

Final Examination (Close Book)

Exam date & time: June 07, 2016, 10:20AM–12:00PM (100 minutes)

Do the following problems. The points are specified in the brackets (i.e., []). There are 100 points in total. Each of problems 1 to 5 consists of one mathematical statement and two questions. For the first question, you have to answer \bigcirc for “true” or \times for “false” to indicate your judgement on the validity of the given statement. For the second question, you have to prove the statement if your answer is \bigcirc for the first question or disprove the statement if your answer is \times . If the answer for the first question is incorrect, there will be no points awarded to the second question (as well as the first question).

In the second question, you do not need to prove any of the mathematical statements that have been proved in the textbook as theorems, corollaries, and lemmas.

1. [10] If $A \leq_m B$ and $B \leq_m C$, then $A \leq_m C$.
 - (a) [3] True (\bigcirc) or false (\times)?
 - (b) [7] Show that your answer for (a) is correct.
2. [10] If $A \leq_m B$ and B is regular, then A is also regular.
 - (a) [3] True (\bigcirc) or false (\times)?
 - (b) [7] Show that your answer for (a) is correct.
3. [10] A_{TM} is mapping reducible to E_{TM} . (Hint: $\overline{E_{\text{TM}}}$ is Turing-recognizable.)
 - (a) [3] True (\bigcirc) or false (\times)?
 - (b) [7] Show that your answer for (a) is correct.
4. [10] Let $\text{DOUBLE-SAT} = \{\langle \phi \rangle \mid \phi \text{ has at least two satisfying assignments}\}$ is NP-complete.
 - (a) [3] True (\bigcirc) or false (\times)?
 - (b) [7] Show that your answer for (a) is correct.

5. [10] NP is closed under union.

(a) [3] True (\bigcirc) or false (\times)?

(b) [7] Show that your answer for (a) is correct.

6. [10] Reduce the following problem instance of SAT:

$$\phi_6 = (x_2 \vee \overline{x_3} \vee \overline{x_5} \vee x_6) \wedge (\overline{x_1} \vee x_3 \vee \overline{x_4} \vee x_5 \vee \overline{x_6})$$

to one of 3SAT.

7. [10] Reduce the following problem instance of 3SAT:

$$\phi_7 = (\overline{x_1} \vee \overline{x_2} \vee x_3) \wedge (x_1 \vee x_3 \vee \overline{x_4}) \wedge (x_2 \vee \overline{x_4} \vee x_5) \wedge (\overline{x_3} \vee x_4 \vee x_5)$$

to one of CLIQUE.

8. [10] Reduce the following problem instance of 3SAT:

$$\phi_8 = (\overline{x_1} \vee \overline{x_2} \vee x_3) \wedge (x_1 \vee x_3 \vee \overline{x_4}) \wedge (x_2 \vee \overline{x_4} \vee x_5) \wedge (\overline{x_3} \vee x_4 \vee x_5)$$

to one of VERTEX-COVER.

9. [10] Reduce the following problem instance of 3SAT:

$$\phi_9 = (\overline{x_1} \vee \overline{x_2} \vee x_3) \wedge (x_1 \vee x_3 \vee \overline{x_4}) \wedge (x_2 \vee \overline{x_4} \vee x_5) \wedge (\overline{x_3} \vee x_4 \vee x_5)$$

to one of SUBSET-SUM.

10. [10] Reduce the following problem instance of 3SAT:

$$\phi_{10} = (\overline{x_1} \vee \overline{x_2} \vee x_3) \wedge (x_1 \vee x_3 \vee \overline{x_4}) \wedge (x_2 \vee \overline{x_4} \vee x_5)$$

to one of HAMPATH.