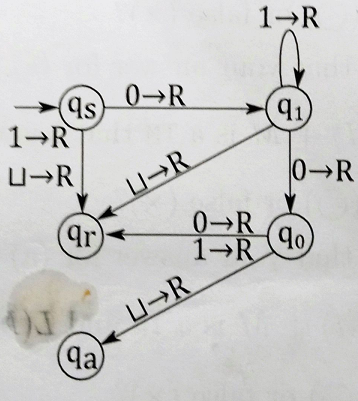
Midterm Examination (Close Book)

Exam date & time: May 14, 2019, 10:10AM–11:50AM (100 minutes)

Do the following problems. The points are specified in the brackets (i.e., []). There are 100 points in total. If a problem consists of one mathematical statement and two questions. For the first question, you have to answer ○ for "**true**" or × for "**false**" to indicate your judgement on the validity of the given statement. For the second question, you have to prove the statement if your answer is ○ for the first question or disprove the statement if your answer is ×. If the answer for the first question is incorrect, there will be no points awarded to the second question (as well as the first question).

All of the mathematical statements that have been proved in the textbook as theorems, corollaries, and lemmas can be used.

1. [30] For Turing machine $M_1 = (Q, \Sigma, \Gamma, \delta, q_s, q_a, q_r)$:
   - $Q = \{ q_s, q_a, q_r, q_0, q_1 \}$, $\Sigma = \{ \texttt{0}, \texttt{1} \}$, $\Gamma = \{ \texttt{0}, \texttt{1}, \sqcup \}$,
   - and δ is described with the following state diagram:



   (a) [10] Give the sequence of configurations that M_1 enters when started on the input string: `01101`

   (b) [10] Reduce $\langle M_1, \texttt{01100} \rangle$ to an *MPCP* instance. You need to give only the dominos constructed in parts 1 (the starting configuration), 2 (transitions moving the head right), and 3 (transitions moving the head left). Others are $\left[\frac{\texttt{0}}{\texttt{0}}\right]$, $\left[\frac{\texttt{1}}{\texttt{1}}\right]$, $\left[\frac{\sqcup}{\sqcup}\right]$, $\left[\frac{\#}{\#}\right]$, $\left[\frac{\#}{\sqcup\#}\right]$, $\left[\frac{\texttt{0}q_{accept}}{q_{accept}}\right]$, $\left[\frac{q_{accept}\texttt{0}}{q_{accept}}\right]$, $\left[\frac{\texttt{1}q_{accept}}{q_{accept}}\right]$, $\left[\frac{q_{accept}\texttt{1}}{q_{accept}}\right]$, $\left[\frac{\sqcup q_{accept}}{q_{accept}}\right]$, $\left[\frac{q_{accept}\sqcup}{q_{accept}}\right]$, $\left[\frac{q_{accept}\#\#}{\#}\right]$.

   (c) [10] Does the *MPCP* instance in (b) have a match? If so, give one match; if not, give an explanation.

2. [10] The collection of decidable languages is closed under the operation of star.

   (a) [3] True (◯) or false (×)?

   (b) [7] Show that your answer for (a) is correct.

3. [10] The collection of Turing-recognizable languages is closed under the operation of union.

   (a) [3] True (◯) or false (×)?

   (b) [7] Show that your answer for (a) is correct.

4. [10] $L_4 = \{ \langle M \rangle \mid M \text{ is a } \texttt{DFA} \text{ that accepts } w^{\mathcal{R}} \text{ whenever it accepts } w \}$ is a decidable language, where $w^{\mathcal{R}}$ is the ***reverse*** of w.

   (a) [3] True (◯) or false (×)?

   (b) [7] Show that your answer for (a) is correct.

5. [10] $L_5 = \{ \langle M \rangle \mid M \text{ is a } \texttt{TM} \text{ that accepts } w^{\mathcal{R}} \text{ whenever it accepts } w \}$ is a decidable language, where $w^{\mathcal{R}}$ is the ***reverse*** of w.

   (a) [3] True (◯) or false (×)?

   (b) [7] Show that your answer for (a) is correct.

6. [10] $L_6 = \{ \langle M \rangle \mid M \text{ is a } \texttt{TM} \text{ that has useless states } \}$ is decidable.

   (a) [3] True (◯) or false (×)?

   (b) [7] Show that your answer for (a) is correct.

7. [10] $E_{\texttt{TM}} = \{ \langle M \rangle \mid M \text{ is a } \texttt{TM} \text{ and } L(M) = \emptyset \}$ is Turing-recognizable.

   (a) [3] True (◯) or false (×)?

   (b) [7] Show that your answer for (a) is correct.

8. [10] $\overline{E_{\texttt{TM}}}$ is Turing-recognizable.

   (a) [3] True (◯) or false (×)?

   (b) [7] Show that your answer for (a) is correct.