Midterm Examination (Close Book)

Exam date & time: October 31, 2019, 10:10AM–11:50AM (100 minutes)

Notations:

- $\Sigma_{01} = \{ 0, 1 \}$.
- $\#_a(w)$ is the number of symbol a in string w.

Do the following problems. The points are specified in the brackets (i.e., []). There are **100** points in total.

1. [15] For NFA $N_1 = (\{ q_0, q_1 \}, \Sigma_{01}, \delta_1, q_0, \{ q_0 \})$, where δ_1 is

	ε	0	1
q_0	$\emptyset$	$\{ q_1 \}$	$\{ q_0, q_1 \}$
q_1	$\emptyset$	$\{ q_0, q_1 \}$	$\{ q_0 \}$

   (a) [5] Draw the state diagram of N_1.

   (b) [10] Convert N_1 to an equivalent DFA.

2. [15] For DFA $D_2 = (\{ q_0, q_1 \}, \Sigma_{01}, \delta_2, q_0, \{ q_1 \})$, where δ_2 is

	0	1
q_0	q_0	q_1
q_1	q_0	q_1

   (a) [5] Draw the state diagram of D_2.

   (b) [10] Convert D_2 to an equivalent regular expression by firstly eliminating q_0 and then q_1.

3. [30] For $w \in \Sigma_{01}^*$, prove or disprove the following statements:

   (a) [15] $L_{3a} = \{ w \mid \#_0(w) = \#_1(w) \}$ is regular.

   (b) [15] $L_{3b} = \{ x \mid x \neq ww, x \in \Sigma_{01}^*, w \in \Sigma_{01}^* \}$ is context-free.

4. [10] Convert PDA $M_4 = (Q_4 = \{ q_1, q_2, q_3, q_4, q_5 \}, \Sigma_{01}, \Sigma_{01} \cup \{ \$ \}, \delta_4, q_1, \{ q_5 \})$, where δ_4 is given by the following table, to a CFG. (All the rules have to be written out except for $p, q, r \in Q_4$, $A_{pq} \to A_{pr}A_{rq}$.)

Input:	ε				0				1			
Stack:	ε	\$	0	1	ε	\$	0	1	ε	\$	0	1
q_1	$\{(q_2, \$)\}$											
q_2	$\{(q_4, 0)\}$				$\{(q_2, 0)\}$				$\{(q_2, 1)\}$			
q_3		$\{(q_5, \varepsilon)\}$					$\{(q_3, \varepsilon)\}$					$\{(q_3, \varepsilon)\}$
q_4			$\{(q_3, \varepsilon)\}$									

5. [10] For CFG $G_5 = (\{ S \}, \{ \mathtt{a}, \mathtt{b} \}, R, S)$, where $R =$

$$S \rightarrow \mathtt{ab} \mid \mathtt{a}S\mathtt{b}$$

Convert G_5 to a PDA. All the states have to be written out.

6. [10] Convert G_5 to an equivalent CFG G_6 in Chomsky normal form step by step.

7. [10] For CFG $G_7 = (\{ S, A, B, C, D \}, \Sigma_{01}, R, S)$, where $R =$

$$\begin{aligned} S &\rightarrow AB \mid CB \\ A &\rightarrow \mathtt{0} \\ B &\rightarrow \mathtt{1} \\ C &\rightarrow AD \\ D &\rightarrow AB \mid CB \end{aligned}$$

Use CYK algorithm to check whether or not each of the following strings can be generated by G_7.

(a) [5] 00011.

(b) [5] 000111.