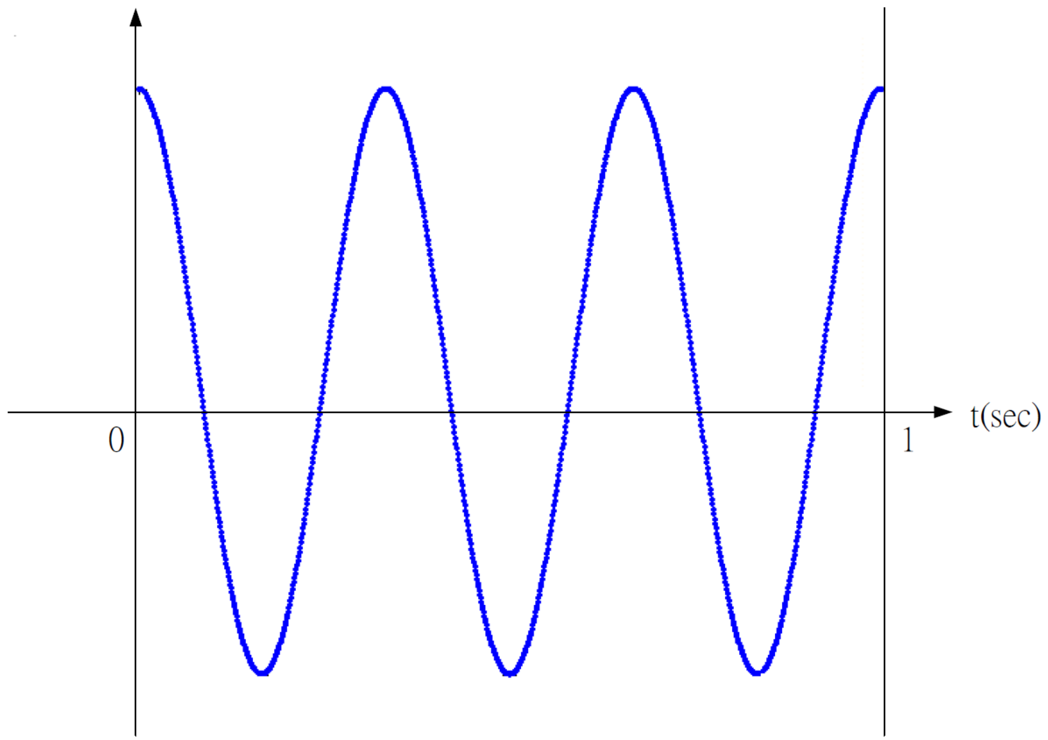
Modulation

Fourier Transform v.s. Communication (1/8)

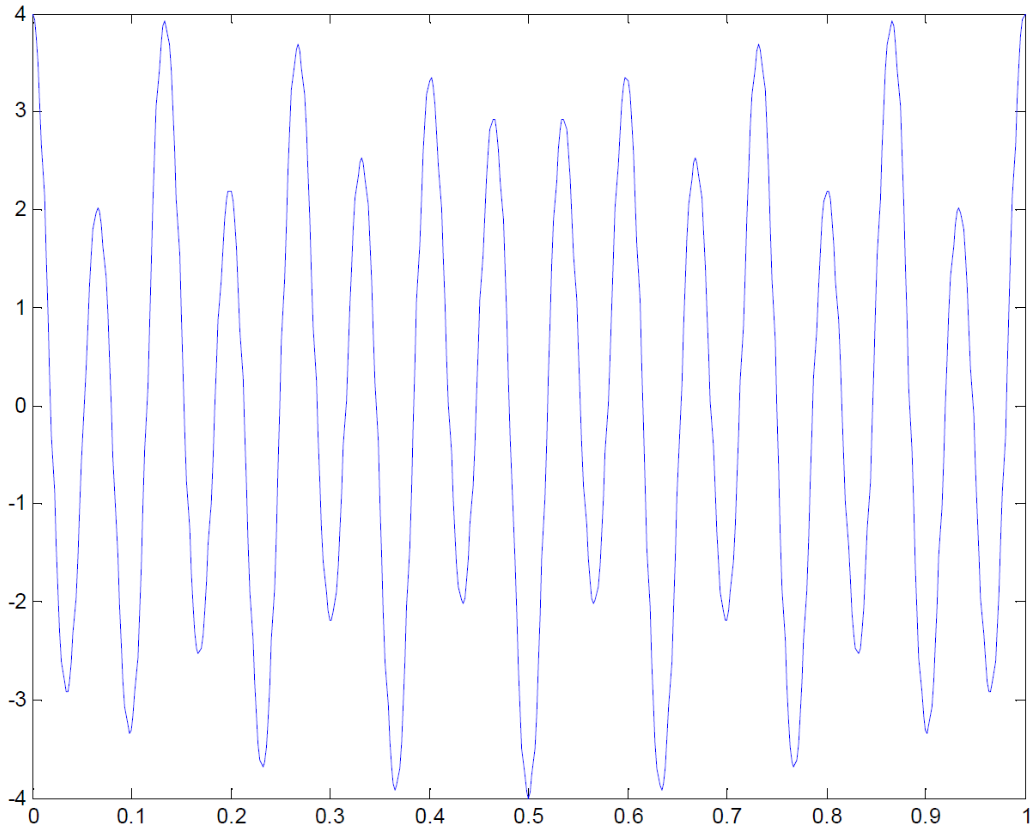
- Look at the following signal, it's cosine with frequency 3



圖一. 頻率爲 3 的 cosine 函數

Fourier Transform v.s. Communication (2/8)

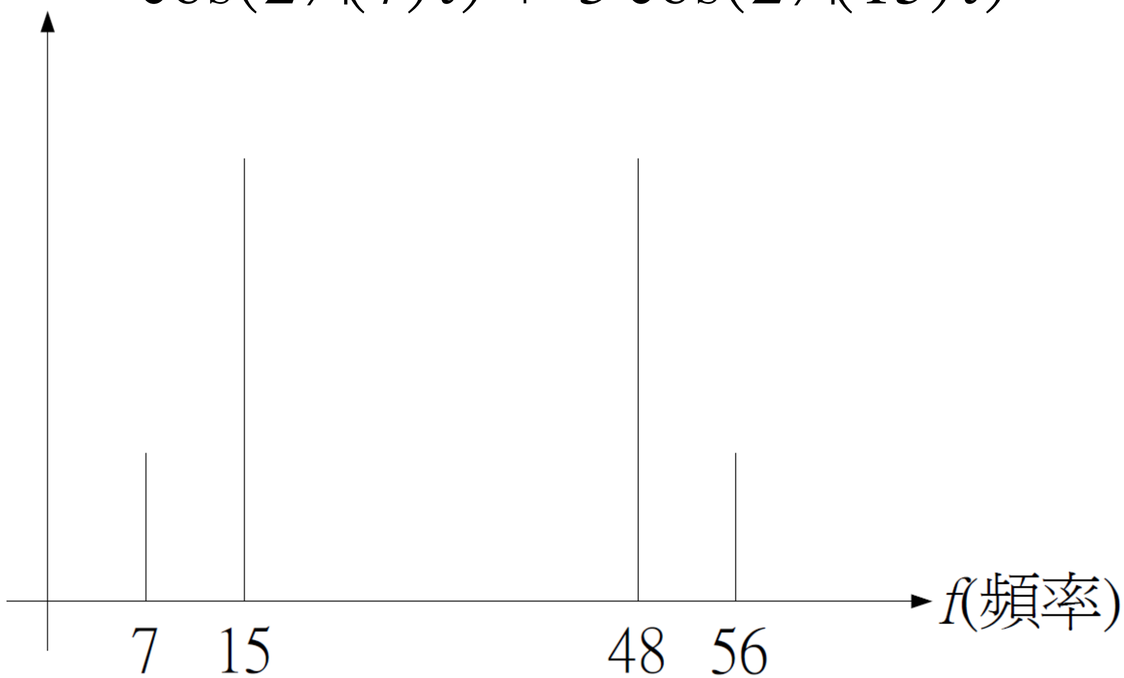
- Now look at another signal, we have no idea what it is



Fourier Transform v.s. Communication (3/8)

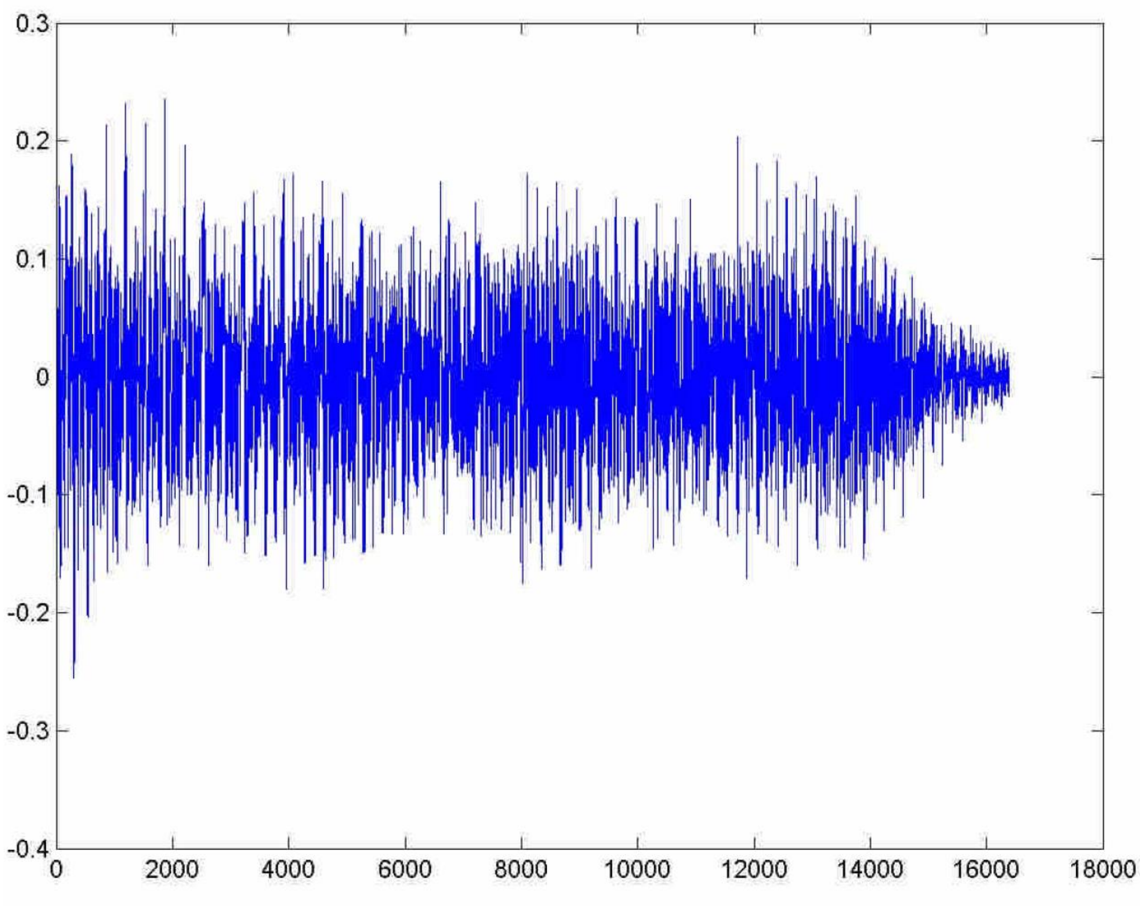
- Using Fourier transform to analyze the signal, we know that the signal is synthesized by two cosine functions

cos(2¼(7)t) + 3 cos(2¼(15)t)



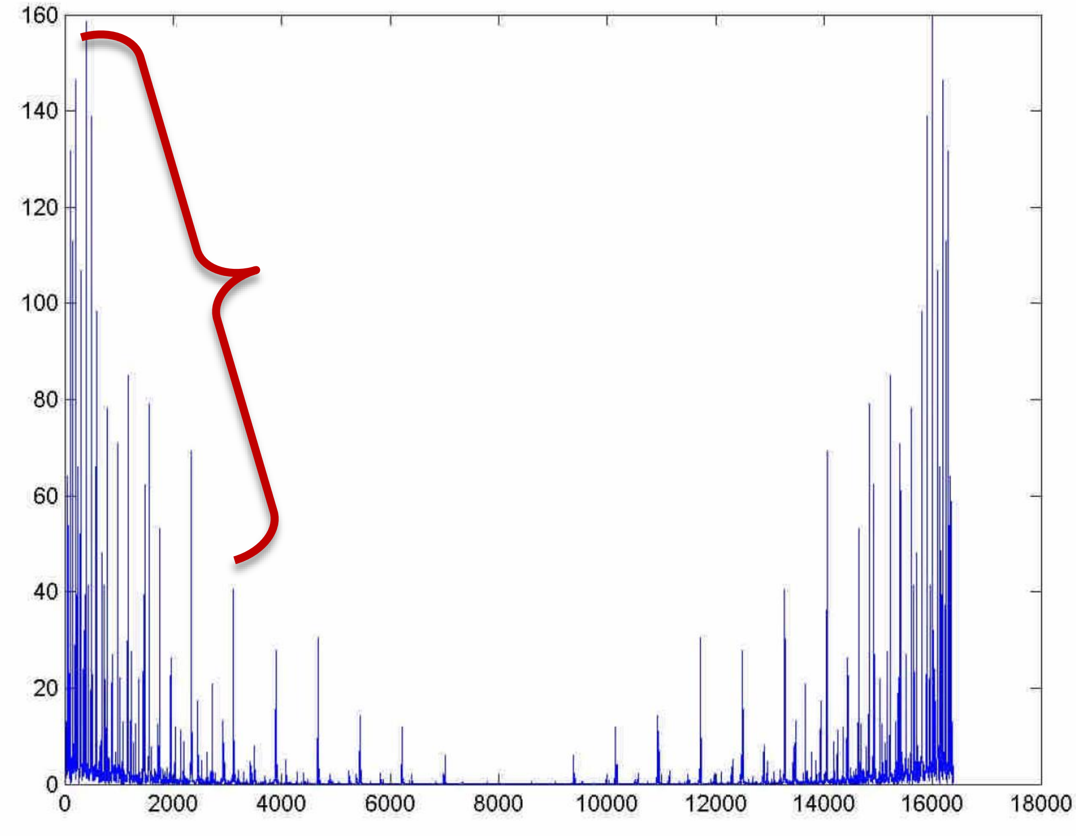
Fourier Transform v.s. Communication (4/8)

- We further analyze 1-sec music signals, as below



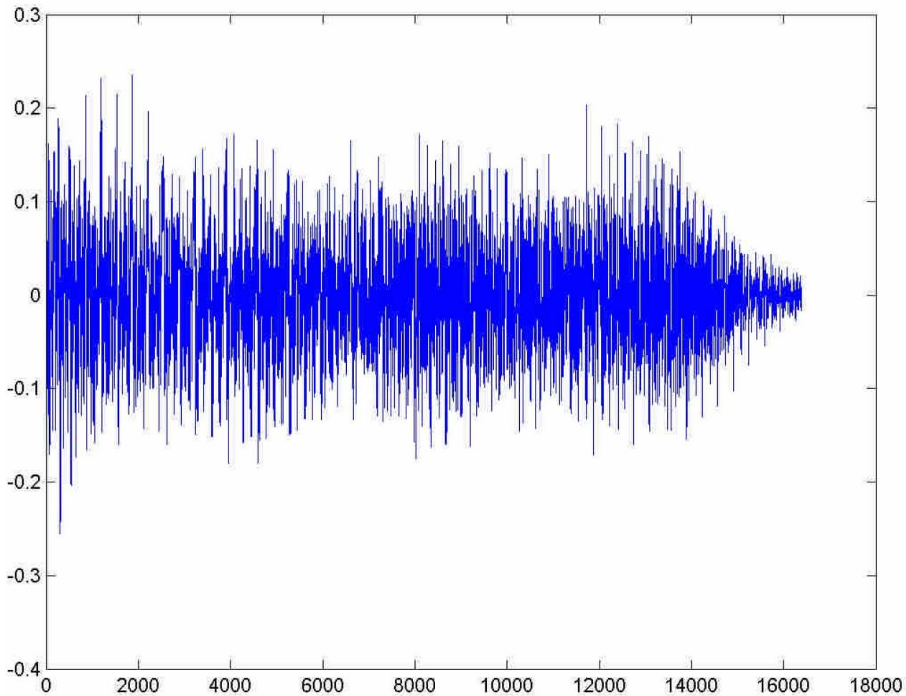
Fourier Transform v.s. Communication (5/8)

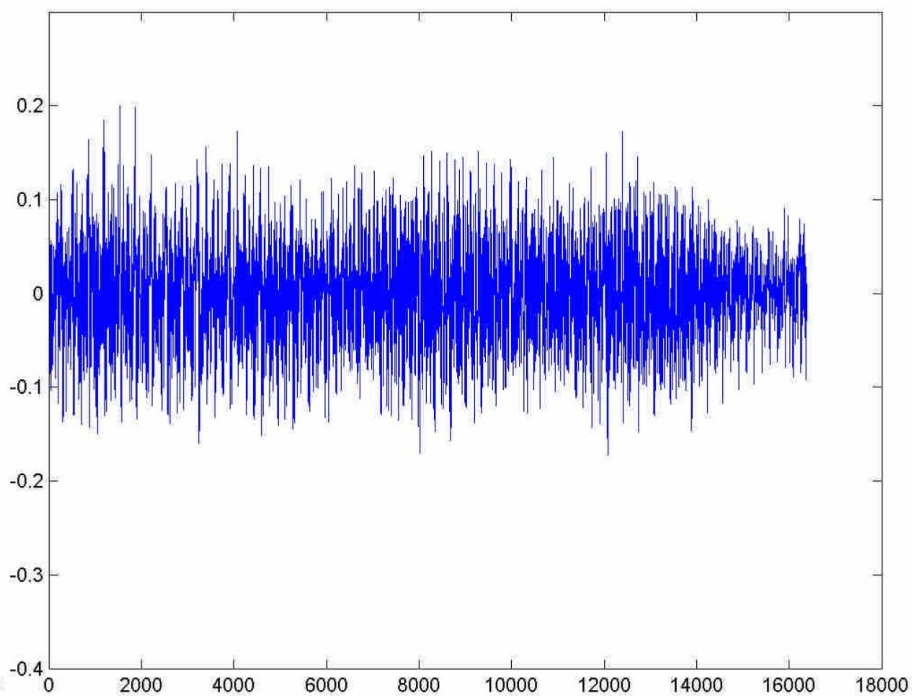
- We analyze the signals through discrete Fourier transform, and found that the voice frequency is below 3000Hz



Fourier Transform v.s. Communication (6/8)

- Using inverse Fourier transform to transform signals from frequency domain to time domain with and without filtering (amplitude < 10)





Fourier Transform v.s. Communication (7/8)

- We know $\lambda = \frac{v}{f}$, if we want to transmit below-3KHz voice signals, wave length is

- The antenna size would be half the wave length, and thus it's 50 km (too ridiculous)
- So we need MODULATION
- How to do that? Multiply with a cosine function

$$s(t) = m(t)\cos(2\pi f_c t)$$

Fourier Transform v.s. Communication (8/8)

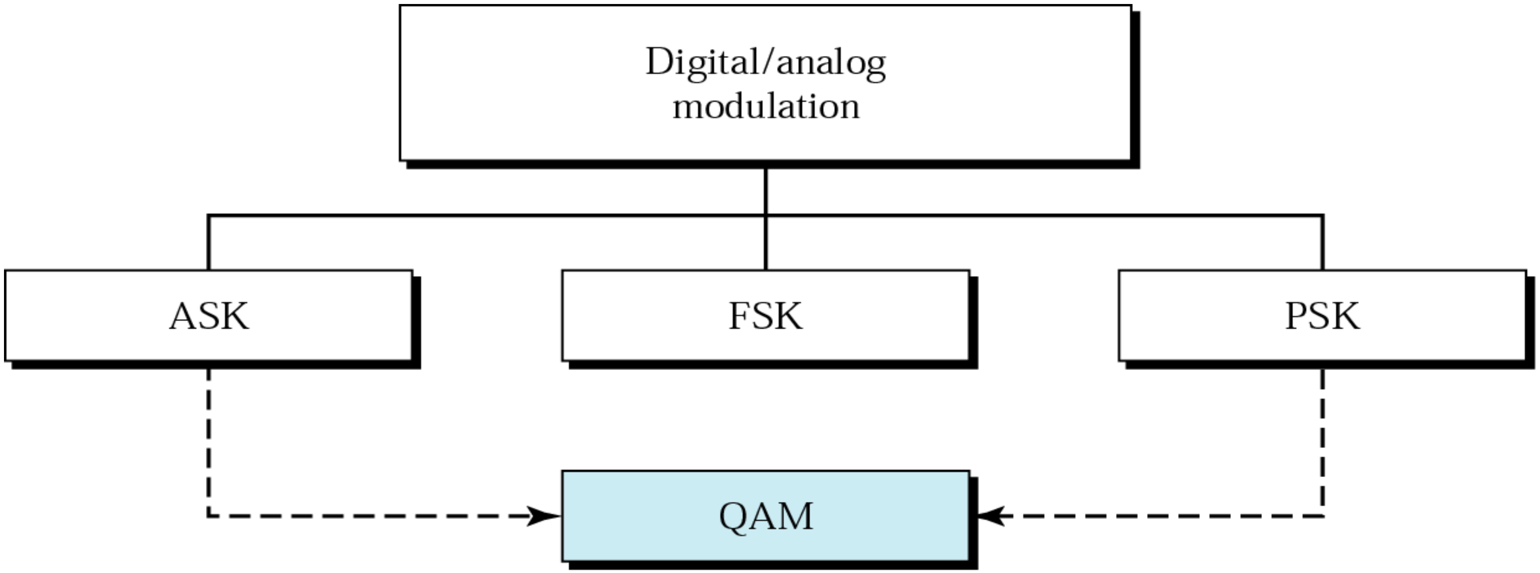
- Why the bandwidth is doubled to transit the modulated signals?
  - Assume the original signal is $m(t) = \cos(2\pi f t)$
  - The modulated signal is $s(t) = \cos(2\pi f t)\cos(2\pi f_c t)$

$$\cos(2\pi f t)\cos(2\pi f_c t) = \frac{1}{2}[(\cos(2\pi(f_c + f)t) + (\cos(2\pi(f_c - f)t)]$$

  - The frequency is modified from f to f+f_c and f-f_c

Modulation Techniques

- Analog modulation
  - AM, FM, PM
- Digital modulation
  - ASK, FSK, PSK, QPSK, QAM



AM (Amplitude Modulation) (1/2)

- The amplitude of a carrier signal with a constant frequency is as varied as the information signal
- The power of the transmitted wave varies in amplitude in accordance with the power of the modulating signal
- Mathematical representation
  - Carrier signal: $A\cos(2\pi f_c t)$
  - Modulating signal: $x(t)$
  - Modulated signal:

    $s(t) = [A + x(t)]\cos(2\pi f_c t)$

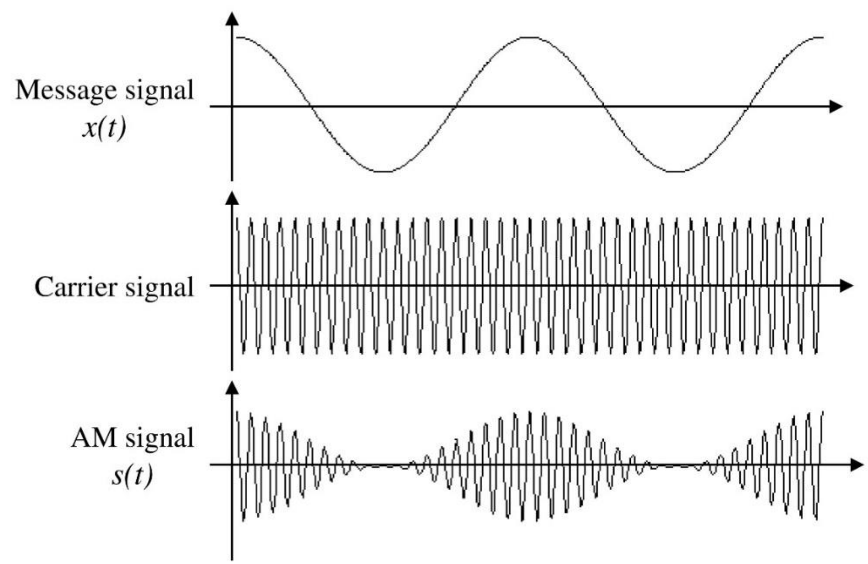


Figure 7.25 Amplitude modulation.

AM (Amplitude Modulation) (2/2)

- Bandwidth of an AM scheme is twice that of the modulating signal (double sideband nature)
- The receiver filters the carrier signal, and rebuild the information
- Since up to 1/3 of the overall signal power is contained in the sidebands, and 2/3 of the signal power is contained in the carrier, AM is an inefficient scheme

FM (Frequency Modulation) (1/2)

- Modulated signal:
  - : peak frequency deviation,
  - Modulation index is defined as [redacted]
  - : max used modulating frequency

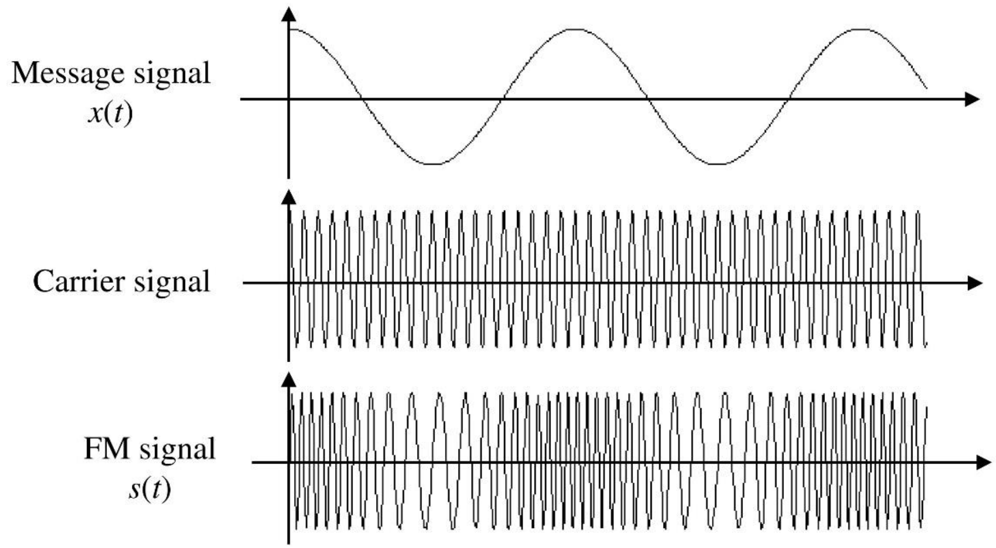


Figure 7.26 Frequency modulation.

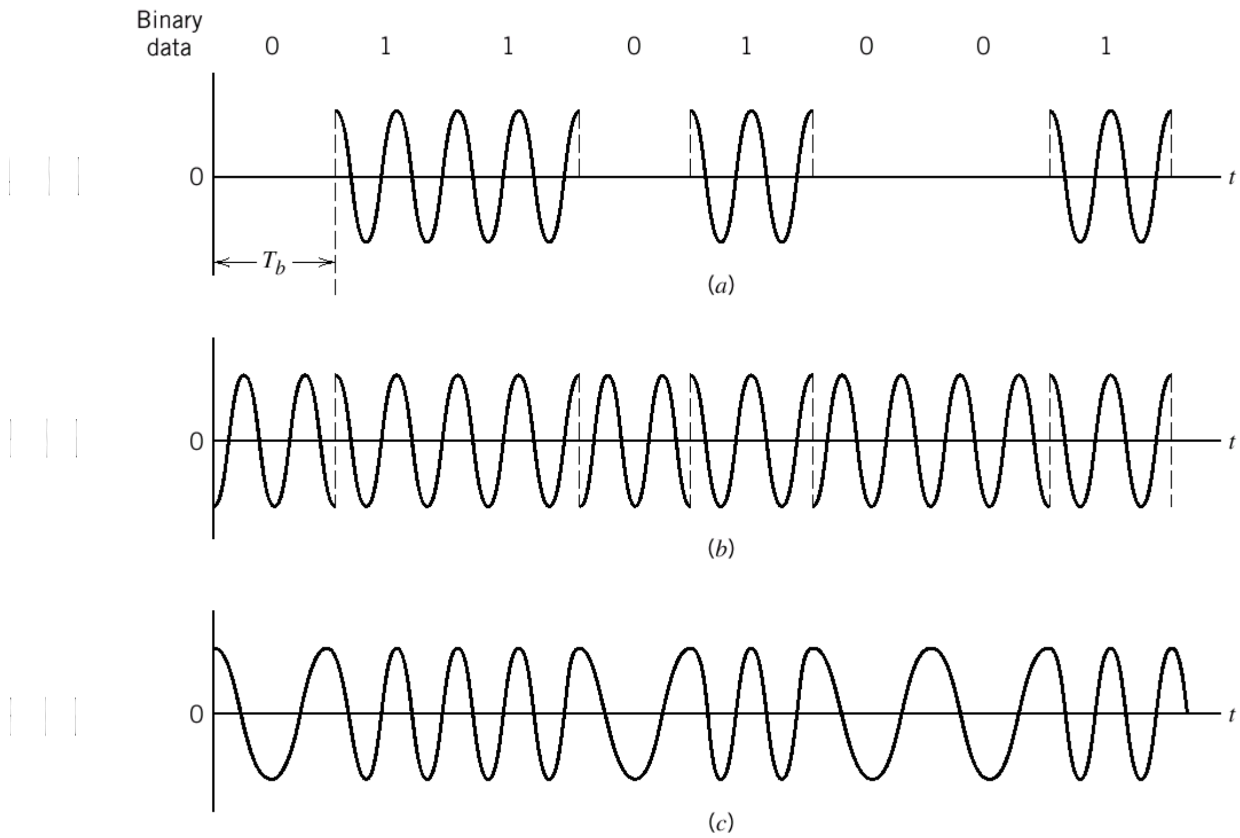
FM (Frequency Modulation) (2/2)

- The carrier frequency varies between the extremes of &
- In FM, the total wave power does not change when the frequency alters
- The recover the signal, the receiver rebuilds the information wave by checking how the known carrier signal has modified the information
- An FM system provides a better SNR than an AM system
- Another advantage of FM is that it needs less radiated power
- However, it does require a larger bandwidth than AM
- The bandwidth of a FM signal may be determined using

$$BW = 2(\beta + 1)f_m$$

Digital Signal Modulation

- Amplitude shift keying (ASK), frequency shift keying (FSK), phase shift keying (PSK)



Carrier signal: $\cos(2\pi f_c t)$

QPSK (Quadrature Phase Shift Keying)

- BPSK: carrying 1 bit, and thus 0 or 1
- QPSK: carrying 2 bits, and thus 00, 01, 10, or 11

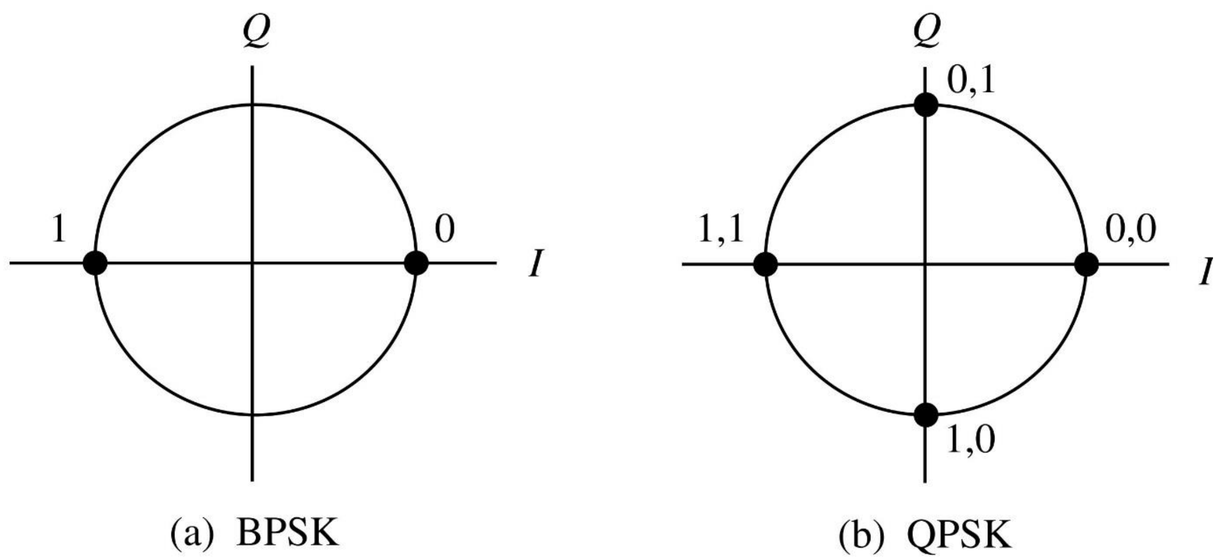


Figure 7.29 Signal constellations of BPSK and QPSK.

QAM (Quadrature Amplitude Modulation)

- ASK+PSK
- For example, a 16QAM uses 12 phases and 3 amplitudes to represent 4-bit symbol

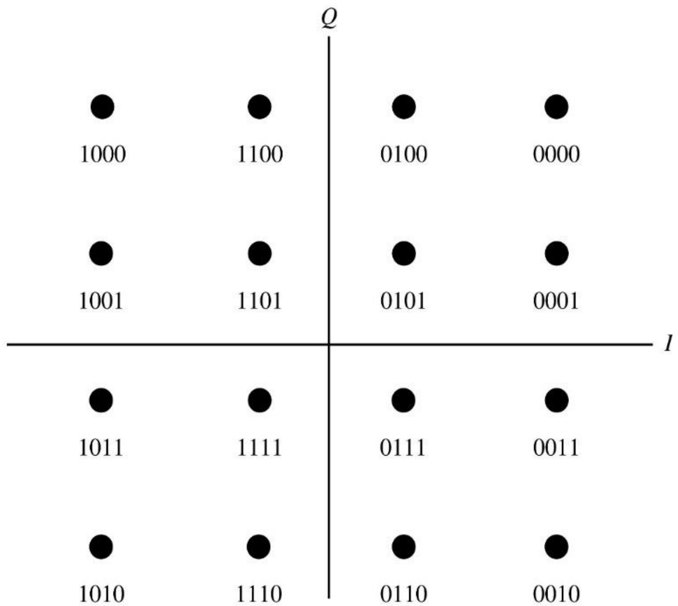


Figure 7.31 Rectangular constellation of 16QAM.

Table 7.2: ▶ A Representative QAM Table

Bit sequence represented	Amplitude	Phase shift
000	1	0
001	2	0
010	1	$\pi/2$
011	2	$\pi/2$
100	1	π
101	2	π
110	1	$3\pi/2$
111	2	$3\pi/2$

MCS Mode Table of IEEE 802.11n & 11ac

MCS Index - 802.11n and 802.11ac

802.11n 802.11ac

HT MCS Index	VHT MCS Index	Spatial Streams	Modulation	Coding	20MHz Data Rate No SGI	20MHz Data Rate SGI	40MHz Data Rate No SGI	40MHz Data Rate SGI	80MHz Data Rate No SGI	80MHz Data Rate SGI	160MHz Data Rate No SGI	160MHz Data Rate SGI
0	0	1	BPSK	1/2	6.5	7.2	13.5	15	29.3	32.5	58.5	65
1	1	1	QPSK	1/2	13	14.4	27	30	58.5	65	117	130
2	2	1	QPSK	3/4	19.5	21.7	40.5	45	87.8	97.5	175.5	195
3	3	1	16-QAM	1/2	26	28.9	54	60	117	130	234	260
4	4	1	16-QAM	3/4	39	43.3	81	90	175.5	195	351	390
5	5	1	64-QAM	2/3	52	57.8	108	120	234	260	468	520
6	6	1	64-QAM	3/4	58.5	65	121.5	135	263.3	292.5	526.5	585
7	7	1	64-QAM	5/6	65	72.2	135	**150**	292.5	325	585	650
	8	1	256-QAM	3/4	78	86.7	162	180	351	390	702	780
	9	1	256-QAM	5/6	n/a	n/a	180	**200**	390	**433.3**	780	866.7
8	0	2	BPSK	1/2	13	14.4	27	30	58.5	65	117	130
9	1	2	QPSK	1/2	26	28.9	54	60	117	130	234	260
10	2	2	QPSK	3/4	39	43.3	81	90	175.5	195	351	390
11	3	2	16-QAM	1/2	52	57.8	108	120	234	260	468	520
12	4	2	16-QAM	3/4	78	86.7	162	180	351	390	702	780
13	5	2	64-QAM	2/3	104	115.6	216	240	468	520	936	1040
14	6	2	64-QAM	3/4	117	130.3	243	270	526.5	585	1053	1170
15	7	2	64-QAM	5/6	130	144.4	270	**300**	585	650	1170	1300
	8	2	256-QAM	3/4	156	173.3	324	360	702	780	1404	1560
	9	2	256-QAM	5/6	n/a	n/a	360	**400**	780	**866.7**	1560	1733.3
16	0	3	BPSK	1/2	19.5	21.7	40.5	45	87.8	97.5	175.5	195
17	1	3	QPSK	1/2	39	43.3	81	90	175.5	195	351	390
18	2	3	QPSK	3/4	58.5	65	121.5	135	263.3	292.5	526.5	585
19	3	3	16-QAM	1/2	78	86.7	162	180	351	390	702	780
20	4	3	16-QAM	3/4	117	130	243	270	526.5	585	1053	1170
21	5	3	64-QAM	2/3	156	173.3	324	360	702	780	1404	1560
22	6	3	64-QAM	3/4	175.5	195	364.5	405	n/a	n/a	1579.5	1755
23	7	3	64-QAM	5/6	195	216.7	405	**450**	877.5	975	1755	1950
	8	3	256-QAM	3/4	234	260	486	540	1053	1170	2106	2340
	9	3	256-QAM	5/6	260	288.9	540	**600**	1170	**1300**	n/a	n/a

π /4 QPSK

- π/4 QPSK consists of two QPSK
- When performing modulation, switch between these two QPSK on symbol basis
- Possible phase differences: ± (π /4), ±(3 π /4)
- An example: symbols are 01 10 00 11
  - Initial state is 11 as marked
  - The phase differences of modulated signal are [-π/4, +3π/4, +3π/4, +3π/4]

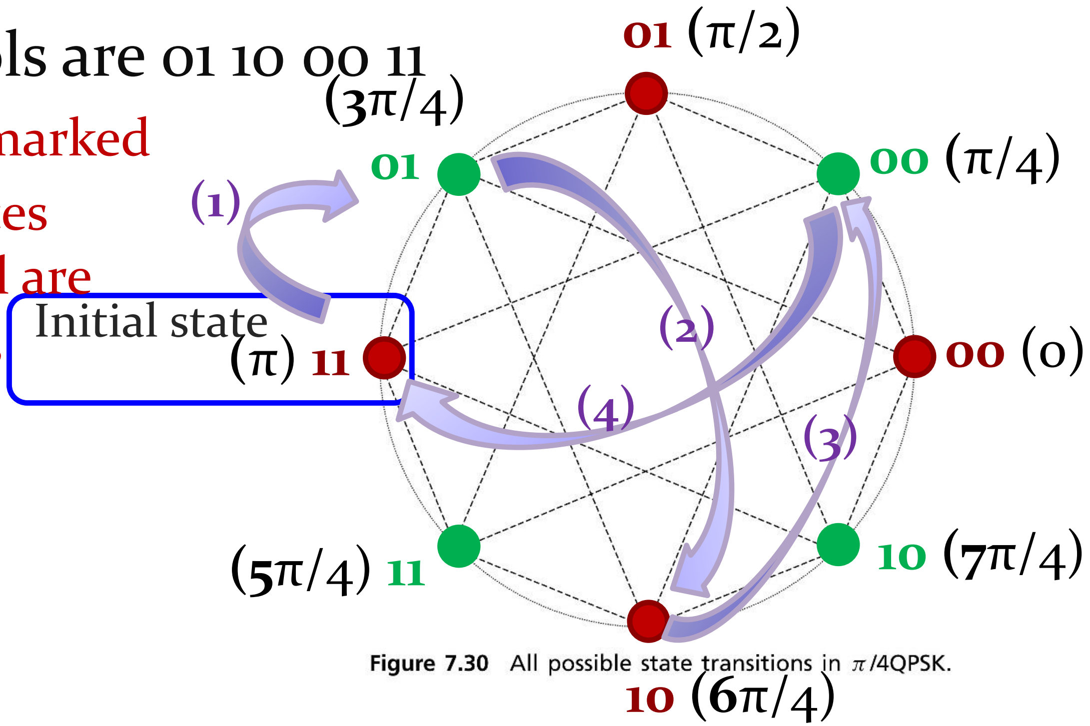


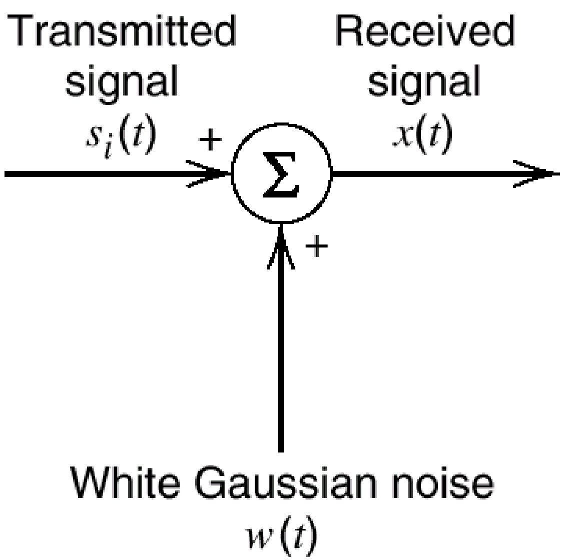
Figure 7.30 All possible state transitions in π/4QPSK.

Signal representations and transmissions

- How do we transmit messages to others?
  - Start by defining a set of characters(symbols), m_1, m_2,…,m_M
  - Represent the symbols with certain signal formats, s_1,…,s_M
  - Signal s_i (t) is transmitted over a channel at a speed of $1/T$ *sec*
  - Suppose the channel has the following two characteristics:
    - The channel is linear, with a bandwidth that is wide enough to accommodate the transmission of signal with negligible distortion
    - The channel is corrupted by an additive noise $w(t)$, which is the sample function of a *zero-mean white Gaussian noise process*
  - We refer to this channel as an additive white Gaussian noise (AWGN) channel, and express the received signal x(t) as

$$x(t) = s_i(t) + w(t), \qquad \begin{Bmatrix} 0 \le t \le T \\ i = 1,2, \dots M \end{Bmatrix}$$

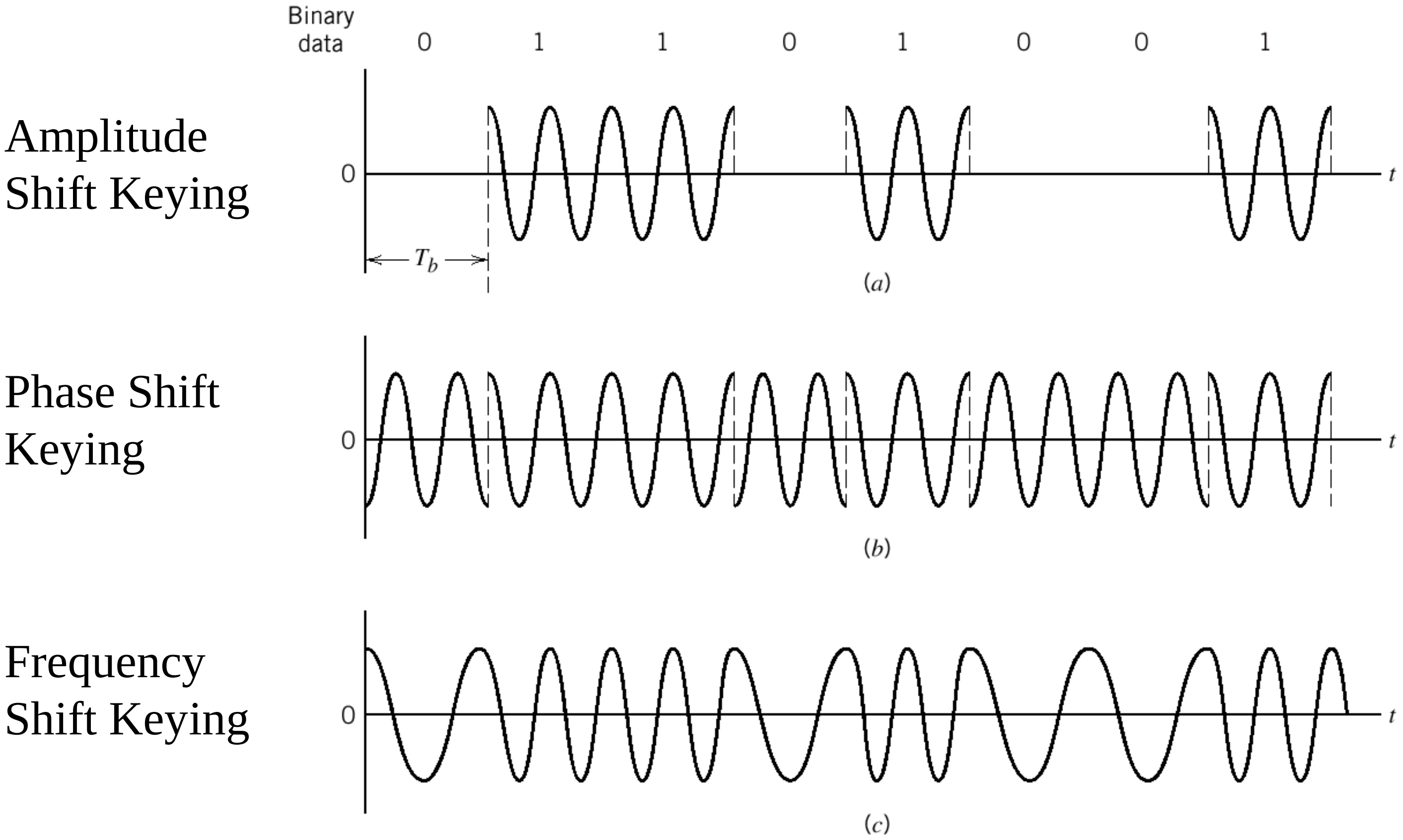
- The received signal can thus be modeled as



– What does the receiver do?
  - Observes the received signal x(t) for a duration of T
  - Makes a best estimate of the transmitted signal s_i (t), or m_i, such that $P_e = \sum_{i=1}^{M} p_i P(\hat{m} \neq m_i | m_i)$ is minimized

– What are the basic types of signals for data transmissions?

- Consider a simple sinusoidal wave $s(t) = \cos(2\pi f_c t)$
- What can we do to transmit a binary data stream with $s(t)$



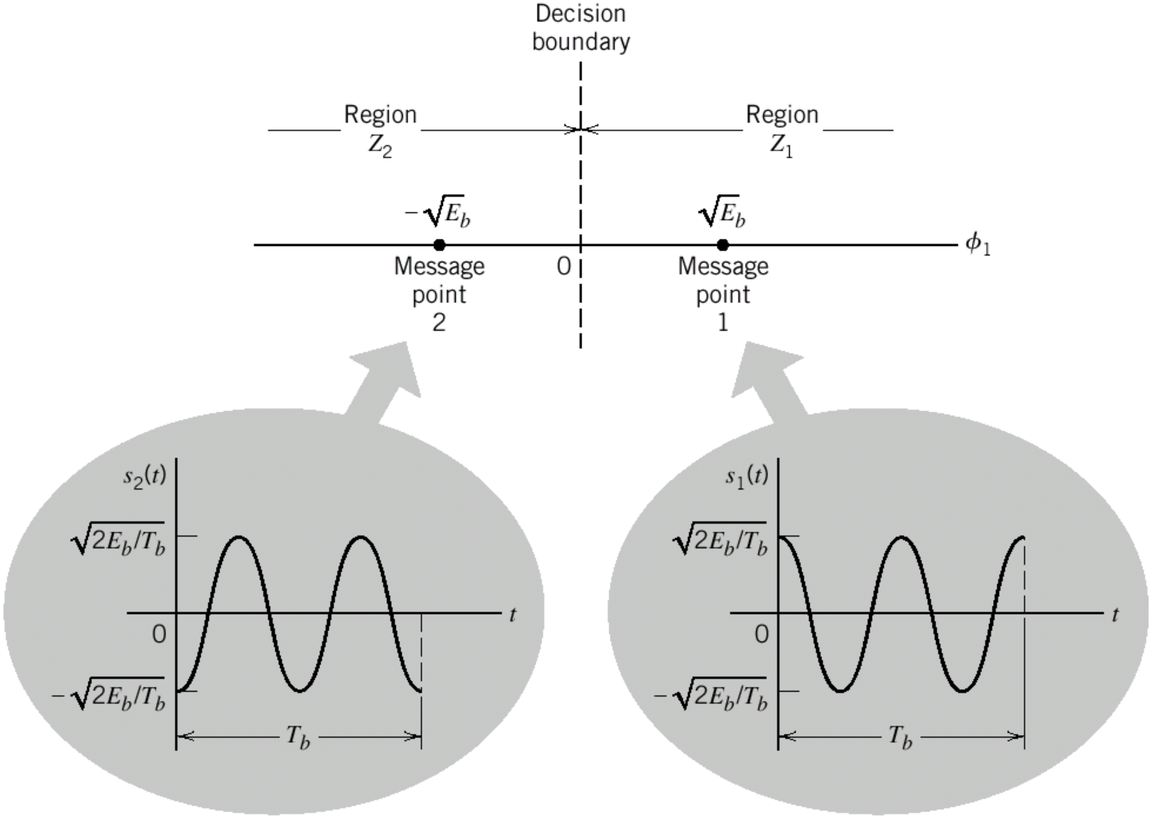
- Take PSK as an example, we may have
  - $s_1(t) = \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t)$ to represent a binary symbol 1
  - $s_2(t) = \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t + \pi)$ to represent a binary symbol 0

  where $0 \leq t \leq T_b$, and E_b is the energy per bit
  - To ensure that each transmitted bit contains an integral number of cycles of the carrier wave, we have $f_c = n_c / T_b$, with $n_c \in \mathbb{N}$
  - Two sinusoidal waves that differ only in a relative phase of 180° are referred to as *antipodal signals*
  - This type of modulation is referred to as binary PSK (BPSK)
  - Define a basis function $\phi_1(t) = \sqrt{\frac{2}{T_b}}\cos(2\pi f_c t), \quad 0 \leq t < T_b$
  - Then $s_1(t) = \sqrt{E_b}\phi_1(t)$ and $s_2(t) = -\sqrt{E_b}\phi_1(t), \quad 0 \leq t < T_b$

- A coherent BPSK signal is therefore characterized *by having a one-dimensional signal space* $\phi_1(t)$, with a signal constellation consisting of two message points, i.e. M=2
- The coordinate of the two message points of BPSK on $\phi_1(t)$ are

$$s_{11} = \int_0^{T_b} s_1(t)\phi_1(t)dt = +\sqrt{E_b}$$

$$s_{21} = \int_0^{T_b} s_2(t)\phi_1(t)dt = -\sqrt{E_b}$$



- Recall the BPSK modulation
  - $s_1(t) = \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t)$ for the message bit $m_1 = 1$
  - $s_2(t) = \sqrt{\frac{2E_b}{T_b}}\cos(2\pi f_c t + \pi)$ for the message bit $m_2 = 0$

    where $0 \leq t \leq T_b$, and E_b is the energy per bit
- How do we do detection? Remember that we have noise!!
  - The received signal is given by $x(t) = s_i(t) + w(t)$, and

    $x_1 = \int_0^{T_b} x(t)\phi_1(t)dt = \pm\sqrt{E_b} + \int_0^{T_b} w(t)\phi_1(t)dt$
  - The sampled noise

    $$w_1 = \int_0^{T_b} w(t)\phi_1(t)dt$$

    is Gaussian given that $w(t)$ is a Gaussian random process

– Define a random variable W_1 and a random process $W(t)$ for $w_1$and $w(t)$, respectively. The variance of W_1 is

- $$\sigma_{x_1}{}^2 = E\left[\int_0^{T_b} W(t)\phi_1(t)dt \int_0^{T_b} W(u)\phi_1(u)du\right]$$
$$= E\left[\int_0^{T_b}\int_0^{T_b} \phi_1(t)\phi_1(u)W(t)W(u)dtdu\right]$$
$$= \int_0^{T_b}\int_0^{T_b} \phi_1(t)\phi_1(u)E[W(t)W(u)]dtdu$$
$$= \int_0^{T_b}\int_0^{T_b} \phi_1(t)\phi_1(u)\frac{N_0}{2}\delta(t-u)dtdu$$
$$= \frac{N_0}{2}\int_0^{T_b} \phi_1{}^2(t)dt = \frac{N_0}{2}$$

- Similarly, it is easy to show that

$$\mathrm{cov}[X_iX_j] = \frac{N_0}{2}\int_0^{T_b} \phi_i(t)\phi_j(t)dt = 0, \qquad i \neq j$$

– Therefore, the received samples of signals are modeled as

$$x_1 = \pm\sqrt{E_b} + w_1 \qquad \text{with} \qquad w_1 \sim \mathcal{N}(0, N_0/2)$$

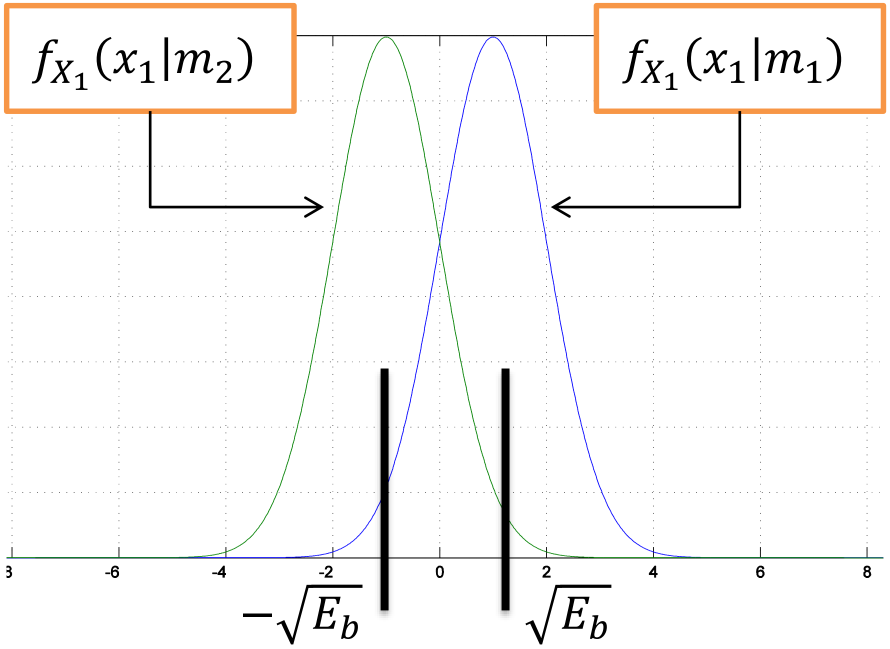
– The conditional probability density function (PDF) of x_1 is

$$f_{X_1}(x_1|m_i) = \frac{1}{\sqrt{\pi N_0}} \exp\left[-\frac{1}{N_0}(x_1 - s_{i1})^2\right],\ s_{i1} = \pm\sqrt{E_b}$$

– Suppose the probability mass function (PMF), $p(m_1)$=$p(m_2)$=0.5

– The decision rule follows as

$$f_{X_1}(x_1|m_2) \underset{m_2}{\overset{m_1}{\lesseqgtr}} f_{X_1}(x_1|m_1)$$



- The probability of mistaking 1 a 0 is, therefore, given by

$$p_{10} = \int_0^\infty f_{x_1}(x_1|m_2 = 0)dx_1$$

$$= \frac{1}{\sqrt{\pi N_0}} \int_0^\infty \exp\left[-\frac{1}{N_0}(x_1 + \sqrt{E_b})^2\right] dx_1 = \frac{1}{\sqrt{\pi}} \int_{\sqrt{E_b/N_0}}^\infty \exp[-z^2] dz$$

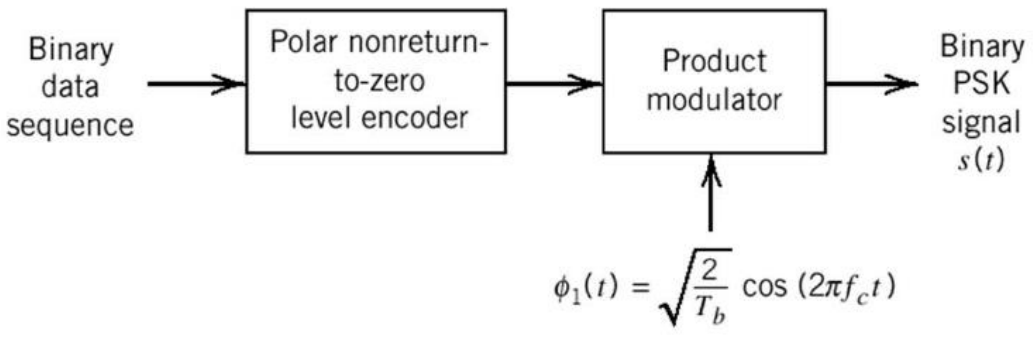
$$= \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

- Since $p_e = p_{10}p(0) + p_{01}p(1)$ and $p_{01} = p_{10}$ with $p(0) = p(1) = 0.5$
- Finally, we have the bit error rate (BER) of BPSK as follows

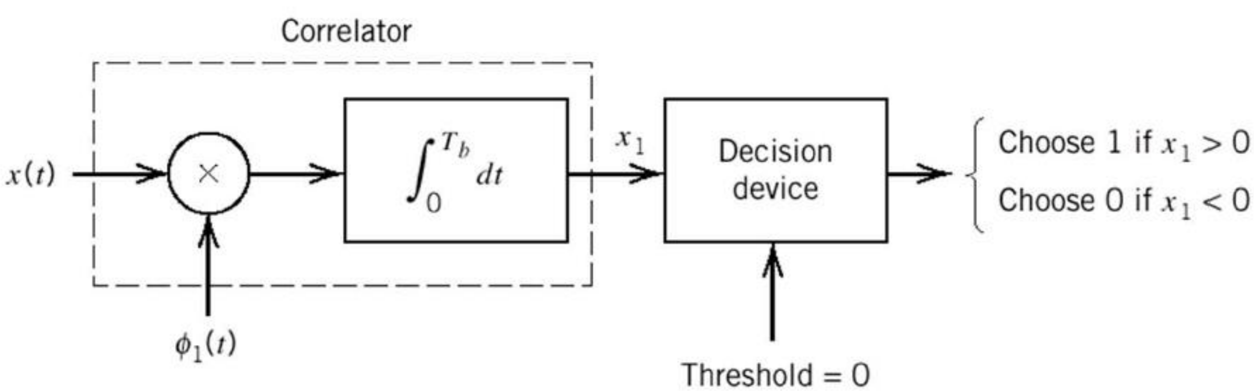
$$p_e = \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

– Generation and detection of BPSK

- Polar nonreturn-to-zero (NRZ) level encoder:



(a)



(b)

HW2

- Generate a series of binary random numbers
- Modulate the binary numbers with BPSK, given that the carrier frequency f_c is 1 MHz, and the symbol energy E_b is 10 dB and frequency is 1 KHz
- Demodulate the transmitted signal using the block diagram on pp. 27
- Add the demodulated samples by AWGN noise $\mathcal{N}(0, N_0/2)$, with N_0=1
- Do symbol detection on the resultant samples
- Calculate the BER
- Redo the same experiments with 0 dB$\leq E_b \leq$ 30 dB
- Draw and compare the BER with the theoretical values