymbol{A} + r\boldsymbol{B}$
- $\boldsymbol{A}(r\boldsymbol{B}) = r(\boldsymbol{A}\boldsymbol{B}) = (r\boldsymbol{A})\boldsymbol{B}$

Properties of matrix multiplication

Let **A**, **B** and **C** are matrices with appropriate dimensions.

- $\boldsymbol{A}(\boldsymbol{BC}) = (\boldsymbol{AB})\boldsymbol{C}$ (Associative law)
- $\boldsymbol{A}(\boldsymbol{B}+\boldsymbol{C}) = \boldsymbol{AB}+\boldsymbol{AC}$ (Distributive law)
- $(\boldsymbol{A}+\boldsymbol{B})\boldsymbol{C} = \boldsymbol{AC}+\boldsymbol{BC}$ (Distributive law)
- There are unique matrices $\boldsymbol{I}_m$ and $\boldsymbol{I}_n$ such that $\boldsymbol{I}_m\boldsymbol{A} = \boldsymbol{A}\boldsymbol{I}_n = \boldsymbol{A}$ (Multiplicative identity)

Matrix transpose

Transpose of a matrix is where the rows and columns are interchanged.

$$\boldsymbol{B} = \boldsymbol{A}^T \dashrightarrow b_{ij} = a_{ji}$$

Ex:-

$$\boldsymbol{A} = \begin{bmatrix} 2 & 7 & 1 \\ 8 & 6 & 4 \end{bmatrix} \qquad\qquad \boldsymbol{A}^T =$$

Properties of matrix transpose

- $(\boldsymbol{A}^T)^T = \boldsymbol{A}$
- For a symmetric matrix, $\boldsymbol{A}^T = \boldsymbol{A}$
- $(\boldsymbol{A} + \boldsymbol{B})^T = \boldsymbol{A}^T + \boldsymbol{B}^T$
- $(\boldsymbol{AB})^T = \boldsymbol{B}^T\boldsymbol{A}^T$
- $(r\boldsymbol{B})^T = r\boldsymbol{B}^T$

Ex:-

$$\boldsymbol{A} = \begin{bmatrix} 2 & 7 & 1 \\ 8 & 6 & 4 \end{bmatrix} \qquad \boldsymbol{A}^T = \begin{bmatrix} 2 & 8 \\ 7 & 6 \\ 1 & 4 \end{bmatrix}$$

Determinant of a matrix

Determinant is defined only for a square matrix.

$$|\boldsymbol{A}| = det(\boldsymbol{A})$$

- For a scalar matrix,

$$\boldsymbol{A} = a \dashrightarrow |\boldsymbol{A}| = a$$

- In 2×2 case,

$$\boldsymbol{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \dashrightarrow |\boldsymbol{A}| = ad - bc$$

- In 3×3 case

$$\boldsymbol{A} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \dashrightarrow |\boldsymbol{A}| = a(ei - fh) - b(di - gf) + c(dh - ge)$$

Ex:- Find the determinant of following matrices.

$$\boldsymbol{A} = \begin{bmatrix} 2 & 4 \\ -3 & 2 \end{bmatrix}$$

$$\boldsymbol{B} = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 0 & 1 \\ 1 & -1 & 3 \end{bmatrix}$$

Ex:- Find the determinant of following matrices.

$$\boldsymbol{A} = \begin{bmatrix} 2 & 4 \\ -3 & 2 \end{bmatrix} -----\rightarrow \det(A) = 16$$

$$\boldsymbol{B} = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 0 & 1 \\ 1 & -1 & 3 \end{bmatrix} -----\rightarrow \det(B) = 1(0+1) - 2(9-1) + 1(-1-0) = -16$$

Trace of a matrix

Sum of the diagonal elements of a matrix is called the trace of the matrix.

Ex:-

$$\boldsymbol{A} = \begin{bmatrix} 3 & 7 & 2 \\ -1 & 6 & 4 \\ 9 & 0 & -5 \end{bmatrix} \qquad tr(\boldsymbol{A}) =$$

Properties of the trace of a matrix

- If A I scalar $tr(\boldsymbol{A}) = \boldsymbol{A}$
- $tr(\boldsymbol{A}^T) = tr(\boldsymbol{A})$
- $tr(k\boldsymbol{A}) = k \times tr(\boldsymbol{A})$ when k is a real value
- $tr(\boldsymbol{I_n}) = n$
- $tr(\boldsymbol{A} \pm \boldsymbol{B}) = tr(\boldsymbol{A}) \pm tr(\boldsymbol{B})$
- $tr(\boldsymbol{AB}) = tr(\boldsymbol{BA})$

Ex:-

$$\boldsymbol{A} = \begin{bmatrix} 3 & 7 & 2 \\ -1 & 6 & 4 \\ 9 & 0 & -5 \end{bmatrix} \qquad tr(\boldsymbol{A}) = 4$$

Inverse of a matrix

The inverse of a matrix A is denoted by $\boldsymbol{A}^{-1}$, such that,

$$\boldsymbol{A}\boldsymbol{A}^{-1} = \boldsymbol{A}^{-1}\boldsymbol{A} = \boldsymbol{I}$$

To have an inverse of a matrix, the column vectors should be independent.

Ex:- Check whether matrix **A** has independent column vectors.

$$\boldsymbol{A} = \begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix}$$

Adjoint of a matrix

Let $\boldsymbol{A} = [a_{ij}]$ be a square matrix of order n. The adjoint of a matrix **A** is the transpose of cofactor matrix of **A**

$$Adj(\boldsymbol{A}) = [cof(a_{ij})]^T$$

$$cof(a_{ij}) = (-1)^{i+j} \det(M_{ij})$$

Ex:- Find the adjoint of the following matrix.

$$\boldsymbol{A} = \begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix}$$

Ex:- Find the adjoint of the following matrix.

$$\boldsymbol{A} = \begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix}$$

$$\boldsymbol{Cofactor\ matrix\ of\ A} = \begin{bmatrix} (-1)^{1+1}(0-6) & (-1)^{1+2}(-4-0) & (-1)^{1+3}(4-0) \\ 1 & -1 & -1 \\ -6 & 2 & 4 \end{bmatrix}$$

$$\boldsymbol{Cofactor\ matrix\ of\ A} = \begin{bmatrix} -6 & 4 & 4 \\ 1 & -1 & -1 \\ -6 & 2 & 4 \end{bmatrix}$$

$$\boldsymbol{Adj}(\boldsymbol{A}) = \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$$

Matrix inverse through adjoint

The inverse of the matrix can be found through,

$$A^{-1} = \frac{Adj(A)}{|A|}$$

To have an inverse the matrix should be a non-singular matrix.

Ex:- Find the inverse of following matrix,

$$\boldsymbol{B} = \begin{bmatrix} 1 & 5 & 2 \\ 0 & -1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

Ex:-

$$\boldsymbol{A} = \begin{bmatrix} 1 & -1 & 2 \\ 4 & 0 & 6 \\ 0 & 1 & -1 \end{bmatrix}$$

$$\boldsymbol{det}(\boldsymbol{A}) = -2$$

$$\boldsymbol{Adj}(\boldsymbol{A}) = \begin{bmatrix} -6 & 1 & -6 \\ 4 & -1 & 2 \\ 4 & -1 & 4 \end{bmatrix}$$

$$A^{-1} = \frac{Adj(A)}{|A|}$$

$$A^{-1} = \begin{bmatrix} 3 & -0.5 & 2 \\ -2 & 0.5 & -1 \\ -2 & 0.5 & -2 \end{bmatrix}$$

- For a 2×2 matrix the inverse is particularly simple.

$$\boldsymbol{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

The inverse is,

$$\boldsymbol{A}^{-1} = \frac{1}{\det(\boldsymbol{A})} \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}$$

Ex

$$\boldsymbol{B} = \begin{bmatrix} 12 & 10 \\ 3 & 7 \end{bmatrix}$$

- If A is a diagonal matrix, Suppose **A** is given as follows,

$$\boldsymbol{A} = \begin{bmatrix} 9 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 7 \end{bmatrix}$$

$$\boldsymbol{A}^{-1} = \begin{bmatrix} \frac{1}{9} & 0 & 0 \\ 0 & \frac{1}{6} & 0 \\ 0 & 0 & \frac{1}{7} \end{bmatrix}$$

Properties of matrix inverse

- $I^{-1} = I$
- $(A^{-1})^{-1} = A$
- $(A^T)^{-1} = (A^{-1})^T$
- $(AB)^{-1} = B^{-1}A^{-1}$

Eigenvalues & Eigenvectors

The Eigenvalues of a square matrix **A** are describe the solutions for the λ in the equation,

$$|\boldsymbol{A} - \lambda \boldsymbol{I}| = 0$$

There is a non zero vector **v** which is the Eigenvector of **A** if there is a scalar λ such that,

$$\boldsymbol{Av} = \lambda \boldsymbol{v}$$

The scalar λ is said to be the Eigenvalue of **A** corresponding to **v**.

Ex:- Find the Eigen values for the following matrix **P** and corresponding Eigenvectors.

$$P = \begin{bmatrix} 1 & -5 \\ -5 & 1 \end{bmatrix}$$

If $\boldsymbol{v}^T \boldsymbol{v} = 1$ then **v** is called as a Normalized EigenVector. Normalized Eigenvector is generally denoted as **e**.

Ex:- If you get a normalized Eigenvector when $\lambda = 6$ in the previous example, find that normalized Eigenvector

Ex:- Find the Eigenvalues for the following matrix **P** and corresponding Eigenvectors.

$$P = \begin{bmatrix} 1 & -5 \\ -5 & 1 \end{bmatrix}$$

$$|P - \lambda I| = 0$$

$$\left| \begin{bmatrix} 1 & -5 \\ -5 & 1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| = 0$$

$$\left| \begin{bmatrix} 1-\lambda & -5 \\ -5 & 1-\lambda \end{bmatrix} \right| = 0$$

$$(1-\lambda)^2 - 25 = 0$$

$$(1-\lambda) \; - 5 = 0 \text{ or} (1-\lambda) + 5 = 0$$

$$\lambda = -4 \; or \; \lambda = 6$$

Ex:- Find the Eigenvalues for the following matrix **P** and corresponding Eigenvectors.

$$P = \begin{bmatrix} 1 & -5 \\ -5 & 1 \end{bmatrix}$$

$$\lambda = -4 \; or \; \lambda = 6$$

When $\lambda = 6$,

$$\begin{bmatrix} 1 & -5 \\ -5 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 6\begin{bmatrix} x \\ y \end{bmatrix}$$

$$x - 5y = 6x$$
$$-5x + y = 6y$$

$$x = -y$$

$$Eigenvector = \begin{bmatrix} -t \\ t \end{bmatrix} \text{ where t} \in R$$

Find the Eigenvector where $\lambda = -4$.

Ex:- If you get a normalized Eigen vector when $\lambda = 6$ in the previous example, find that normalized Eigen vector

$$\begin{bmatrix} -t & t \end{bmatrix} \begin{bmatrix} -t \\ t \end{bmatrix} = 1$$

$$2t^2 = 1$$

$$t = \pm \frac{1}{\sqrt{2}}$$

$$Normalized\ Eigenvector = \begin{bmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$$

Numpy library

- NumPy is a library for the Python programming language, adding support for large, multi-dimensional arrays and matrices, along with a large collection of high-level mathematical functions to operate on these arrays.
- It also has functions for working in domain of linear algebra, fourier transform, and matrices.
- It provides large number of functions for working with numerical arrays.