1. Demonstrate whether each of these statements is true or false for polynomials over a field.

    (a) The product of monic polynomials is monic. (5%)

    (b) The product of polynomials of degrees m and n has degree $m + n$. (5%)

    (c) The sum of polynomials of degrees m and n has degree $\max\{m, n\}$. (5%)

2. Determine which of the following polynomials are reducible over $GF(2)$.

    (a) $x^2 + 1$. (5%)

    (b) $x^2 + x + 1$. (5%)

    (c) $x^4 + x + 1$. (5%)

3. Determine the gcd of the following pairs of polynomials.

    (a) $(x^3 + x + 1)$ and $(x^2 + 1)$ over $GF(3)$. (5%)

    (b) $(x^3 - 2x + 1)$ and $(x^2 - x - 2)$ over $GF(5)$. (5%)

4. Determine the multiplicative inverse of $x^2 + 1$ in $GF(2^3)$ with $m(x) = x^3 + x - 1$. (10%)

5. Develop a set of tables similar to Table 5.3 for $GF(4)$ with $m(x) = x^2 + x + 1$. (10%)