# Introduction to Formal Language
# Chapter 2 Practice Solutions

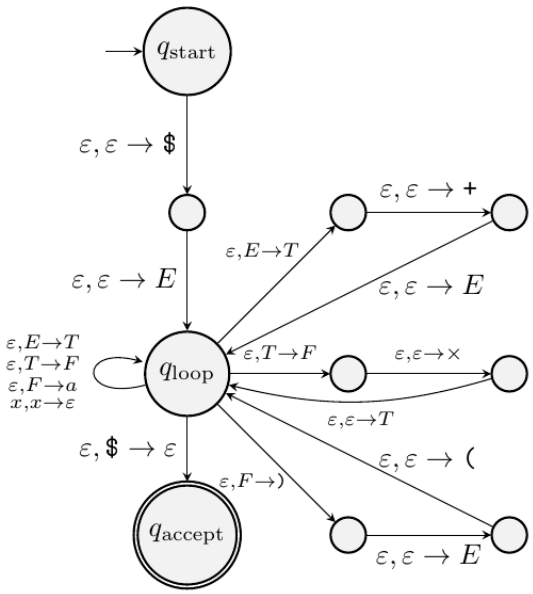
April. 20, 2020

**2.4** which includes 2 subproblems.

**a.** in the p.160

**d.** in the p.160

**2.11**




**2.14** We follow the template from *Example 2.10*, coming immediately after **Theorem 2.9**.

1. First we add a new start variable $A_0$.

$$A \to BAB \mid B \mid \varepsilon \qquad \mathbf{A_0 \to A}$$
$$B \to 00 \mid \varepsilon \qquad A \to BAB \mid B \mid \varepsilon$$
$$B \to 00 \mid \varepsilon$$

2. Secondly we remove $\varepsilon$-rules. (Note that in performing this step we exclude strings from language of grammar. This is the only string we lose in the process).

$$A_0 \to A \qquad A_0 \to A$$
$$A \to BAB \mid B \mid \varepsilon \qquad A \to BAB \mid \mathbf{BB} \mid B \mid \mathbf{AB} \mid \mathbf{BA} \mid \mathbf{A}$$
$$B \to 00 \mid \varepsilon \qquad B \to 00 \mid \varepsilon$$

3. Thirdly we remove all unit rules.

$$\begin{array}{ll} A_0 \to A & A_0 \to A \mid \mathbf{BAB} \mid \mathbf{BB} \mid 00 \mid \mathbf{AB} \mid \mathbf{BA} \\ A \to BAB \mid BB \mid B \mid 00 \mid AB \mid BA \mid A & A \to BAB \mid BB \mid 00 \mid AB \mid BA \mid A \\ B \to 00 & B \to 00 \end{array}$$

4. Lastly we add additional variables, to obtain the equivalent grammar in Chomsky form.

$$\begin{aligned} A_0 &\to BA_1 \mid BB \mid ZZ \mid AB \mid BA \\ A &\to BA_1 \mid BB \mid ZZ \mid AB \mid BA \\ B &\to ZZ \\ Z &\to 0 \\ A_1 &\to AB \end{aligned}$$

**2.16** Union and concatenation are trivial. Assume that we are given languages $L_1$ and $L_2$ generated by context-free grammars $G_1$ and $G_2$ respectively.
Then grammar which includes all rules of both grammar $G_1$ and $G_2$ and one additional rule $S \to S_1 \mid S_2$, where $S$ is the new starting variable of grammar $G$, and $S_1$ and $S_2$ are old starting variables of grammars $G_1$ and $G_2$. recognizes exactly union $L_1 \cup L_2$. Concatenation is obtained analogously, by adding rule $S \to S_1S_2$. Star is also not difficult, but last Exercise shows that we need to be careful. This time we wisely add new starting variable $S_0$ and add rules $S_0 \to SS \mid \varepsilon$ to new grammar, where $S$ is the old starting variable. Note the analogy with adding a new initial state to NFA which recognizes star of language.

**2.42** which includes 2 subproblems.

**b.** in the p.162

**c.** in the p.162