MATLAB functions

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Help

- In the command window, type

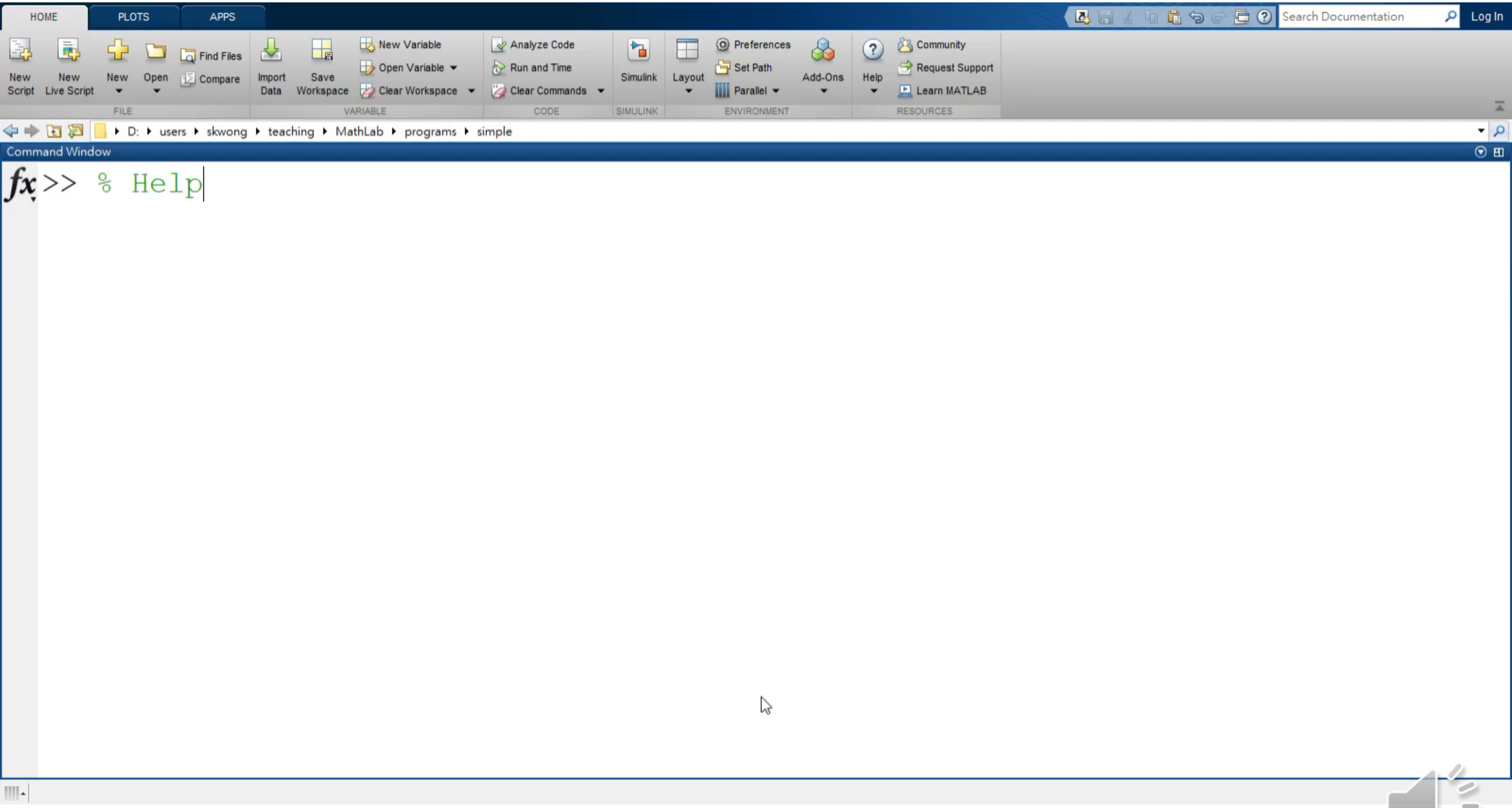
  help

- A list of topics is shown, e.g.,

HELP topics:

Documents\MATLAB	- (No table of contents file)
matlab\datafun	- Data analysis and Fourier transforms.
matlab\datatypes	- Data types and structures.
matlab\elfun	- Elementary math functions.

MATLAB Help (0:33)



Built-in functions

A built-in function is part of the MATLAB executable.

Most built-in functions have a .m file associated with them.

The file supplies the help documentation for the function.

Functions

- A function is a group of statements that perform a task.
- It takes one or more input values and produces output.
- A functions has 3 components:

  1) input, 2) output, and 3) name

```
b = tan(x)   % in radians
```

- **`x:`** the input, or argument(s)
- **`tan(x):`** tan(x) returns an output and we assign it to b.
- **`tan:`** the tangent function name

MATLAB functions

- A function call:

  variable = function(argument list)

  argument list := numbers or variables

The following items are required for a function:

- ➢The function name
- ➢the input values
- ➢the function purpose
- ➢the outputs

- For example,

  x = sin(t)

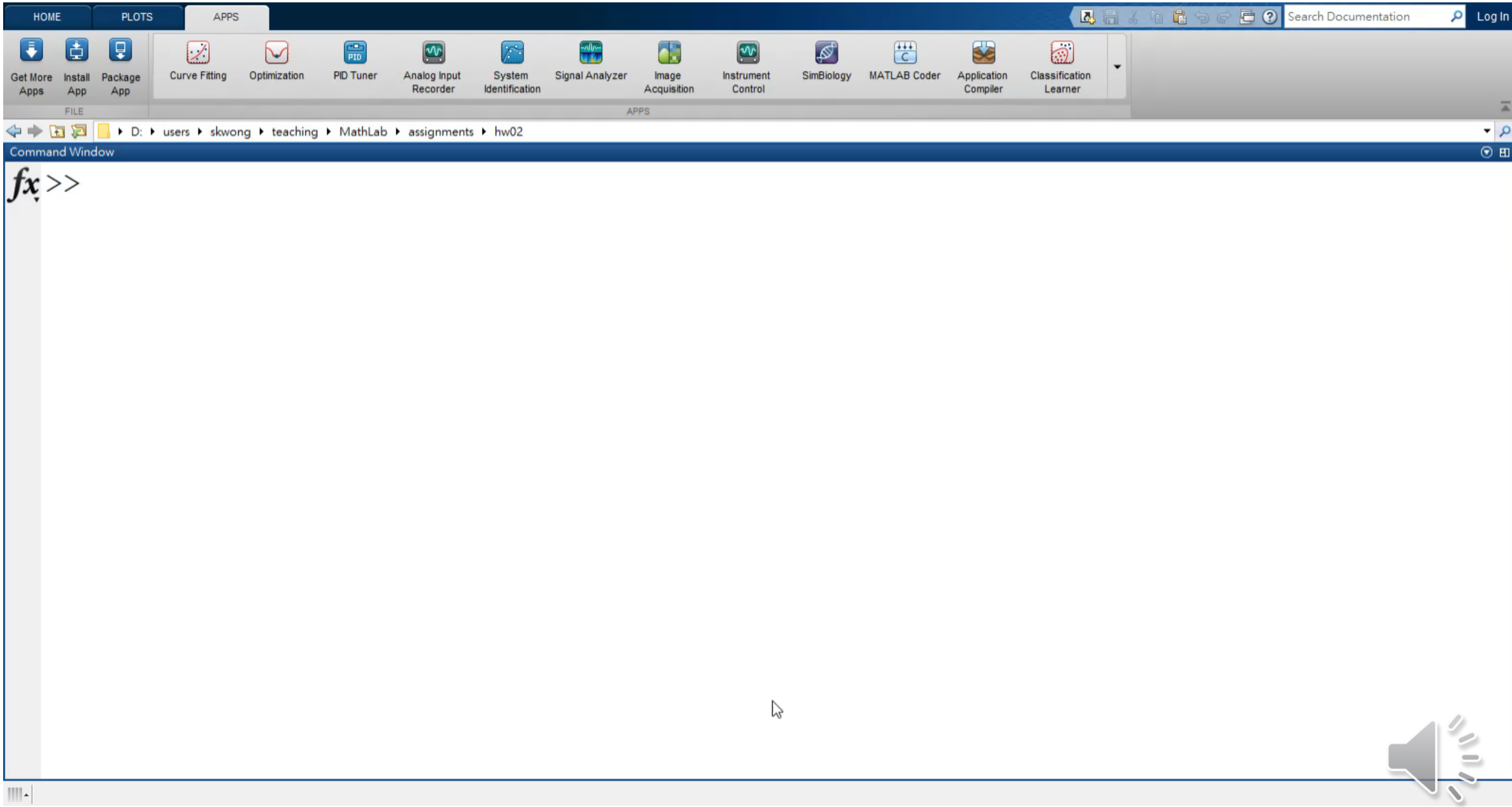
HELP

- Matlab has many built-in functions.
- Type

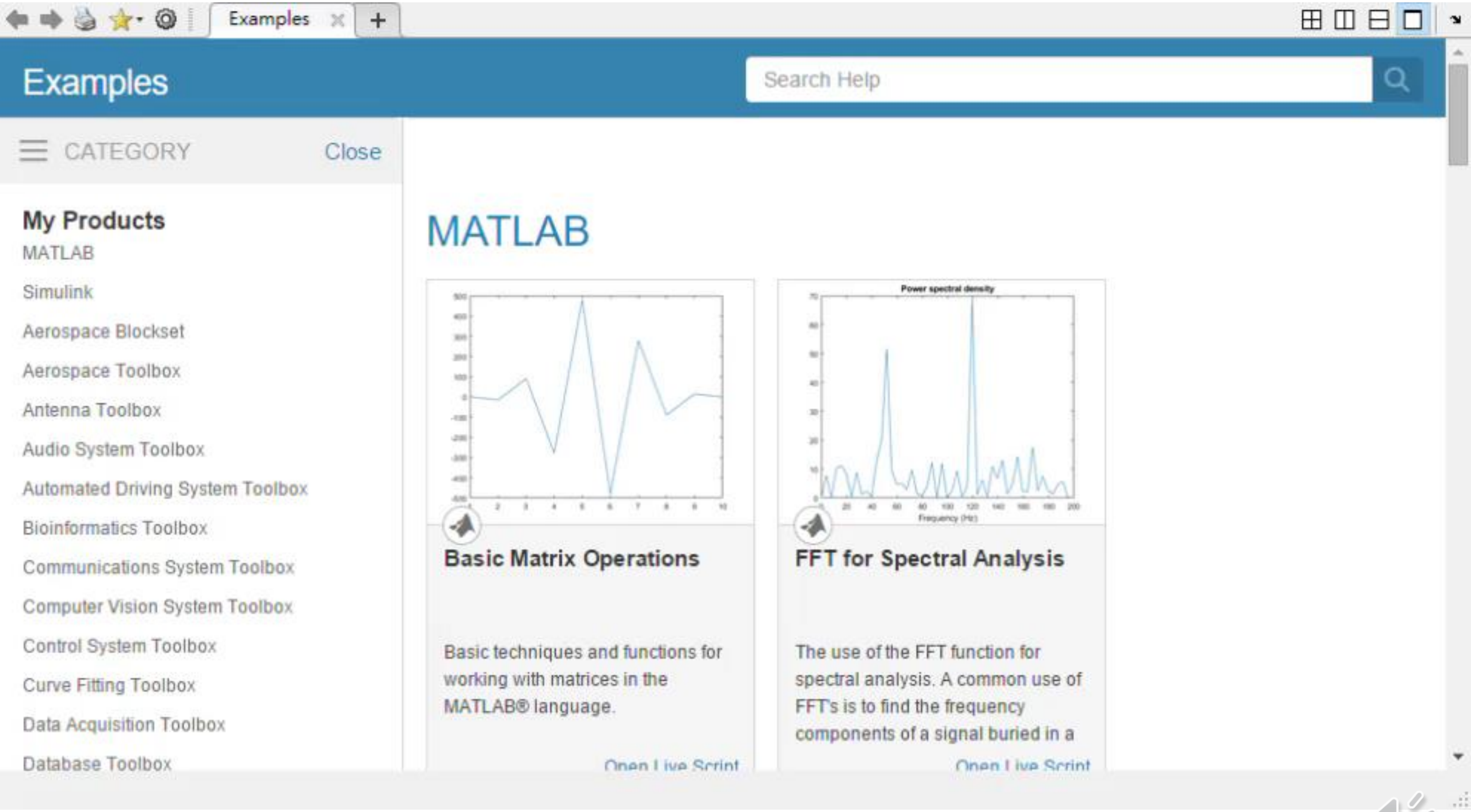
    help functionName

  to show the details of the function

Demo (2:23)
Type some functions to see how to use them



Help: Examples



Rounding functions

round

fix

floor

ceil

How to learn them?

Rounding functions

round

fix

floor

ceil

How to learn them?

help round

help fix

help floor

help ceil

Rounding functions

Function	Description
round(X)	rounds each element of X to the nearest integer.
fix(X)	rounds the elements of X to the nearest integers towards zero.
floor(X)	rounds the elements of X to the nearest integers towards minus infinity.
ceil(X)	rounds the elements of X to the nearest integers towards infinity.

Rounding functions

A = [-2:0.4:2];

A = -2 -1.6 -1.2 -0.8 -0.4 0 0.4 0.8 1.2 1.6 2

round(A) =

-2 -2 -1 -1 0 0 0 1 1 2 2

fix(A) =

-2 -1 -1 0 0 0 0 0 1 1 2

floor(A) =

-2 -2 -2 -1 -1 0 0 0 1 1 2

ceil(A) =

-2 -1 -1 0 0 0 1 1 2 2 2

Rounding functions

A = [-2:0.4:2];

A = -2 -1.6 -1.2 -0.8   -0.4   0   0.4 0.8 1.2   1.6   2

round(A) =

-2 -2 -1 -1 0 0 0 1 1 2 2

fix(A) = %towards zero

-2 -1 -1 0 0 0 0 0 1 1 2

floor(A) =

-2 -2 -2 -1 -1 0 0 0 1 1 2

ceil(A) =

-2 -1 -1 0 0 0 1 1 2 2 2

Discrete-mathematics functions

Function	Description
`factor(N)`	returns a vector containing the prime factors of N.
`rats(X,LEN)`	uses RAT to display rational approximations to the elements of X.
`factorial(N)`	factorial(N) for scalar N, is the product of all the integers from 1 to N, i.e. prod(1:N).
`gcd(A,B)`	G = gcd(A,B) is the greatest common divisor of corresponding elements of A and B.
`lcm(A,B)`	lcm(A,B) is the least common multiple of corresponding elements of A and B.

Discrete-mathematics functions

Function	Description
`factor(N)`	factor(24) %the prime factors ans = 2 2 2 3
`rats(X,LEN)`	rats(3.5122, 14) % rational approximations ans = ' 17561/5000 '
`factorial(N)`	factorial(5) % 1*2*…*5 ans = 120
`gcd(A,B)`	gcd(15, 24) %the greatest common divisor ans = 3
`lcm(A,B)`	lcm(15,24) %least common multiple ans = 120

Trigonometric functions

- Trigonometric functions accept angles in radians.

  Conversion formula: 180 degrees $= \pi$ radians

- Example: 8 degrees = 8*(pi/180) radians

- **pi** is a built-in constant
- sin, cos, tan, acos, asin, atan

Hyperbolic functions

- Hyperbolic sine: the odd part of the exponential function, that is

$$\sinh x = \frac{e^x - e^{-x}}{2} = \frac{e^{2x} - 1}{2e^x} = \frac{1 - e^{-2x}}{2e^{-x}}.$$

- Hyperbolic cosine: the even part of the exponential function, that is

$$\cosh x = \frac{e^x + e^{-x}}{2} = \frac{e^{2x} + 1}{2e^x} = \frac{1 + e^{-2x}}{2e^{-x}}.$$

- Hyperbolic tangent:

$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{e^{2x} - 1}{e^{2x} + 1}.$$

- Hyperbolic cotangent: for $x \neq 0$,

$$\coth x = \frac{\cosh x}{\sinh x} = \frac{e^x + e^{-x}}{e^x - e^{-x}} = \frac{e^{2x} + 1}{e^{2x} - 1}.$$

- Hyperbolic secant:

$$\operatorname{sech} x = \frac{1}{\cosh x} = \frac{2}{e^x + e^{-x}} = \frac{2e^x}{e^{2x} + 1}.$$

- Hyperbolic cosecant: for $x \neq 0$,

$$\operatorname{csch} x = \frac{1}{\sinh x} = \frac{2}{e^x - e^{-x}} = \frac{2e^x}{e^{2x} - 1}.$$

Hyperbolic functions
Plot all the functions.

- Hyperbolic sine: the odd part of the exponential function, that is

$$\sinh x = \frac{e^x - e^{-x}}{2} = \frac{e^{2x} - 1}{2e^x} = \frac{1 - e^{-2x}}{2e^{-x}}.$$

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$$\operatorname{sech} x = \frac{1}{\cosh x} = \frac{2}{e^x + e^{-x}} = \frac{2e^x}{e^{2x} + 1}.$$

- Hyperbolic cosecant: for $x \neq 0$,

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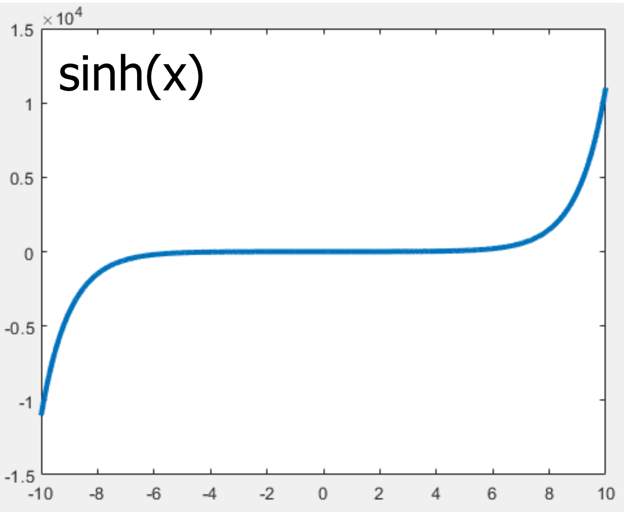
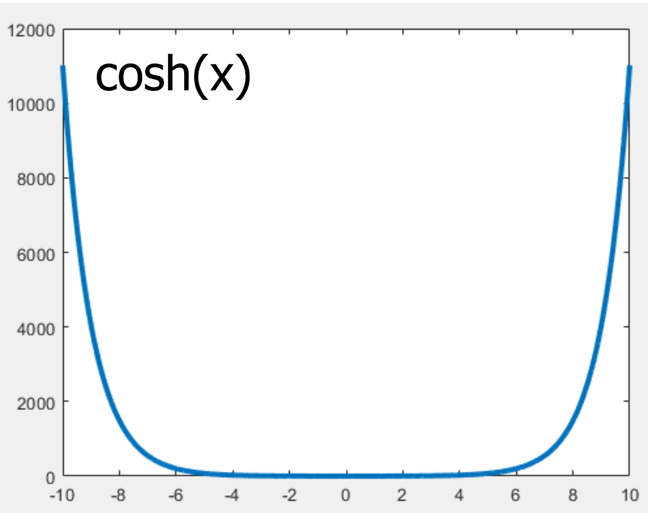
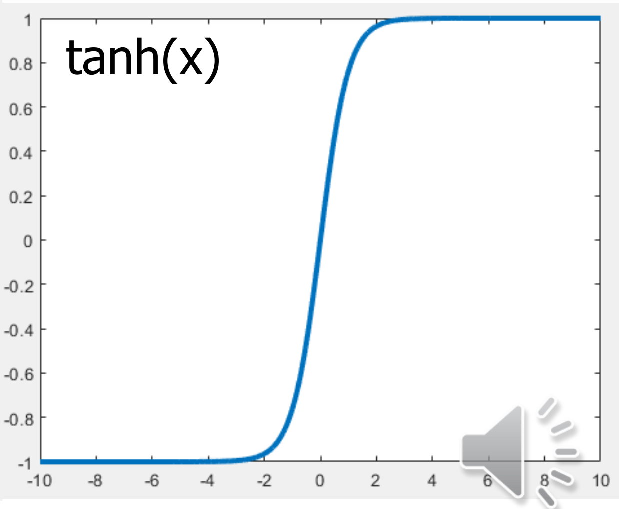






Hyperbolic functions
Plot all the functions.

Mirror? upside down?

- Hyperbolic sine: the odd part of the exponential function, that is

$$\sinh x = \frac{e^x - e^{-x}}{2} = \frac{e^{2x} - 1}{2e^x} = \frac{1 - e^{-2x}}{2e^{-x}}.$$

sinh x = - sinh -x

- Hyperbolic cosine: the even part of the exponential function, that is

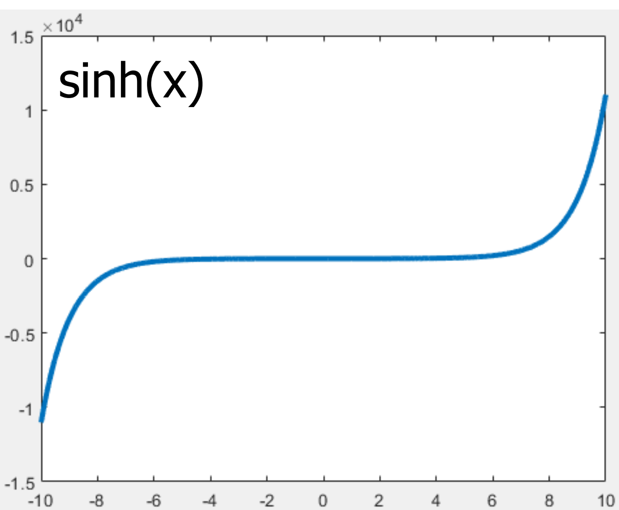
$$\cosh x = \frac{e^x + e^{-x}}{2} = \frac{e^{2x} + 1}{2e^x} = \frac{1 + e^{-2x}}{2e^{-x}}.$$

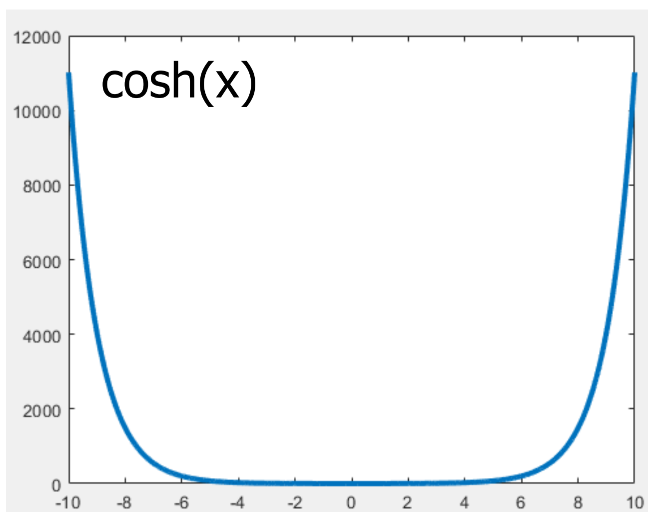
cosh x = cosh -x

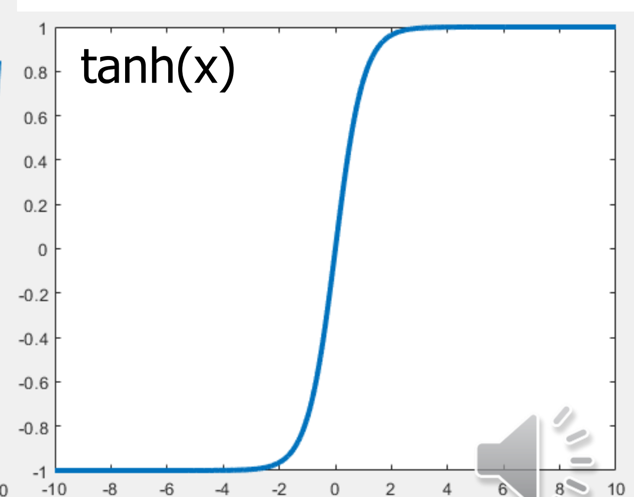
- Hyperbolic tangent:

$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{e^{2x} - 1}{e^{2x} + 1}.$$

tanh x = - tanh -x





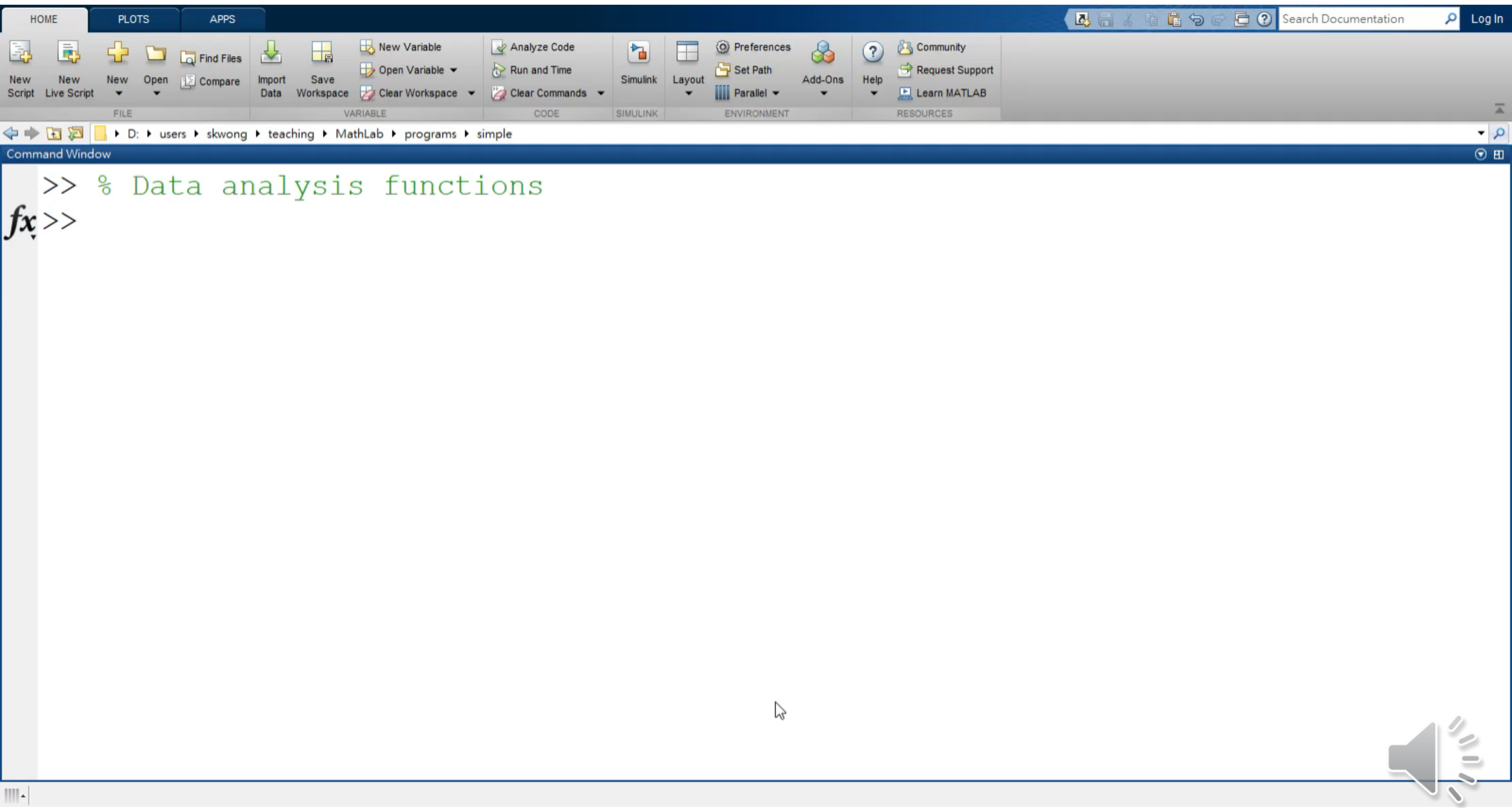


Data analysis functions

- Analyze data statistically.
- MATLAB has many statistical functions:

max()	sum()	size()
min()	prod()	length()
mean()	sort()	std()
median	sortrows	var()

Demo (2:59)
Data analysis functions



Data analysis practice

```
A =
     1     2     3
     1     1     5
     5     1     1
     1     5     5

sortrows(A)
ans =

     1     1     5
     1     2     3
     1     5     5
     5     1     1
```

help sortrows

B = sortrows(A) sorts the rows of matrix A in ascending order as a group. B has the same size and type as A. A must be a 2-D matrix.

Data analysis practice

```
A =
      1     2     3
      1     1     5
      5     1     1
      1     5     5

>> sortrows(A,2)

ans =
      1     1     5
      5     1     1
      1     2     3
      1     5     5
```

B = sortrows(A,COL) sorts the matrix according to the columns specified by the vector COL.

If an element of COL is positive, the corresponding column in A is sorted in ascending order; if an element of COL is negative, the corresponding column in A is sorted in descending order.

Data analysis practice

```
A =
     1     2     3
     1     1     5
     5     1     1
     1     5     5

>> sortrows(A,-2)
ans =

     1     5     5
     1     2     3
     1     1     5
     5     1     1
```

B = sortrows(A,COL) sorts the matrix according to the columns specified by the vector COL.

If an element of COL is positive, the corresponding column in A is sorted in ascending order; if an element of COL is negative, the corresponding column in A is sorted in descending order.

Data analysis practice

```
A =
      1     2     3
      1     1     5
      5     1     1
      1     5     5

>> sortrows(A, [-2 3])
ans =
      1     5     5
      1     2     3
      5     1     1
      1     1     5
```

B = sortrows(A,COL) sorts the matrix according to the columns specified by the vector COL.

If an element of COL is positive, the corresponding column in A is sorted in ascending order; if an element of COL is negative, the corresponding column in A is sorted in descending order.

Data analysis practice

```
A =
     1     2     3
     1     1     5
     5     1     1
     1     5     5

>> sortrows(A, [-2 -3])
ans =
     1     5     5
     1     2     3
     1     1     5
     5     1     1
```

B = sortrows(A,COL) sorts the matrix according to the columns specified by the vector COL.

If an element of COL is positive, the corresponding column in A is sorted in ascending order; if an element of COL is negative, the corresponding column in A is sorted in descending order.

Exercises

v = [4 3 2 1 6 5 4]

1. Find the size of v.
2. Sort v in ascending order.
3. Sort v in descending order.
4. Find the mean of v.
5. Find the median of v.
6. Find the standard deviation of v.
7. Find the cumulative product of v.

Exercises

v = [4 3 2 1 6 5 4]

1. Find the size of v.
2. Sort v in ascending order.
3. Sort v in descending order.
4. Find the mean of v.
5. Find the median of v.
6. Find the standard deviation of v.
7. Find the cumulative product of v.

```
size(v)

ans =

     1     7
```

Exercises

v = [4 3 2 1 6 5 4]

1. Find the size of v.
2. Sort v in ascending order.
3. Sort v in descending order.
4. Find the mean of v.
5. Find the median of v.
6. Find the standard deviation of v.
7. Find the cumulative product of v.

```
sort(v)

ans =

     1     2     3     4     4     5     6
```

Exercises

help sort

B = sort(A,DIRECTION)
and B = sort(A,DIM,DIRECTION) also specify the sort direction. DIRECTION must be:
'ascend' - (default) Sorts in ascending order.

'descend' - Sorts in descending order.

v = [4 3 2 1 6 5 4]

1. Find the size of v.
2. Sort v in ascending order.
3. Sort v in descending order.
4. Find the mean of v.
5. Find the median of v.
6. Find the standard deviation of v.
7. Find the cumulative product of v.

```
sort(v, 'descend')

ans =

    6     5     4     4     3     2     1
```

Exercises

v = [4 3 2 1 6 5 4]

1. Find the size of v.
2. Sort v in ascending order.
3. Sort v in descending order.
4. Find the mean of v.
5. Find the median of v.
6. Find the standard deviation of v.
7. Find the cumulative product of v.

mean(v)

median(v)

std(v)

prod(v)

Exercises

v =

4	4
3	3
2	2
1	1
6	2
5	8
4	9

Sort the rows of x based on the 2nd column in ascending order..

Sort the rows of x based on the first column in descending order.

Compute the average of each row.

Compute the average of each column.

Exercises

v =

4	4
3	3
2	2
1	1
6	2
5	8
4	9

Sort the rows of x based on the 2nd column in ascending order..

sortrows(v,2)

ans =

1	1
2	2
6	2
3	3
4	4
5	8
4	9

Exercises

v =

4	4
3	3
2	2
1	1
6	2
5	8
4	9

Sort the rows of x based on the first column in descending order.

```
sortrows(v,-1)

ans =

     6     2
     5     8
     4     4
     4     9
     3     3
     2     2
     1     1
```

Exercises

v =

4	4
3	3
2	2
1	1
6	2
5	8
4	9

Compute the average of each row.

```
mean(v,2)

ans =

    4.0000
    3.0000
    2.0000
    1.0000
    4.0000
    6.5000
    6.5000
```

mean(X,DIM) takes the mean along the dimension DIM of X.

Exercises

v =

4	4
3	3
2	2
1	1
6	2
5	8
4	9

Compute the average of each column.

```
mean(v,1)

ans =

    3.5714    4.1429
```

mean(X,DIM) takes the mean along the dimension DIM of X.

Exercises

```
v =

     4     4
     3     3
     2     2
     1     1
     6     2
     5     8
     4     9
```

Compute the average of v.

```
mean(v)  % same as mean(v,1)

ans =

    3.5714    4.1429

>> mean(mean(v))

ans =

    3.8571
```

Exercises

v = [4 3 7 5 4 9 2; 4 5 3 9 7 5 1; (7:-1:1)]

v = ?

Exercises

```
v = [ 4 3 7 5 4 9 2; 4 5 3 9 7 5 1; (7:-1:1)]

v =

    4    3    7    5    4    9    2

    4    5    3    9    7    5    1

    7    6    5    4    3    2    1

>> sort(v)

ans = ?
```

Exercises

v = [4 3 7 5 4 9 2; 4 5 3 9 7 5 1; (7:-1:1)]

v =

4	3	7	5	4	9	2
4	5	3	9	7	5	1
7	6	5	4	3	2	1

>> sort(v)

ans =

4	3	3	4	3	2	1
4	5	5	5	4	5	1
7	6	7	9	7	9	2

Exercises

```
v =

    4    3    7    5    4    9    2

    4    5    3    9    7    5    1

    7    6    5    4    3    2    1


>> prod(v)

ans = ?
```

Exercises

```
v =

     4     3     7     5     4     9     2
     4     5     3     9     7     5     1
     7     6     5     4     3     2     1

>> prod(v)

ans =

   112    90   105   180    84    90     2
```

S = prod(X) is the product of the elements of the vector X.

If X is a matrix, S is a row vector with the product over each column.

For N-D arrays, prod(X) operates on the first non-singleton dimension.

Exercises

v =

4	3	7	5	4	9	2
4	5	3	9	7	5	1
7	6	5	4	3	2	1

S = prod(X) is the product of the elements of the vector X.

If X is a matrix, S is a row vector with the product over each column.

For N-D arrays, prod(X) operates on the first non-singleton dimension.

```
>> prod(v)
ans =
   112    90   105   180    84    90     2
```

```
prod(prod(v))
ans =
  2.8805e+12
```

Probability

Probability is the measure of the likelihood that an event will occur.

Probability quantifies as a number between 0 and 1, where, loosely speaking,

0 indicates impossibility and

1 indicates certainty.

A simple example is the tossing of a fair (unbiased) coin. Since the coin is fair, the two outcomes ("heads" and "tails") are both equally probable (which could be written as 0.5 or 50%).

https://en.wikipedia.org/wiki/Probability

Probability

Example:

Throw a fair dice with six sides twice.

What is the probability that we have

A) the two numbers are: 1 and 2 (can be in any order)?

B) the sum of the two numbers is smaller than or equal to 4?

Probability

Example:

Throw a fair dice with six sides twice.

What is the probability that we have

A) the two numbers are: 1 and 2 (can be in any order)?

B) the sum of the two numbers is smaller than or equal to 4?

List all the possible outcomes:

(1,1), (2, 1), (3, 1), (4, 1), (5, 1), (6, 1)
(1,2), (2, 2), (3, 2), (4, 2), (5, 2), (6, 2)
(1,3), (2, 3), (3, 3), (4, 3), (5, 3), (6, 3)
(1,4), (2, 4), (3, 4), (4, 4), (5, 4), (6, 4)
(1,5), (2, 5), (3, 5), (4, 5), (5, 5), (6, 5)
(1,6), (2, 6), (3, 6), (4, 6), (5, 6), (6, 6)

Probability

Example:
Throw a fair dice with six sides twice.
What is the probability that we have

A) the two numbers are: 1 and 2
(can be in any order)?

B) the sum of the two numbers is smaller than or equal to 4?

List all the possible outcomes:
(1,1), **(2, 1)**, (3, 1), (4, 1), (5, 1), (6, 1)
(1,2), (2, 2), (3, 2), (4, 2), (5, 2), (6, 2)
(1,3), (2, 3), (3, 3), (4, 3), (5, 3), (6, 3)
(1,4), (2, 4), (3, 4), (4, 4), (5, 4), (6, 4)
(1,5), (2, 5), (3, 5), (4, 5), (5, 5), (6, 5)
(1,6), (2, 6), (3, 6), (4, 6), (5, 6), (6, 6)

$P((i = 1, j = 2)$ or $(i=2, j = 1))$

$= P(i=1,j=2) + P(i=2,j=1)$

Probability

Example:

Throw a fair dice with six sides twice.

What is the probability that we have

A) the two numbers are: 1 and 2
(can be in any order)?

B) the sum of the two numbers is smaller than or equal to 4?

List all the possible outcomes:

(1,1), (2, 1), (3, 1), (4, 1), (5, 1), (6, 1)
(1,2), (2, 2), (3, 2), (4, 2), (5, 2), (6, 2)
(1,3), (2, 3), (3, 3), (4, 3), (5, 3), (6, 3)
(1,4), (2, 4), (3, 4), (4, 4), (5, 4), (6, 4)
(1,5), (2, 5), (3, 5), (4, 5), (5, 5), (6, 5)
(1,6), (2, 6), (3, 6), (4, 6), (5, 6), (6, 6)

Probability

Example:

Throw a fair dice with six sides twice.

What is the probability that we have

A) the two numbers are: 1 and 2 (can be in any order)?

B) the sum of the two numbers, i and j, is smaller than or equal to 4?

List all the possible outcomes:

(1,1), (2, 1), (3, 1), (4, 1), (5, 1), (6, 1)
(1,2), (2, 2), (3, 2), (4, 2), (5, 2), (6, 2)
(1,3), (2, 3), (3, 3), (4, 3), (5, 3), (6, 3)
(1,4), (2, 4), (3, 4), (4, 4), (5, 4), (6, 4)
(1,5), (2, 5), (3, 5), (4, 5), (5, 5), (6, 5)
(1,6), (2, 6), (3, 6), (4, 6), (5, 6), (6, 6)

P(i+j <= 4)

Probability

Example:

Throw a fair dice with six sides twice.

What is the probability that we have

C) the sum of the two numbers is: 4 or 8

List all the possible outcomes:

(1,1), (2, 1), (3, 1), (4, 1), (5, 1), (6, 1)
(1,2), (2, 2), (3, 2), (4, 2), (5, 2), (6, 2)
(1,3), (2, 3), (3, 3), (4, 3), (5, 3), (6, 3)
(1,4), (2, 4), (3, 4), (4, 4), (5, 4), (6, 4)
(1,5), (2, 5), (3, 5), (4, 5), (5, 5), (6, 5)
(1,6), (2, 6), (3, 6), (4, 6), (5, 6), (6, 6)

Probability

Example:
Throw a fair dice with six sides twice.
What is the probability that we have

C) the sum of the two numbers is: 4 or 8

List all the possible outcomes:
(1,1), (2, 1), **(3, 1)**, (4, 1), (5, 1), (6, 1)
(1,2), **(2, 2)**, (3, 2), (4, 2), (5, 2), **(6, 2)**
(1,3), (2, 3), (3, 3), (4, 3), **(5, 3)**, (6, 3)
(1,4), (2, 4), (3, 4), **(4, 4)**, (5, 4), (6, 4)
(1,5), (2, 5), **(3, 5)**, (4, 5), (5, 5), (6, 5)
(1,6), **(2, 6)**, (3, 6), (4, 6), (5, 6), (6, 6)

Probability

Example:

Throw a fair dice with six sides twice.

What is the probability that we have

C) the sum of the two numbers is: 4 or 8

List all the possible outcomes:

(1,1), (2, 1), **(3, 1)**, (4, 1), (5, 1), (6, 1)
(1,2), **(2, 2)**, (3, 2), (4, 2), (5, 2), **(6, 2)**
(1,3), (2, 3), (3, 3), (4, 3), **(5, 3)**, (6, 3)
(1,4), (2, 4), (3, 4), **(4, 4)**, (5, 4), (6, 4)
(1,5), (2, 5), **(3, 5)**, (4, 5), (5, 5), (6, 5)
(1,6), **(2, 6)**, (3, 6), (4, 6), (5, 6), (6, 6)

$P(\ i+j = 4 \text{ or } i+j = 8)$

$= P(i+j = 4)$
$\quad + P(i+j=8)$

Probability

P(A or B) =
P(A) + P(B)

Example:

Throw a fair dice with six sides twice.

What is the probability that we have

C) the sum of the two numbers is: 4 or 8

List all the possible outcomes:

(1,1), (2, 1), **(3, 1)**, (4, 1), (5, 1), (6, 1)
(1,2), **(2, 2)**, (3, 2), (4, 2), (5, 2), **(6, 2)**
(1,3), (2, 3), (3, 3), (4, 3), **(5, 3)**, (6, 3)
(1,4), (2, 4), (3, 4), **(4, 4)**, (5, 4), (6, 4)
(1,5), (2, 5), **(3, 5)**, (4, 5), (5, 5), (6, 5)
(1,6), **(2, 6)**, (3, 6), (4, 6), (5, 6), (6, 6)

P(i+j = 4 or i+j = 8)

= P(i+j =4)
+ P(i+j=8)

Independent events

Probability

Example:
Throw a fair dice with six sides twice.
What is the probability that we have

D) the product of the two numbers is: >= 8
or their sum is >= 8

List all the possible outcomes:
(1,1), (2, 1), (3, 1), (4, 1), (5, 1), (6, 1)
(1,2), (2, 2), (3, 2), (4, 2), (5, 2), (6, 2)
(1,3), (2, 3), (3, 3), (4, 3), (5, 3), (6, 3)
(1,4), (2, 4), (3, 4), (4, 4), (5, 4), (6, 4)
(1,5), (2, 5), (3, 5), (4, 5), (5, 5), (6, 5)
(1,6), (2, 6), (3, 6), (4, 6), (5, 6), (6, 6)

Probability

Example:

Throw a fair dice with six sides twice.

What is the probability that we have

D) the product of the two numbers is: >= 8

or their sum is >= 8

List all the possible outcomes:

(1,1), (2, 1), (3, 1), (4, 1), (5, 1), (6, 1)
(1,2), (2, 2), (3, 2), (4, 2), (5, 2), (6, 2)
(1,3), (2, 3), (3, 3), (4, 3), (5, 3), (6, 3)
(1,4), (2, 4), (3, 4), (4, 4), (5, 4), (6, 4)
(1,5), (2, 5), (3, 5), (4, 5), (5, 5), (6, 5)
(1,6), (2, 6), (3, 6), (4, 6), (5, 6), (6, 6)

P(i+j >= 8 or i*j >= 8)

= P(i+j >= 8)
+ P(i*j>=8) ??

Probability

Example:
Throw a fair dice with six sides twice.
What is the probability that we have

D) the product of the two numbers is: >= 8
or their sum is >= 8

List all the possible outcomes:
(1,1), (2, 1), (3, 1), (4, 1), (5, 1), (6, 1)
(1,2), (2, 2), (3, 2), (4, 2), (5, 2), **(6, 2)**
(1,3), (2, 3), (3, 3), (4, 3), **(5, 3), (6, 3)**
(1,4), (2, 4), (3, 4), **(4, 4), (5, 4), (6, 4)**
(1,5), (2, 5), **(3, 5), (4, 5), (5, 5), (6, 5)**
(1,6), **(2, 6)**, **(3, 6), (4, 6), (5, 6), (6, 6)**

P(i+j >= 8 or i*j >= 8)

= **P(i+j >= 8)**
+ P(i*j>=8) **%wrong**

Dependent events

Probability

Example:

Throw a fair dice with six sides twice.

What is the probability that we have

D) the product of the two numbers is: >= 8
or their sum is >= 8

List all the possible outcomes:

(1,1), (2, 1), (3, 1), (4, 1), (5, 1), (6, 1)
(1,2), (2, 2), (3, 2), **(4, 2), (5, 2)**, **(6, 2)**
(1,3), (2, 3), **(3, 3), (4, 3), (5, 3), (6, 3)**
(1,4), **(2, 4), (3, 4), (4, 4), (5, 4), (6, 4)**
(1,5), **(2, 5), (3, 5), (4, 5), (5, 5), (6, 5)**
(1,6), **(2, 6)**, **(3, 6), (4, 6), (5, 6), (6, 6)**

P(i+j >= 8 or i*j >= 8)

= P(i+j >= 8)
+ **P(i*j>=8)**

%wrong

Dependent

Probability

Example:

Throw a fair dice with six sides twice.

What is the probability that we have

D) the product of the two numbers is: >= 8
or their sum is >= 8

List all the possible outcomes:

(1,1), (2, 1), (3, 1), (4, 1), (5, 1), (6, 1)
(1,2), (2, 2), (3, 2), **(4, 2), (5, 2)**, **(6, 2)**
(1,3), (2, 3), **(3, 3), (4, 3), (5, 3), (6, 3)**
(1,4), **(2, 4), (3, 4), (4, 4), (5, 4), (6, 4)**
(1,5), **(2, 5), (3, 5), (4, 5), (5, 5), (6, 5)**
(1,6), **(2, 6)**, **(3, 6), (4, 6), (5, 6), (6, 6)**

P(i+j >= 8 or i*j >= 8)

= 22/36 = 11/18

Dependent

Probability

Example:
Throw a fair dice with six sides twice.
What is the probability that we have
D) the product of the two numbers is: >= 8
or their sum is >= 8

List all the possible outcomes:
(1,1), (2, 1), (3, 1), (4, 1), (5, 1), (6, 1)
(1,2), (2, 2), (3, 2), **(4, 2), (5, 2)**, **(6, 2)**
(1,3), (2, 3), **(3, 3), (4, 3), (5, 3), (6, 3)**
(1,4), **(2, 4), (3, 4), (4, 4), (5, 4), (6, 4)**
(1,5), **(2, 5), (3, 5), (4, 5), (5, 5), (6, 5)**
(1,6), **(2, 6)**, **(3, 6), (4, 6), (5, 6), (6, 6)**

P(A or B) =
P(A) + P(B) – P(A and B)

P(i+j >= 8 or i*j >= 8)

= P(i+j >=8)
+ P(i*j >=8)
- P(i+j >= 8 and i*j >= 8)

Dependent events

Random numbers

A numeric sequence is said to be statistically random when it contains no recognizable patterns or regularities; sequences such as the results of an ideal dice roll or the digits of π exhibit statistical randomness.

e.g., for a dice,

we have:1, 1, 1, (the next number?)

https://en.wikipedia.org/wiki/Statistical_randomness

Tests for randomness

- Kendall and Smith's original four tests were hypothesis tests, which took as their null hypothesis the idea that each number in a given random sequence had an equal chance of occurring, and that various other patterns in the data should be also distributed equiprobably.

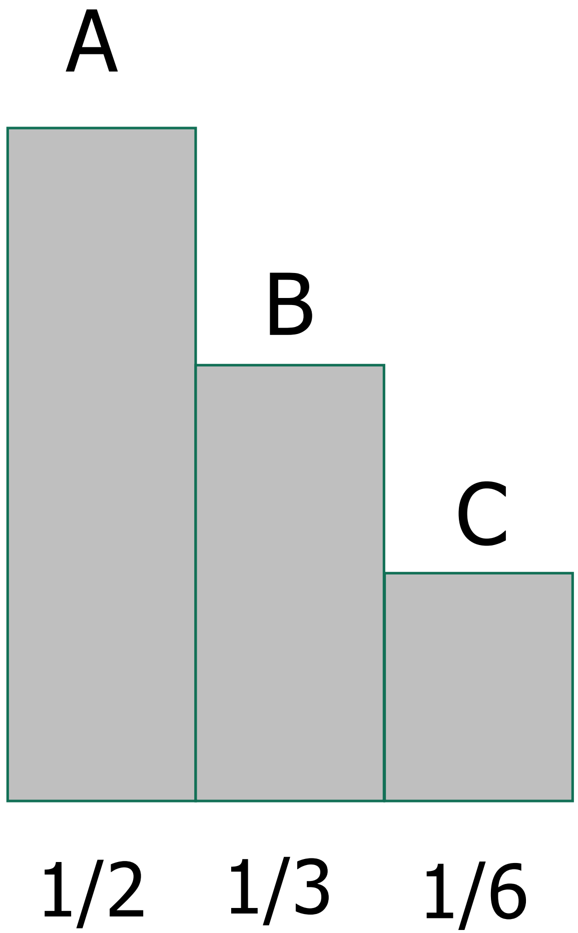
https://en.wikipedia.org/wiki/Statistical_randomness

Kendall and Smith's original four tests

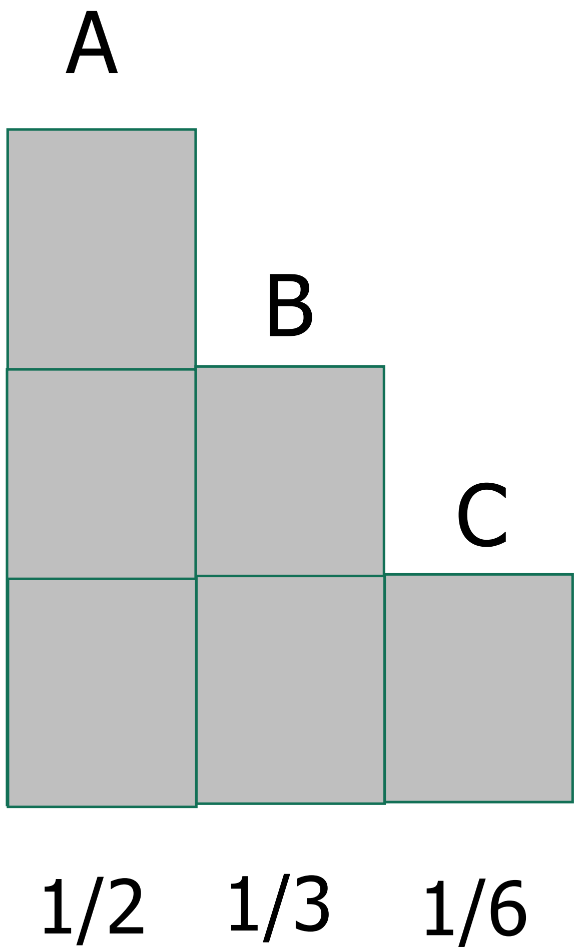
- The **frequency test**: check to make sure that there were roughly the same number of 0s, 1s, 2s, 3s, etc.
- The **serial test: check** for sequences of two digits at a time (00, 01, 02, etc.), comparing their observed frequencies with their hypothetical predictions were they equally distributed.
- The **poker test**, tested for certain sequences of five numbers at a time (AAAAA, AAAAB, AAABB, etc.) based on hands in the game poker.
- The **gap test**, looked at the distances between zeroes (00 would be a distance of 0, 030 would be a distance of 1, 02250 would be a distance of 3, etc.).

- https://en.wikipedia.org/wiki/Statistical_randomness

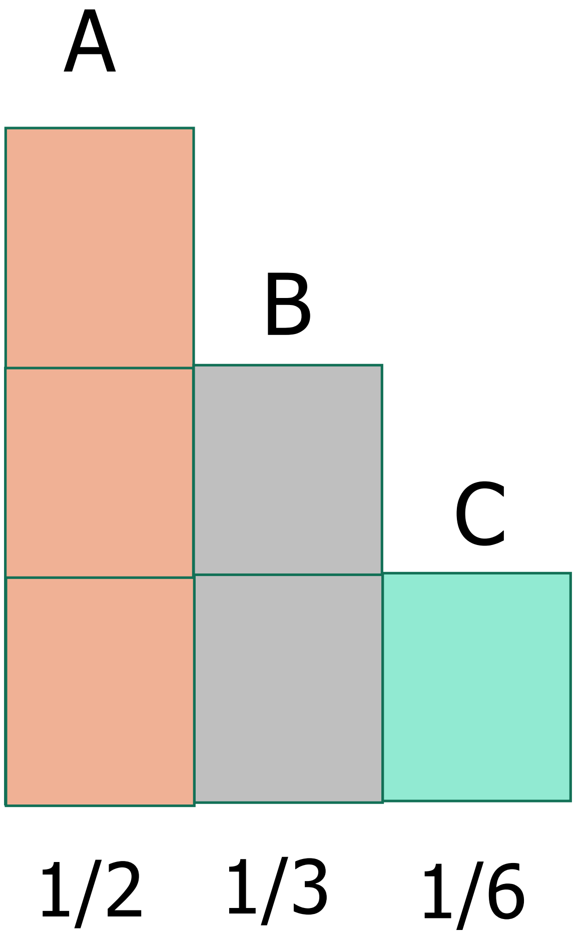
Probability



Probability

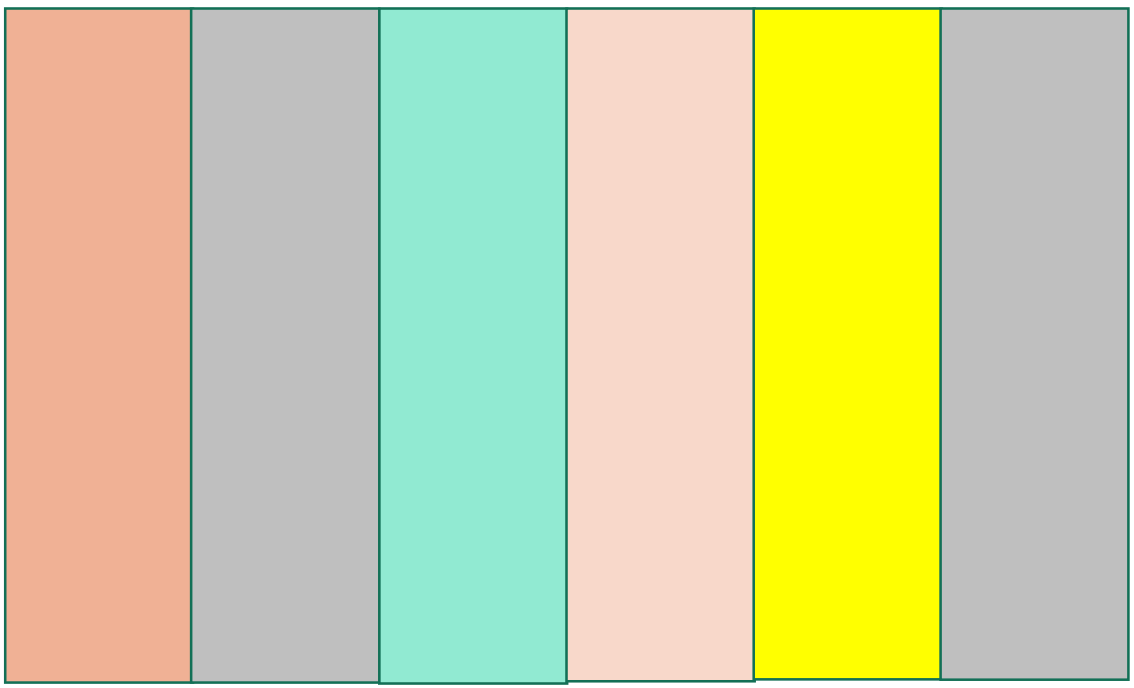


Probability

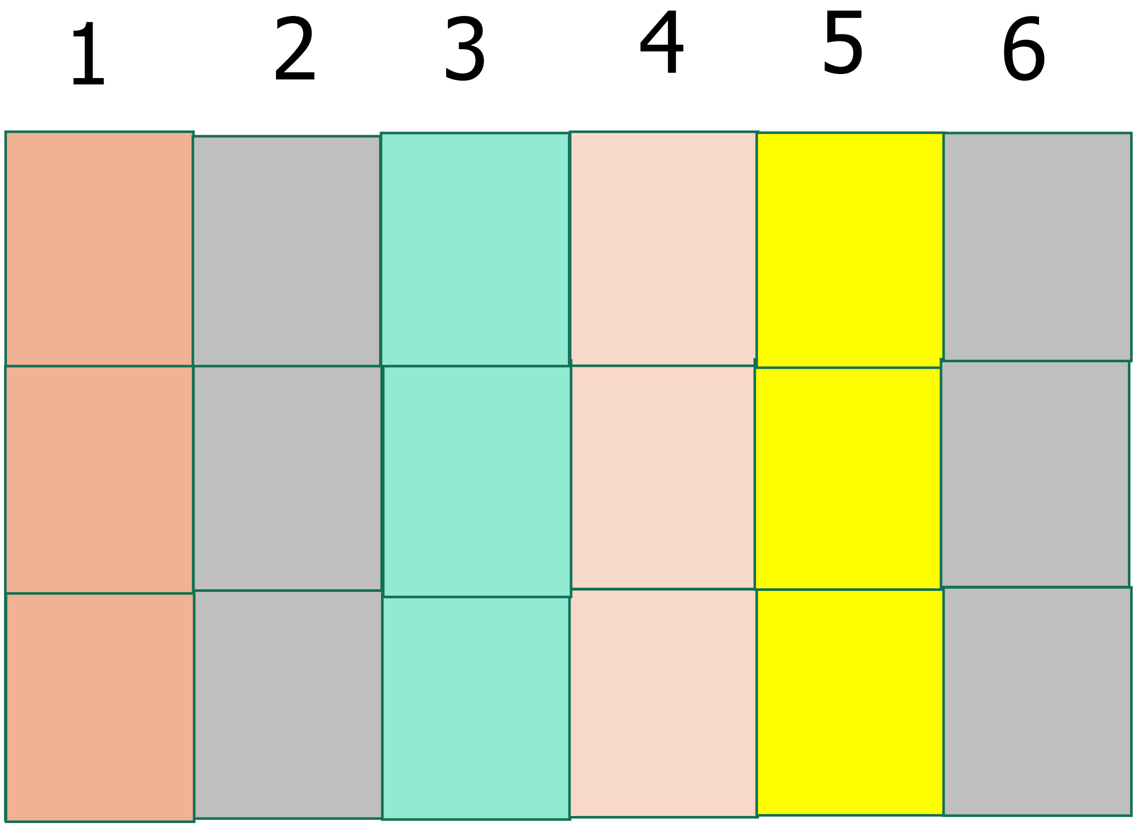


Probability

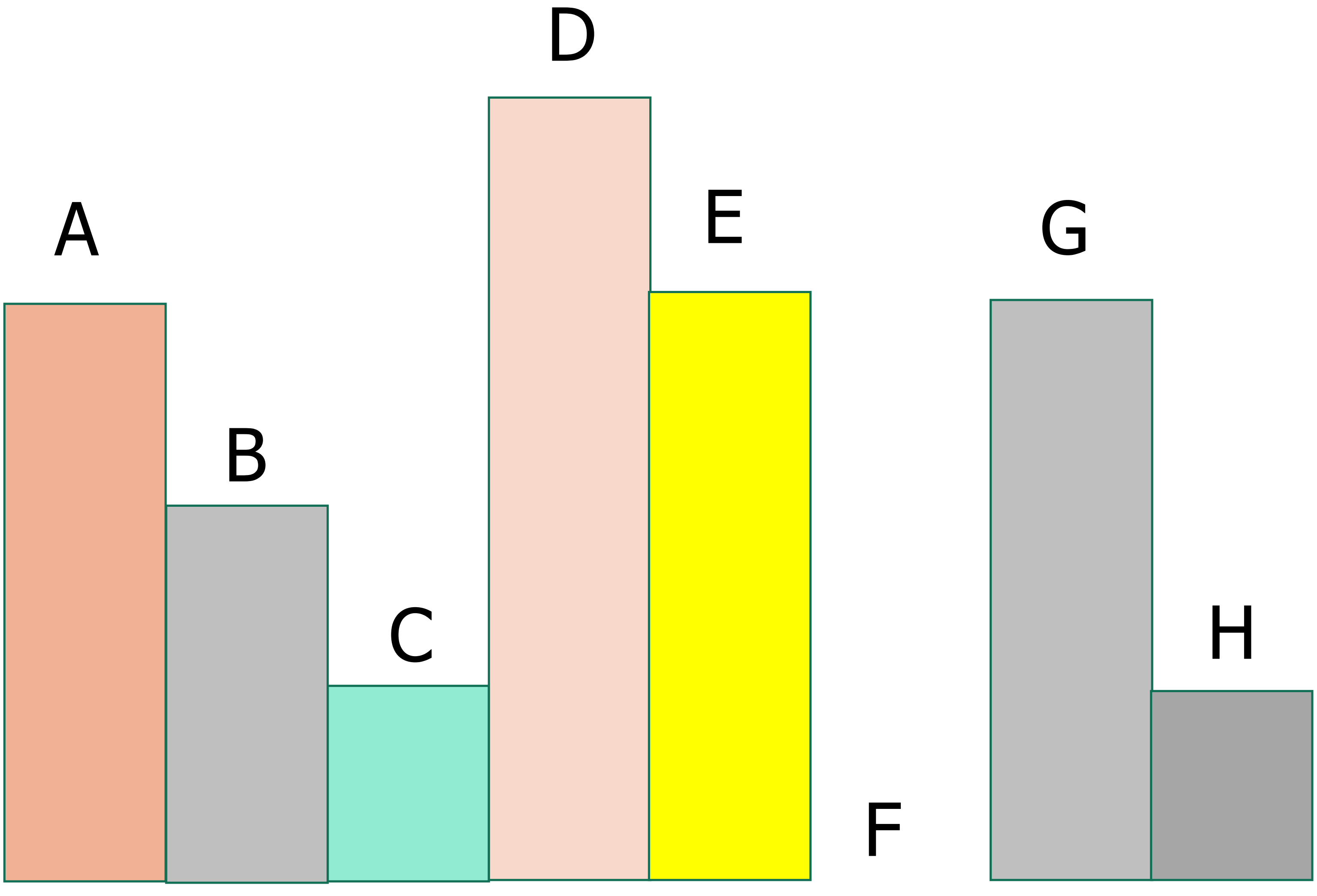
1 2 3 4 5 6



Probability

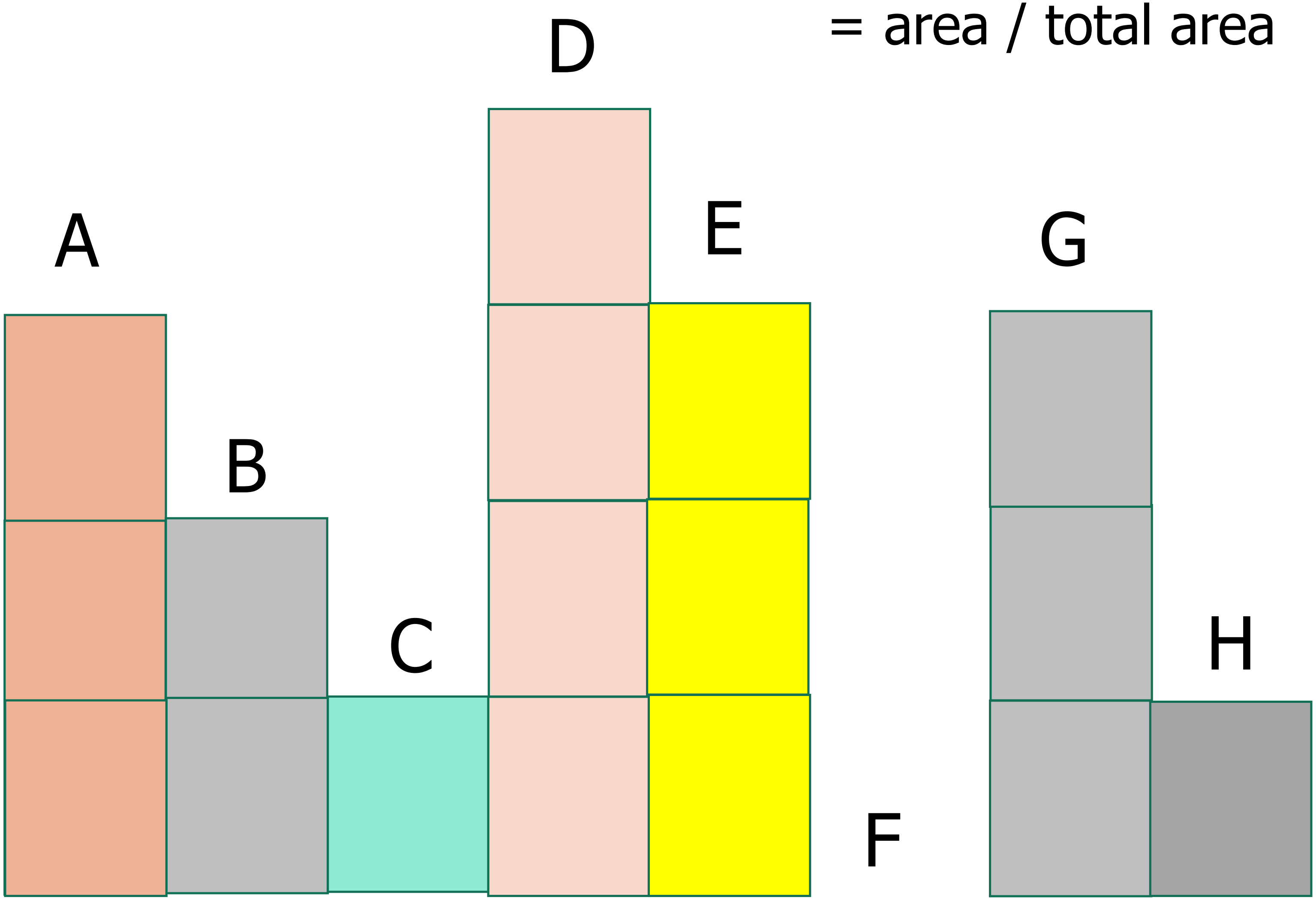


Probability

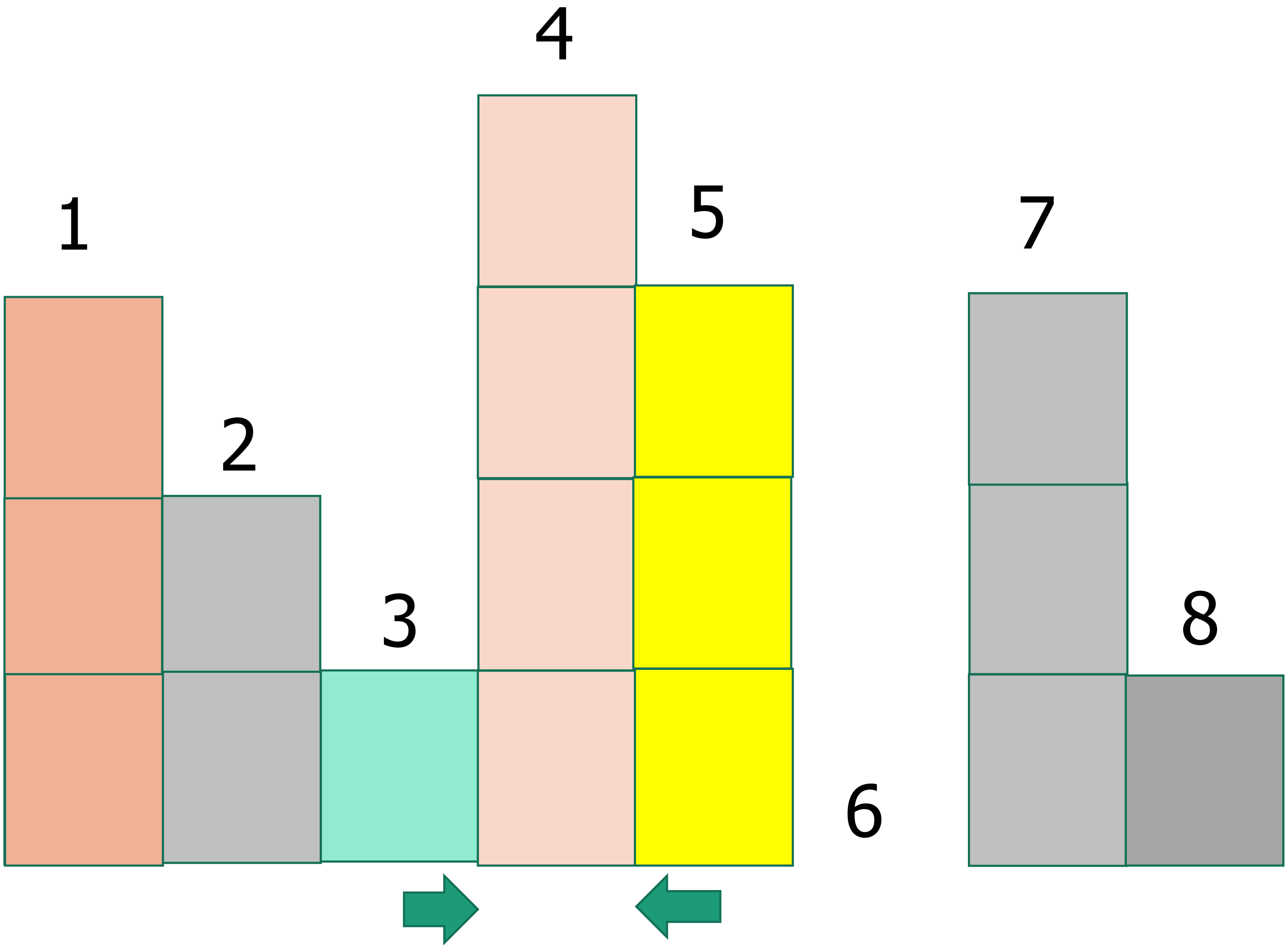


Probability

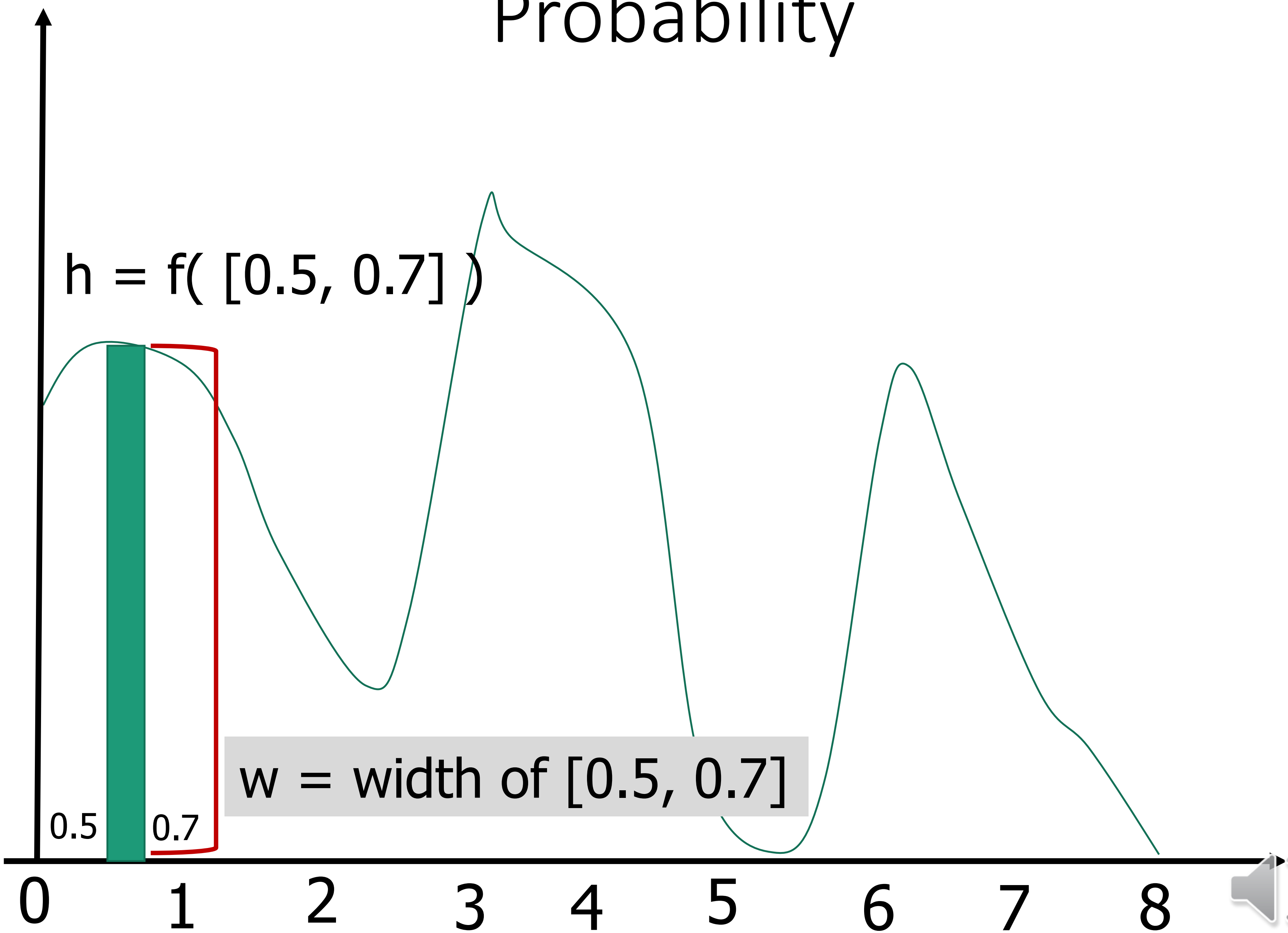
#blocks / #total blocks
= area / total area



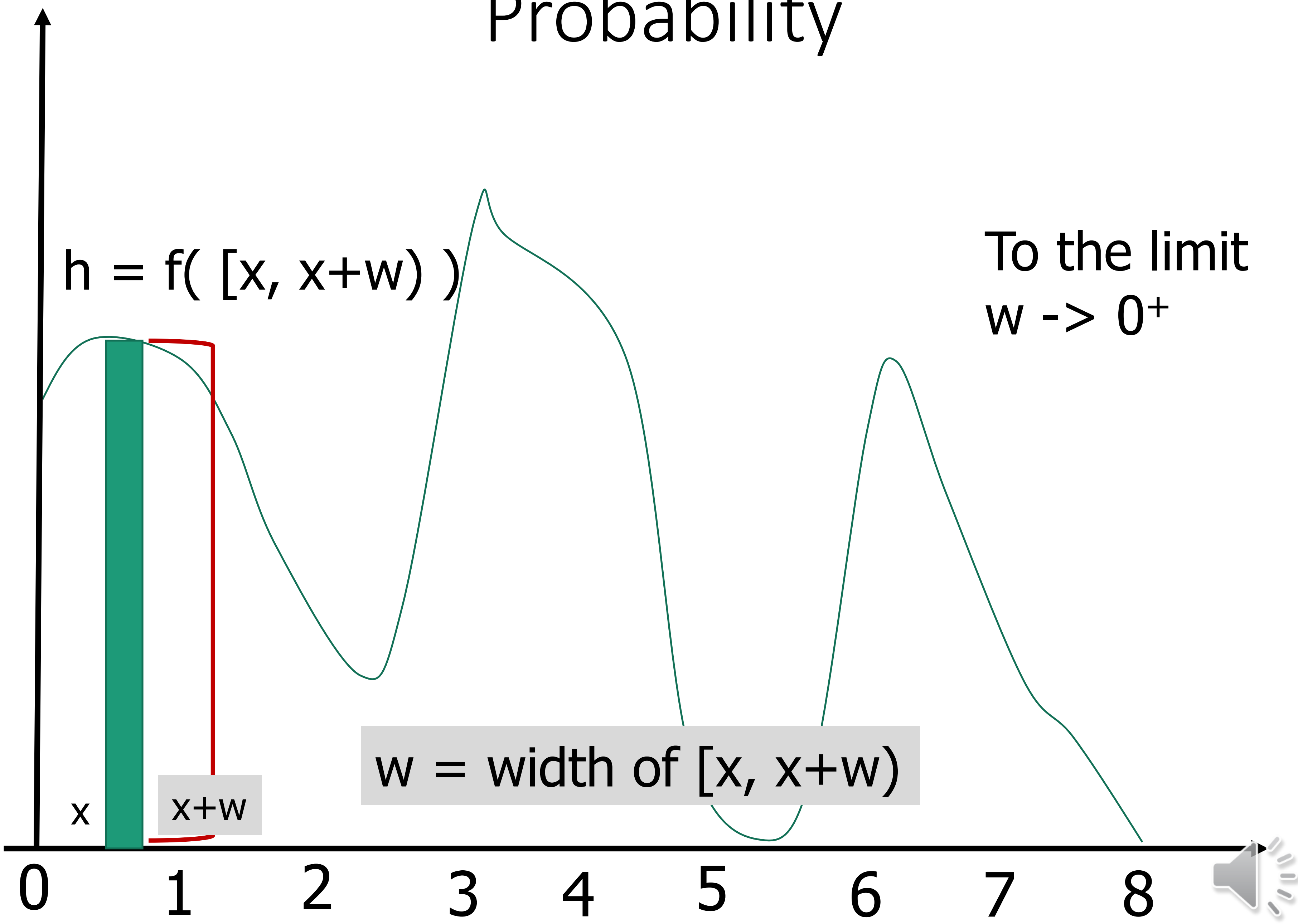
Probability



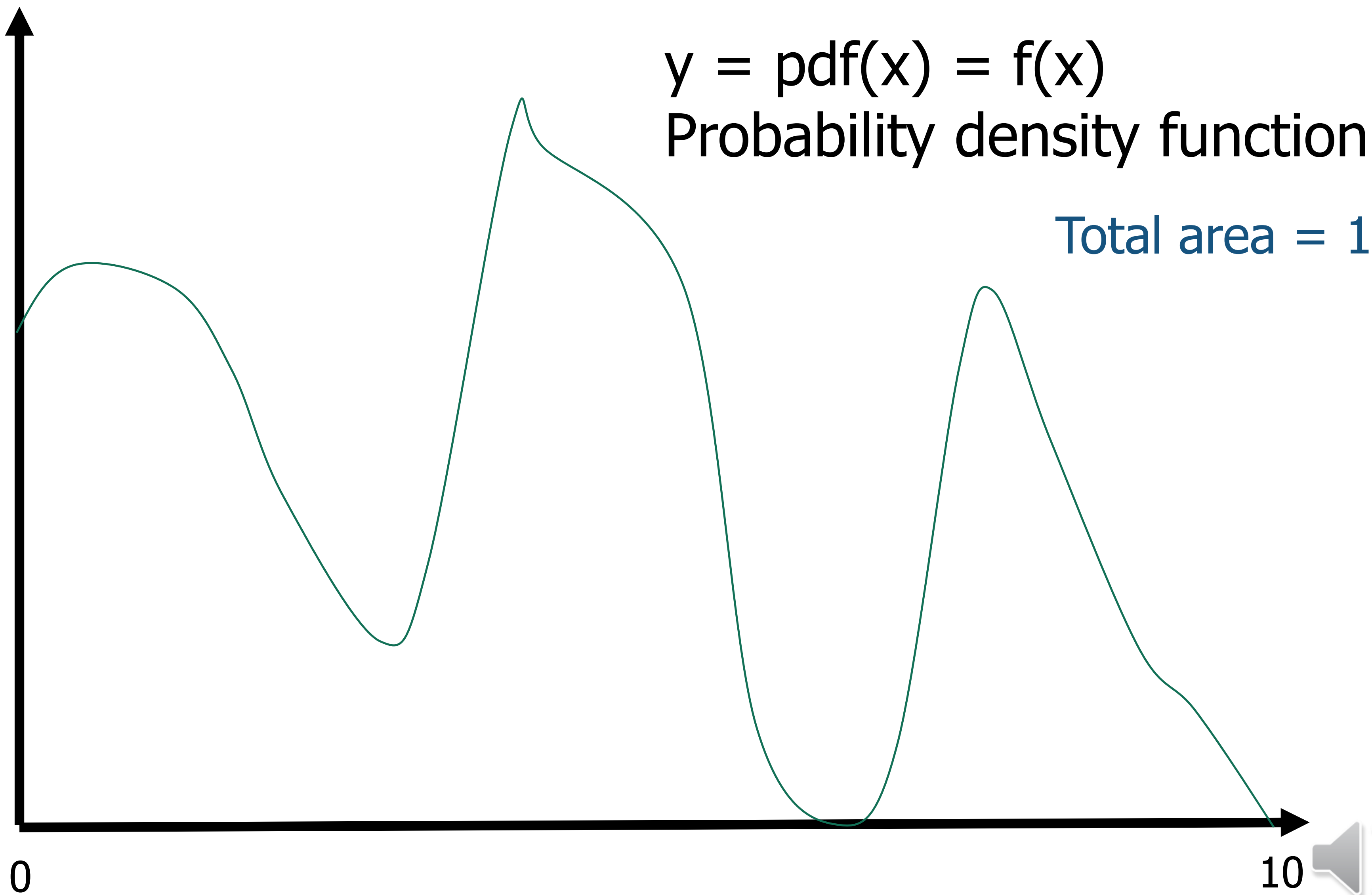
Probability



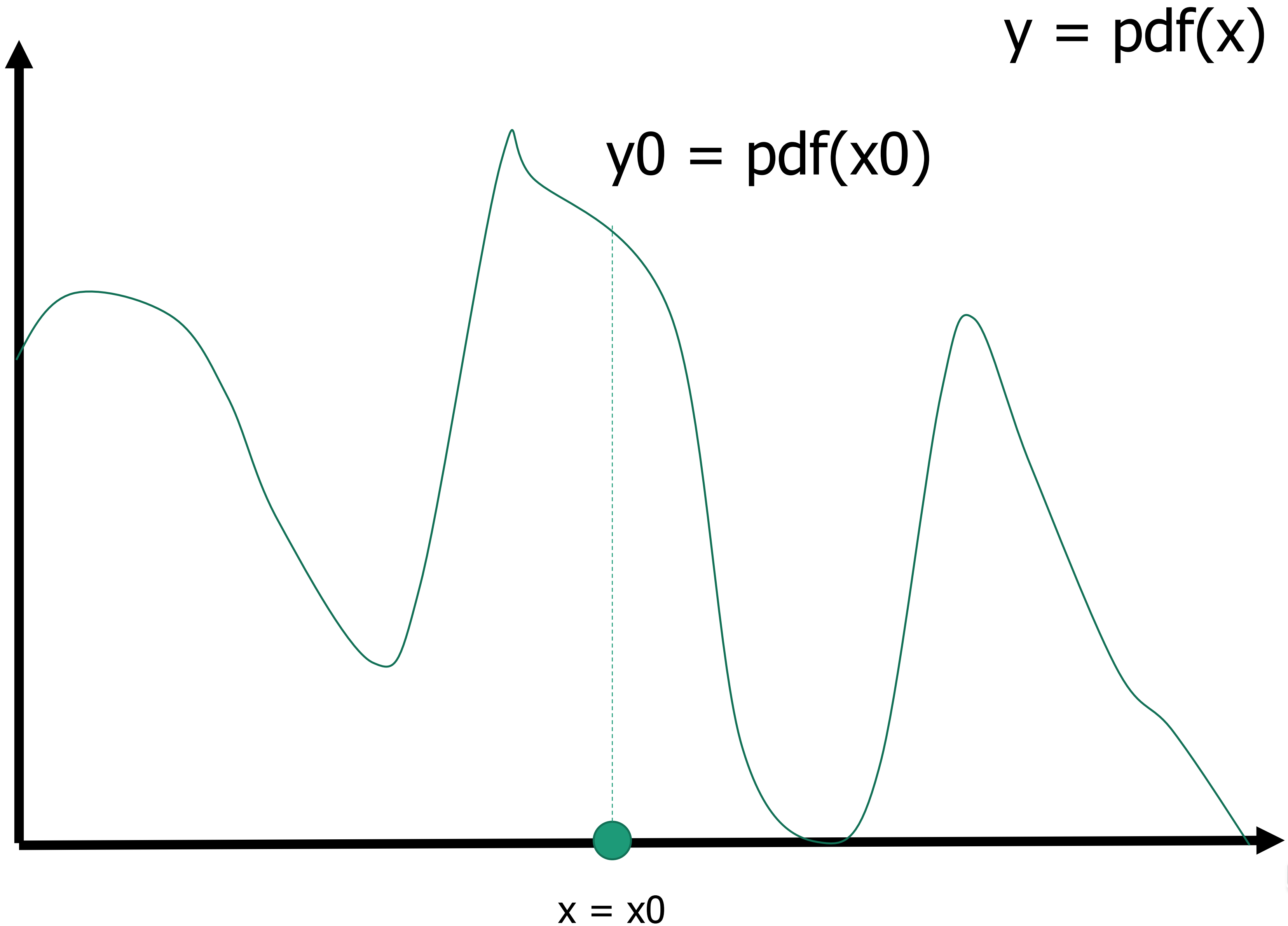
Probability



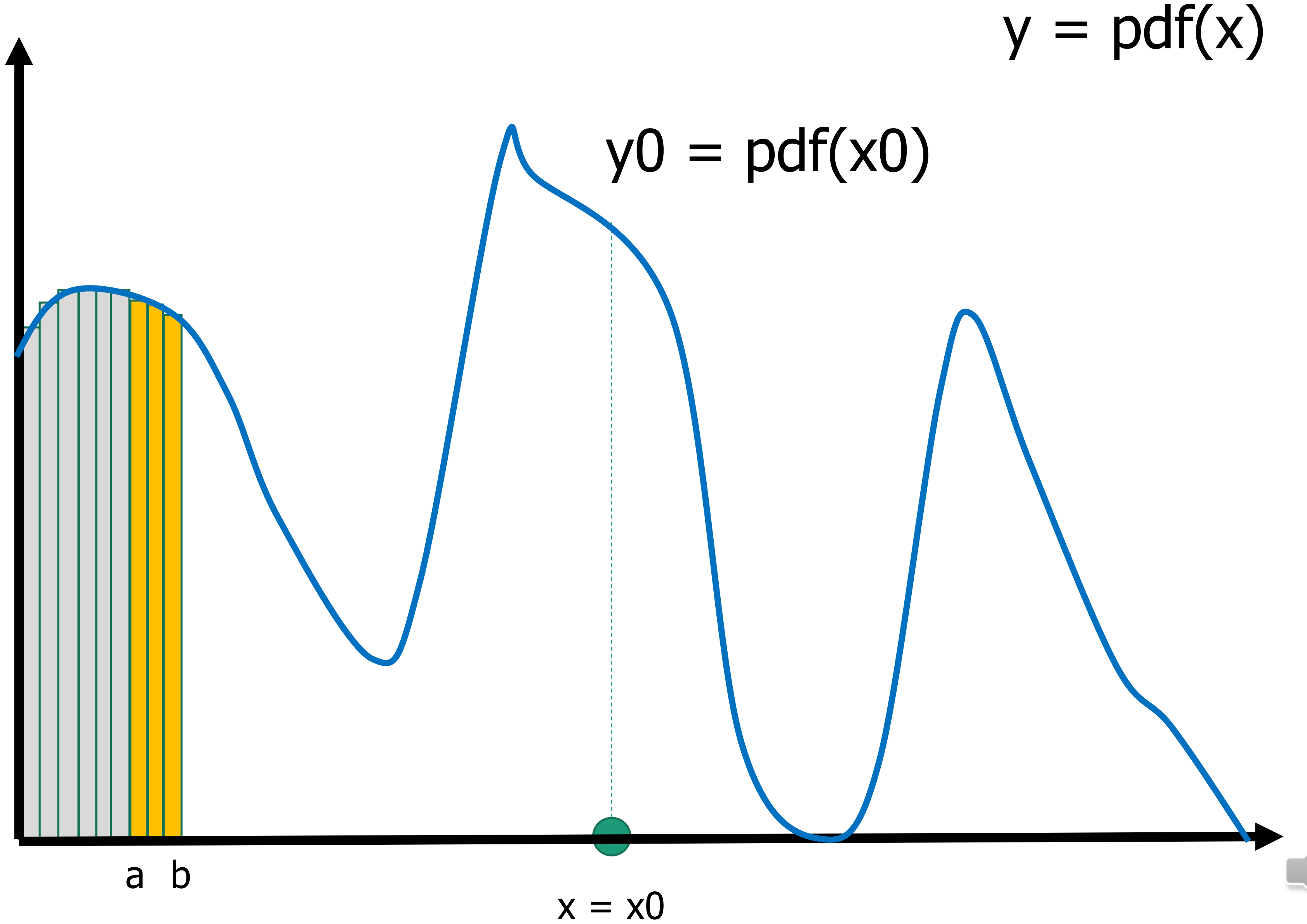
Probability density function



Probability density function

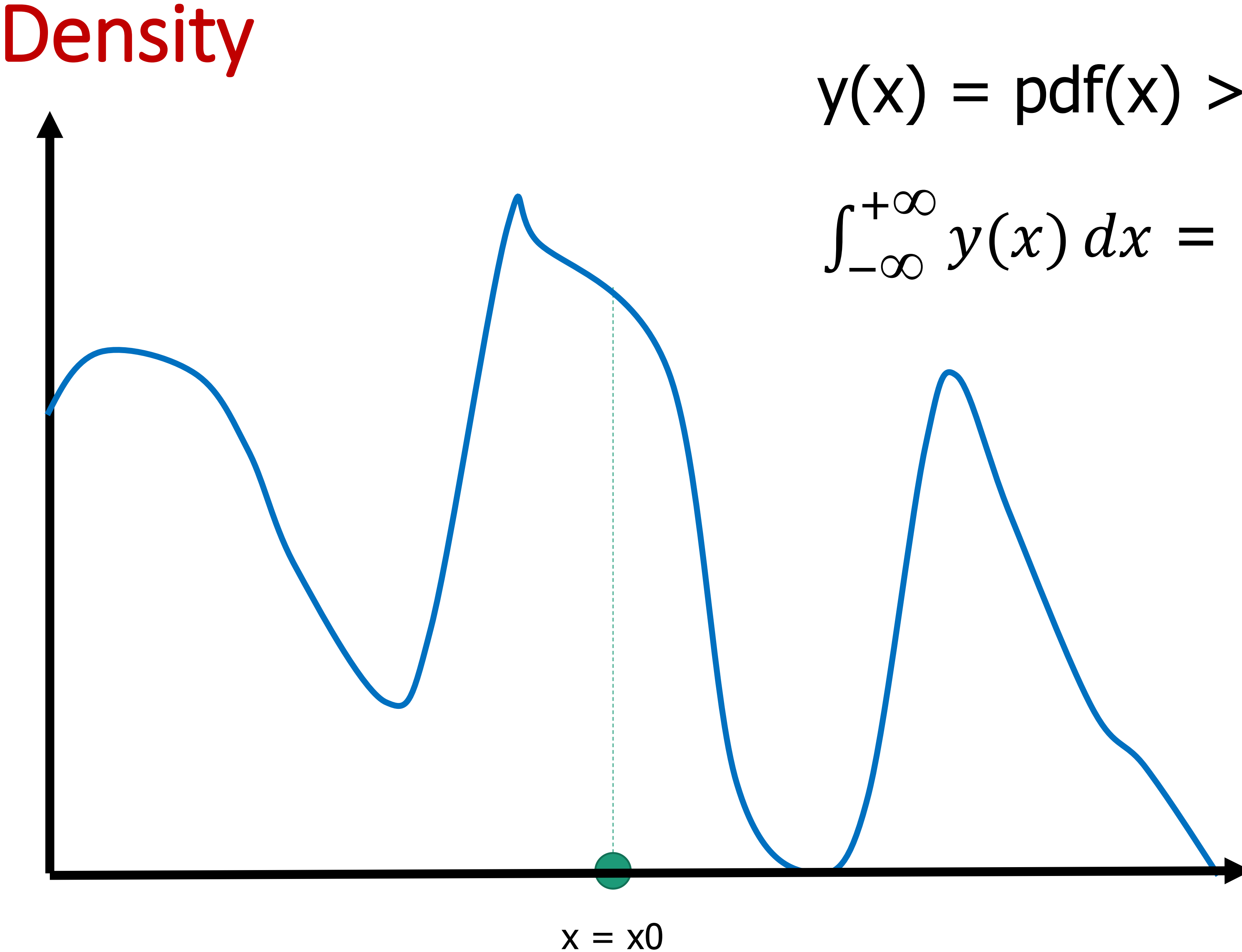


Probability density function



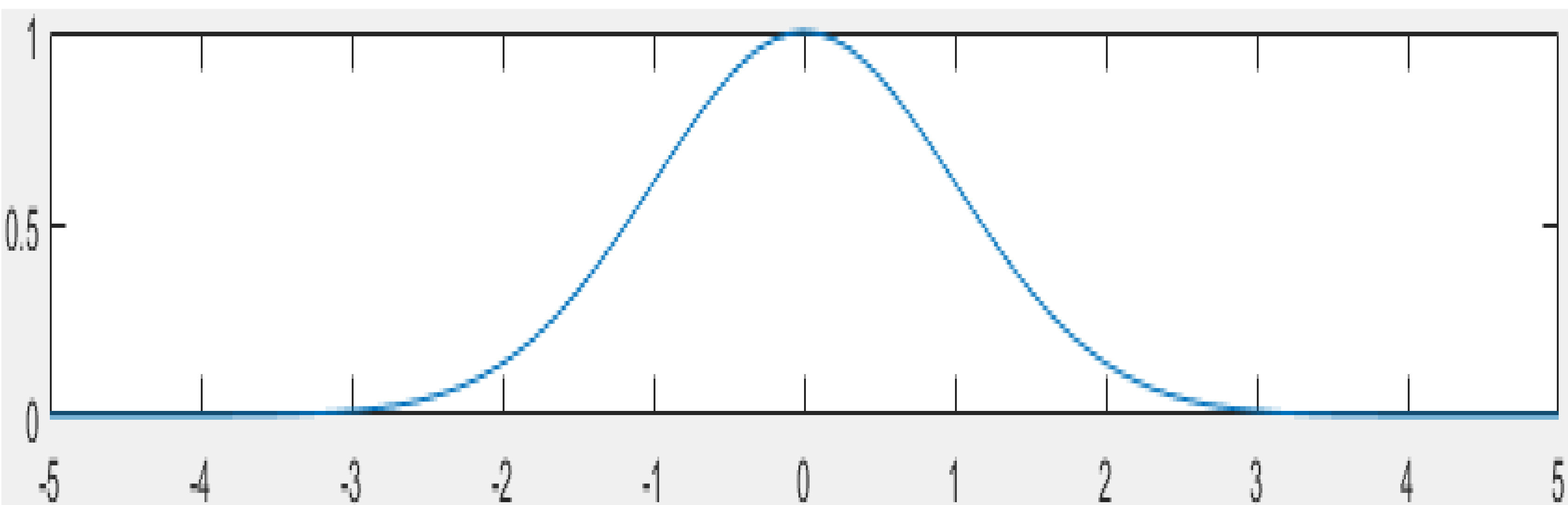
$$y(x) = \mathrm{pdf}(x) >= 0$$

$$\int_{-\infty}^{+\infty} y(x)\, dx = 1$$



Gaussian Probability Density Function

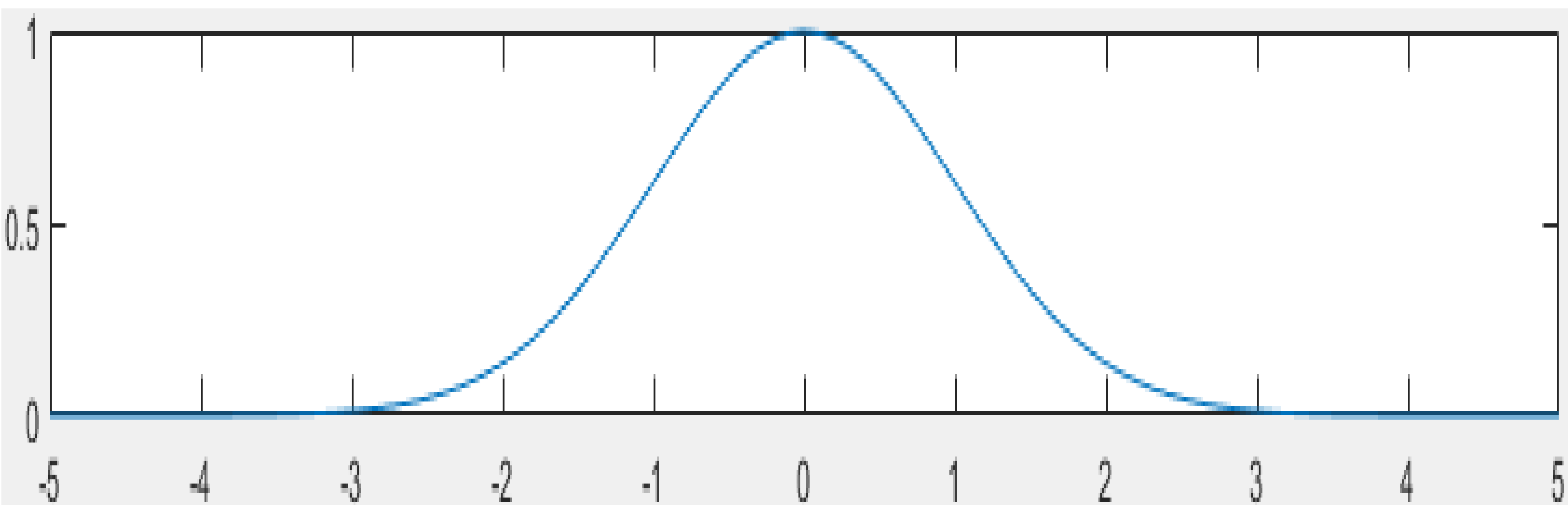
$$f(x; \mu, \sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$



Normal (or Gaussian) Probability Density Function

$$f(x; \underbrace{\mu, \sigma}_{\text{parameters}}) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

μ : Mean
σ : Standard deviation



Generation of random numbers

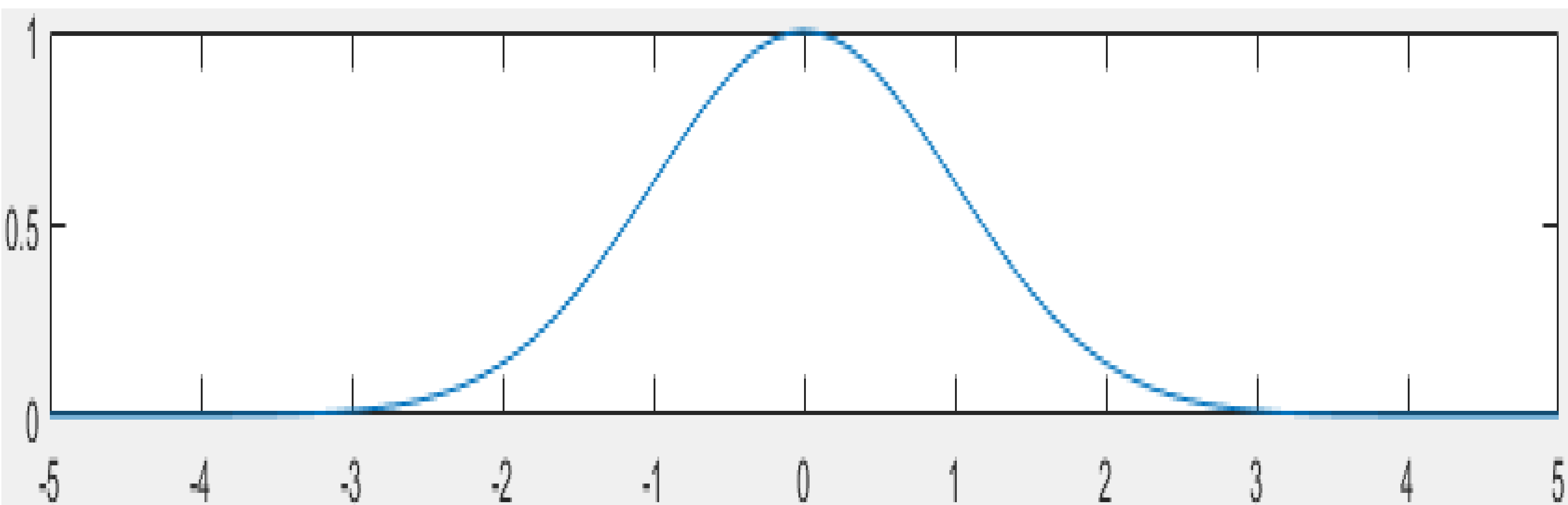
```
rng('default') % for reproducibility
r = normrnd(0,1)

% Save the current state of the random number generator.
s = rng;
r = normrnd(0,1)

%Restore the state of the random number generator to s
rng(s);
r = normrnd(0,1)      % same set of numbers are generated
```

Generation of random numbers

$$f(x;\underbrace{\mu,\sigma}_{\text{parameters}}) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

μ : Mean

σ : Standard deviation



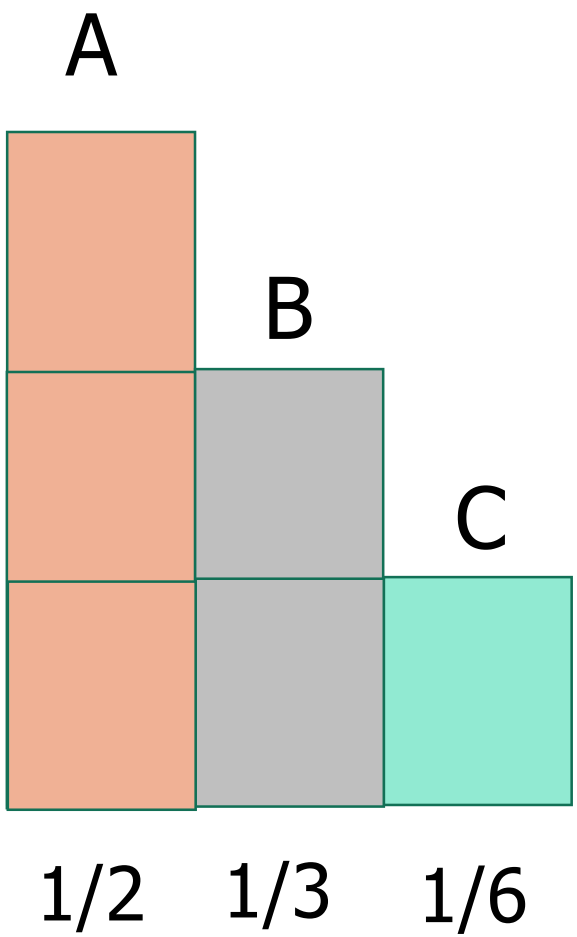
Generation of random numbers

- rand(n) produces an n×n matrix of random numbers from 0 to 1.
- rand(n,m) produces an n×m matrix of random numbers between 0 and 1.

Produce a random number between 0 and 30, do the following:

```
w = 30.*rand(1)
```

Probability

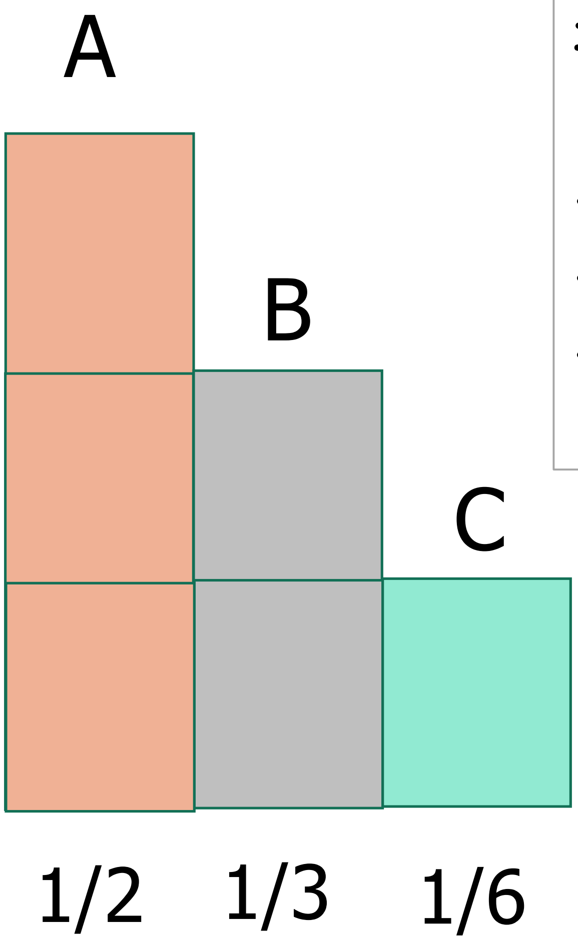


There are three letters which are inside a box.
We pick one letter from the box.

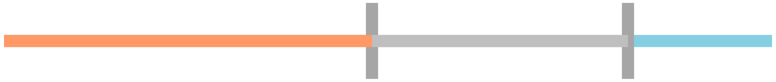
How can we use a uniform random number generator to produce a letter?

The probability that a letter is picked.

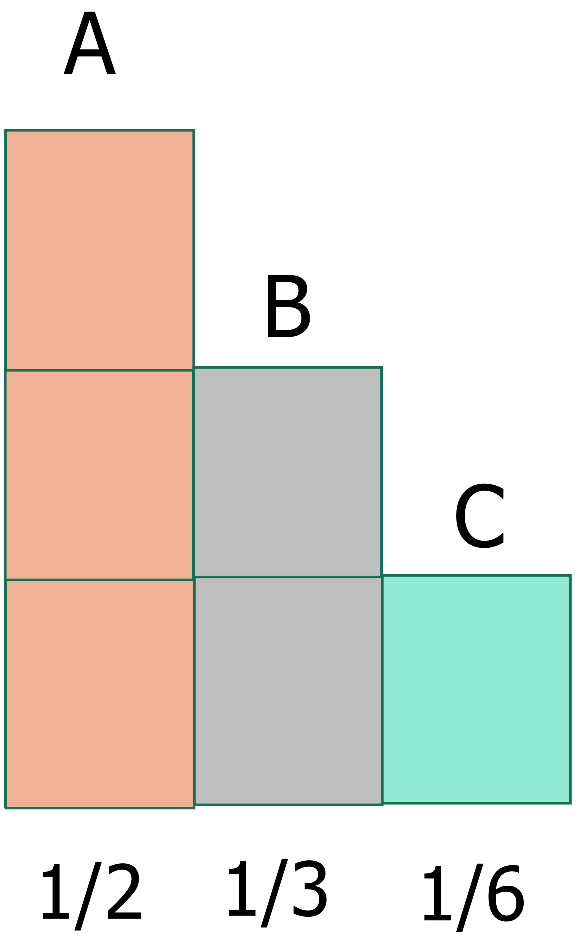
Probability




```
x = rand(1)

if ( x < 1/2) then A
if ( 1/2 <=x < 1/2+1/3) then B
if (1/2+1/3 <= x ) then C
```

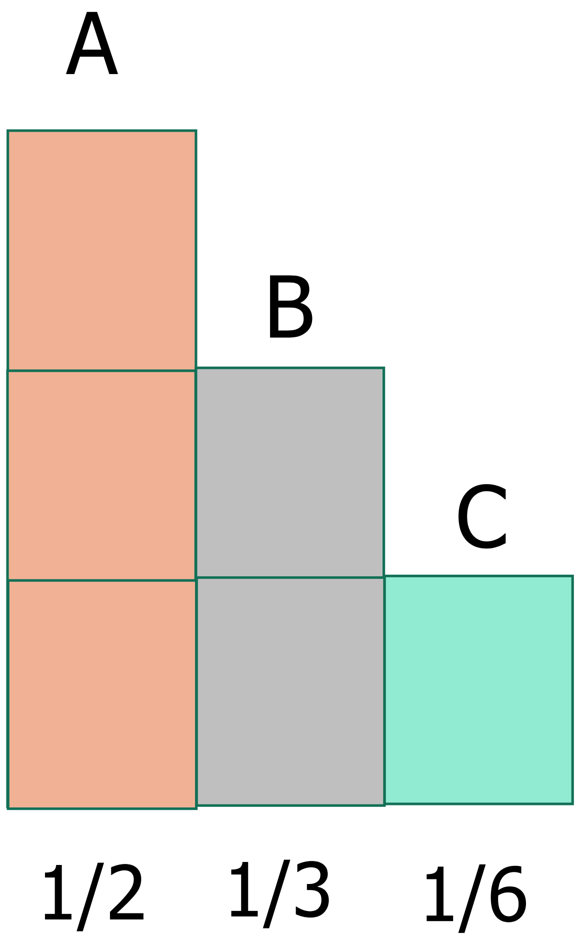


Probability



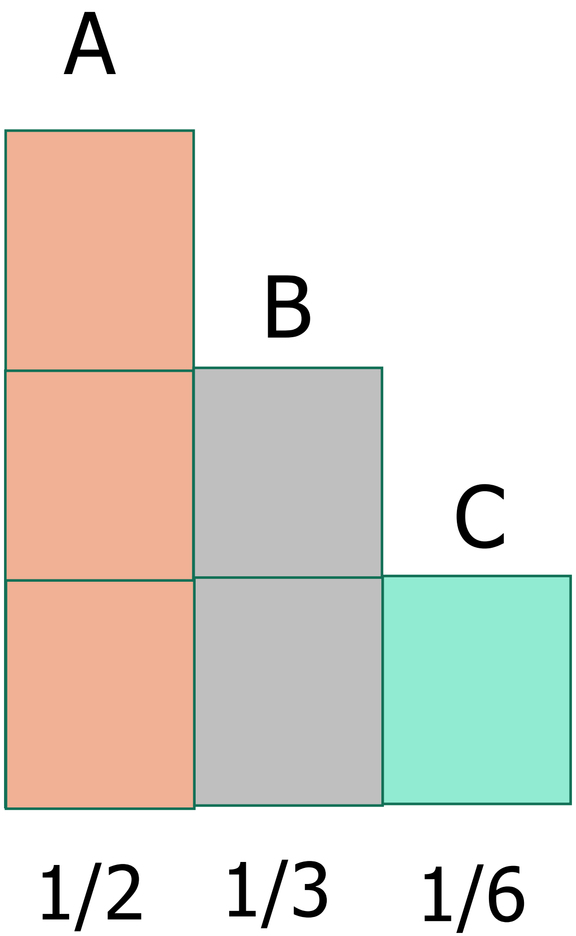

```
x = rand(1)

if ( x < 1/6) then C
if ( 1/6 <=x < 1/6+1/3) then B
if (1/6+1/3 <= x ) then A
```

Probability




```
x = rand(1)

if ( x < 1/6) then C
if ( 1/6 <=x < 1/6+1/3) then B
if (1/6+1/3 <= x ) then A

Cumulative probability:
The value of a random variable
falls within a range.
```

Probability




```
x = rand(1)

if ( x < 1/6) then C
if ( 1/6 <=x < 3/6) then B
if ( 1/2 <= x ) then A
```

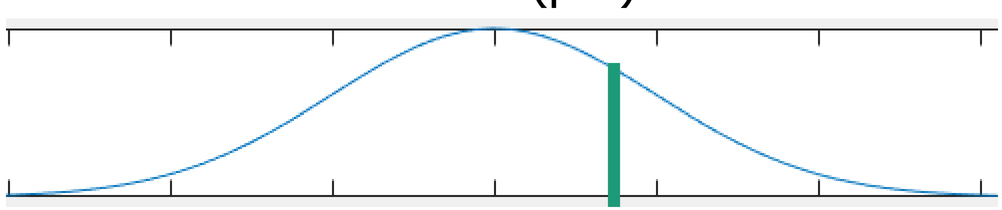
Generation of random numbers

- Normal random numbers

$$f(x;\sigma,c) = e^{\frac{-(x-c)^2}{2\sigma^2}}$$

- r = normrnd(c, sigma)
- Generates a random number from the normal distribution
- c: mean; sigma: standard deviation parameter sigma.

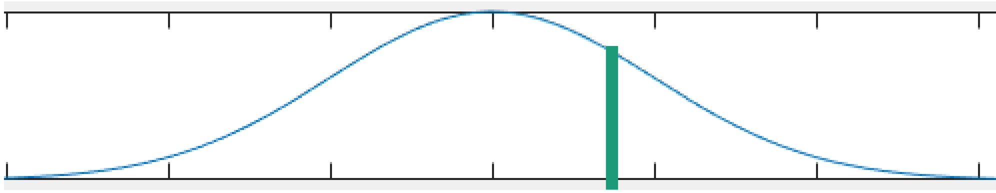
probability density function (pdf)

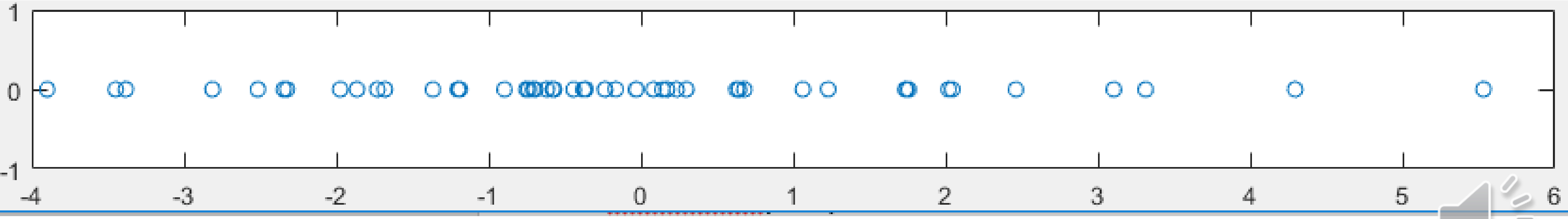


Normal random numbers

```
% create a new 1-by-n vector
% of random numbers
n = 50;
y = zeros(1,n);
c = 0;
sigma = 2;
r  = normrnd( c, sigma, [1,n] );
plot(r,y,'o')
```

probability density function (pdf)

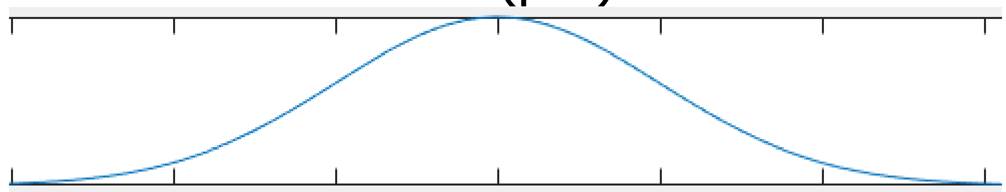


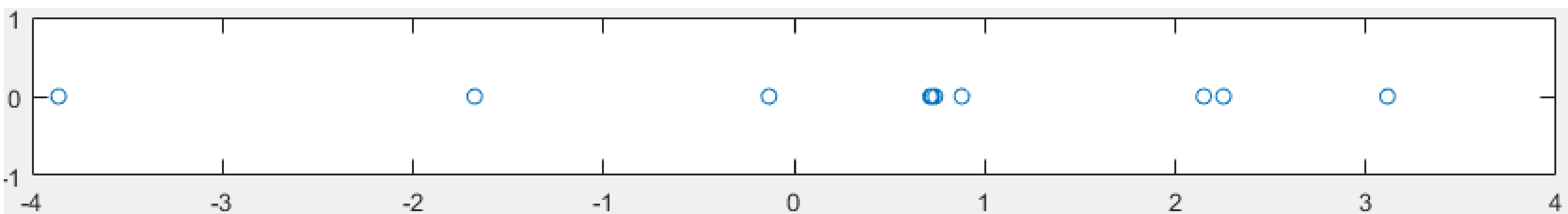


Normal random numbers

```
y = zeros(1,n);
c = 1;    % mean
sigma = 2;
r  = normrnd( c, sigma, [1,n] );
plot(r,y,'o')
```

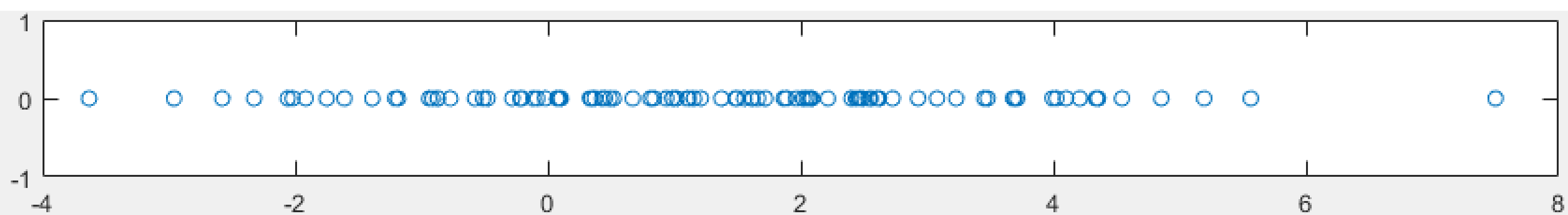
probability density function (pdf)





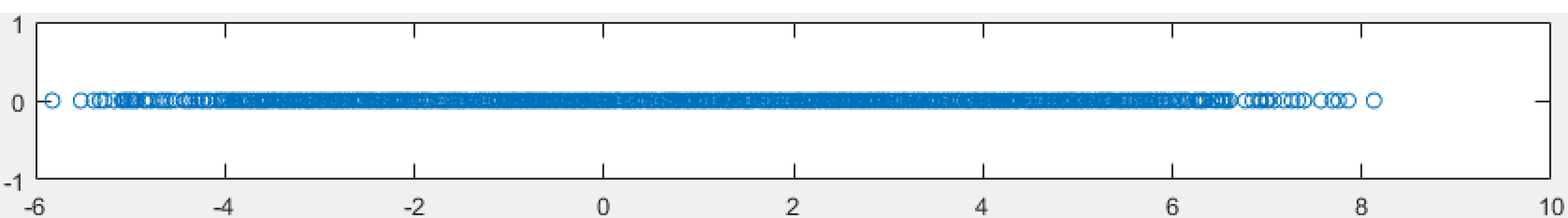
n=10

Mean 0.4906



n=100

Mean 1.2677



n=10000

Mean 1.0196

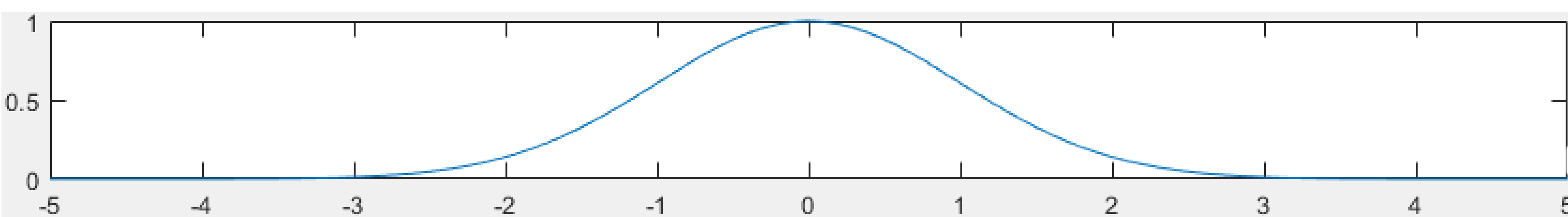
Gaussian curve membership function

gaussmf

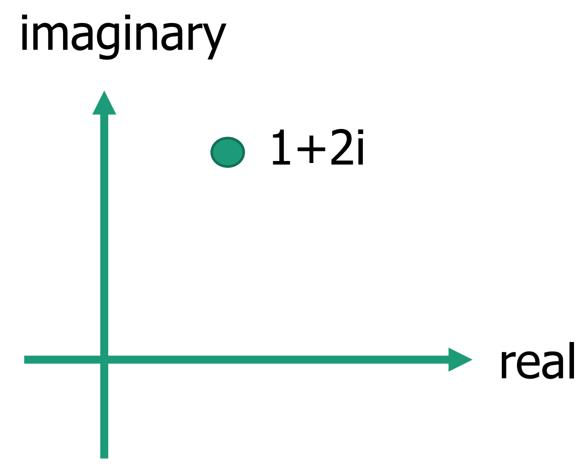
y = gaussmf(x,[sig c])

$$f(x;\sigma,c) = e^{\frac{-(x-c)^2}{2\sigma^2}}$$

```
x = -5:0.1:5
sig = 1; c = 0;
y = gaussmf(x, [sig c]);
plot(x,y)
```



Complex numbers



- Complex numbers: a + b i:
  - a is the real part
  - b is the imaginary part

  where i = sqrt(-1).
- Complex numbers can be assigned in MATLAB on command lines as follows:

```
>> a = 2; b = 3;          % Must assign a and b before
>> c = a+b*i              %  assigning c as shown
>> c = complex(a,b)       % or this other way.
```

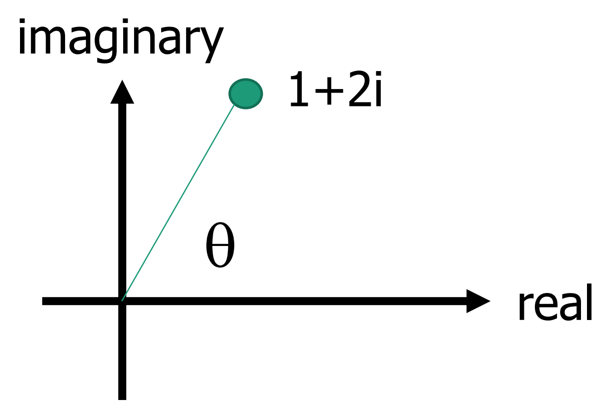
Complex numbers

real(c): real component of a complex number

imag(c): imaginary component of a complex number

abs(c): absolute value or modulus of a complex number. c = a + bi. abs(c) = sqrt(a^2+b^2)

angle(c): angle or argument expressed in radians of a complex number



Other useful functions

- clock: produces an array that tells year, month, day, hour, min, sec.
- date: tells date
- pi: the number pi (3.141592653589…..)
- i: imaginary number % i = sqrt(-1)
- j: imaginary number % j = sqrt(-1)

clock

clock

```
ans =
   1.0e+03 *

    2.0190    0.0030    0.0130    0.0200    0.0440    0.0507
```

clock

clock

ans =

1.0e+03 *

2.0190   0.0030   0.0130   0.0200   0.0440   0.0507

```
clock  Current date and time as date vector.
   C = clock returns a six element date vector containing the current time
   and date in decimal form:

      [year month day hour minute seconds]
```

clock

clock

ans =

1.0e+03 *

2.0190 0.0030 0.0130 0.0200 0.0440 0.0507

[year month day hour minute seconds]

```
clock  Current date and time as date vector.
    C = clock returns a six element date vector containing the current time
    and date in decimal form:

      [year month day hour minute seconds]
```