Probability Density Function

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Uniformly distributed random numbers

- rand(n): produce an n×n matrix of random numbers inside [0,1].
- rand(n,m) produces an n×m matrix of random numbers inside [0,1].

Uniformly distributed random numbers

Problem: Generate a 10-by-1 column vector of uniformly distributed numbers in the interval [-5,5].

```
n = 10;

x = -5 + (5+5)*rand(n,1)

x =

   3.1472
   4.0579
  -3.7301
   4.1338
   1.3236
  -4.0246
  -2.2150
   0.4688
   4.5751
   4.6489
```

Uniformly distributed random numbers

Problem: Generate a 10-by-3 column vector of uniformly distributed numbers in the interval [-5,5].

```
n = 10;

x = -5 + (5+5)*rand(n,3)

x =

  -3.4239    1.5574    2.0605
   4.7059   -4.6429   -4.6817
   4.5717    3.4913   -2.2308
  -0.1462    4.3399   -4.5383
   3.0028    1.7874   -4.0287
  -3.5811    2.5774    3.2346
  -0.7824    2.4313    1.9483
   4.1574   -1.0777   -1.8290
   2.9221    1.5548    4.5022
   4.5949   -3.2881   -4.6555
```

How to check visually the result?
Use histogram

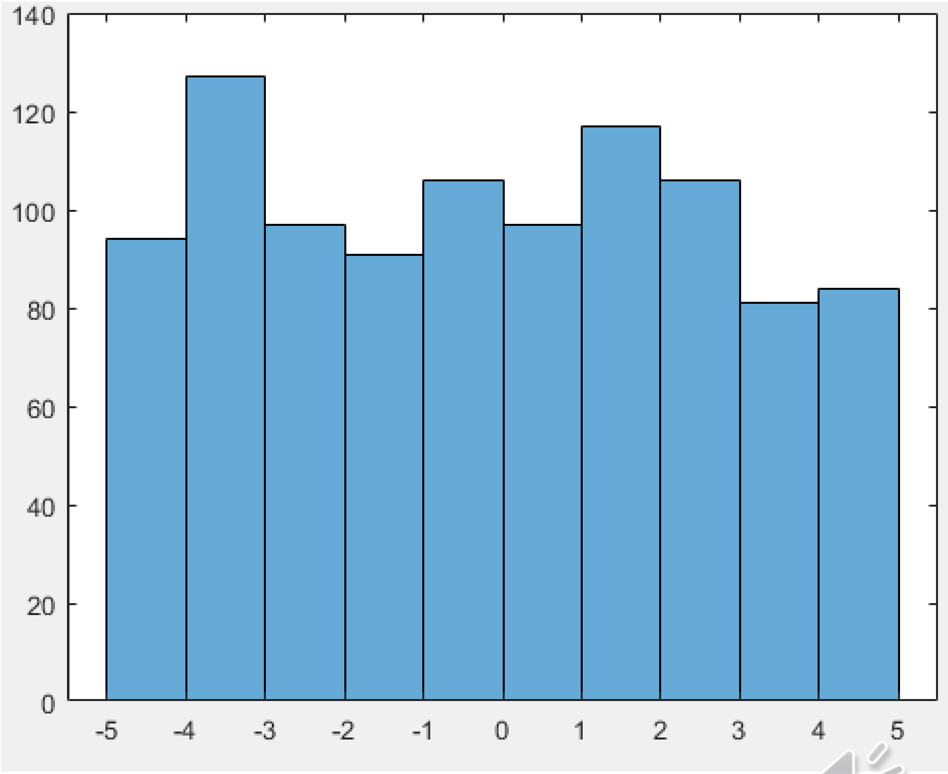
Need to validate the correctness of our results.

Uniformly distributed random numbers

Problem: Generate a n-by-1 column vector of uniformly distributed numbers in the interval [-5,5].

Assume n = 1000.

```
n = 1000;
x = -5 + (5+5)*rand(n,1);
h = histogram(x);
```



histogram(x):

- automatically compute an appropriate number of bins
- cover the range of values in x
- show the shape of the underlying distribution.

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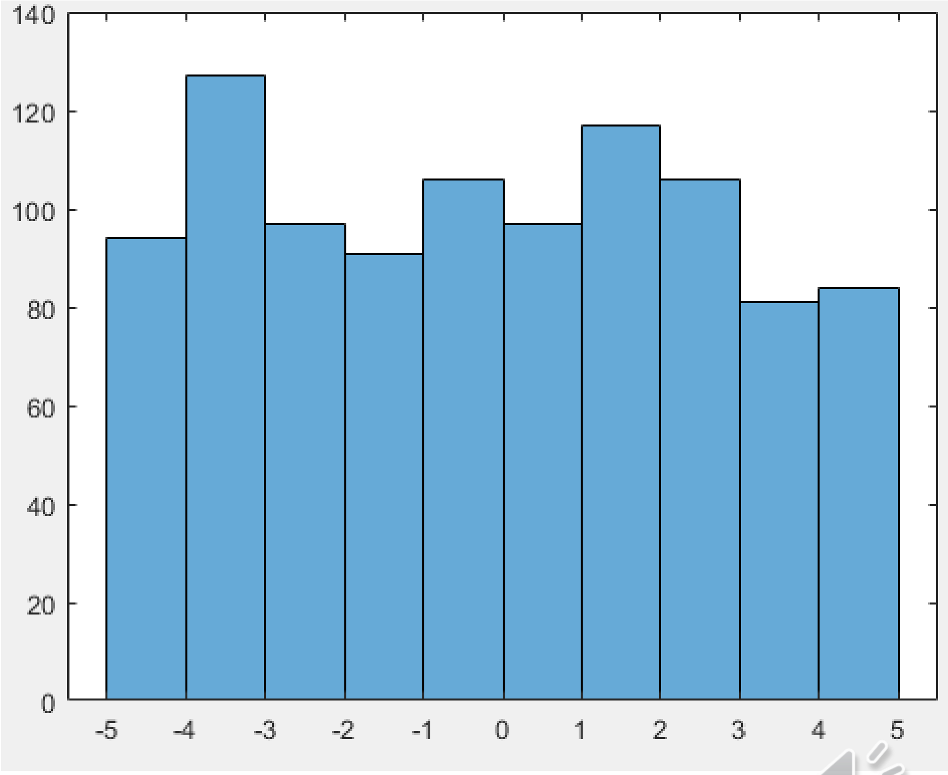


histogram(x):

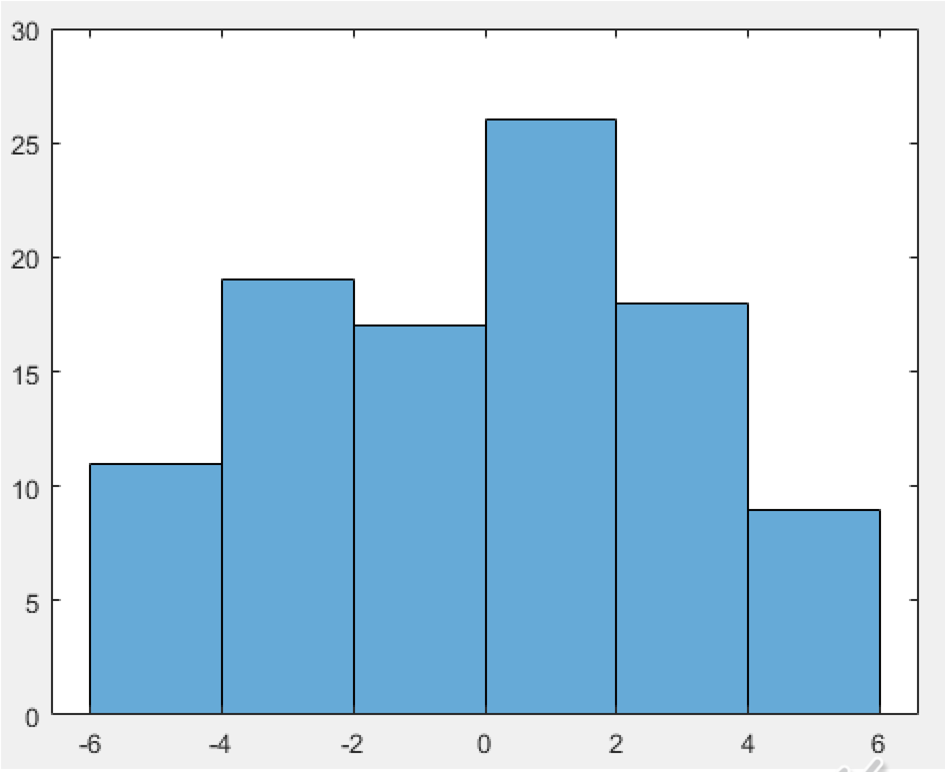
- automatically compute an appropriate number of bins
- cover the range of values in x
- show the shape of the underlying distribution.

Uniformly distributed random numbers

```
n = 100;
x = -5 + (5+5)*rand(n,1);
h = histogram(x);


h.Values
ans =

    11    19    17    26    18    9
```



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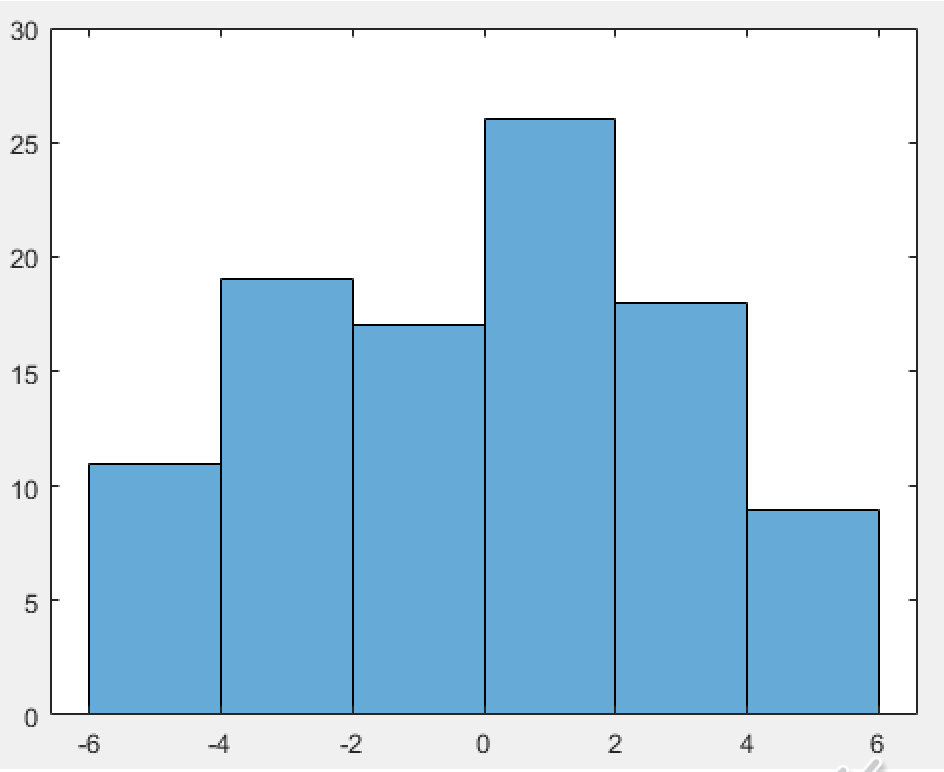

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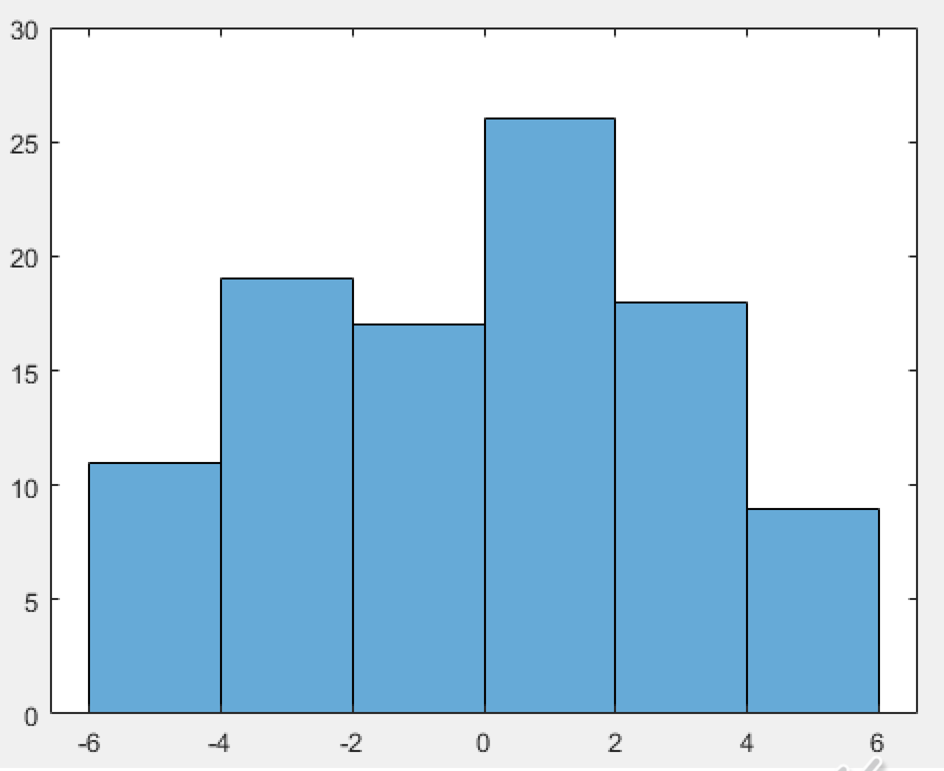


Uniformly distributed random numbers

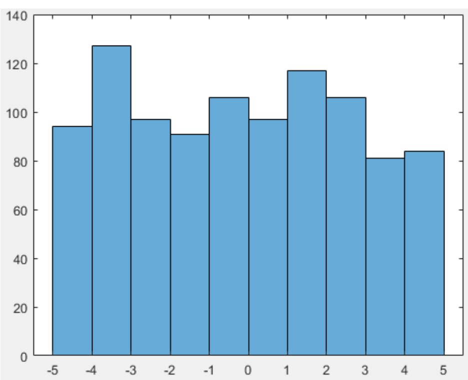
x = -5 + (5+5)*rand(n,1);

h = histogram(x);

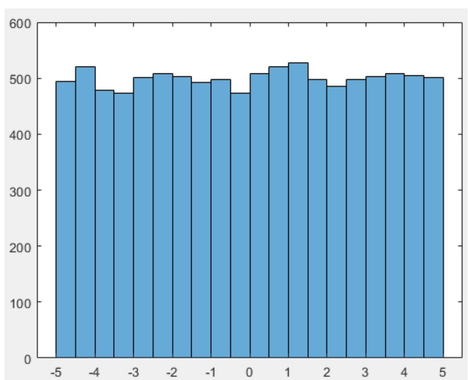
hmax = max(h.Values);

hmin = min(h.Values);

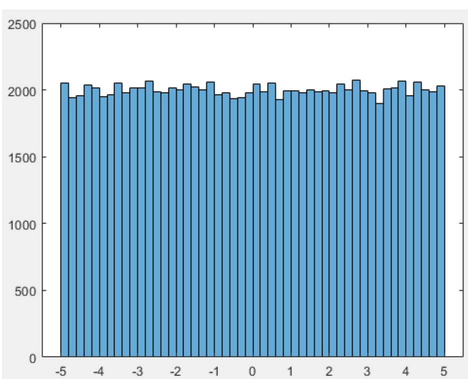
rv = (hmax-hmin)/hmin   %relative difference w.r.t. the minimum number



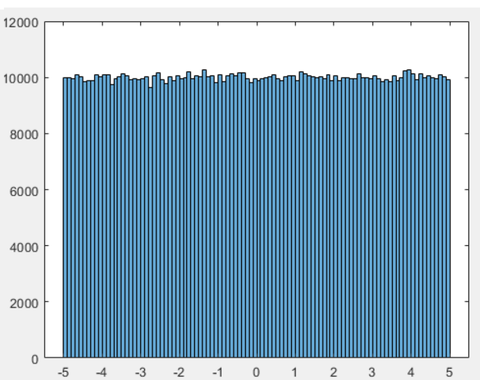
n = 1000
rv = 0.4524



n = 10000
rv = 0.1490



n = 100000
rv = 0.1012



n = 1,000,000
rv = 0.0646

Uniformly distributed random numbers

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x = -5 + (5+5)*rand(n,1);

h = histogram(x);

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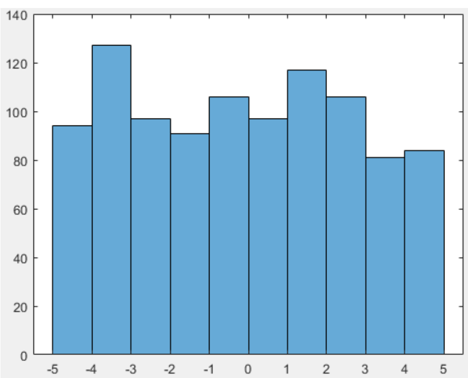

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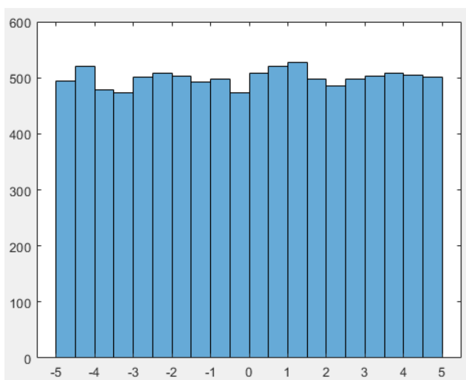

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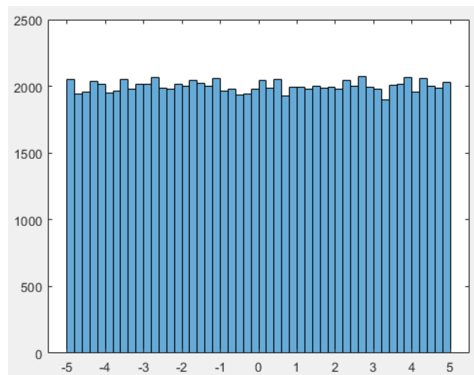

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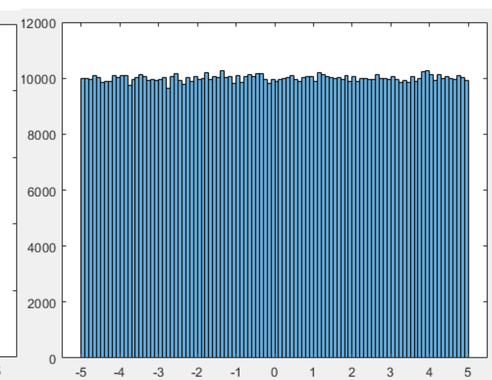

n = 1,000,000
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Uniformly distributed random numbers

```
n = 100000;
y = -5 + (5+5)*rand(n,1);
x = y.^2;
h = histogram(x);
% what do we get?
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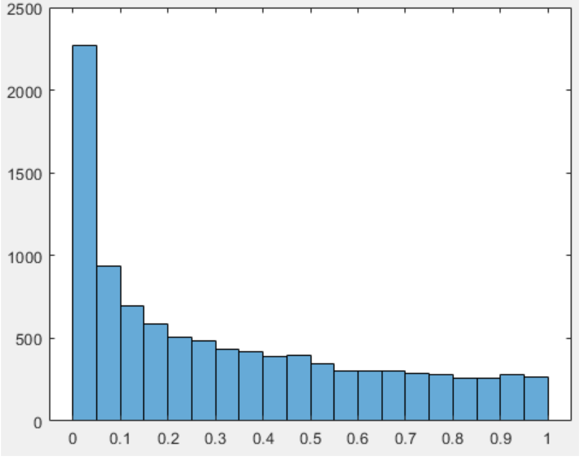

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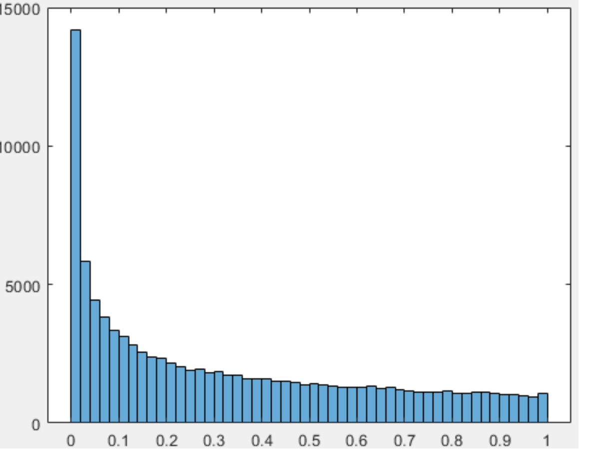

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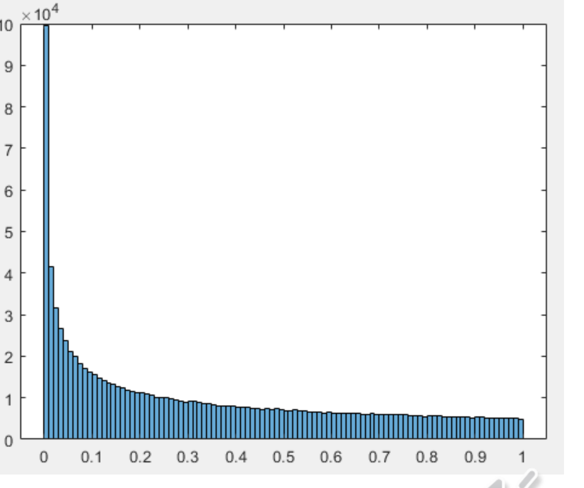
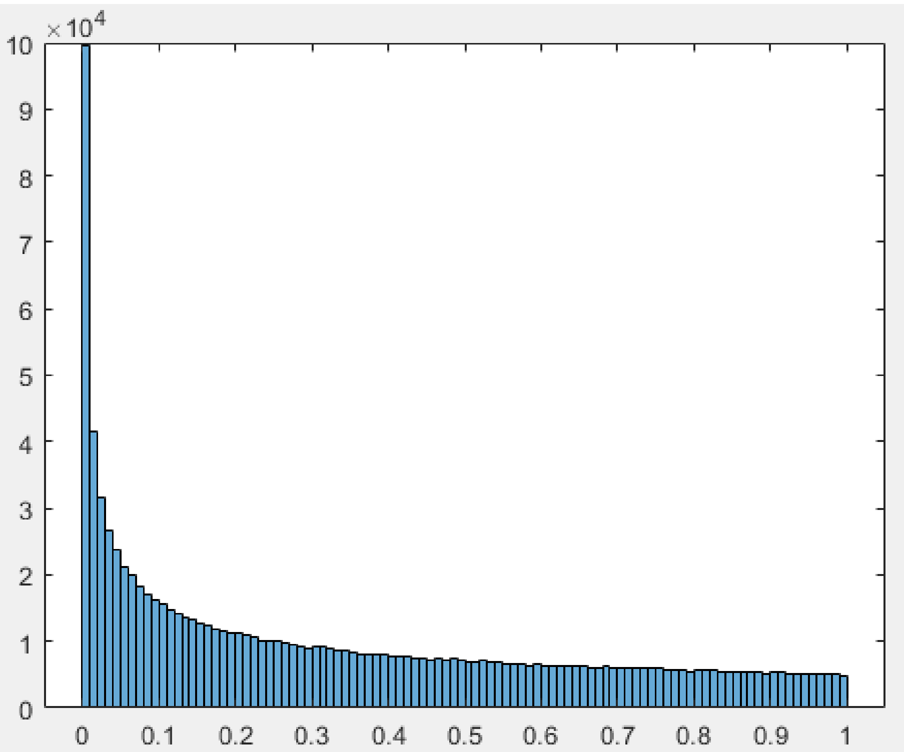


n = 1000000;

A histogram using the 'probability' normalization

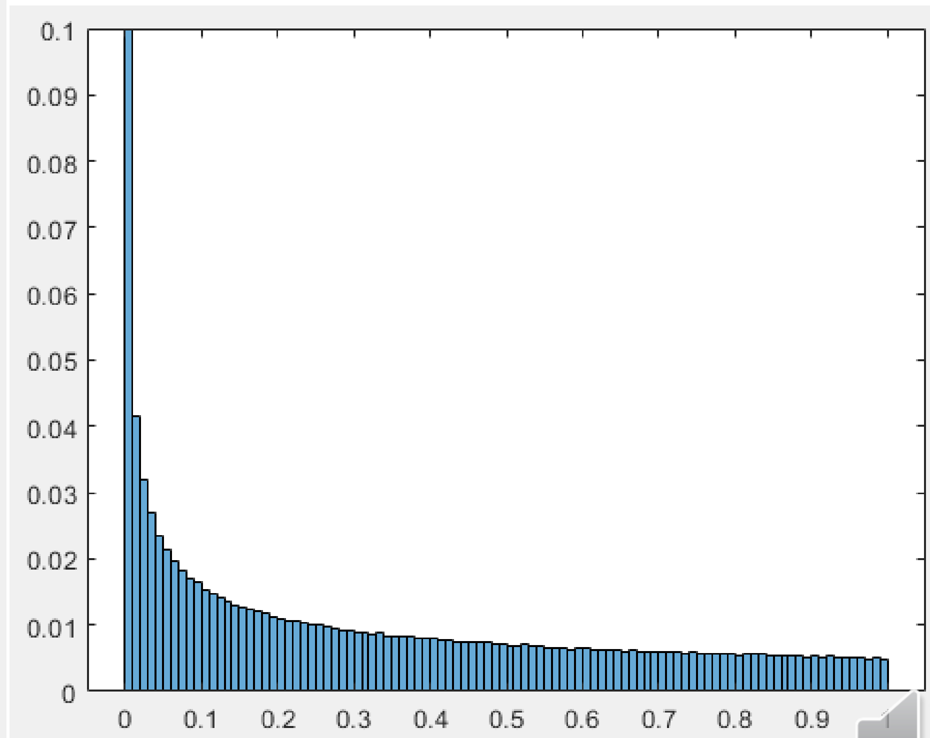
```
y = -5 + (5+5)*rand(n,1); x = y.^2;
h = histogram(x,'Normalization','probability')
```

Number of outcomes



h = histogram(x);

probability



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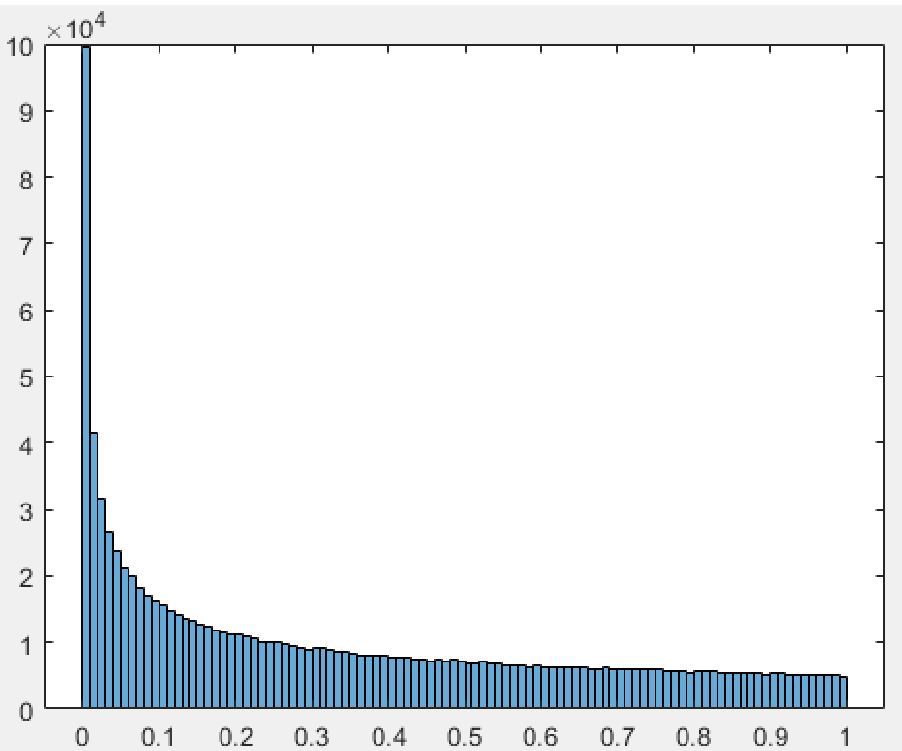

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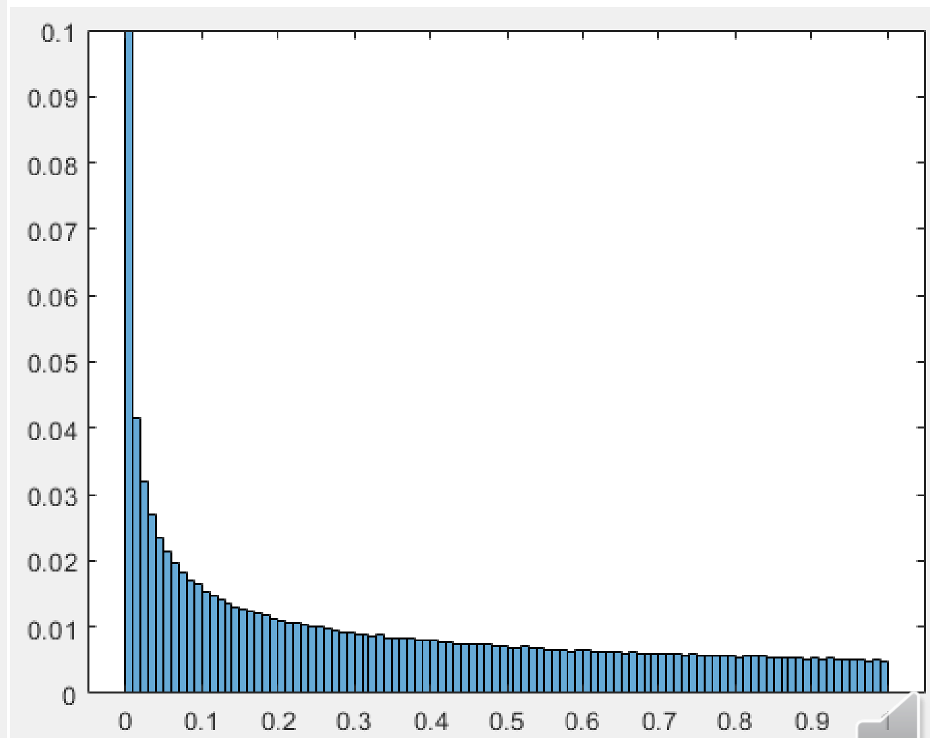


histogram(x,'Normalization','probability')

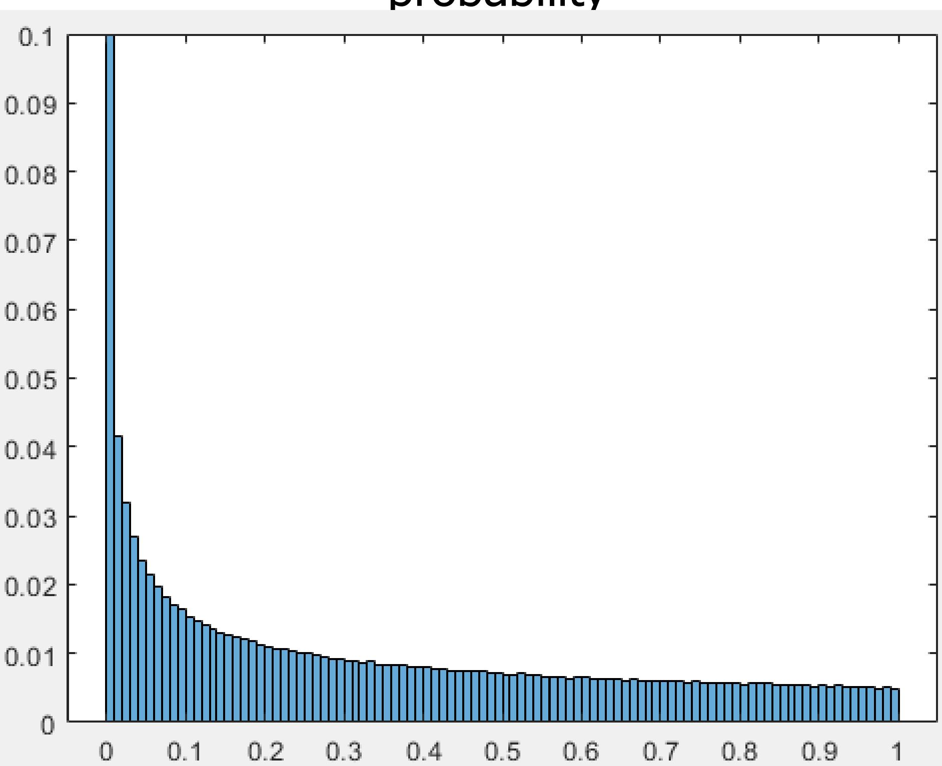
A histogram using the probability density function

y = -5 + (5+5)*rand(n,1); x = y.^2;

h = histogram(x,'Normalization', 'pdf')

Note: the value of the pdf can be larger than 1. We use the bounded area to determine probability.

probability



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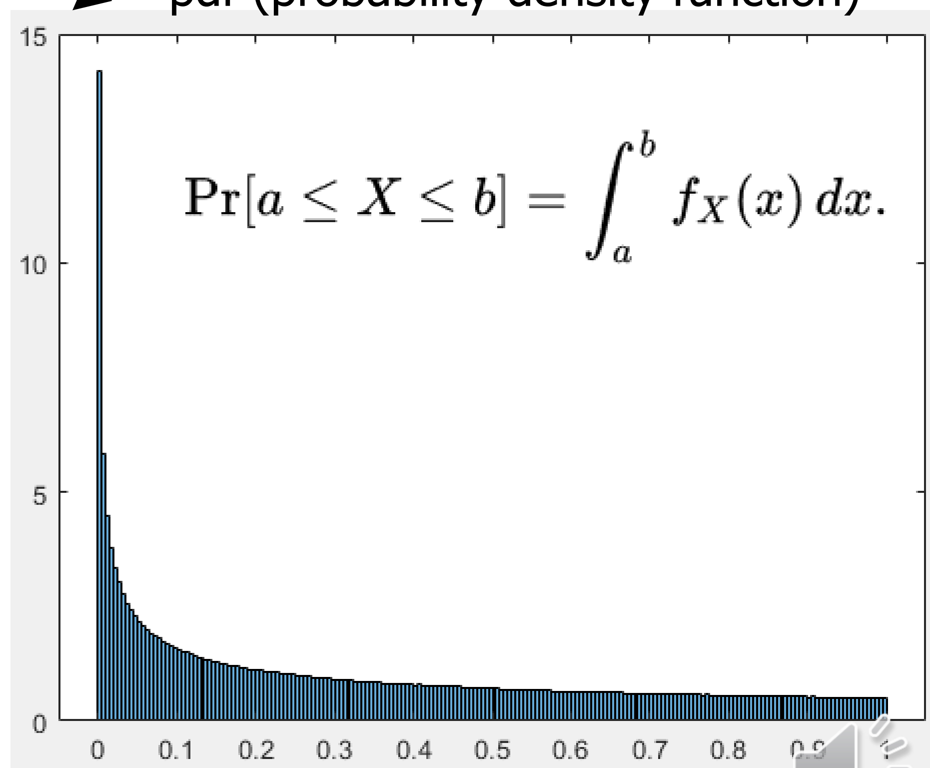
pdf (probability density function)



$$\Pr[a \le X \le b] = \int_a^b f_X(x)\,dx.$$

h = histogram(x,'Normalization', 'pdf')

How to use the graph of a pdf?

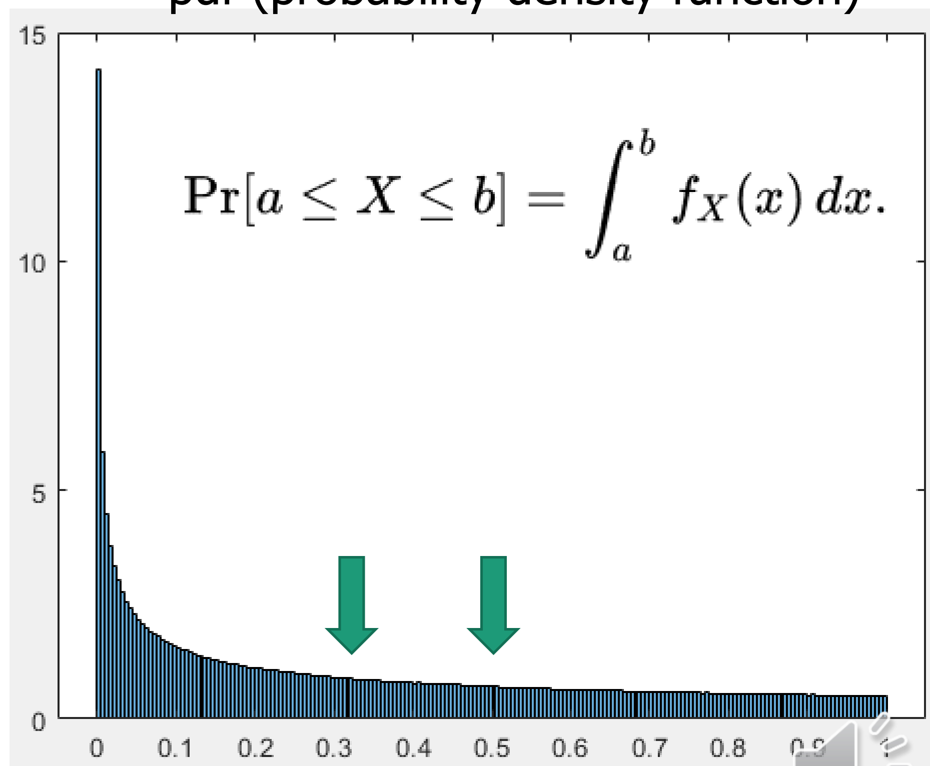
Determine the probability that a number is generated inside an **interval**?

The area of the pdf bounded inside the interval is the probability.

Example:

What is the probability that a number is generated inside [0.3, 0.5]?

pdf (probability density function)



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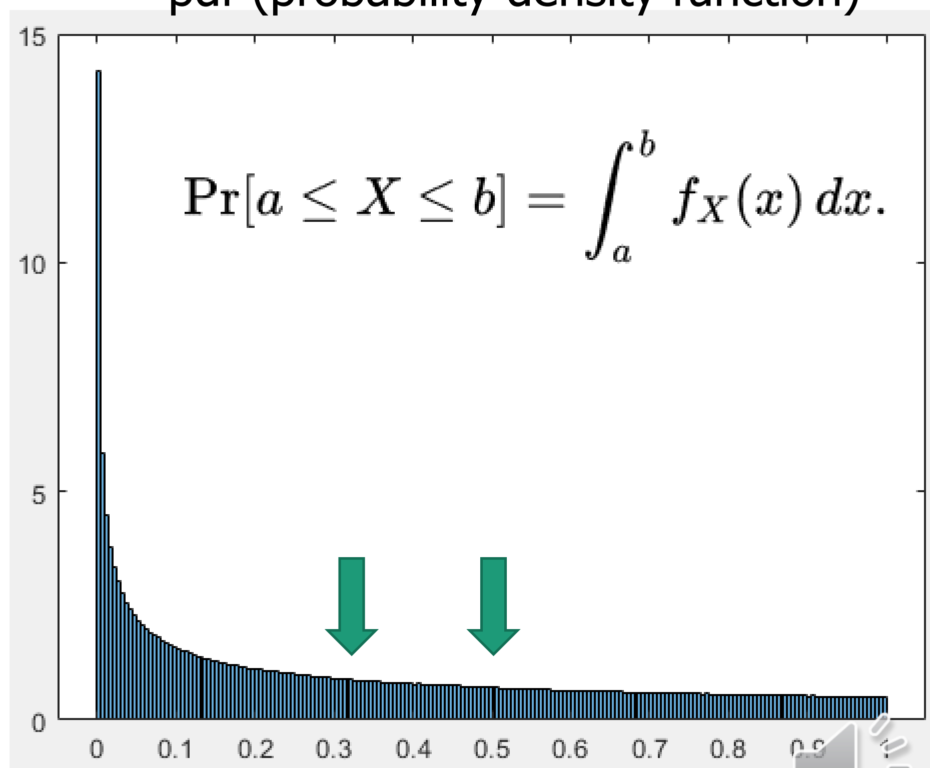
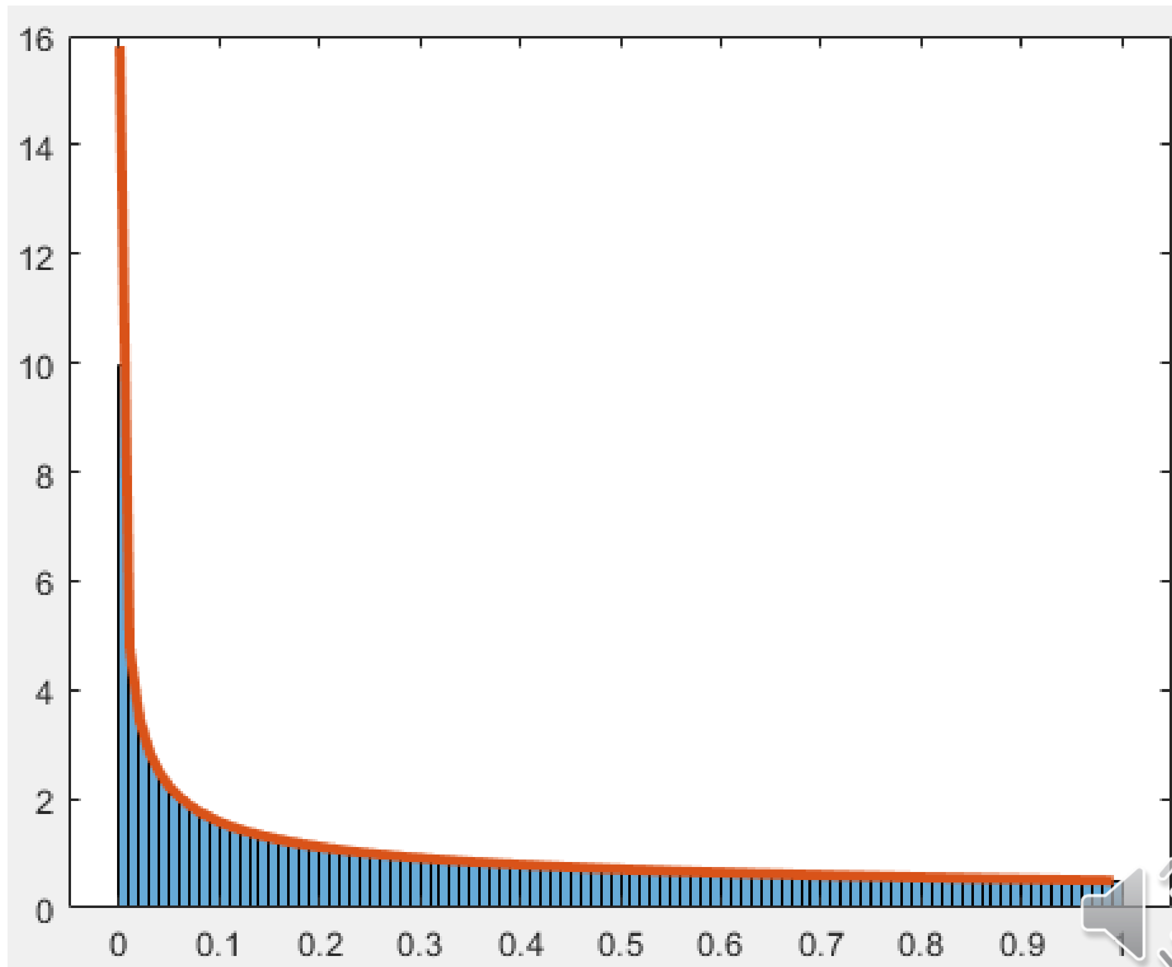


$$\Pr[a \le X \le b] = \int_a^b f_X(x)\,dx.$$

h = histogram(x,'Normalization', 'pdf')

```
clf;                              %Clear current figure window
n = 10000000; y = rand(n,1);
x = y.^2;
h = histogram(x,'Normalization','pdf')
hold on
z = 0.001:0.01:1;
f = 1./sqrt(z)./2;
plot(z,f,'LineWidth',3);
```



The pdf is:

$$f(x) = 1/(2\ x^{1/2})$$

Example

Assume that y is generated randomly in a uniform manner in [0,1].

Let x = sqrt(y).

What is the pdf of x?

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The pdf is:

$f(x) = 2x$

Write a program to compute n samples of y.

Compute x for all samples of y.

Show the histogram of x in pdf.

Plot the pdf, f(x). Use the proper ranges for the axes.

```
clf;
n = 10000000;
y = rand(n,1);
x = sqrt(y);
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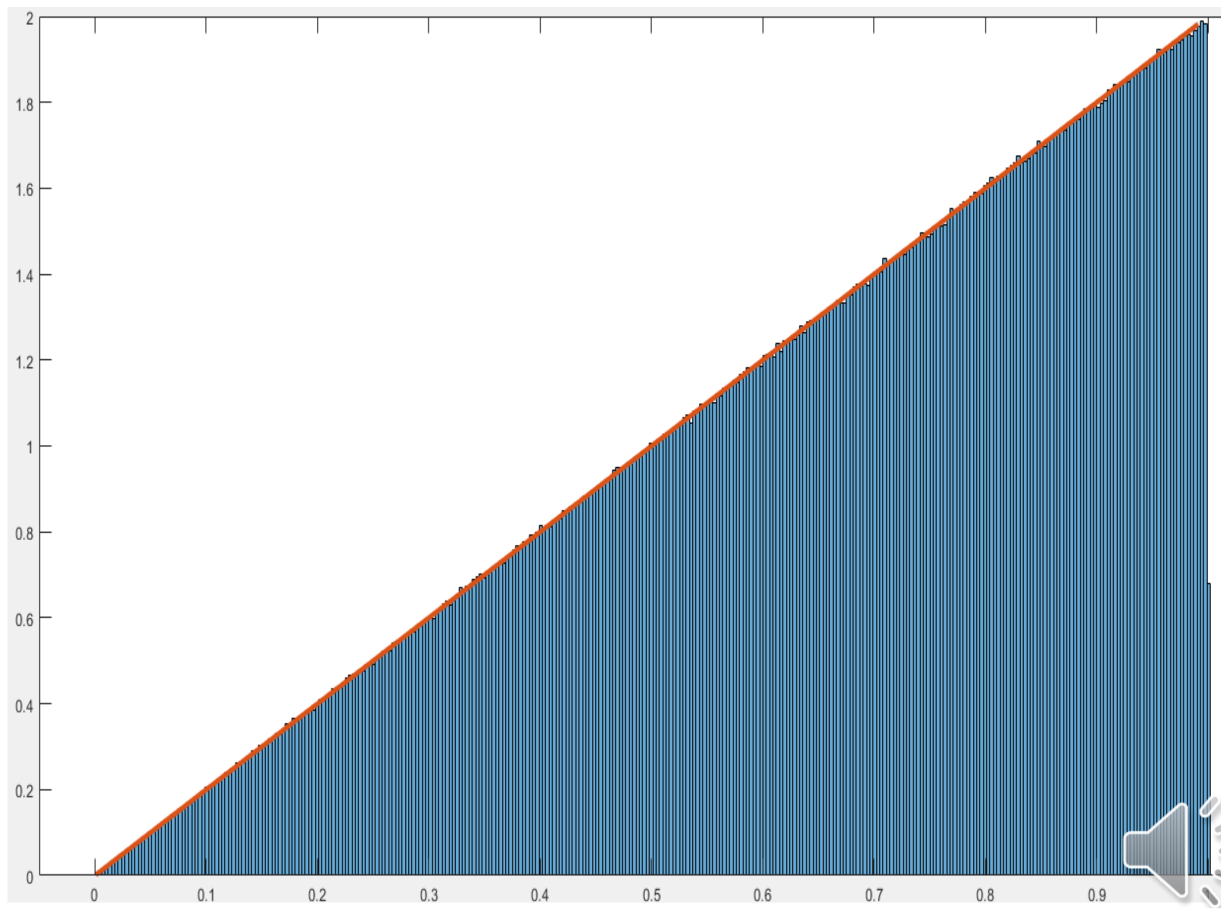
hold on
z = 0.001:0.01:1;
f = 2.*z;
plot(z,f,'LineWidth',3);
```



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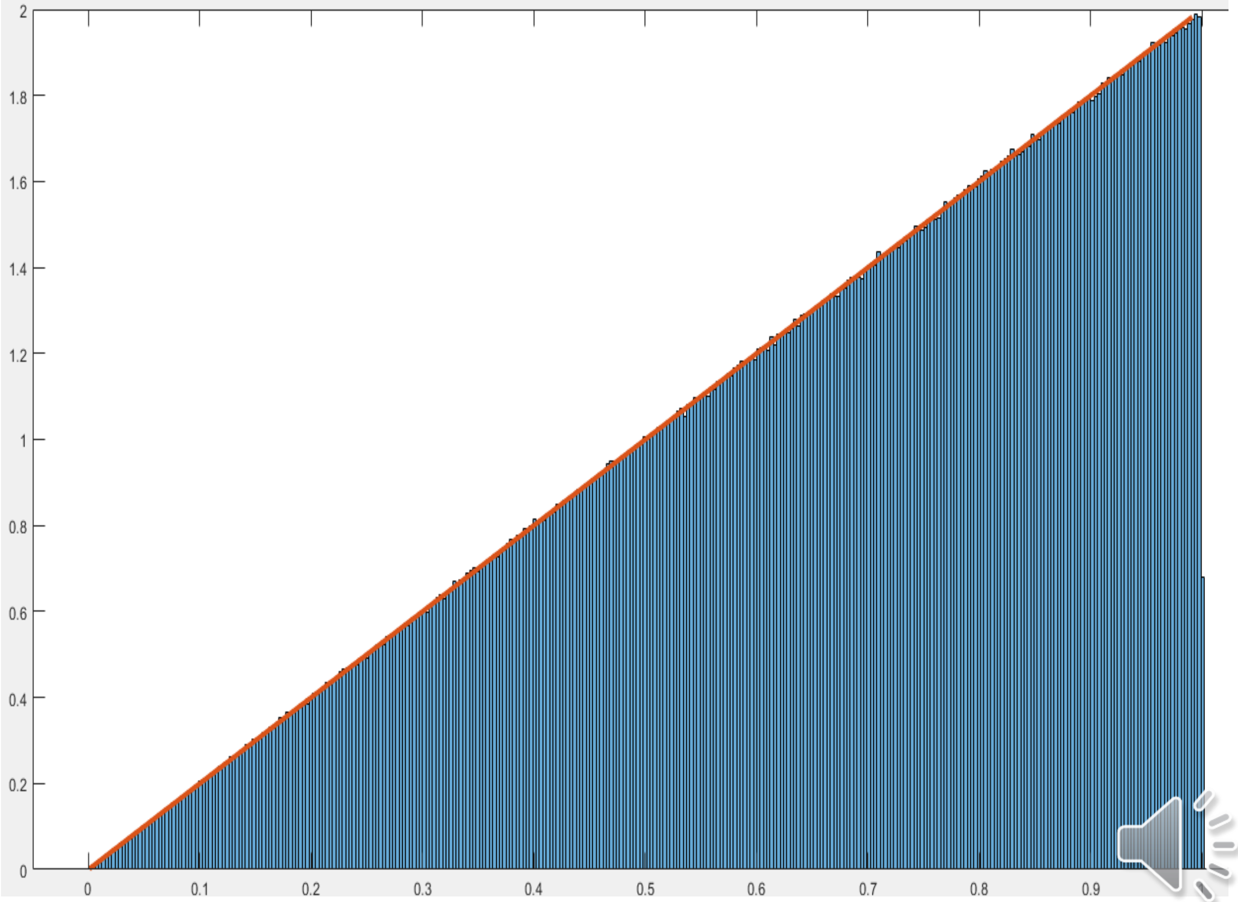
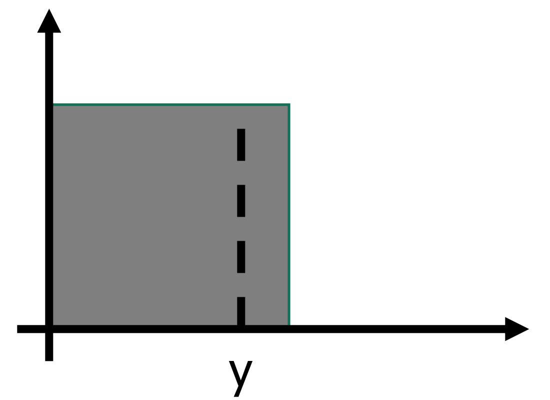


Probability density function conversion

Assume that Y is generated randomly in a uniform manner inside [0,1].

Then we have the cumulative probability

$p(Y \leq y) = y$



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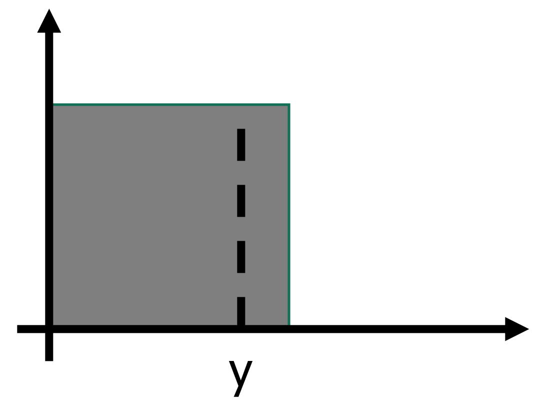

Probability density function conversion

Assume that y_0 is generated randomly in a uniform manner inside [0,1].

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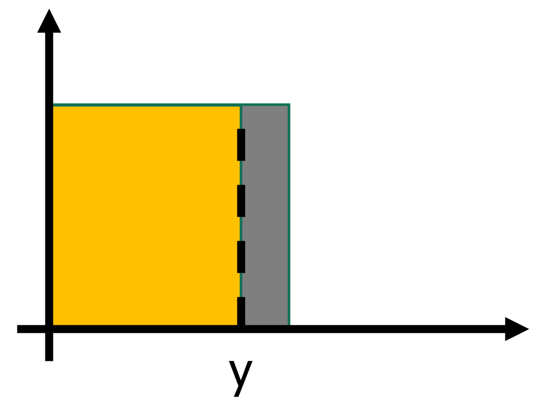
$F_Y(y) = p\ (Y \leq y) = y$



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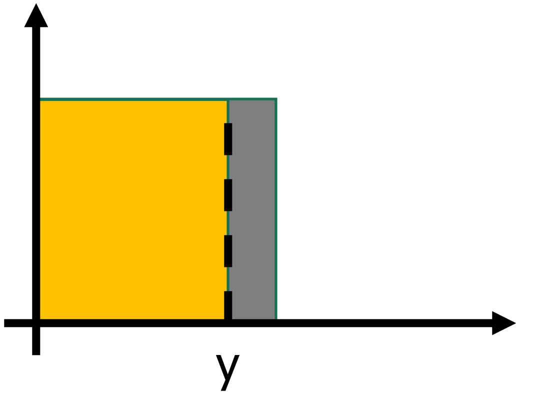
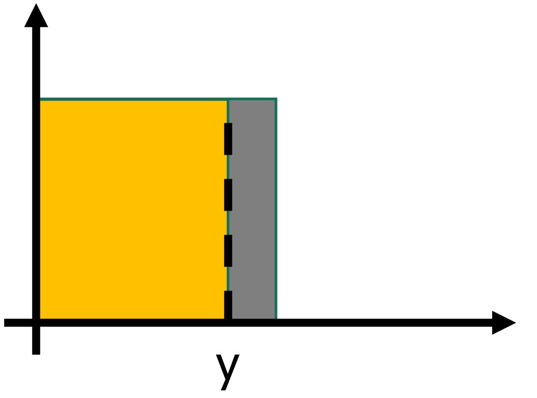


Probability density function conversion

Assume that y_0 is generated randomly in a uniform manner inside [0,1].

Then we have the cumulative probability

$F_Y(y) = p\,(Y \leq y) = y$

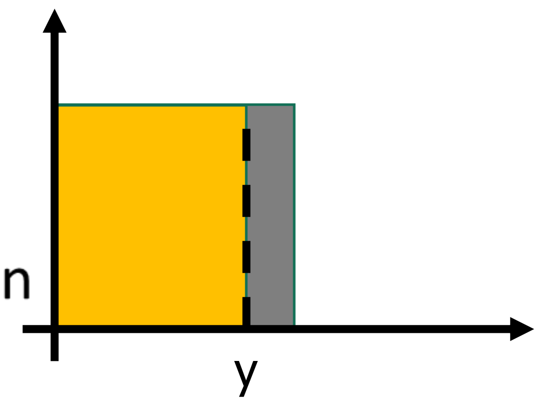


Probability density function conversion

Assume that Y is generated randomly in a uniform manner inside [0,1].

Then we have the cumulative probability

$F_Y(y) = p\ (Y \leq y) = y$



Assume f(Y) is the probability density function of Y. We have

$$p(\ Y\ \leq\ y) = \int_0^y f(t)dt = y$$

$$f(Y) = 1$$

Probability density function conversion

Assume that Y is generated randomly in a uniform manner inside [0,1].

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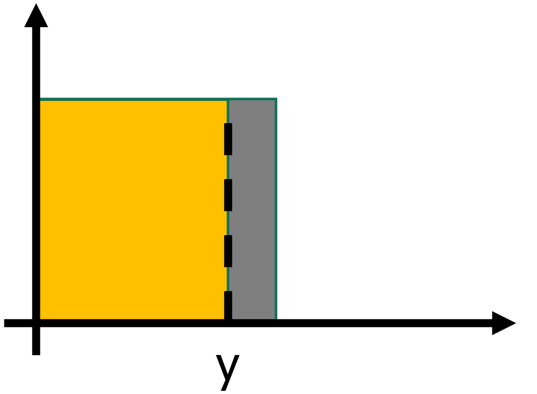

The probability density function of Y:

$f(Y) = 1$

Now, let X = F(Y). What is $p\ (X \leq x)$?

Probability density function conversion

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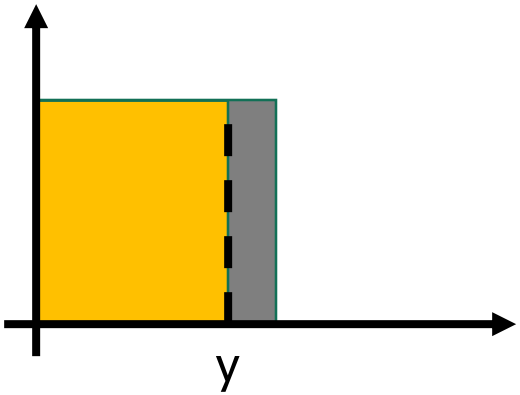

Now, let X = F(Y). What is $p\ (X \leq x)$?

Example:

$X = Y^2$

What is $p(Y^2 \leq x)$?

Probability density function conversion

Assume that Y is generated randomly in a uniform manner inside [0,1].

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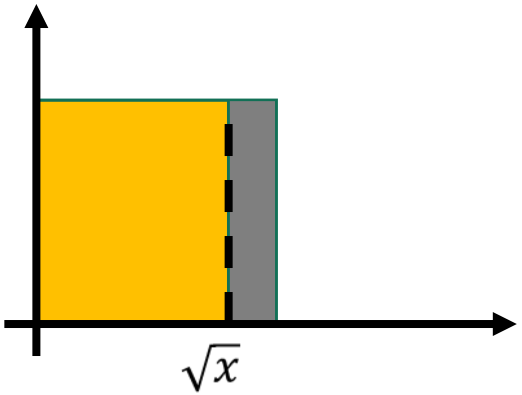

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What is $p(Y^2 \leq x)$*?*

$$p(Y^2 \leq x)$$
$$= p(Y \leq \sqrt{x})$$
$$= \sqrt{x}$$

Probability density function conversion

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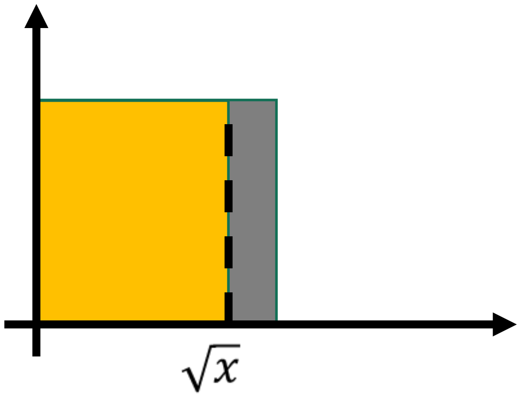

Now, let X = F(Y). What is *p* $(X \leq x)$?

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What is $p(Y^2 \leq x)$*?*

$p(X \leq x) = \sqrt{x}$

This is the cumulative probability of X.

What is the pdf of X?

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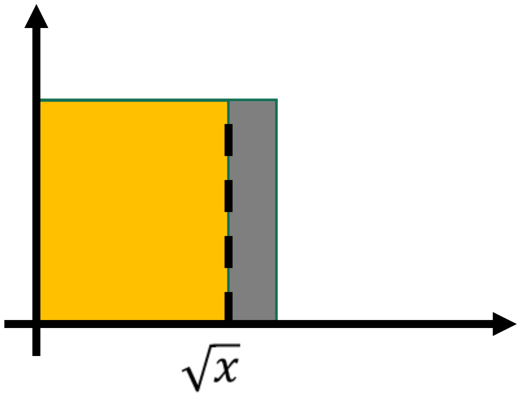

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Example:

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What is $p(Y^2 \leq x)$*?*

$F_X(x) = p(X \leq x) = \sqrt{x}$

This is the cumulative probability of X.

What is the pdf of X?

The derivative of $F_X(x)$ with respect to x, which is

$f(x) = \frac{1}{2\sqrt{x}}$

$$
\begin{aligned}
& p(Y^2 \leq x) \\
&= p(Y \leq \sqrt{x}) \\
&= \sqrt{x}
\end{aligned}
$$

Probability density function conversion

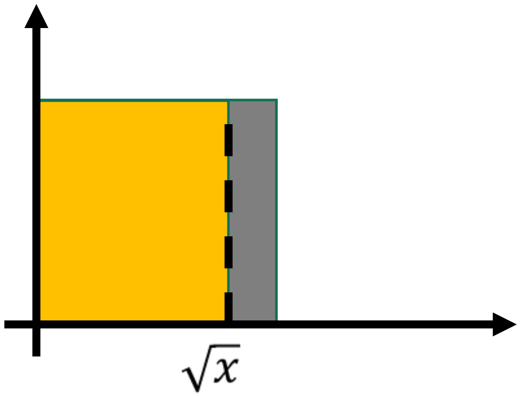
Assume that Y is generated randomly in a uniform manner inside [0,1].

Now, let X = F(Y). What is *p* $(X \leq x)$?

Example:

X = Y+Y²

Note: this idea can be applied to increasing functions only.



What is $p(Y+Y^2 \leq x)$?

$F_X(x) = p(X \leq x) = \frac{-1+\sqrt{1+4x}}{2}$

This is the cumulative probability of X.

What is the pdf of X?

The derivative of $F_X(x)$ with respect to x:

$f(x) = \frac{1}{\sqrt{1+4x}}$

$$p(Y + Y^2 \leq x)$$
$$= p\left(Y \leq \frac{-1+\sqrt{1+4x}}{2}\right)$$
$$= \frac{-1+\sqrt{1+4x}}{2}$$

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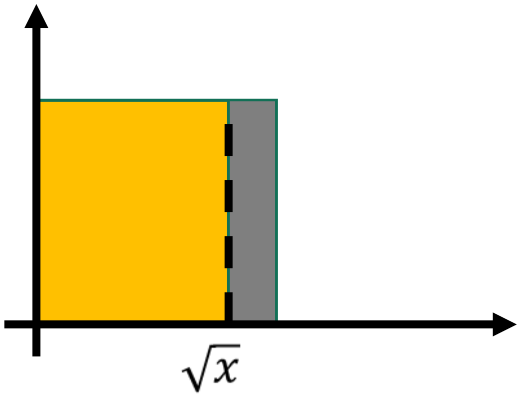

What is $p(Y+Y^2 \le x)$*?*

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This is the cumulative probability of X.

What is the pdf of X?

The derivative of $F_X(x)$ with respect to x:

$f(x) = \frac{1}{\sqrt{1+4x}}$

$$p(Y + Y^2 \le x)$$
$$= p\left(Y \le \frac{-1+\sqrt{1+4x}}{2}\right)$$
$$= \frac{-1+\sqrt{1+4x}}{2}$$

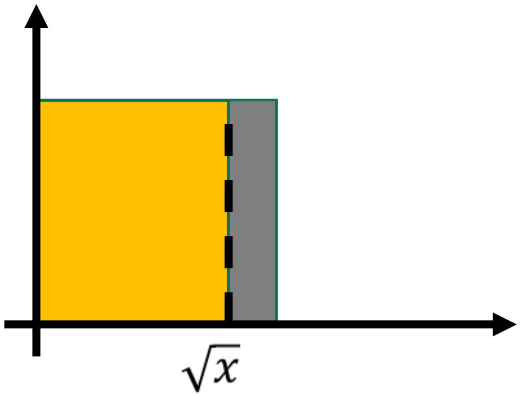
Probability density function conversion

```
clear
syms x; syms y;
solve(y+y^2 == x, y)

ans =
 - (4*x + 1)^(1/2)/2 - 1/2
   (4*x + 1)^(1/2)/2 - 1/2

cpf = (-1+sqrt(1+4*x))/2;
diff(cpf)

ans=
1/(4*x + 1)^(1/2)
```



$$p(Y + Y^2 \le x)$$
$$= p\left(Y \le \frac{-1+\sqrt{1+4x}}{2}\right)$$
$$= \frac{-1+\sqrt{1+4x}}{2}$$

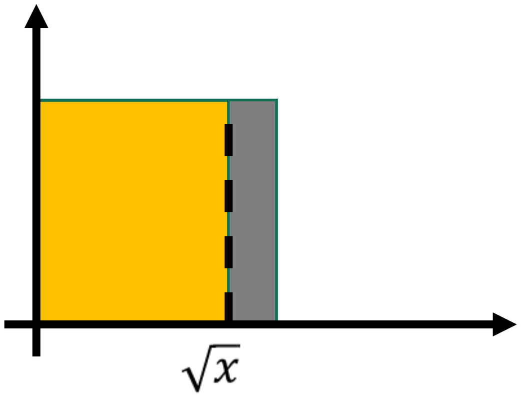
Probability density function conversion

```
clear
syms x; syms y;
solve(y+y^2 == x, y)

ans =
 - (4*x + 1)^(1/2)/2 - 1/2
   (4*x + 1)^(1/2)/2 - 1/2

cpf = (-1+sqrt(1+4*x))/2;
diff(cpf)

ans=
1/(4*x + 1)^(1/2)
```



$$p(Y + Y^2 \leq x)$$
$$= p\left(Y \leq \frac{-1+\sqrt{1+4x}}{2}\right)$$
$$= \frac{-1+\sqrt{1+4x}}{2}$$

Probability density function conversion
Fix the bug

```
%
% find and fix the bugs
%
% x = y + y^2
%
close all; clear;clf;
n = 10000000; y = rand(n,1);
x = y + y^2;
h = histogram(x,'Normalization',‘cdf')
hold on
x = 0.001:0.01:1;
f = 1/(4*x + 1)^(1/2);
plot(x,f,'LineWidth',3);
```

```
clear
syms x; syms y;
solve(y+y^2 == x, y)

ans =
 - (4*x + 1)^(1/2)/2 - 1/2
   (4*x + 1)^(1/2)/2 - 1/2

cpf = (-1+sqrt(1+4*x))/2;
diff(cpf)

ans=
1/(4*x + 1)^(1/2)
```

Probability density function conversion
Fix the bug (1 min)

```
%
% find and fix the bugs
%
% x = y + y^2
%
close all; clear;clf;
n = 10000000; y = rand(n,1);
x = y + y^2;
h = histogram(x,'Normalization',‘cdf')
hold on
x = 0.001:0.01:1;
f = 1/(4*x + 1)^(1/2);
plot(x,f,'LineWidth',3);
```

```
clear
syms x; syms y;
solve(y+y^2 == x, y)

ans =
 - (4*x + 1)^(1/2)/2 - 1/2
   (4*x + 1)^(1/2)/2 - 1/2

cpf = (-1+sqrt(1+4*x))/2;
diff(cpf)

ans=
1/(4*x + 1)^(1/2)
```

Probability density function conversion
Fix the bug

```
%
% find and fix the bugs
%
% x = y + y^2
%
close all; clear;clf;
n = 10000000; y = rand(n,1);
x = y + y^2;
h = histogram(x,'Normalization',‘cdf')
hold on
x = 0.001:0.01:1;
f = 1/(4*x + 1)^(1/2);
plot(x,f,'LineWidth',3);
```

```
clear
syms x; syms y;
solve(y+y^2 == x, y)

ans =
 - (4*x + 1)^(1/2)/2 - 1/2
   (4*x + 1)^(1/2)/2 - 1/2

cpf = (-1+sqrt(1+4*x))/2;
diff(cpf)

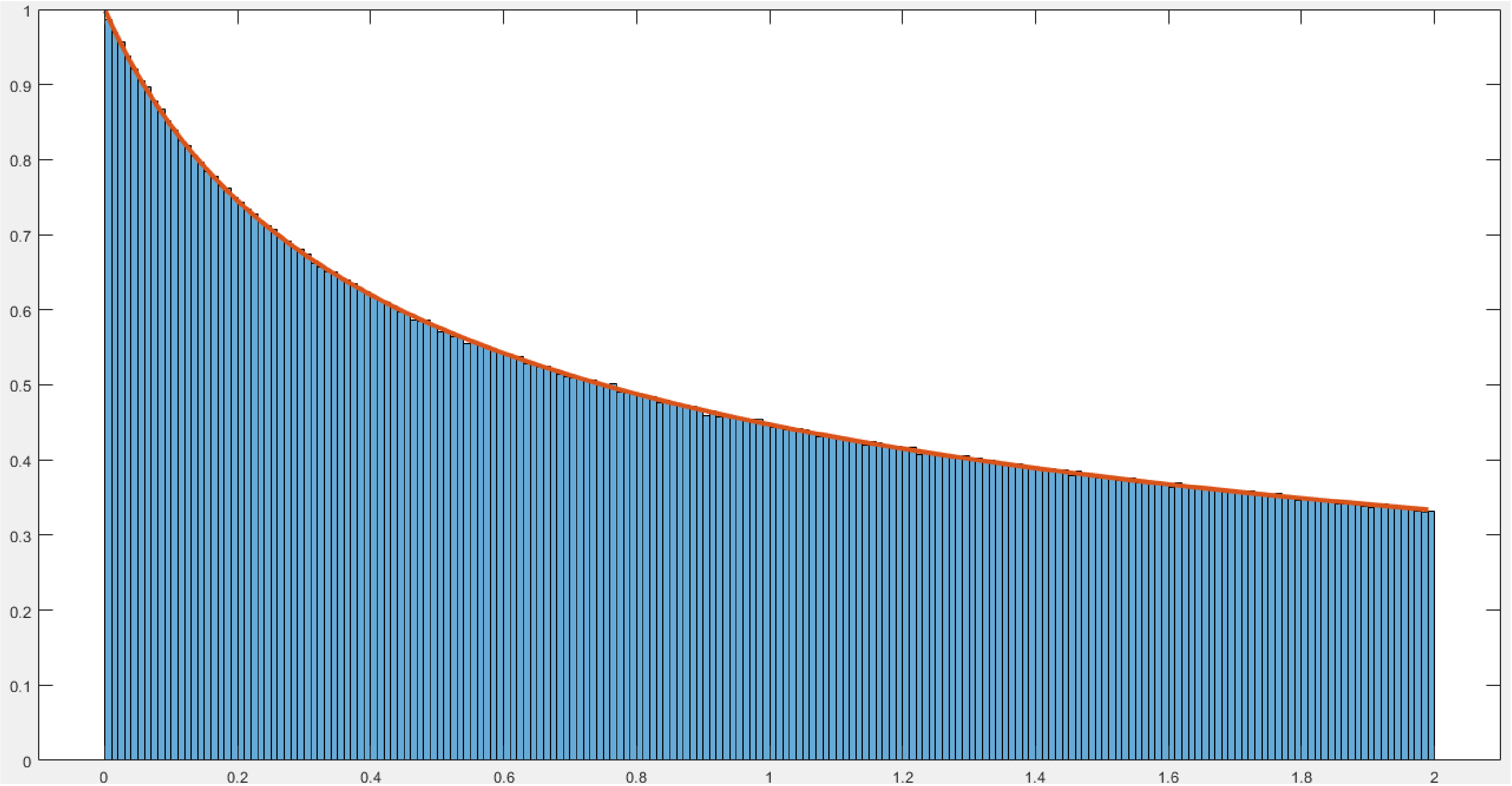
ans=
1/(4*x + 1)^(1/2)
```

Probability density function conversion

```
close all; clear;clf;
n = 10000000; y = rand(n,1);
x = y + y.^2;
h =
histogram(x,'Normalization','pdf')
hold on
x = 0.001:0.01:2;
f = 1./(4.*x + 1).^(1/2);
plot(x,f,'LineWidth',3);
```

```
close all; clear;clf;
n = 10000000; y = rand(n,1);
x = y + y.^2;
h =
histogram(x,'Normalization','pdf')
hold on
x = 0.001:0.01:1;
f = 1/(4*x + 1)^(1/2);
plot(x,f,'LineWidth',3);
```

Result



Probability density function conversion

Y is a random variable.

Y is generated randomly in a uniform manner inside [0,1].

Let $X = 1 - Y$.

The cpf is

$P(X <= x)$

$= P(1-Y <= x)$

$= P(1-x <= Y)$

$= P(Y >= 1-x)$

$= 1 – P(Y < 1- x)$

Probability density function conversion

$P(Y >= 1-x)$

$P(Y <= 1-x)$

Y is a random variable.

Y is generated randomly in a uniform manner inside [0,1].

Let $X = 1-Y$.

$P(X <= x)$

$= P(1-Y <= x)$

$= P(1-x <= Y)$

$= P(Y >= 1-x)$

$= 1 – P(Y < 1-x)$

```
clear; syms x; syms y;
func = 1-y;
s = solve(x == func, y)
ty0 = 0.5;                      % testing
tx0 = subs(func, y, ty0);
d_func = diff(func);

cpf = s(1);
if (subs(d_func, y, ty0) < 0)
    cpf = 1 - cpf;
end

my_pdf = diff(cpf);
```

So? What is the cpf of X? what is the pdf of X?

Probability density function conversion

P(Y >= 1-x)

P(Y <= 1-x)

Y is a random variable.

Y is generated randomly in a uniform manner inside [0,1].

Let X = 1-Y.

$P(X <= x)$

$= P(1-Y <= x)$

$= P(1-x <= Y)$

$= P(Y >= 1-x)$

$= 1 – P(Y < 1-x)$

```
clear; syms x; syms y;
func = 1-y;
s = solve(x == func, y)
ty0 = 0.5;                      % testing
tx0 = subs(func, y, ty0);
d_func = diff(func);
% check if d_func is increasing…
cpf = s(1);
if (subs(d_func, y, ty0) < 0)
    cpf = 1 - cpf;
end

my_pdf = diff(cpf);
```

So? What is the cpf of X? what is the pdf of X?

Probability density function conversion

P(Y >= 1-x)

P(Y <= 1-x)

Y is a random variable.

Y is generated randomly in a uniform manner inside [0,1].

Let X = 1-Y.

$P(X <= x)$

$= P(1-Y <= x)$

$= P(1-x <= Y)$

$= P(Y >= 1-x)$

$= 1 – P(Y < 1-x)$

```
clear; syms x; syms y;
func = 1-y;
s = solve(x == func, y)
ty0 = 0.5;                      % testing
tx0 = subs(func, y, ty0);
d_func = diff(func);
% check if d_func is increasing…
cpf = s(1);
if (subs(d_func, y, ty0) < 0)
    cpf = 1 - cpf;
end

my_pdf = diff(cpf);
```

So? What is the cpf of X? what is the pdf of X?

Probability density function conversion

An increasing function:

f(h2) >= f(h1), for h2 > h1
f'(h) >=0, for any h

Y is a random variable.

Y is generated randomly in a uniform manner inside [0,1].

Let X = 1-Y.

P(X <= x)

= P(1-Y <= x)

= P(1-x <= Y)

= P(Y >= 1-x)

= 1 – P(Y < 1-x)

```
clear; syms x; syms y;
func = 1-y;
s = solve(x == func, y)
ty0 = 0.5;                        % testing
tx0 = subs(func, y, ty0);
d_func = diff(func);
% check if d_func is increasing…
cpf = s(1);
if (subs(d_func, y, ty0) < 0)
    cpf = 1 - cpf;
end

my_pdf = diff(cpf);
```

So? What is the cpf of X? what is the pdf of X?

Probability density function conversion

```
clear; syms x; syms y;
func = 1-y;
s = solve(x == func, y)
ty0 = 0.5;                                    %
testing
tx0 = subs(func, y, ty0);
d_func = diff(func);

cpf = s(1);
if (subs(d_func, y, ty0) < 0)
    cpf = 1 - cpf;
end

my_pdf = diff(cpf);
```

```
close all;
n = 10000000; yv = rand(n,1);
%x1 = double(subs(func, y, yv));
x1 = 1 - yv;
h = histogram(x1,...
   'Normalization','pdf')
hold on
dx = 1/500;
x0 = 0:dx:1;
f = double(subs(my_pdf, x, x0));
plot(x0,f,'LineWidth',3);
set(gca, 'Fontsize',15);
```

The pdf of X is 1, i.e., a uniform distribution.

Probability density function conversion

```
clear; syms x; syms y;
func = 1-y;
s = solve(x == func, y)
ty0 = 0.5;                                  %
testing
tx0 = subs(func, y, ty0);
d_func = diff(func);

cpf = s(1);
if (subs(d_func, y, ty0) < 0)
    cpf = 1 - cpf;
end

my_pdf = diff(cpf);
```

```
close all;
n = 10000000; yv = rand(n,1);
%x1 = double(subs(func, y, yv));
x1 = 1 - yv;
h = histogram(x1,...
    'Normalization','pdf')
hold on
dx = 1/500;
x0 = 0:dx:1;
f = double(subs(my_pdf, x, x0));
plot(x0,f,'LineWidth',3);
set(gca, 'Fontsize',15);
```

%x1 = double(subs(func, y, yv));
Take too long time to evaluate.

Probability density function conversion

```
clear; syms x; syms y;
func = 1-y;
s = solve(x == func, y)
ty0 = 0.5;                              %
testing
tx0 = subs(func, y, ty0);
d_func = diff(func);

cpf = s(1);
if (subs(d_func, y, ty0) < 0)
    cpf = 1 - cpf;
end

my_pdf = diff(cpf);
```

```
close all;
n = 10000000; yv = rand(n,1);
%x1 = double(subs(func, y, yv));
x1 = 1 - yv;
h = histogram(x1,...
   'Normalization','pdf')
hold on
dx = 1/500;
x0 = 0:dx:1;
f = double(subs(my_pdf, x, x0));
plot(x0,f,'LineWidth',3);
set(gca, 'Fontsize',15);
```

%x1 = double(subs(func, y, yv));
Take too long time to evaluate.

Probability density function conversion

Y is a random variable.

Y is generated randomly in a uniform manner inside [0,1].

Let $X = 1-Y^2$.

The cpf is:

$P(X <= x)$

$= P(1-Y^2 <= x)$

$= P(1-x <= Y^2)$

$= P(Y >= sqrt(1-x) \text{ or } Y <= -sqrt(1-x))$

$= P(Y >= sqrt(1-x))$

$=1-P(Y < sqrt(1-x))$

Probability density function conversion

Y is a random variable.

Y is generated randomly in a uniform manner inside [0,1].

Let $X = 1-Y^2$.

The cpf is:

$P(X <= x)$

$= P(1-Y^2 <= x)$

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$= P(Y >= sqrt(1-x) \text{ or } Y <= -sqrt(1-x))$

$= P(Y >= sqrt(1-x))$

$=1-P(Y < sqrt(1-x))$

Probability density function conversion

Y is a random variable.

Y is generated randomly in a uniform manner inside [0,1].

Let $X = 1-Y^2$.

The cpf is:

$P(X <= x)$

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$= P(Y >= sqrt(1-x) \text{ or } Y <= -sqrt(1-x))$

$= P(Y >= sqrt(1-x))$

$=1-P(Y < sqrt(1-x))$

Probability density function conversion

Y is a random variable.

Y is generated randomly in a uniform manner inside [0,1].

Let $X = 1-Y^2$.

The cpf is:

$P(X <= x)$

$= P(1-Y^2 <= x)$

$= P(1-x <= Y^2)$

$= P(Y >= sqrt(1-x) \text{ or } Y <= -sqrt(1-x))$

$= P(Y >= sqrt(1-x))$

$=1-P(Y < sqrt(1-x)) = 1- sqrt(1-x)$

Probability density function conversion

Let $X = 1-Y^2$.

$P(X <= x)$

$= P(1-Y^2 <= x)$

$= P(1-x <= Y^2)$

$= P(Y >= sqrt(1-x)$

$\quad or \ \ Y <= -sqrt(1-x))$

$= P(Y >= sqrt(1-x))$

$=1-P(Y < sqrt(1-x))$

```
%Let X = 1-Y^2
clear; syms x; syms y;
func = 1-y^2;
s = solve(x == func, y)
d_func = diff(func);
ty0 = 0.5;                          % testing
tx0 = double(subs(func, y, ty0));
for i = 1:2
    if (double(subs(s(i), tx0))>0)
        cpf = s(i);
        break;
    end
end
if (double(subs(d_func, y, ty0)) < 0)
    cpf = 1 - cpf;
end
my_pdf = diff(cpf);
```

Probability density function conversion

Let $X = 1-Y^2$.

$P(X <= x)$

$= P(1-Y^2 <= x)$

$= P(1-x <= Y^2)$

$= P(Y >= sqrt(1-x)$

$\quad or \ \ Y <= -sqrt(1-x))$

$= P(Y >= sqrt(1-x))$

$=1-P(Y < sqrt(1-x))$

```
%Let X = 1-Y^2
clear; syms x; syms y;
func = 1-y^2;
s = solve(x == func, y)
d_func = diff(func);
ty0 = 0.5;                    % testing
tx0 = double(subs(func, y, ty0));
for i = 1:2
    if (double(subs(s(i), tx0))>0)
        cpf = s(i);
        break;
    end
end
if (double(subs(d_func, y, ty0)) < 0)
    cpf = 1 - cpf;
end
my_pdf = diff(cpf);
```

Probability density function conversion

Let $X = 1\text{-}Y^2$.

$P(X <= x)$

$= P(1\text{-}Y^2 <= x)$

$= P(\ 1\text{-}x <= Y^2)$

= P(Y >= sqrt(1-x)

or Y <= -sqrt(1-x))

= P(Y >= sqrt(1-x))

=1-P(Y < sqrt(1-x))

```
%Let X = 1-Y^2
clear; syms x; syms y;
func = 1-y^2;
s = solve(x == func, y)
d_func = diff(func);
ty0 = 0.5;                          % testing
tx0 = double(subs(func, y, ty0));
for i = 1:2
    if (double(subs(s(i), tx0))>0)
        cpf = s(i);
        break;
    end
end
if (double(subs(d_func, y, ty0)) < 0)
    cpf = 1 - cpf;
end
my_pdf = diff(cpf);
```

Probability density function conversion

```
%Let X = 1-Y^2
clear; syms x; syms y;
func = 1-y^2;
s = solve(x == func, y)
d_func = diff(func);
ty0 = 0.5;        % testing
tx0 = double(...
   subs(func, y, ty0));
for i = 1:2
    if (double(...
        subs(s(i), tx0))>0)
         cpf = s(i);
         break;
    end
end
if (double(...
    subs(d_func, y, ty0)) < 0)
    cpf = 1 - cpf;
end
my_pdf = diff(cpf);
```

```
close all;
n = 10000000; y = rand(n,1);
x1 = 1-y.^2;
h = histogram(x1,...
   'Normalization','pdf')
hold on
dx = 1/500;
x0 = 0:dx:0.99999;
f = double(...
    subs(my_pdf, x, x0));
plot(x0,f,'LineWidth',3);
set(gca, 'Fontsize',15);
```

Summary

1. Construct a symbolic equation.
2. Solve for y.
3. Pick the right root of y to define the cumulative probability function CPF.
4. Compute the pdf which is equal to the first derivative of the CPF.
5. Generate samples for Y.
6. Compute all X.
7. Draw the histogram for all X.
8. Set the range for X properly.
9. Plot the pdf.
10. Beautify the figure.

```
clear

syms x; syms y;

solve(y+y^2 == x, y)


ans =

 - (4*x + 1)^(1/2)/2 - 1/2
   (4*x + 1)^(1/2)/2 - 1/2


cpf = (-1+sqrt(1+4*x))/2;

diff(cpf)


ans=

1/(4*x + 1)^(1/2)
```

```
close all; clear;clf;

n = 10000000;

y = rand(n,1);

x = y + y.^2;

h = histogram(x,...
   'Normalization','pdf')

hold on

x = 0.001:0.01:2;

f = 1./(4.*x + 1).^(1/2);

plot(x,f,'LineWidth',3);

%need more for beatifying the figure
```

Summary

1. Construct a symbolic equation.
2. Solve for y.
3. Pick the right root of y to define the cumulative probability function CPF.
4. Compute the pdf which is equal to the first derivative of the CPF.
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```
clear
syms x; syms y;
solve(y+y^2 == x, y)

ans =
 - (4*x + 1)^(1/2)/2 - 1/2
   (4*x + 1)^(1/2)/2 - 1/2

cpf = (-1+sqrt(1+4*x))/2;
diff(cpf)

ans=
1/(4*x + 1)^(1/2)
```

```
close all; clear;clf;

n = 10000000;
y = rand(n,1);
x = y + y.^2;
h = histogram(x,...
  'Normalization','pdf')
hold on
x = 0.001:0.01:2;
f = 1./(4.*x + 1).^(1/2);
plot(x,f,'LineWidth',3);
%need more for beatifying the figure
```

Summary

1. Construct a symbolic equation.
2. Solve for y.
3. Pick the right root of y to define the cumulative probability function CPF.
4. Compute the pdf which is equal to the first derivative of the CPF.
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8. Set the range for X properly.
9. Plot the pdf.
10. Beautify the figure.

```
clear

syms x; syms y;

solve(y+y^2 == x, y)


ans =

 - (4*x + 1)^(1/2)/2 - 1/2

   (4*x + 1)^(1/2)/2 - 1/2


cpf = (-1+sqrt(1+4*x))/2;

diff(cpf)


ans=

1/(4*x + 1)^(1/2)
```

```
close all; clear;clf;

n = 10000000;

y = rand(n,1);

x = y + y.^2;

h = histogram(x,...

  'Normalization','pdf')

hold on

x = 0.001:0.01:2;

f = 1./(4.*x + 1).^(1/2);

plot(x,f,'LineWidth',3);

%need more for beatifying the figure

// subs: too long time
```

Exercise

Y is a random variable.

Y is generated randomly in a uniform manner inside **[0,1].**

Let $X = \cos((Y^2+aY)/2)$

Write a program to produce one figure. The system specification is as follows.

Show student name, ID, and email address.

Ask to input n. Ask to input *a* which is inside [0,1].

Generate n sample points of X.

Draw the histogram of X.

Draw the pdf of X.

Beautify the figure. including: show axis labels and show legend.

Shows the title as: pdf for $X = \cos((Y^2+aY)/2)$; $a = \ldots$

Exercise

Y is a random variable.

Y is generated randomly in a uniform manner inside **[0,1].**

Let $X = \cos((Y^2+aY)/2)$

Write a program to produce one figure. The system specification is as follows.

Show student name, ID, and email address.

Ask to input n. Ask to input *a* which is inside [0,1].

Generate n sample points of X.

Draw the histogram of X.

Draw the pdf of X.

Beautify the figure. including: show axis labels and show legend.

Shows the title as: pdf for $X = \cos((Y^2+aY)/2)$; $a = ...$

Y is a random variable. Y is generated randomly in a uniform manner inside **[0,1]**. *Let* $X = \cos((Y^2+aY)/2)$

```
clear
syms x; syms y;
a = input('Input a:');
s = solve(x == cos(...
   (y^2+a*y)/2), y)
s =
 (1/4 - 2*acos(x))^(1/2) - 1/2     %1
  (2*acos(x) + 1/4)^(1/2) - 1/2    %2
- (1/4 - 2*acos(x))^(1/2) - 1/2    %3
- (2*acos(x) + 1/4)^(1/2) - 1/2    %4
cpf = ?
```

Y is a random variable. Y is generated randomly in a uniform manner inside **[0,1]**. *Let* $X = \cos((Y^2+aY)/2)$ *(1 min)*

```
clear
syms x; syms y;
a = input('Input a:');
s = solve(x == cos(...
   (y^2+a*y)/2), y)
s =
 (1/4 - 2*acos(x))^(1/2) - 1/2     %1
  (2*acos(x) + 1/4)^(1/2) - 1/2    %2
 - (1/4 - 2*acos(x))^(1/2) - 1/2   %3
 - (2*acos(x) + 1/4)^(1/2) - 1/2   %4
cpf = ?
```

Y is a random variable. Y is generated randomly in a uniform manner inside **[0,1]**. *Let* $X = \cos((Y^2+aY)/2)$ *(Idea)*

```
clear
syms x; syms y;
a = input('Input a:');
s = solve(x == cos(...
   (y^2+a*y)/2), y)
s =
 (1/4 - 2*acos(x))^(1/2) - 1/2     %1
  (2*acos(x) + 1/4)^(1/2) - 1/2    %2
- (1/4 - 2*acos(x))^(1/2) - 1/2    %3
- (2*acos(x) + 1/4)^(1/2) - 1/2    %4
cpf = ?
```

Y is a random variable. Y is generated randomly in a uniform manner inside **[0,1]**. *Let X = cos((Y²+aY)/2).* ***We can subsitute values***.

```
clear
syms x; syms y;
a = input('Input a:');
s = solve(x == cos(...
  (y^2+a*y)/2), y)
s =
 (1/4 - 2*acos(x))^(1/2) - 1/2     %1
  (2*acos(x) + 1/4)^(1/2) - 1/2    %2
 - (1/4 - 2*acos(x))^(1/2) - 1/2   %3
 - (2*acos(x) + 1/4)^(1/2) - 1/2   %4
cpf = ?
```

```
clear
syms x; syms y;
a = input('Input a:');
s = solve(x == cos(...
  (y^2+a*y)/2), y)

subs(s(1), x,2)
ans = (1/4 - 2*acos(2))^(1/2) - 1/2
subs(s(2), 2)
ans = (2*acos(2) + 1/4)^(1/2) - 1/2
subs(s(3), 2)
ans = - (1/4 - 2*acos(2))^(1/2) - 1/2
subs(s(4), 2)
ans = - (2*acos(2) + 1/4)^(1/2) - 1/2
```

Y is a random variable. Y is generated randomly in a uniform manner inside **[0,1]**. *Let $X = cos((Y^2+aY)/2)$.* ***We can subsitute values***.

```
clear
syms x; syms y;
a = input('Input a:');
s = solve(x == cos(...
    (y^2+a*y)/2), y)
s =
 (1/4 - 2*acos(x))^(1/2) - 1/2      %1
  (2*acos(x) + 1/4)^(1/2) - 1/2     %2
 - (1/4 - 2*acos(x))^(1/2) - 1/2    %3
 - (2*acos(x) + 1/4)^(1/2) - 1/2    %4
cpf = ?
```

```
clear
syms x; syms y;
a = input('Input a:');
s = solve(x == cos(...
    (y^2+a*y)/2), y)

subs(s(1), x,2)
ans = (1/4 - 2*acos(2))^(1/2) - 1/2
subs(s(2), 2)
ans = (2*acos(2) + 1/4)^(1/2) - 1/2
subs(s(3), 2)
ans = - (1/4 - 2*acos(2))^(1/2) - 1/2
subs(s(4), 2)
ans = - (2*acos(2) + 1/4)^(1/2) - 1/2
```

WRONG!

Y is a random variable. Y is generated randomly in a uniform manner inside **[0,1].** *Let $X = cos((Y^2+aY)/2)$.* ***We can subsitute values.***

```
clear
syms x; syms y;
a = input('Input a:');
s = solve(x == cos(...
   (y^2+a*y)/2), y)
s =
 (1/4 - 2*acos(x))^(1/2) - 1/2     %1
  (2*acos(x) + 1/4)^(1/2) - 1/2    %2
 - (1/4 - 2*acos(x))^(1/2) - 1/2   %3
 - (2*acos(x) + 1/4)^(1/2) - 1/2   %4
cpf = ?
```

NOTE:
x is inside
[cos ((1+a)/2), cos 0]

```
clear
syms x; syms y;
a = input('Input a:');
s = solve(x == cos(...
   (y^2+a*y)/2), y)

subs(s(1), x,2)
ans = (1/4 - 2*acos(2))^(1/2) - 1/2
subs(s(2), 2)
ans = (2*acos(2) + 1/4)^(1/2) - 1/2
subs(s(3), 2)
ans = - (1/4 - 2*acos(2))^(1/2) - 1/2
subs(s(4), 2)
ans = - (2*acos(2) + 1/4)^(1/2) - 1/2
```

Y is a random variable. Y is generated randomly in a uniform manner inside **[0,1]**. *Let* $X = cos((Y^2+aY)/2)$

```
clear

syms x; syms y;

a = input('Input a:');

s = solve(x == cos(...
   (y^2+a*y)/2), y)

s =
 (1/4 - 2*acos(x))^(1/2) - 1/2     %1
  (2*acos(x) + 1/4)^(1/2) - 1/2    %2
 - (1/4 - 2*acos(x))^(1/2) - 1/2   %3
 - (2*acos(x) + 1/4)^(1/2) - 1/2   %4

cpf = ?
```

NOTE:
x is inside
[cos ((1+a)/2), cos 0]

```
clear

syms x; syms y;

a = input('Input a:');

s = solve(x == cos(...
   (y^2+a*y)/2), y)


subs(s(1),0.5)
ans = (1/4 - (2*pi)/3)^(1/2) - 1/2
subs(s(2), 0.5)
ans = ((2*pi)/3 + 1/4)^(1/2) - 1/2
subs(s(3), 0.5)
ans = - (1/4 - (2*pi)/3)^(1/2) - 1/2
subs(s(4), 0.5)
ans = - ((2*pi)/3 + 1/4)^(1/2) - 1/2
```

Y is a random variable. Y is generated randomly in a uniform manner inside **[0,1]**. *Let* $X = \cos((Y^2+aY)/2)$

```
clear
syms x; syms y;
a = input('Input a:');
s = solve(x == cos(...
   (y^2+a*y)/2), y)

subs(s(1),0.5)
ans = (1/4 - (2*pi)/3)^(1/2) - 1/2
subs(s(2), 0.5)
ans = ((2*pi)/3 + 1/4)^(1/2) - 1/2
subs(s(3), 0.5)
ans = - (1/4 - (2*pi)/3)^(1/2) - 1/2
subs(s(4), 0.5)
ans = - ((2*pi)/3 + 1/4)^(1/2) - 1/2
```

```
clear
syms x; syms y;
a = input('Input a:');
s = solve(x == cos(...
   (y^2+a*y)/2), y)

double(subs(s(1),0.5))
ans = -0.5000 + 1.3581i
double(subs(s(2), 0.5))
ans = 1.0311
double(subs(s(3), 0.5))
ans = -0.5000 - 1.3581i
double(subs(s(4), 0.5))
ans = -2.0311
```

1i imaginery unit

What is the problem? None of the value inside [0,1]!

Y is a random variable. Y is generated randomly in a uniform manner inside **[0,1].** *Let* $X = \cos((Y^2+aY)/2)$

NOTE:
x is inside
[cos ((1+a)/2), cos 0]

If a = 1,
[cos(1), 1];
= [0.5403, 1].

```
Clear; syms x; syms y;
a = input('Input a:');
s = solve(x == cos(...
   (y^2+a*y)/2), y)

double(subs(s(1),0.5))
ans = -0.5000 + 1.3581i
double(subs(s(2), 0.5))
ans = 1.0311
double(subs(s(3), 0.5))
ans = -0.5000 - 1.3581i
doubles(subs(s(4), 0.5))
ans = -2.0311
```

```
double(subs(s(1), (cos(1)+1)/2))
ans = -0.5000 + 1.0646i

double(subs(s(2), (cos(1)+1)/2))
ans = 0.7781

double(subs(s(3), (cos(1)+1)/2))
ans = -0.5000 - 1.0646i

double(subs(s(4), (cos(1)+1)/2))
ans = -1.7781
```

Y is a random variable. Y is generated randomly in a uniform manner inside **[0,1].** *Let* $X = \cos((Y^2+aY)/2)$

```
Clear; syms x; syms y;
a = input('Input a:');
s = solve(x == cos(...
   (y^2+a*y)/2), y)


double(subs(s(1),0.5))
ans = -0.5000 + 1.3581i
double(subs(s(2), 0.5))
ans = 1.0311
double(subs(s(3), 0.5))
ans = -0.5000 - 1.3581i
doubles(subs(s(4), 0.5))
ans = -2.0311
```

So
cpf = s(2)

NOTE:
x is inside
[cos ((1+a)/2), cos 0]

If a = 1,
[cos(1), 1];
= [0.5403, 1].

```
double(subs(s(1), (cos(1)+1)/2))
ans = -0.5000 + 1.0646i

double(subs(s(2), (cos(1)+1)/2))
ans = 0.7781

double(subs(s(3), (cos(1)+1)/2))
ans = -0.5000 - 1.0646i

double(subs(s(4), (cos(1)+1)/2))
ans = -1.7781
```

Y is a random variable. Y is generated randomly in a uniform manner inside **[0,1]**. *Let X = cos((Y^2+aY)/2). Robust implementation*

```
clear; syms x; syms y;
a = input('Input a:');
func = cos((y^2+a*y)/2);
d_func = diff(func, y);
s = solve(x == func, y)
ty0 = 0.5;
tx0 = subs(func,y,ty0);
```

```
for i = 1:4
  if (double(subs(s(i), x, tx0))>0)
      cpf = s(i);
      break;
  end
end
if (subs(d_func, y, ty0)<0)
    cpf = 1 - cpf;
end
my_pdf = diff(cpf);
```

```
close all;
n = 10000000; y = rand(n,1);
x1 = cos((y.^2+a.*y)./2);
h = histogram(x1,'Normalization','pdf')
hold on
dx = (1-cos((1+a)/2))/500;
x0 = cos((1+a)/2):dx:0.99999;
f = double(subs(my_pdf, x, x0));
plot(x0,f,'LineWidth',3);
set(gca, 'Fontsize',15);
```

Y is a random variable. Y is generated randomly in a uniform manner inside **[0,1]**. *Let X = cos((Y^2+aY)/2). Robust implementation*

```
clear; syms x; syms y;
a = input('Input a:');
func = cos((y^2+a*y)/2);
d_func = diff(func, y);
s = solve(x == func, y)
ty0 = 0.5;
tx0 = subs(func,y,ty0);
```

```
for i = 1:4
  if (double(subs(s(i), x, tx0))>0)
      cpf = s(i);
      break;
  end
end
if (subs(d_func, y, ty0)<0)
    cpf = 1 - cpf;
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func = cos((y^2+a*y)/2);
d_func = diff(func, y);
s = solve(x == func, y)
ty0 = 0.5;
tx0 = subs(func,y,ty0);
```

```
for i = 1:4
  if (double(subs(s(i), x, tx0))>0)
      cpf = s(i);
      break;
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end
if (subs(d_func, y, ty0)<0)
    cpf = 1 - cpf;
end
my_pdf = diff(cpf);
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dx = (1-cos((1+a)/2))/500;
x0 = cos((1+a)/2):dx:0.99999;
f = double(subs(my_pdf, x, x0));
plot(x0,f,'LineWidth',3);
set(gca, 'Fontsize',15);
```

Y is a random variable. Y is generated randomly in a uniform manner inside **[0,1]**. *Let* $X = \cos((Y^2+aY)/2)$. *Robust implementation*

```
clear; syms x; syms y;
a = input('Input a:');
func = cos((y^2+a*y)/2);
d_func = diff(func, y);
s = solve(x == func, y)
ty0 = 0.5;
tx0 = subs(func,y,ty0);
```

```
for i = 1:4
  if (double(subs(s(i), x, tx0))>0)
      cpf = s(i);
      break;
  end
end
if (subs(d_func, y, ty0)<0)
    cpf = 1 - cpf;
end
my_pdf = diff(cpf);
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x1 = cos((y.^2+a.*y)./2);
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dx = (1-cos((1+a)/2))/500;
x0 = cos((1+a)/2):dx:0.99999;
f = double(subs(my_pdf, x, x0));
plot(x0,f,'LineWidth',3);
set(gca, 'Fontsize',15);
```

Y is a random variable. Y is generated randomly in a uniform manner inside **[0,1]**. *Let X = cos((Y²+aY)/2). Robust implementation*

```
clear; syms x; syms y;
a = input('Input a:');
func = cos((y^2+a*y)/2);
d_func = diff(func, y);
s = solve(x == func, y)
ty0 = 0.5;
tx0 = subs(func,y,ty0);
```

```
for i = 1:4
   if (double(subs(s(i), x, tx0))>0)
        cpf = s(i);
        break;
   end
end
if (subs(d_func, y, ty0)<0)
     cpf = 1 - cpf;
end
my_pdf = diff(cpf);
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Y is a random variable. Y is generated randomly in a uniform manner inside **[0,1]**. *Let X = cos((Y²+aY)/2). Robust implementation*

```
clear; syms x; syms y;
a = input('Input a:');
func = cos((y^2+a*y)/2);
d_func = diff(func, y);
s = solve(x == func, y)
ty0 = 0.5;
tx0 = subs(func,y,ty0);
```

```
for i = 1:4
  if (double(subs(s(i), x, tx0))>0)
      cpf = s(i);
      break;
  end
end
if (subs(d_func, y, ty0)<0)
    cpf = 1 - cpf;
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hold on
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x0 = cos((1+a)/2):dx:0.99999;
f = double(subs(my_pdf, x, x0));
plot(x0,f,'LineWidth',3);
set(gca, 'Fontsize',15);
```
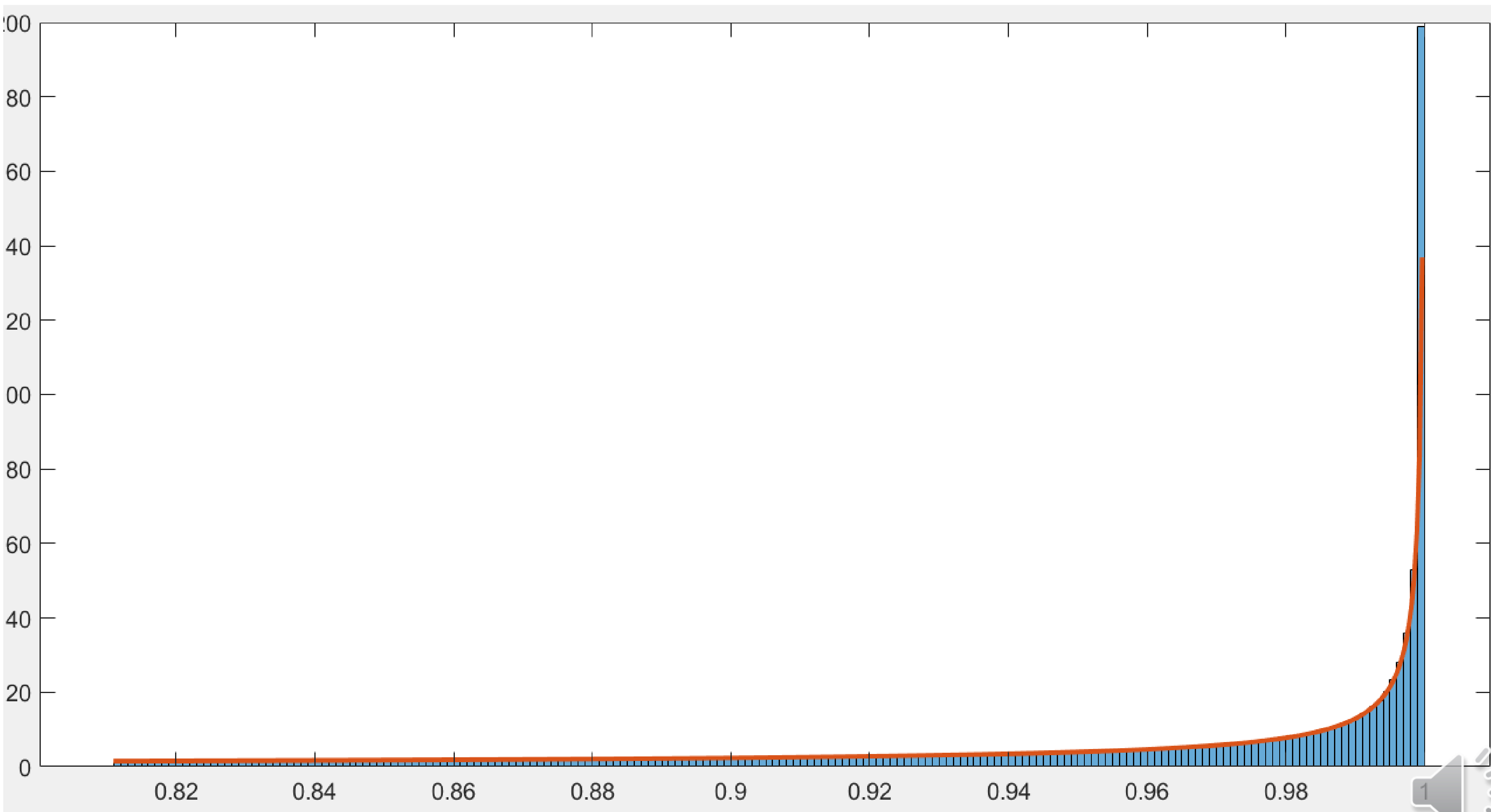
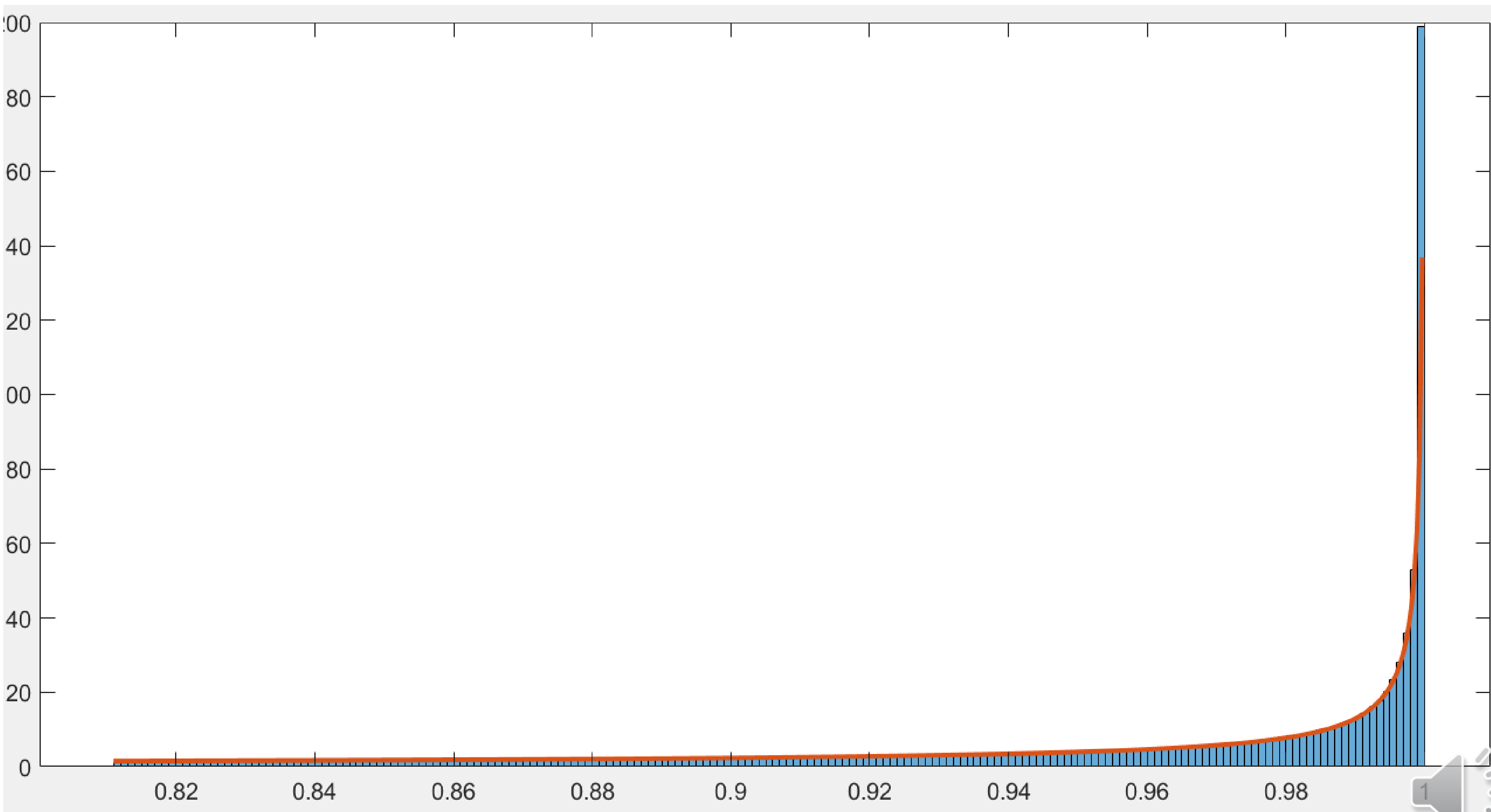

Y is a random variable. Y is generated randomly in a uniform manner inside **[0,1].** *Let* $X = \cos((Y^2+aY)/2)$*. Robust implementation*

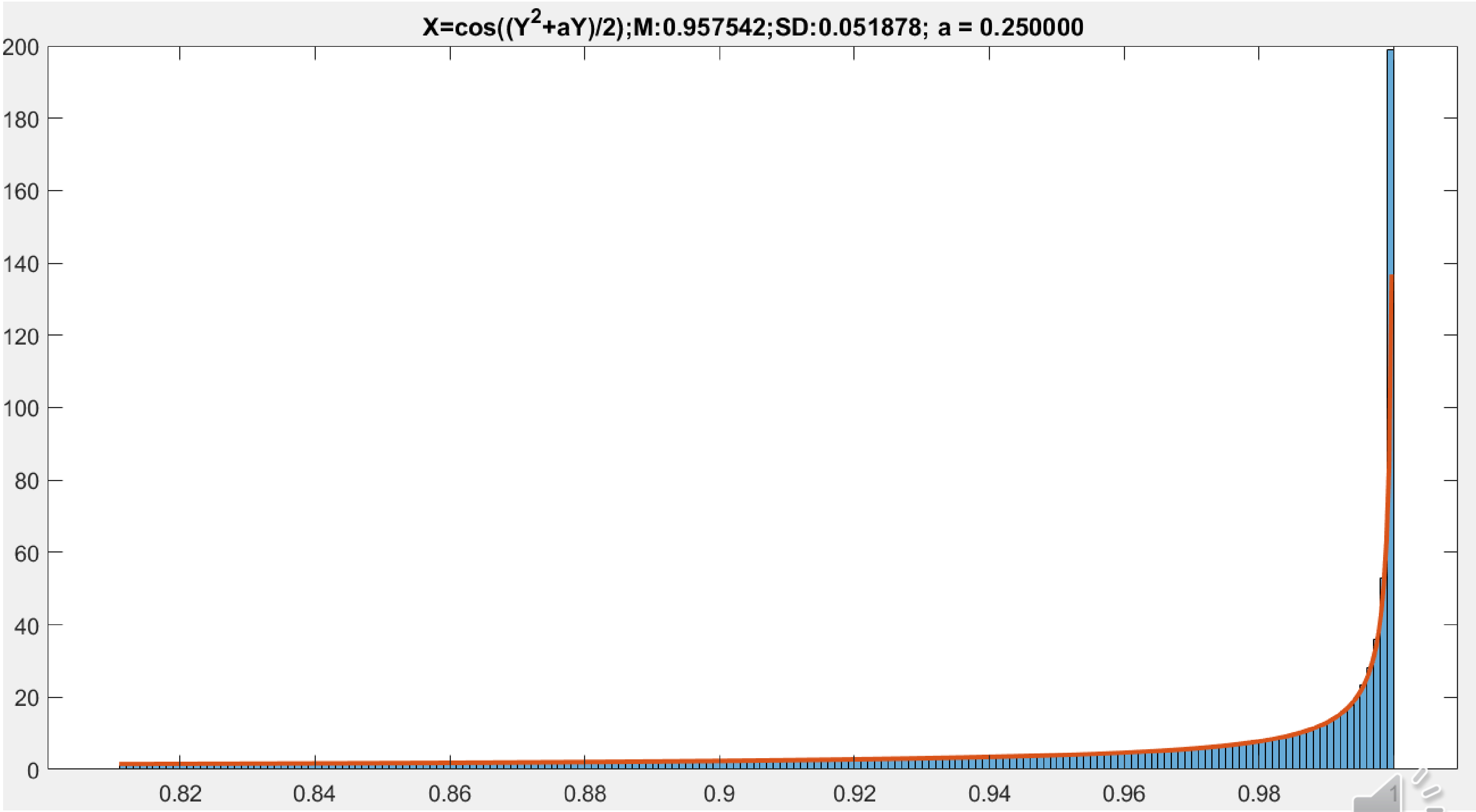


Y is a random variable. Y is generated randomly in a uniform manner inside **[0,1]**. *Let* $X = \cos((Y^2+aY)/2)$. *Robust implementation*

Make a title?

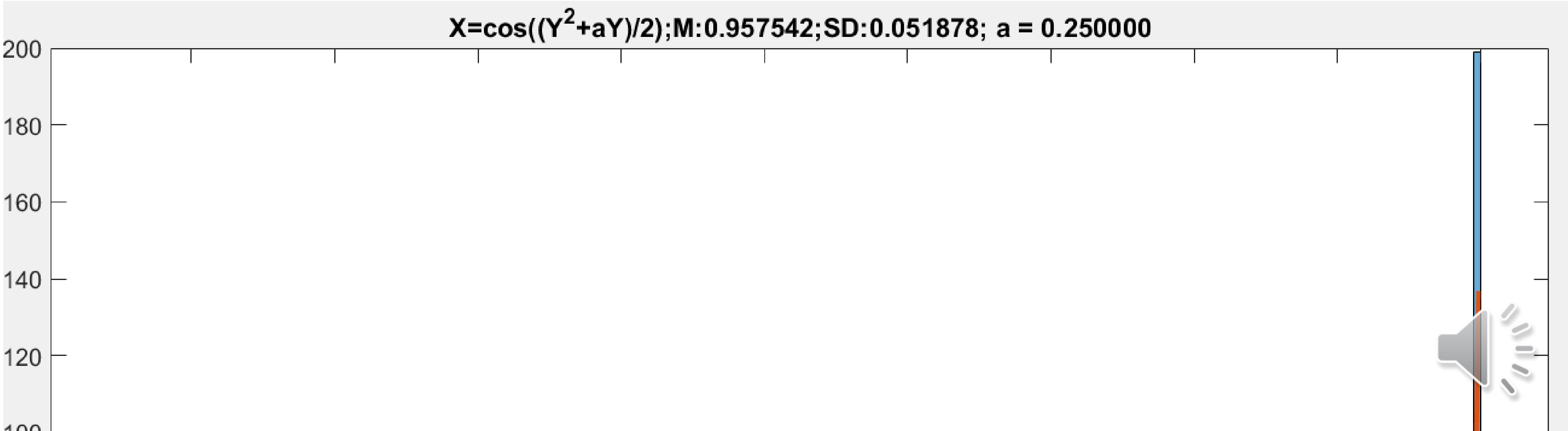


Y is a random variable. Y is generated randomly in a uniform manner inside **[0,1].** *Let* $X = \cos((Y^2+aY)/2)$. *Robust implementation*



Y is a random variable. Y is generated randomly in a uniform manner inside **[0,1]**. *Let X = cos((Y*^{2}*+aY)/2). Robust implementation*

```
str = "X=cos((Y^2+aY)/2);"+ ...
"M:%f;SD:%f; a = %f";
tmsg = sprintf(str, mean(x1),
std(x1), a);
title(tmsg);
```



Exercise

Y is a random variable.

Y is generated randomly in a uniform manner inside [0,1].

Let $X = -Y-Y^2$.

Write a program to produce one figure. The system specification is as follows.

Show student name, ID, and email address.

Ask to input n.

Generate n sample points of X.

Draw the histogram of X.

Draw the pdf of X.

Beautify the figure. including: show axis labels and show legend.

Exercise

Y is a random variable.

Y is generated randomly in a uniform manner inside **[1,2].**

Let $X = Y^2 - Y$.

Write a program to produce one figure. The system specification is as follows.

Show student name, ID, and email address.

Ask to input n.

Generate n sample points of X.

Draw the histogram of X.

Draw the pdf of X.

Beautify the figure. including: show axis labels and show legend.

Exercise

Y is a random variable.

Y is generated randomly in a uniform manner inside [0,1].

Let $X = 3\sin(Y^2)$.

Write a program to produce one figure. The system specification is as follows.

Show student name, ID, and email address.

Ask to input n.

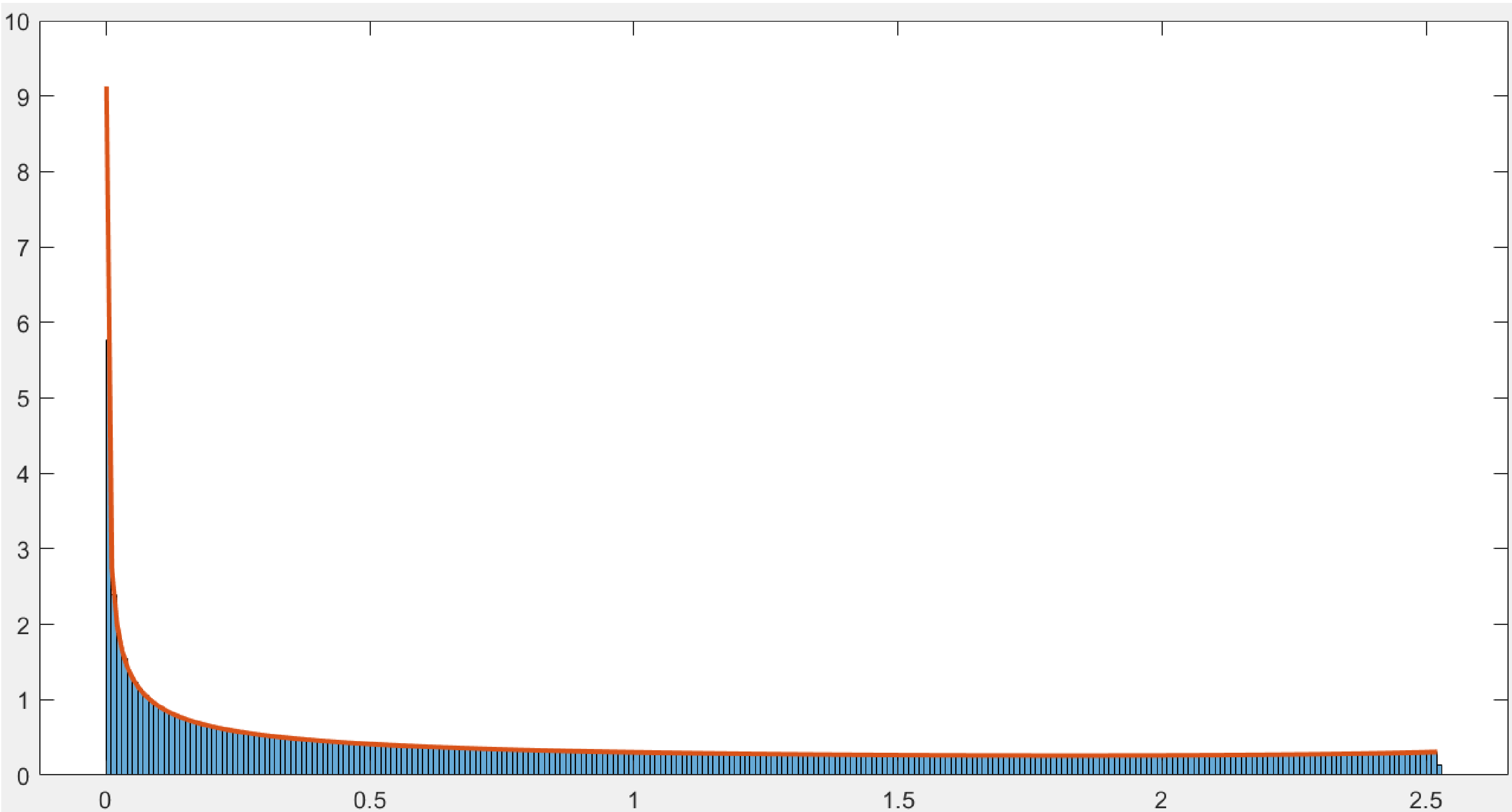
Generate n sample points of X.

Draw the histogram of X.

Draw the pdf of X.

Beautify the figure, including: show axis labels and show legend.

$X = 3\sin(Y^2)$



Don't include the two end points because of division by zero.

Exercise

Y is a random variable.

Y is generated randomly in a uniform manner inside [0,1].

Let $X = e^{Y}-1$.

Write a program to produce one figure. The system specification is as follows.

Show student name, ID, and email address.

Ask to input n.

Generate n sample points of X.

Draw the histogram of X.

Draw the pdf of X.

Beautify the figure. including: show axis labels and show legend.

$X = e^{Y}-1$