Final Examination (Close Book)

Exam date & time: June 11, 2019, 10:10AM–11:50AM (100 minutes)

Do the following problems. The points are specified in the brackets (i.e., []). There are 100 points in total. Each of problems 1 to 5 consists of one mathematical statement and two questions. For the first question, you have to answer ○ for "**true**" or × for "**false**" to indicate your judgement on the validity of the given statement. For the second question, you have to prove the statement if your answer is ○ for the first question or disprove the statement if your answer is ×. If the answer for the first question is incorrect, there will be no points awarded to the second question (as well as the first question).

In the second question, you do not need to prove any of the mathematical statements that have been proved in the textbook as theorems, corollaries, and lemmas.

1. [10] P is closed under complement.
   (a) [3] True (○) or false (×)?
   (b) [7] Show that your answer for (a) is correct.
2. [10] NP is closed under union.
   (a) [3] True (○) or false (×)?
   (b) [7] Show that your answer for (a) is correct.
3. [10] Let $TRIANGLE = \{\langle G\rangle \mid G$ is an undirected graph and contains a triangle, i.e., a 3-clique $\}$. $TRIANGLE$ is in P.
   (a) [3] True (○) or false (×)?
   (b) [7] Show that your answer for (a) is correct.
4. [10] Let $CNF_k = \{\langle\phi\rangle \mid \phi$ is a satisfiable cnf-formula where each variable appears in at most k places$\}$. CNF_2 is in P.
   (a) [3] True (○) or false (×)?
   (b) [7] Show that your answer for (a) is correct.

5. [10] Let $HALF\text{-}CLIQUE = \{\langle G\rangle \mid G$ is an undirected graph and contains a complete sub-graph with at least $m/2$ nodes, where m is the number of nodes in $G\}$. $HALF\text{-}CLIQUE$ is NP-complete.

   (a) [3] True (○) or false (×)?

   (b) [7] Show that your answer for (a) is correct.

6. [10] Reduce the following problem instance of SAT:

$$\phi_6 = (\overline{x_1} \vee x_3 \vee x_5 \vee \overline{x_6}) \wedge (x_1 \vee \overline{x_2} \vee \overline{x_3} \vee x_4 \vee \overline{x_6})$$

   to one of $3SAT$.

7. [10] Reduce the following problem instance of $3SAT$:

$$\phi_7 = (x_1 \vee \overline{x_2} \vee x_3) \wedge (x_2 \vee \overline{x_3} \vee \overline{x_4}) \wedge (\overline{x_2} \vee x_4 \vee \overline{x_5}) \wedge (x_3 \vee \overline{x_4} \vee x_5)$$

   to one of $CLIQUE$.

8. [10] Reduce the following problem instance of $3SAT$:

$$\phi_8 = (x_1 \vee \overline{x_2} \vee x_3) \wedge (x_2 \vee \overline{x_3} \vee \overline{x_4}) \wedge (\overline{x_2} \vee x_4 \vee \overline{x_5}) \wedge (x_3 \vee \overline{x_4} \vee x_5)$$

   to one of $VERTEX\text{-}COVER$.

9. [10] Reduce the following problem instance of $3SAT$:

$$\phi_9 = (x_1 \vee \overline{x_2} \vee x_3) \wedge (x_2 \vee \overline{x_3} \vee \overline{x_4}) \wedge (\overline{x_2} \vee x_4 \vee \overline{x_5}) \wedge (x_3 \vee \overline{x_4} \vee x_5)$$

   to one of $SUBSET\text{-}SUM$.

10. [10] Reduce the following problem instance of $3SAT$:

$$\phi_{10} = (\overline{x_1} \vee \overline{x_2} \vee x_3) \wedge (x_1 \vee x_3 \vee \overline{x_4}) \wedge (x_2 \vee \overline{x_4} \vee x_5)$$

   to one of $HAMPATH$.