Matlab 计算不定积分

(1) $\displaystyle\int x^2 dx = \frac{x^3}{3} + C$

(2) $\displaystyle\int \frac{1}{x} dx = \ln|x| + C$

(3) $\displaystyle\int \frac{x^2}{1+x^2} dx = \int \frac{1+x^2-1}{1+x^2} dx = \int \left(1 - \frac{1}{1+x^2}\right) dx = x - \arctan x + C$
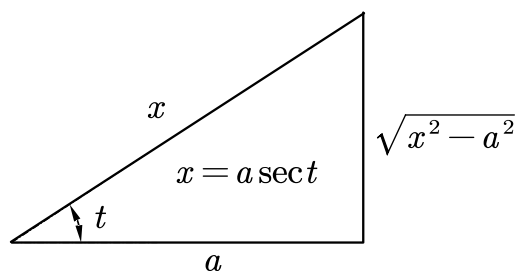
(4) $\displaystyle\int \frac{1}{e^x+1} dx = \int \frac{1+e^x-e^x}{e^x+1} dx = \int \left(1 - \frac{e^x}{e^x+1}\right) dx = x - \int \frac{1}{e^x+1} d(e^x) = x - \ln(e^x+1) + C$

(5) 求 $\displaystyle\int \frac{1}{\sqrt{x^2-a^2}} dx$.

令 $x = a\sec t \left(0 < t < \dfrac{\pi}{2}\right)$, 则 $dx = d(a\sec t) = a\sec t\tan t\, dt$, $x^2 - a^2 = a^2\sec^2 t - a^2 = a^2\tan^2 t$

所以原式 $= \displaystyle\int \frac{a\sec t\tan t}{a\tan t} dt = \int \sec t\, dt = \ln|\sec t + \tan t| + C_1$

现在我们利用三角形将 $\sec t$ 和 $\tan t$ 还原成 x 的函数:

$\sec t = \dfrac{1}{\cos t} = \dfrac{x}{a}$, $\tan t = \dfrac{\sqrt{x^2-a^2}}{a}$

所以 $\displaystyle\int \frac{1}{\sqrt{x^2-a^2}} dx = \ln\left|\frac{x}{a} + \frac{\sqrt{x^2-a^2}}{a}\right| + C_1 = \ln\left|x + \sqrt{x^2-a^2}\right| + C_1 - \ln a = \ln\left|x + \sqrt{x^2-a^2}\right| + C$

Matlab 计算定积分

(1) $\displaystyle\int_0^{\frac{\pi}{2}} \sin x\, dx = -\cos x\Big|_0^{\frac{\pi}{2}} = 1$

(2) $\displaystyle\int_a^b e^x dx = e^x\Big|_a^b = e^b - e^a$

(3) 若 $\displaystyle\int_0^{+\infty} \frac{\sin x}{x} dx = \frac{\pi}{2}$, 求 $I = \displaystyle\int_0^{+\infty} \frac{\sin^2 x}{x^2} dx$.

$I = \displaystyle\int_0^{+\infty} \frac{\sin^2 x}{x^2} dx = \int_0^{+\infty} \sin^2 x\, d\left(-\frac{1}{x}\right) = -\frac{\sin^2 x}{x}\Big|_0^{+\infty} - \int_0^{+\infty} -\frac{1}{x}\cdot 2\sin x\cos x\, dx$

$= \displaystyle\int_0^{+\infty} \frac{\sin 2x}{x} dx = \int_0^{+\infty} \frac{\sin 2x}{2x} d(2x) = \frac{\pi}{2}$