Midterm Examination (Close Book)

Exam date & time: April 12, 2016, 10:10AM–11:50AM (100 minutes)

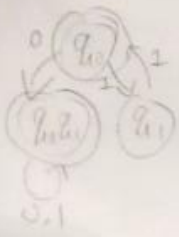
Notations:

- $\Sigma_{01} = \{ 0, 1 \}$.
- w^R is the reversal of string w.

Do the following problems. The points are specified in the brackets (i.e., []). There are **100** points in total.

1. [15] For NFA $N_1 = (\{ q_0, q_1 \}, \Sigma_{01}, \delta_1, q_0, \{ q_0 \})$, where δ_1 is

	ε	0	1
q_0	$\emptyset$	$\{ q_0, q_1 \}$	$\{ q_1 \}$
q_1	$\emptyset$	$\emptyset$	$\{ q_0 \}$



   (a) [5] Draw the state diagram of N_1.

   (b) [10] Convert N_1 to an equivalent DFA.

2. [15] For DFA $D_2 = (\{ q_0, q_1 \}, \Sigma_{01}, \delta_2, q_0, \{ q_1 \})$, where δ_2 is

	0	1
q_0	q_0	q_1
q_1	q_1	q_1

   (a) [5] Draw the state diagram of D_2.

   (b) [10] Convert D_2 to an equivalent regular expression by firstly eliminating q_0 and then q_1.

3. [30] Prove or disprove each of the following languages is regular:

   (a) [15] Let $\Sigma_{3a} = \left\{ \begin{bmatrix}0\\0\\0\end{bmatrix}, \begin{bmatrix}0\\0\\1\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}, \begin{bmatrix}0\\1\\1\end{bmatrix}, \begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}1\\0\\1\end{bmatrix}, \begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}1\\1\\1\end{bmatrix} \right\}$. A string of symbols in Σ_{3a} gives three rows of 0s and 1s. Consider each row to be a binary number and let

   $$L_{3a} = \{ w \in \Sigma_{3a}^* \mid \text{the bottom row of } w \text{ is the sum of the top two rows} \} .$$

   For example,

   $$\begin{bmatrix}0\\0\\1\end{bmatrix}\begin{bmatrix}1\\0\\0\end{bmatrix}\begin{bmatrix}1\\1\\0\end{bmatrix} \in L_{3a} , \qquad \text{but} \qquad \begin{bmatrix}0\\0\\1\end{bmatrix}\begin{bmatrix}1\\0\\1\end{bmatrix} \notin L_{3a} .$$

(b) [15] $L_{3b} = \{ 0^i 1^j \mid j > i \geq 0 \}$.

4. [15] Prove or disprove the following languages is context-free:

$$L_4 = \{ w \mid w = w^R, w \in \Sigma_{01}^* \} .$$

5. [10] For CFG $G_5 = (\{ E, T, F \}, \{ \mathsf{a}, (,), +, \times \}, R, E)$, where $R =$

$$\begin{array}{lcl} E & \rightarrow & E + T \mid T \\ T & \rightarrow & T \times F \mid F \\ F & \rightarrow & (E) \mid \mathsf{a} \end{array}$$

(a) [5] Give the parse tree and derivation for string $\mathsf{a} + \mathsf{a} + \mathsf{a}$.

(b) [5] Convert G_5 to a PDA.

6. [15] For CFG $G_6 = (\{ A, B \}, \{ \mathsf{a} \}, R, A)$, where $R =$

$$\begin{array}{lcl} A & \rightarrow & BAB \mid B \mid \varepsilon \\ B & \rightarrow & \mathsf{aa} \mid \varepsilon \end{array}$$

(a) [10] Convert G_6 to an equivalent CFG G_6' in Chomsky normal form step by step.

(b) [5] Use CYK algorithm to check whether or not string aaaaa can be generated by G_6'.