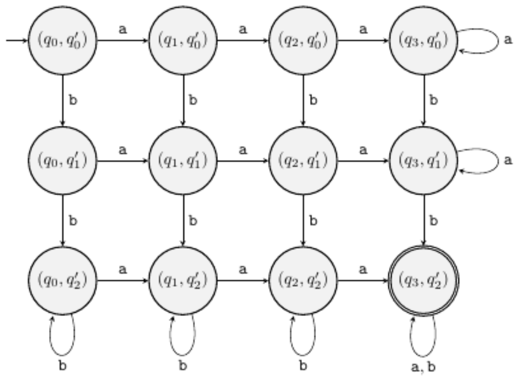
Introduction to Formal Language
Chapter 1 Practice Solutions

April. 27, 2020

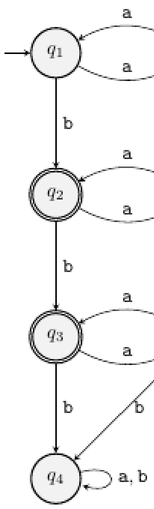
1.4 which includes 7 subproblems.

a.



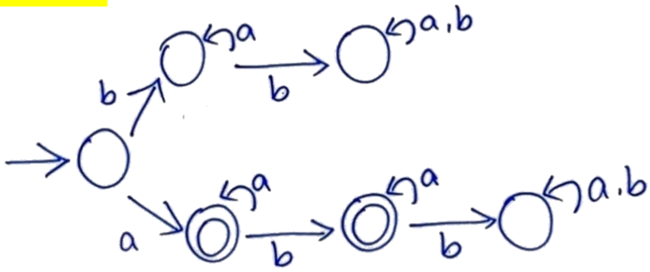
b. in the p.94

c.

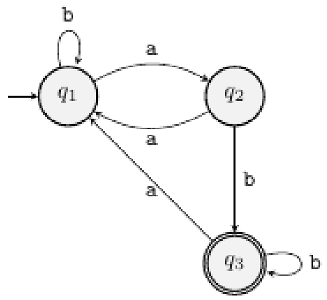


d. in the p.95

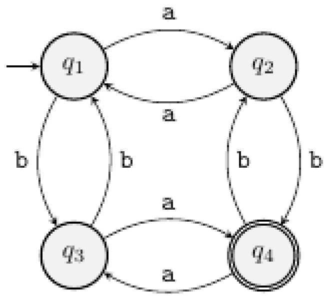
e. updated



f.



g.

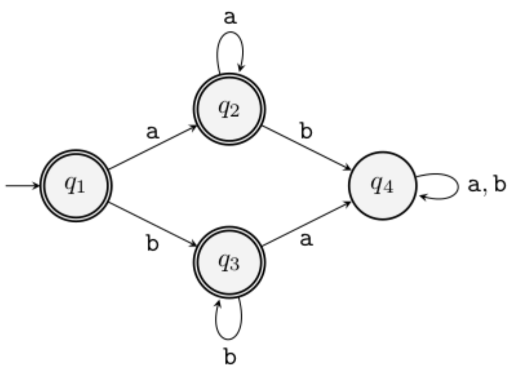


1.5 which includes 8 subproblems.

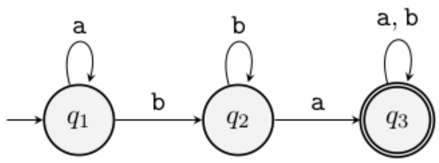
a. in the p.96

b. in the p.96

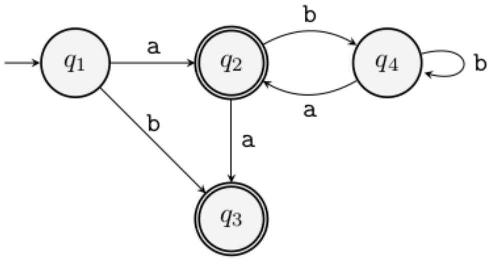
c.



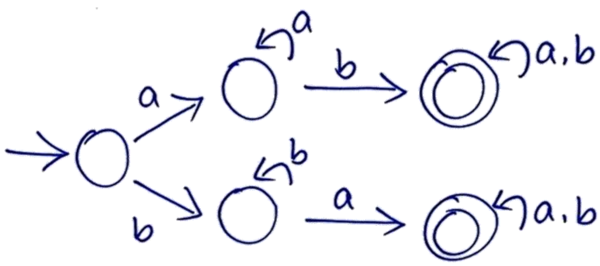
d.



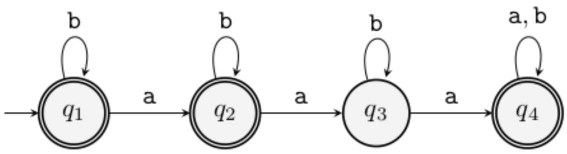
e.



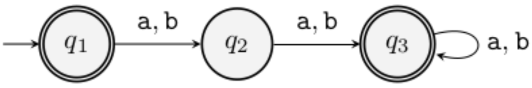
f. updated



g.



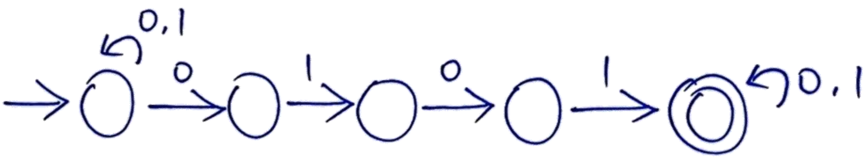
h.



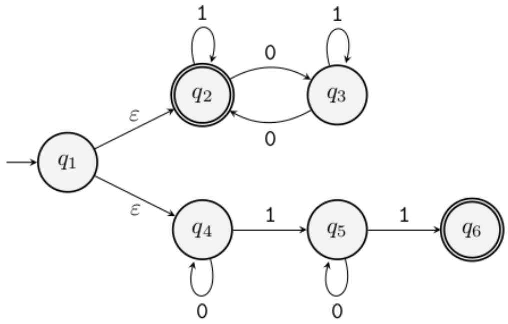
1.7 which includes 8 subproblems.

a. in the p.96

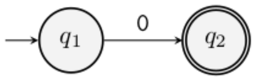
b. updated



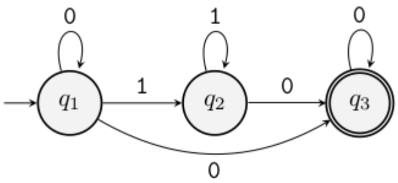
c.



d.

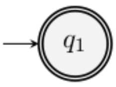


e.

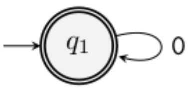


f. in the p.96

g.



h.



1.11 in the p.96

1.14 which includes 2 subproblems.

a. Language is regular iff some DFA recognizes it. The conclusion above guarantees that if there is DFA which recognizes a language, then there must be a DFA which recognizes its complement. Hence, complement of any regular language is also regular, which exactly means that class of regular languages is closed under complement.

*Another solution about this *proof*:

Suppose language B over alphabet Σ has a DFA

$$M = (Q, \Sigma, \delta, q_1, F).$$

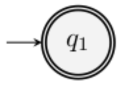
Then, a DFA for the complementary language $\overline{B}$ is

$$\overline{M} = (Q, \Sigma, \delta, q_1, Q - F).$$

The reason why $\overline{M}$ recognizes $\overline{B}$ is as follows. First note that M and $\overline{M}$ have the same transition function δ. Thus, since M is deterministic, $\overline{M}$ is also deterministic. Now consider any string $w \in \Sigma^*$. Running M on input string w will result in M ending in some state $r \in Q$. Since M is deterministic, there is only one possible state that M can end in on input w. If we run $\overline{M}$ on the same input w, then $\overline{M}$ will end in the same state r since M and $\overline{M}$ have the same transition function. Also, since $\overline{M}$ is deterministic, there is only one possible ending state that $\overline{M}$ can be in on input w.

Now suppose that $w \in B$. Then M will accept w, which means that the ending state $r \in F$, i.e., r is an accept state of M. But then $r \notin Q - F$, so $\overline{M}$ does not accept w since $\overline{M}$ has $Q - F$ as its set of accept states. Similarly, suppose that $w \notin B$. Then M will not accept w, which means that the ending state $r \notin F$. But then $r \in Q - F$, so $\overline{M}$ accepts w. Therefore, $\overline{M}$ accepts string w if and only M does not accept string w, so $\overline{M}$ recognizes language $\overline{B}$. Hence, the class of regular languages is closed under complement.

b. Let's take the simplest possible example. Consider the following NFA:

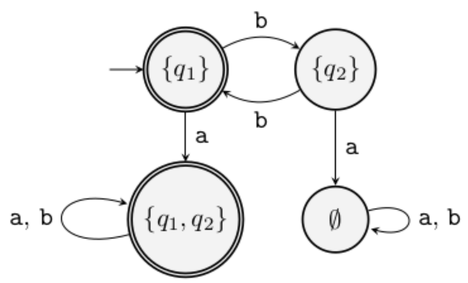


which language does this NFA recognize? Well, there is really no way to go from start state, so it can only accept string which doesn't make it go anywhere. This can only be empty string ε. Hence the language which this NFA recognizes is $\{\varepsilon\}$. What happens to this NFA if we swap its accepting and non-accepting states? Not much of course, it just loses its only accept state. That means that it cannot accept any string, so the language it recognizes is empty set $\emptyset$. Since this is obviously not the complement of previous language, we must admit that our construction did not produce the expected result.

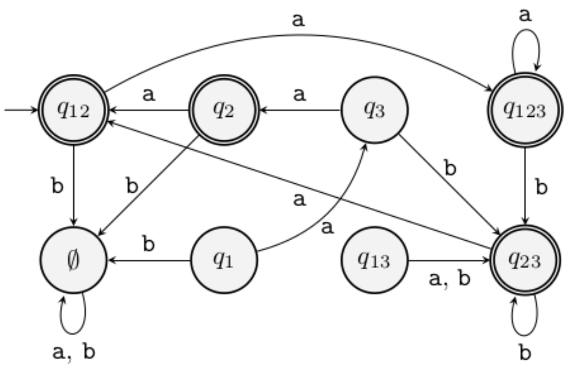
Does this mean that we can say that class of languages recognized by NFAS is not closed under complement? Not so fast. Remember magnificent **Theorem 1.39**, which implies that this class of languages is the same as the class from part **(a)**. Hence the conclusion from above is valid here as well.

1.16 which includes 2 subproblems.

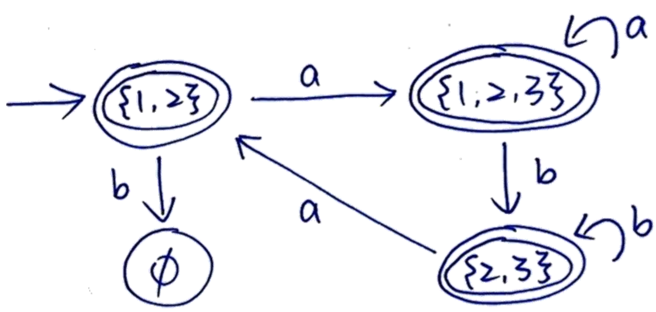
a.



b.

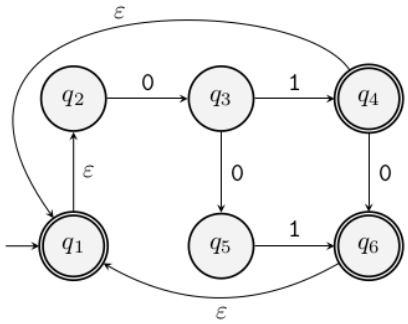


you should simplify it to

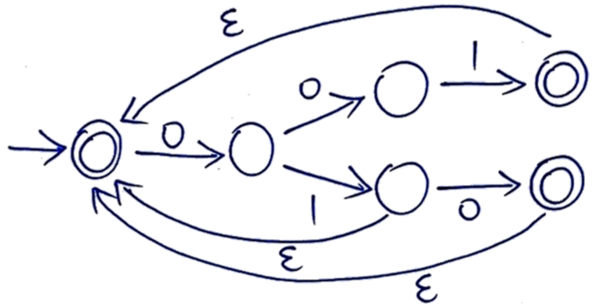


1.17 which includes 2 subproblems.

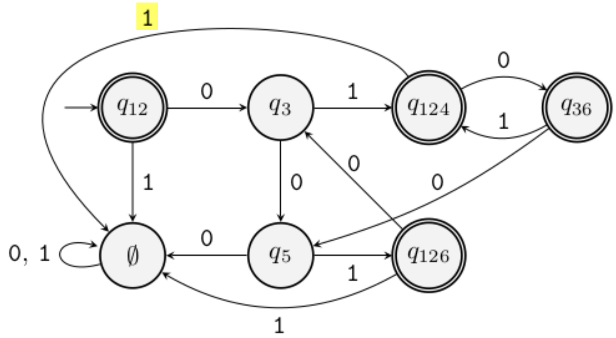
a. [sol. 1]



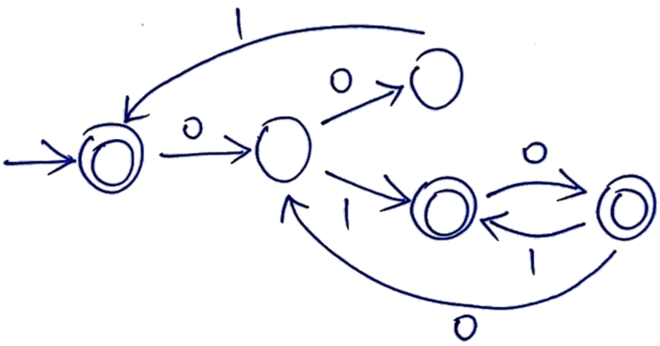
[sol. 2]



b. [sol. 1]



[sol. 2]



1.21 which includes 2 subproblems.

a. (a*b)(a∪ba*b)*
b. **ε** ∪(a∪b)a*b(b∪a (a∪b)a*b)*(ε ∪a)

1.29 which includes 2 subproblems.

a. in the p.97
c. in the p.97

1.34 $\boxed{\text{1.44 on p.90 for the } \textit{hardcover textbook}}$