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T3 ________	Problem Chosen	F3 ________
T4 ________	**B**	F4 ________

2016
Mathematical Contest in Modeling (MCM/ICM) Summary Sheet

Summary

In order to examine the performance of alternatives handling the space debris problem, we construct three types of models.

The first type of model discusses a method to quantitively analyze different approaches to removing space debris. We define Gain Ratio to be the ratio of benefit and cost, which together with Risk serves as the judging criteria of alternative's performance. Firstly, we utilize grades standardization model to normalize the representative value of Gain Ratio and Risk. Secondly, we apply analytic hierachy process to endow both criteria with weight. Finally, the weighted and normalized Gain Ratio and Risk are combined to demonstrate the quantized evaluation of specific alternatives. We term this model as assessment model.

The second type of model illustrates the debris distribution in space (DDS model). Our DDS model is based on ORDEM 2000 model, which describes the general time dependency of the debris motions and positions. Moreover, the model predicts that the density of debris is always higher in some particular latitude, being independent of time. Such characteristics has provided us with a stepping point to effective assessment.

The third type of model represents the methods of removing space debris (RSD model). This model is put in the condition of DDS model and to be assessed by the assessment model. In this paper, we construct two RSD models, namely Water Jets Model and Net catching Model.

In the later part of this paper, specific simulations are carried out to determine the best RSD model. We aim at determining the potential existence of viable economical opportunity.

Economical Opportunity from Space

February 2, 2016

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Keywords: assessment model; DDS model; ORDEM 2000; RSD model

Economical Opportunity from Space

1 Introduction

Space debris is a generic term representing the wastes floating around the orbitals. Eliminating or reducing space debris has been a growing concern due to its potential threats to space exploration. The type of debris ranges widely in size and mass, which necessarily leads to a variety of methods to mitigate or remove the particular debris. Among all methods, some may provide a commercial opportunity to the potential private firms. In this paper, we construct a time-dependent model to assess and compare alternatives for handling space debris problem, specifically, water jets and catching nets models, so as to attempt to find an economically attractive opportunity.

1.1 Restatement of the Problem

We are required to develop a time-dependent mathematical model to assess alternatives for addressing the space debris problem and judge whether an economically attractive opportunity exists or not. We define the subproblems as:

- Construct models describing the approaches of removing space debris (RSD models).
- Build a mathematical model to demonstrate the debris distribution in space (DDS model).
- Propose a time-dependent mathematical model for assessing the performance of RSD models in the condition of DDS model based on risk and the ratio of benefit and cost. Provide a criteria of economically satisfactory performance of RSD models and thereby determine the existence of viable opportunity (We affirm the existence of such an opportunity).
- Compare the RSD models and give the specific recommendation.
- Consider necessary "What if?" problems and optimize the models.

2 Assumptions and Justifications

- The orbits in which the debris motions take place around the Earth are perfect circles.
- Momentum and energy are conserved throughout the space.
- Space debris is shaped sphere.
- When observed from the North pole, all space debris are moving counterclockwise.
- The space debris freely moving around the Earth would not bump each other.
- In the orbits which is relatively close to the surface of the Earth there exists air which results in air resistance.
- There are only three kinds of debris classified by size: Large, Medium, and Small.

3 RSD Models

Most research typically focus on two methods to remove the space debris, either by destroying debris directly or moving the debris to outer, useless orbits. Since the latter method is always accompanied by higher energy requirements and risks, it is not suitable in our discussion based on economical opportunity. Therefore, the following models focus on removing debris by destruction. More specifically, we assume the debris would be finally moved to earth atmosphere and burnt up completely.

We consider two RSD models for later simulation, namely water jets model and net catching model. Two models are to serve as a stepping stone for our study and give us insights in model assessments.

3.1 Water Jets Model

Intuitively, water jets model is based on four actions:

- Move the space craft carrying the water to the same orbit as the one in which the target debris is moving,
- The space craft ejects water to the debris when they approach,
- The debris is pushed to the lower orbit and
- Under the influence of air resistance, the orbit of the debris gradually becomes lower and would be burned up in the earth's atmosphere.

Assumptions

- The mass of water is negligible compared to the mass of space craft, $\mathbf{\Delta m_{ij} \ll m}$.
- The space-based water ejected remains to be liquid in space.
- Water will scatter after hitting the target debris.
- The speed of the ejected water relative to the space craft is only dependent on the equippment on the space craft, so $\mathbf{\Delta v = constant.}$

Table 1: **Symbol Table for RSD Models**

Symbol	Definition
G	Gravitational Constant
M_i	The mass of space debris
m	The mass of space craft
m_e	The mass of the Earth
m_n	The mass of net
Δm_{ij}	The mass of ejected water for small debris
Δm_{i1}	The mass of ejected water for medium debris
Δm_{i2}	The mass of ejected water for large debris
Δm_{i3}	The mass of ejected water
m_{fuel}	The total mass of fuel that the space craft carries
m_{water}	The total mass of water that the space craft carries
m_{max}	The mass carrying capacity of the space craft
R_e	The radius of the Earth
R_i	The radius of an orbit in which the space craft moves
$R_i\prime$	The orbit in which the debris moves
R_j	The radius of another orbit in which the space craft moves
R_0	The radius of the orbit lower than R_i
v_i	The speed of space craft in the orbital of radius R_i
$v_i\prime$	The speed of space craft in the orbital of radius R_i after deceleration
v_0	The speed of space craft in the orbital of radius R_0
$v_0\prime$	The speed of space craft in the orbital of radius R_0 after acceleration
v_j	The speed of space craft in the orbital of radius R_j
v_d	The speed of debris after hit by water
Δv	The speed of ejected water relative to the space craft
E_e	The mechanical energy of the space craft on Earth
E_i	The mechanical energy of the space craft in the target orbital
E_{launch}	The mechanical energy required for sending the space craft to the target orbital
E_{total}	Total energy required from fuel in the life span of space craft
E_{sd}	The energy of self-destruction of space craft
$\Delta E_{i\to 0}$	Change of energy when moving from orbit R_i to orbit R_0
$\Delta E_{0\to j}$	Change of energy when moving from orbit R_0 to orbit R_j
t_1	The moment when the water is ejected from the space craft
t_2	The moment when the water hits the target debris
t_3	The moment when the space craft brushes past the debris
d	The diameter of the space craft
D	The diameter of the space debris
a	The length of semi-major axis of an ellipse
b	The length of semi-minor axis of an ellipse
c	The half-local length of an ellipse
ρ	Polar radius
$\rho\prime$	The mass density of space debris in certain size
θ	Polar angle

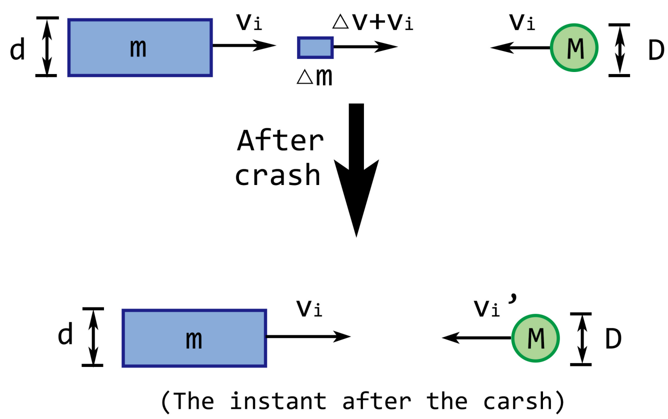

Figure 1: Motional Model.

Model Construction

- **Launch the space craft powered by rocket from the surface of the Earth to the orbit with radius $\mathbf{R_i}$.**
  The energy required in this process is

$$E_{launch} = E_i - E_e = -\frac{Gm_e m}{2R_i} + \frac{Gm_e m}{R_e}$$

- **The space craft and the target debris are both moving in the orbit with radius $\mathbf{R_i}$ and hence with speed $\sqrt{\frac{Gm_e}{R_i}}$. The space craft would eject water at time $\mathbf{t_1}$ and the water would hit the debris at $\mathbf{t_2}$.**
  The conservation of momentum indicates

$$v_d = \frac{M_i \cdot v_i - \Delta m_{ij} \cdot (v_i + \Delta v)}{M_i}$$

Hence $v_d < v_i$ shows that the debris would enter a transitional ellipse orbit, with the distance between the ellipse's perigee and the center of the Earth being R_0.
The conservation of angular momentum indicates that

$$v_0 \cdot R_0 = v_d \cdot R_i$$

Since $v_0 = \sqrt{\frac{Gm_e}{R_0}}$, we obtain the relationship

$$v_d \cdot R_i = \sqrt{Gm_e R_0}$$

This demonstrates the relative relationship of Δm_{ij} and M_i (and v_i), i.e, on the predefined orbit with radius R_i, the larger the mass of debris M_i, the larger the mass of required ejected water Δm_{ij}. Therefore we can define three specific mass of ejected water respectively for Large, Medium, and Small debris: Δm_{i1} for Small, Δm_{i2} for Medium, Δm_{i3} for Large $(\Delta m_{i1} < \Delta m_{i2} < \Delta m_{i3})$

- **At time t_1 when the water is ejected, there exists the minimum of the distance between the space craft and the debris $\mathbf{s_{min}}$.**
  As is illustrated in Figure, since $\Delta\theta \to 0, d_1 \approx d_2$, so the length of the third side of the triangle constructed by d_1, d_2, and $\Delta\theta$ is approximately equal to $d_1 - d_2$. But $d_1 - d_2 \geq \frac{d+D_i}{2}$ and $d_1 = R_i$, so $d_{2_{max}} = R_i - \frac{d+D}{2}$.

  Consider the transitional ellipse. Geometrically, $a = \frac{R_i+R_0}{2}, b = \sqrt{R_i \cdot R_0}, c = \frac{R_i-R_0}{2}$.
  The distance between the left directrix and the left focal point $p = \frac{a^2}{c} - c$, and eccentricity $e = \frac{c}{a}$. Then $\rho = \frac{ep}{1-ecos\theta}$, from which we can obtain $\Delta\theta$. During time t_2 and t_3, the debris moves along the ellipse orbital, where $\rho_1 = d_1, \rho_2 = d_2$.

  Proper approximation yields the area of the sector swept is $S = \frac{1}{2}d_1 d_2 sin\Delta\theta$.

  Kepler's third law indicates
  $$\frac{(t_3 - t_2)_{min}}{T} = \frac{S}{\pi ab},$$
  and
  $$S_{min} = v_i \cdot (t_3 - t_2)_{min} + \Delta s + 2v_i \cdot \frac{S_{min}}{v_i + \Delta v}$$
  From $\Delta\theta \to 0, \Delta s \approx \Delta s\prime$. Using cosine theorem it is clear that $\Delta s\prime = \sqrt{d_1^2 + d_2^2 - 2d_1 d_2 cos\Delta\theta}$, from which we can obtain s_{min}.

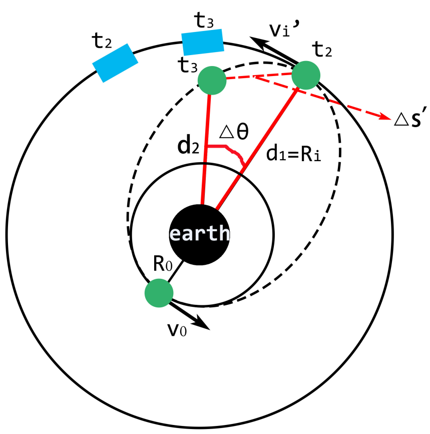

Figure 2: Orbit Transformation II.

- **The total energy required for the whole process from launching to self-destruction consists of the enrgy of fuel and water, which can be quantized in terms of mass.**
  For fuel,
  $$E_{total} = E_{launch} + \sum \Delta E_{i\to j} + E_{sd},$$
  where $\Delta E_{i\to j} = -\frac{Gm_e m}{2R_i} + \frac{Gm_e m}{2R_j}$. We can derive m_{fuel} from E_{total}.

  For water, $m_{water} = \sum \Delta m_{i1} + \sum \Delta m_{i2} + \sum \Delta m_{i3}$.

Considering the the mass carrying capacity of the space craft, i.e., $m_{fuel} + m_{water} <= m_{max}$, we can determine the energy assumption in the life span of the space craft.

3.2 Net Catching Model

Intuitively, net catching model is based on four actions:

- Move the space craft carrying the net to the orbit lower than the one in which the target debris is moving,
- Capture the debris by net when the Earth's core, space craft, and debris is colinear,
- After capturing, the space craft and debris move together toward lower orbit, and
- Release the debris, which would be burned up in the earth's atmosphere.

Assumptions

- The mass of net is negligible compared to the mass of space craft, $\mathbf{m_n} \ll \mathbf{m}$.
- The orbit of space craft and the orbit of the debris are close, $\mathbf{R_i} < \mathbf{R_{i'}}$ and $\mathbf{R_i} \approx \mathbf{R_{i'}}$.
- Duration between casting the net and the net's arriving the debris is negligible.

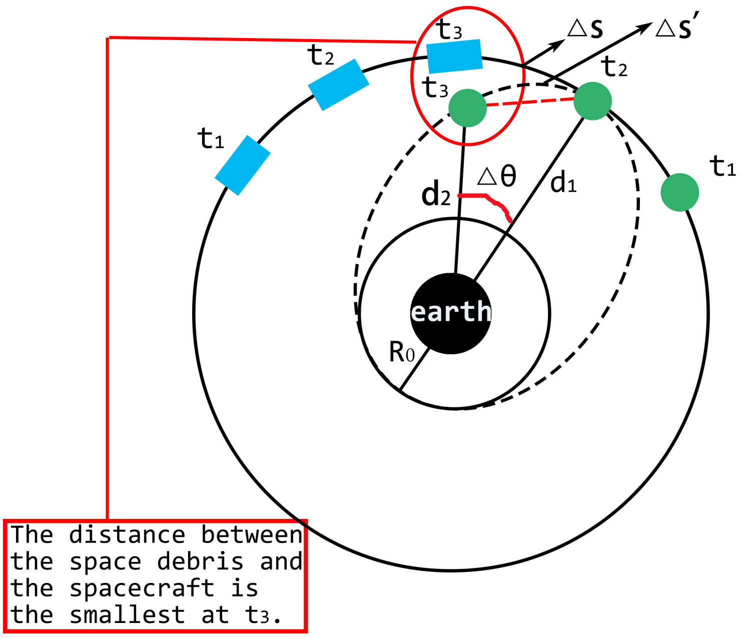

Figure 3: Orbit transformation model.

Model construction

- **Launch the space craft powered by rocket from the surface of the Earth to the orbit with radius R_i, while the orbit in which the target debris is moving has radius $R_{i'}$.**
  The energy required in this process is

$$E_{launch} = E_i - E_e = -\frac{Gm_e m}{2R_i} + \frac{Gm_e m}{R_e}$$

- **When the space craft catches up with the debris(i.e., when the Earth's core, space craft, and debris is colinear), cast the net towards the debris and capture it.**

  Since $R_i < R_i\prime$ and $v_i = \sqrt{\frac{Gm_e}{R_i}}$, the space craft is possible to catch up with the target debris. We assume $R_i \approx R_i\prime$, so $v_i \approx v_i\prime$, which shows after capturing the relative position of space craft and debris would not change.

- **After capturing, the space craft decelerates by burning fuel, and the system of space craft and the debris enters the transitional ellipse orbit. The distance between ellipse's perigee and the center of the Earth is $\mathbf{R_0}$.**

  The conservation of angular momentum indicates

$$v_0 \cdot R_0 = v_i\prime \cdot R_i$$

  Hence the speed of the system of space craft and debris after deceleration is $v_i\prime = \sqrt{\frac{Gm_e R_0}{R_i^2}}$. The required energy for the deceleration is

$$\Delta E_{i\to 0} = \frac{1}{2}(m + M_i)v_i^2 - \frac{1}{2}(m + M_i)v_i\prime^2 = \frac{Gm_e(m + M_i)}{2R_i}(1 - \frac{R_0}{R_i})$$

- **As the system arrives at the perigee of ellipse orbit, the space craft now accelerates by burning fuel, and simultaneously releases the debris.**

  The debris is now exposed to air resistance, decelerate gradually and finally would be burned up in the most ideal case.

  The space craft moves along an ellipse orbit similar but opposite to that of deceleration and finally reaches an orbit with radius R_j.

  The speed of space craft after acceleration is $v_0\prime = \frac{v_j \cdot R_j}{R_0} = \sqrt{\frac{Gm_e R_j}{R_0^2}}$, and the required energy

$$\Delta E_{0\to j} = \frac{1}{2}mv_0\prime^2 - \frac{1}{2}mv_0^2 = \frac{Gm_e m}{2R_0}(\frac{R_j}{R_0} - 1)$$

- **The model predicts that the total energy required for this whole process from launching to the self-destruction of the space craft is**

$$E_{total} = E_{launch} + \sum \Delta E_{i\to 0} + \sum \Delta E_{0\to j} + E_{sd},$$

  where E_{sd} is the energy required for the space craft to move itself from orbit R_j to the atmosphere so as to make itself burnt up.

4 DDS Model

As the time changes, the distribution of the space debris will differ from time to time in space. Here, we set up the DDS model by introducing the ORDEM2000 and the NASA91 space debris model.

4.1 ORDEM2000 space debris model

"ORDEM2000 (ORbital Debris Engineering Model) is a low earth orbit debris model published by NASA in 2000" (Han 32). Enormous database and complicated algorithms are applied to the ORDEM2000 model so that this model "presents the space debris data in the form of software package" (Han 32). The user interface of this software is as Figure XX.

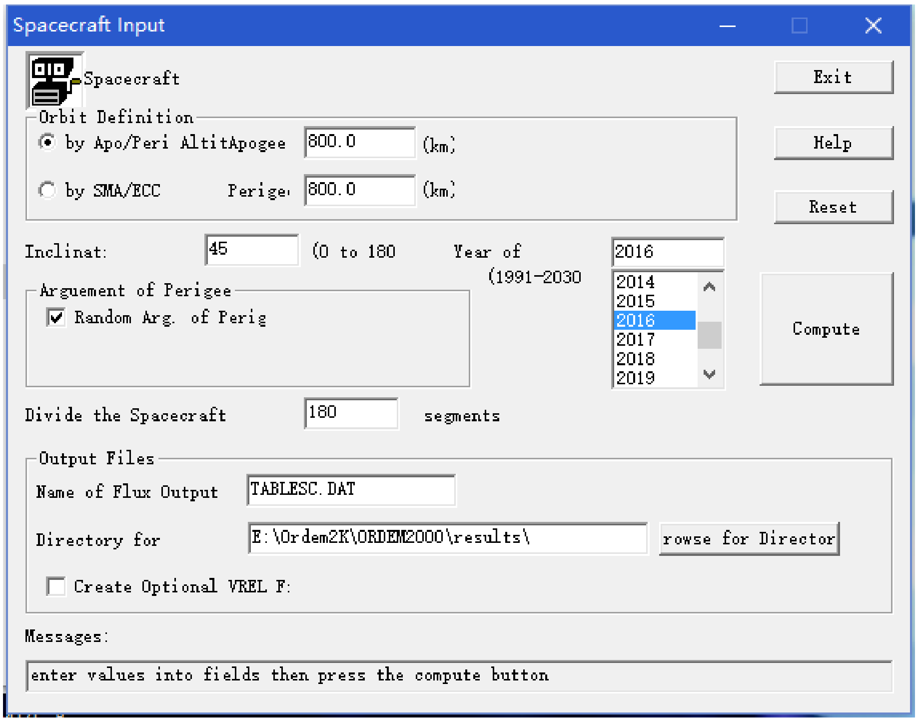

Figure 4: User interface of the software of the ORDEM2000 space debris model.

As long as we input the related data (the altitude of the apogee and perigee, and the inclination angle) of the target orbit, the target year, and the target division of the space according to the latitude, we can get the spatial distribution of the space debris on a certain orbit in a certain year as Figure XX.

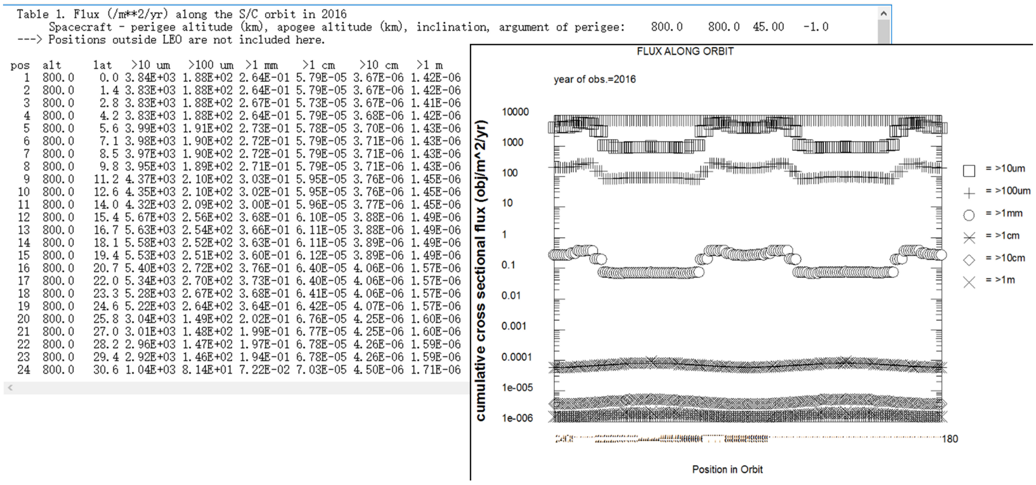

Table 1. Flux (/m**2/yr) along the S/C orbit in 2016
Spacecraft - perigee altitude (km), apogee altitude (km), inclination, argument of perigee: 800.0 800.0 45.00 -1.0
---> Positions outside LEO are not included here.

pos	alt	lat	>10 um	>100 um	>1 mm	>1 cm	>10 cm	>1 m
1	800.0	0.0	3.84E+03	1.88E+02	2.64E-01	5.79E-05	3.67E-06	1.42E-06
2	800.0	1.4	3.83E+03	1.88E+02	2.64E-01	5.79E-05	3.67E-06	1.42E-06
3	800.0	2.8	3.83E+03	1.88E+02	2.67E-01	5.73E-05	3.67E-06	1.41E-06
4	800.0	4.2	3.83E+03	1.88E+02	2.64E-01	5.79E-05	3.68E-06	1.42E-06
5	800.0	5.6	3.99E+03	1.91E+02	2.73E-01	5.78E-05	3.70E-06	1.43E-06
6	800.0	7.1	3.98E+03	1.90E+02	2.72E-01	5.79E-05	3.71E-06	1.43E-06
7	800.0	8.5	3.97E+03	1.90E+02	2.72E-01	5.79E-05	3.71E-06	1.43E-06
8	800.0	9.8	3.95E+03	1.89E+02	2.71E-01	5.79E-05	3.71E-06	1.43E-06
9	800.0	11.2	4.37E+03	2.10E+02	3.03E-01	5.95E-05	3.76E-06	1.45E-06
10	800.0	12.6	4.35E+03	2.10E+02	3.02E-01	5.95E-05	3.76E-06	1.45E-06
11	800.0	14.0	4.32E+03	2.09E+02	3.00E-01	5.96E-05	3.77E-06	1.45E-06
12	800.0	15.4	5.67E+03	2.56E+02	3.68E-01	6.10E-05	3.88E-06	1.49E-06
13	800.0	16.7	5.63E+03	2.54E+02	3.66E-01	6.11E-05	3.88E-06	1.49E-06
14	800.0	18.1	5.58E+03	2.52E+02	3.63E-01	6.11E-05	3.89E-06	1.49E-06
15	800.0	19.4	5.53E+03	2.51E+02	3.60E-01	6.12E-05	3.89E-06	1.49E-06
16	800.0	20.7	5.40E+03	2.72E+02	3.76E-01	6.40E-05	4.06E-06	1.57E-06
17	800.0	22.0	5.34E+03	2.70E+02	3.73E-01	6.40E-05	4.06E-06	1.57E-06
18	800.0	23.3	5.28E+03	2.67E+02	3.68E-01	6.41E-05	4.06E-06	1.57E-06
19	800.0	24.6	5.22E+03	2.64E+02	3.64E-01	6.42E-05	4.07E-06	1.57E-06
20	800.0	25.8	3.04E+03	1.49E+02	2.02E-01	6.76E-05	4.25E-06	1.60E-06
21	800.0	27.0	3.01E+03	1.48E+02	1.99E-01	6.77E-05	4.25E-06	1.60E-06
22	800.0	28.2	2.96E+03	1.47E+02	1.97E-01	6.78E-05	4.26E-06	1.59E-06
23	800.0	29.4	2.92E+03	1.46E+02	1.94E-01	6.78E-05	4.26E-06	1.59E-06
24	800.0	30.6	1.04E+03	8.14E+01	7.22E-02	7.03E-05	4.50E-06	1.71E-06

Figure 5: Output data of the software of the ORDEM2000 space debris model.

4.2 NASA91 space debris model

"NASA91 model is a low earth orbit debris model published by NASA in 1991" (Han 31). The NASA91 space debris model is the base model of the ORDEM2000 space debris model. The main functions of the NASA91 model is similar to the ORDEM2000 model, but the NASA91 model provides an analytic expression for the spatial distribution of the space debris. In addition, a mass density model is included in the NASA91 model. We can apply this mass density model to get the exact mass of the space debris in different sizes.

The mass density model is formulated as:

$$\begin{cases} \rho' = 2.8D^{-0.74}\ [\mathrm{g/cm^3}] & D \geq 0.62[\mathrm{cm}] \\ \rho' = 4.0\ [\mathrm{g/cm^3}] & D < 0.62[\mathrm{cm}] \end{cases} \tag{1}$$

In the above equation, D is the diameter of the space debris, and ρ' is the mass density of the space debris in a certain size.

5 Assessment of Viable Economical Opportunity

It is essential to take the space debris distribution described by ORDEM 2000 model into account when we are to assess the economical viability of a specific RSD models. The time dependency of the debris distribution results in the time dependency of our assessment model and other time-related variants. We explicitly define two factors and propose a criteria to describe how two factors are eventually used to analyze the performance of RSD models.

Variables and Criteria

Considering the interests of private firms or business, our assessment of the potential existence of economically attractive opportunity is based on three variables: Benefit, Cost and Risk. To assess the performance more effectively, we define

$$\text{Gain Ratio (GR)} = \text{Benefit}/\text{Cost}$$

Hence in our model, GR and Risk together compose the criteria for assessment.

Assumptions and Justifications

- The space debris distribution is the function of only time (year), debris' altitude and latitude.
- The time variability of space debris distribution is related to initial year and the duration of the space craft's life span.
- Risk assessment doesn't consider the possibility of collisions between space craft and debris when space craft is changing orbits.
- When the space craft and the debris collide each other, the total energy won't disperse.
- The space craft stays in one orbit for one year.
- Benefit is evaluated by the total mass of debris that RSD models remove.
- Cost is evaluated by the energy assumption (such as fuel assumption) in the life span of RSD models.

Gain Ratio Assessment

To effectively obtain the specific role of each factors (time, altitude, and latitude) in the space debris distribution, we make use of the technique of control variate method.

- **By controlling time and altitude and changing latitude, the simulation in ORDEM 2000 software indicates that there always exists an optimal latitude where GR is maximum.**
  This optimal latitude is time-independent.
- **Fix this optimal latitude for possibly highest Gain Ratio. The simulation in ORDEM 2000 software provides the debris number distribution in different altitudes and year.**

For example, if we define the initial year as year 2016 and the duration of the space craft's life span as three years. Consider the altitude 850, 900, and 950, then we can obtain debris number distribution as:

Height\Size	Large	Medium	Small
850	n_{11}	n_{12}	n_{13}
900	n_{21}	n_{22}	n_{23}
950	n_{31}	n_{32}	n_{33}

Table 2: Debris Number Distribution for year 2016

Similarly for following two years 2017 and 2018 we obtain debris number distributions.

Height\Size	Large	Medium	Small
850	n'_{11}	n'_{12}	n'_{13}
900	n'_{21}	n'_{22}	n'_{23}
950	n'_{31}	n'_{32}	n'_{33}

Table 3: Debris Number Distribution for year 2017

Height\Size	Large	Medium	Small
850	n''_{11}	n''_{12}	n''_{13}
900	n''_{21}	n''_{22}	n''_{23}
950	n''_{31}	n''_{32}	n''_{33}

Table 4: Debris Number Distribution for year 2018

- **According to the defined initial year and the duration of life span, we have different orders of debris removement in terms of orbit altitude.**
  For example, we can first remove the debris in orbit with altitude 850, next in the one with altitude 900, finally 950. Another choice may be first 850, next 950, and finally 900.
  This results in different combinations of debris removing order and thus a set of different Gain Ratios $\{\mathbf{G_1}, \mathbf{G_2}, \dots \mathbf{G_n}\}$.

- **In order to analyze the set of Gain Ratios, we need to standardize and obtain an average Gain Ratio which represents the performance of the RSD model.**
  Specifically, we divide all elements (GRs) in the set by the duration y to get the Gain Ratio per year $\{x_1, x_2, \dots x_n\}$, then curve these x_i to get the standardized representative value.

Risk Assessment

We define the risk to be expressed by the case when the space craft collides with the debris from other orbits, where two orbits coincide.
We use the relative kinetic energy E_r to quantize the risk:

$$E_r = \frac{1}{2}mv_r^2,$$

where m is related to the mass density model proposed by NASA91(Han 32):

$$\text{“}\rho\prime = 2.8d^{-0.74}, d \geq 0.62cm;\ \rho\prime = 4.0g/cm^3, d < 0.62cm.\text{”}$$

- **Since our DDS model can describe the debris distribution in any orbit around the Earth, every debris' relative velocity with respect to the space craft at the point of collisions can be derived.** Then for every v_r, we can obtain $E_r = \frac{1}{2}mv_r^2$ and thus

$$\text{Risk} = \sum E_r = \sum \frac{1}{2}mv_r^2$$

- **Each year's movement in one orbit corresponds to one Risk values $\mathbf{R_i}$, and hence we have a set of Risk value $\{\mathbf{R_1}, \mathbf{R_2}, \ldots \mathbf{R_n}\}$ during the life span of space craft**.
  Risk is time-dependent since our model correlates Risk with relative velocity, which is time-dependent according to DDS model.

- **In order to analyze the set of Risks, we need to standardize and obtain an average Risk which represents the performance of the RSD model.**
  Specifically, we divide all elements (Risks) in the set by the duration y to get the Gain Ratio per year $\{s_1, s_2, \ldots s_n\}$, then curve these s_i to get the standardized representative value.

Construction of Assessment of Economical Opportunity

The constructions for Gain Ratio Assessment and Risk Assessment produce the representative Gain Ratio and Risk values which are of the same standard. In order to determine the best RSD models, we need to obtain the weights of Gain Ratio and Risk in different RSD models.

We apply analytic hierarchy process to establish the weight relationship:

- **The hierarchy is composed of the objective, judgment criteria, and RSD candidates,** which is illustrated in Figure.

- **Based on the objective we determine the relative preference of judgment criteria. Assign fractions to demonstrate such a preference. In our model, we would get a 2 × 2 matrix.**
  For example, considering the objective, if we attach importance to Gain Ratio 5 times less than Risk, then the corresponding fraction is 1/5.

- **Based on each of the judgment criteria we determine the relative preference of RSD candidates. Assign fractions to demonstrate such a preference. For each judgment criteria, we would get a 2 × 2 matrix.**
  For example, considering the factor of Gain Ratio, if we prefer water jets model 9 times more than net catching model, then the corresponding fraction is 9/1.

- **The eigenvector corresponding to the largest eigenvalue for each matrix describes each relative weight.**
  However, we need to check the reliability of this eigenvector.
  Define $CI = \frac{\lambda - n}{n-1}$, and RI to be average of CI of a large number of random matrices.
  If $\frac{CI}{RI} < 0.1$, then we say the eigevector is reliable.

- **Finally, the weight of RSD models with respect to the objective is derived by the matrix product of two matrices composed of largest eigenvectors for each submatrices.**

Hence we can obtain both the weights and representative values for RSD candidates. The product of the weight and the representative value means the final quantitative assessment value, from which we can judge the best alternatives and thus make a proper recommendation.

Table 5: **Symbol Table for Assessment Models**

Symbol	Definition
GR	Gain Ratio
n_{i1}	The number of large debris moving in 2016
n_{i2}	The number of medium debris moving in 2016
n_{i3}	The number of small debris moving in 2016
n'_{ij}	Same for n_{ij}but in 2017
n''_{ij}	Same for n_{ij}but in 2018
G_i	Grain Ratios
x_i	Grain Ratios after unitization
R_i	Risk value
s_i	Risk value after unitization
E_r	Relative kinetic energy
v_r	Relative velocity
$\rho\prime$	The mass density of space debris in certain size
d	The diameter of debris
λ	Largest eigenvalue for a matrix
n	The dimension of a matrix
CI	Coincidence indicator
RI	Random coincidence indicator

6 Model Simulation and Further Exploration

Finding the optimal latitude

In this simulation, we define the interval of angles by 45° and obtain that

Latitude degree	Total mass of debris (Gain Ratio)
0°	8.36×10^5
45°	1.09×10^6
90°	6.54×10^5
135°	1.46×10^6

Table 6: The relationship between Total mass of debris and orbit latitude

We see that the latitude with inclination 135° corresponds to largest Gain Ratio, so this particular latitude is deemed optimal.

What if more accurate judgment of optimal latitude is required?

The 45° interval is convenient to manipulate, but may be too large to lose some accuracy. In fact, if we reduce the interval to 15°, the simulation shows that the optimal latitude approaches 105°. Hence the smaller the interval of angles, the more likely that we can find the most optimal latitude.

Water Jets simulation model (Assessment)

Consider the year t_k and debris in orbit with radius R_i. When hit by water, the debris would decelerate to

$$v_i\prime = \frac{M_j v_i - \Delta m_{ij}(v_i + \Delta v)}{M_j},$$

and since $v_i\prime \cdot R_i = \sqrt{Gm_e R_0}$, we can obtain the mass of water Δm_{ij} required to remove the debris with mass M_j:

$$\Delta m_{ij} = \frac{M_j \sqrt{\frac{Gm_e}{R_i}}(1 - \sqrt{R_0})}{\sqrt{\frac{Gm_e}{R_i}} + \Delta v}$$

Since we can obtain the mass and number distributions of debris, we can know the mass of water required to remove the debris at given orbits and time. Similarly, we can get the mass of space debris that can be removed when the space craft moves to specific orbits at specific time. We can demonstrate by two matrices.

The mass of water used when the space craft goes to the orbit with radius R_i with time t_j:

$$\begin{pmatrix} & R_1 & R_2 & \dots & R_q \\ t_1 & m_{water}(1,1) & m_{water}(1,2) & \dots & m_{water}(1,q) \\ \vdots & & & & \vdots \\ t_p & m_{water}(p,1) & \dots & & m_{water}(p,q) \end{pmatrix}$$

The mass of debris removed when the space craft goes to the orbit with radius R_i with time t_j::

$$\begin{pmatrix} & R_1 & R_2 & \dots & R_q \\ t_1 & m_{debris}(1,1) & m_{debris}(1,2) & \dots & m_{debris}(1,q) \\ \vdots & & & & \vdots \\ t_p & m_{debris}(p,1) & & \dots & m_{debris}(p,q) \end{pmatrix}$$

Here we assume the years that the space craft deals with the debris correspond exactly to the number of orbits that are "cleaned up". We focus on the average value of Gain Ratio with respect to years for assessment.

If the space craft deals with the debris for one year

There are $C_p^1 C_q^1$ choices to determine when and where the space craft go to. In this situation we have the constraints:

- $m_{fuel}(r,s) + m_{water}(r,s) + m_0 \leq m_{max}$, where m_0 denotes the pure mass of space craft.
- The mass of fuel for the condition(t_r, R_s) is

$$m_{fuel}(r,s) = \frac{m_0 + m_{water}(r,s)}{\frac{k_1}{Gm_e(1/R_e+1/(2R_0)-1/R_s)} - 1}, \text{ where } E_{total} = k_1 \cdot m_{fuel}$$

Then Cost(r,s) = $c_1 \cdot m_{fuel}(r,s) + c_2 \cdot m_{water}(r,s)$, where c_1, c_2 are constants.
Benefit(r,s) = $k_2 \cdot m_{debris}(r,s)$, where k_2 is a constant.
Hence we can find the maximum Benefit(r,s) / Cost(r,s).

If the space craft deals with the debris for more than one year

In this case we have some additional constraints:

- The adjacent time is continuous. We have (p-N+1) choices for time.
- The orbits selected must not repeat. We have P_q^N choices for orbits.

Hence we have $(p - N + 1) \cdot P_q^N$ choices overall.
Suppose we choose the states (r, x_1), $(r+1, x_2), \dots$,and $(r+N-1, x_N)$.
Then

$$m_{fuel}(r,x_1)(r+1,x_2)\cdots(r+N-1,x_N) = \frac{m_0 + m_{water}(r,x_1)(r+1,x_2)\cdots(r+N-1,x_N)}{\frac{k_1}{Gm_e(1/R_e+1/(2R_0)-1/R_{x_1})} - 1}.$$

Therefore, we can obtain the benefit and cost, from which we can find the maximum average value (Benefit(r,x_1)(r+1,x_2)$\cdots$(r+N-1,x_N) / Cost(r,x_1)(r+1,x_2)$\cdots$(r+N-1,x_N))/ N, where N is the number of years.

Qualitatively, the equation of m_{fuel} shows that the smaller the length of R_{x_1}, the less the assumption of fuel. Therefore, we may choose the first orbit which has the radius as short as possible.

Net capturing simulation model(Assessment)

Suppose the space craft deals with the debris for N years. Consider the set of states $(t_i, R_{x_1}), (t_{i+1}, R_{x_2}), \ldots, (t_{i+N-1}, R_{x_N})$, which shows the order of space craft's handling debris. Then benefit is quantized by

$$M_2 \cdot (n_{ix_1} + n_{(i+1)x_2} + \cdots + n_{(i+N-1)x_{N_2}})$$

The total energy required in the life span of the space craft is

$$E_{total} = E_{Earth \to x_1} + n_{ix_{12}} E_{x_1 \to 0} + (n_{ix_{12}} - 1) E_{0 \to x_1} + n(i+1)x_2(E_{x_2 \to 0} + E_{0 \to x_2}) \ldots$$

Since $E_{total} \propto m_{fuel}$, we can obtain (Benefit / Cost) / N.

6.1 Simulation of the Performance of RSD Models in Gain Ratio

To simulate the water-jet model and the net-catching model, the proportional constants k_1, k_2, R_0, m_0 and the unit price of water and rocket fuel are reasonably assumed. To simplify the tedious calculation, only three years 2016, 2017, 2018 and three satellite orbits $800km$, $900km$, $1000km$ are considered here.

As a result, the water-jet model has two 3-time-3 matrixes, showing the mass of debris on the orbits at the specific time and the mass of water used to remove these debris.

$$\begin{array}{cc} & \begin{matrix} \text{800km} & \text{900km} & \text{1000km} \end{matrix} \\ \begin{matrix} 2016 \\ 2017 \\ 2018 \end{matrix} & \begin{pmatrix} 35300 & 28500 & 29200 \\ 37500 & 29700 & 30000 \\ 38500 & 30100 & 30500 \end{pmatrix} \end{array} \tag{2}$$

$$\begin{array}{cc} & \begin{matrix} \text{800km} & \text{900km} & \text{1000km} \end{matrix} \\ \begin{matrix} 2016 \\ 2017 \\ 2018 \end{matrix} & \begin{pmatrix} 1780 & 1620 & 1850 \\ 1890 & 1690 & 1900 \\ 1940 & 1710 & 1930 \end{pmatrix} \end{array} \tag{3}$$

Similarly, the net-catching model has one 3-time-3 matrix, showing the mass of medium debris on the orbit at specific time.

$$\begin{array}{cc} & \begin{matrix} \text{800km} & \text{900km} & \text{1000km} \end{matrix} \\ \begin{matrix} 2016 \\ 2017 \\ 2018 \end{matrix} & \begin{pmatrix} 32254.2 & 23750.8 & 26829.6 \\ 34306.7 & 24777.1 & 27416.1 \\ 35186.4 & 25070.3 & 27709.3 \end{pmatrix} \end{array} \tag{4}$$

For the water-jet model, if we take the time duration N as 1. there are 9 possible plans at all. The ratios of benefit and cost per year are listed in the following table.

23.8882	20.0277	18.4612
24.4048	20.3244	18.6379
24.6267	20.4448	18.7533

Table 7: Ratios of benefit and cost per year when $N = 1$.

Obviously, the most optimized ratio is 24.6274.

If we take the time duration N as 2, there are 12 possible plans at all. The ratios of benefit and cost per year are listed in the following table.

13.6433	13.4322	13.0888	12.6458
11.8794	11.6608	13.7864	13.5188
13.2325	12.7177	11.9589	11.7170

Table 8: Ratios of benefit and cost per year when $N = 2$.

So, the most optimized ratio is 13.7864.

If we yak the time duration N as 3, there are 6 possible plans at all. The ratio of benefit and cost per year are listed in the following table.

13.9826	13.9899	13.7370
13.7653	13.4734	13.4940

Table 9: Ratios of benefit and cost per year when $N = 3$.

So, the most optimized ratio is 13.9899.

For the net-catching model, the procedures are the same. The relationship of the most optimized ratio and the time duration N are shown in the following table.

Time Duration N	Best Optimized Ratio $\frac{benefit}{cost}/N$
1	1.5299
2	1.7759
3	1.9053

Table 10: Ratios of benefit and cost per year when $N = 3$.

6.2 Simulation of the Performance of RSD Models in Risk Value

6.2.1 Assumption

- We only consider the damage caused by the collision of the spacecraft and space debris when we calculate the risk value.
- The spacecraft will only collide with the space debris coming from the orbits in other latitudes. To be exact, among these orbits we only consider the orbit which is perpendicular to the motional orbit of the spacecraft.
- The spacecraft will only collide with space debris at four certain positions on the motional orbit of the spacecraft. And these four positions are uniformly distributed on the motional orbit of the spacecraft.
- The space debris in a certain size is also uniformly distributed on its motional orbit.
- The collision between the spacecraft and the space debris is a central collision so that the location of the collision point on the spacecraft won't affect the value of the relative kinetic energy a lot.

- The radiuses of the motional orbits of the spacecraft and the space debris are fixed as a constant R_i throughout the study of risk assessment.

6.2.2 Theoretical Background for Risk Model

According to our assumption, we only need to consider four uniformly distributed collision positions. That means if we see the motional orbit of the spacecraft from the north pole, four positions A, B, C, and D quarter this orbit as Figure XX.

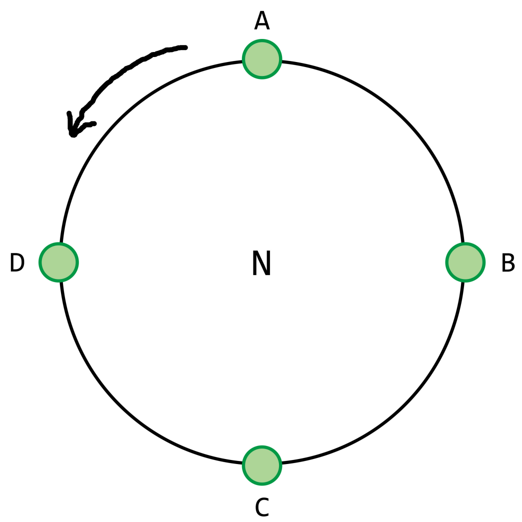

Figure 6: Distribution of four given collision positions.

Also, by our assumption the space debris will only come from the orbit (We call this orbit as debris orbit in the following context.) which is perpendicular to the motional orbit of the spacecraft. Then if we see the motional orbit of the spacecraft from position A to the center of the earth, we can see the figure as Figure XX.

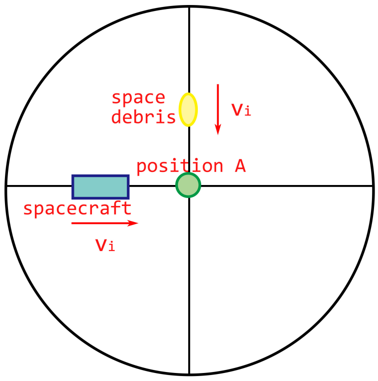

Figure 7: Motion of the spacecraft and space debris seen from position A to the center of the earth.

Then for each orbit on which space debris moves, it have debris in different sizes. For the debris in a certain size, it is uniformly distributed on its orbit according to our assumption. So the debris may be distributed as Figure XX on its orbit.

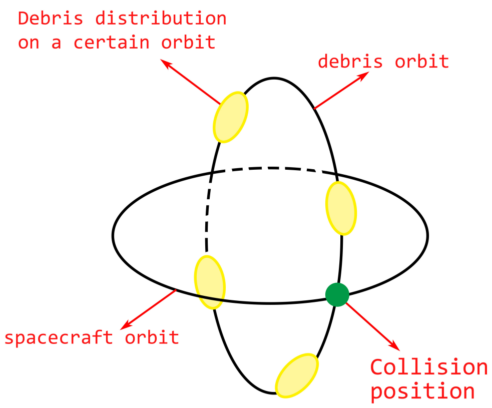

Figure 8: Distribution of the space debris in a certain size on the debris orbit.

Then according to the distribution of the space debris in a certain size on its orbit, we can conclude that there will exist two cases at a certain collision position on the motional orbit of the spacecraft. These two cases are the case that collision happens and the case that collision doesn't happen. And these two cases are vividly shown in Figure XX.

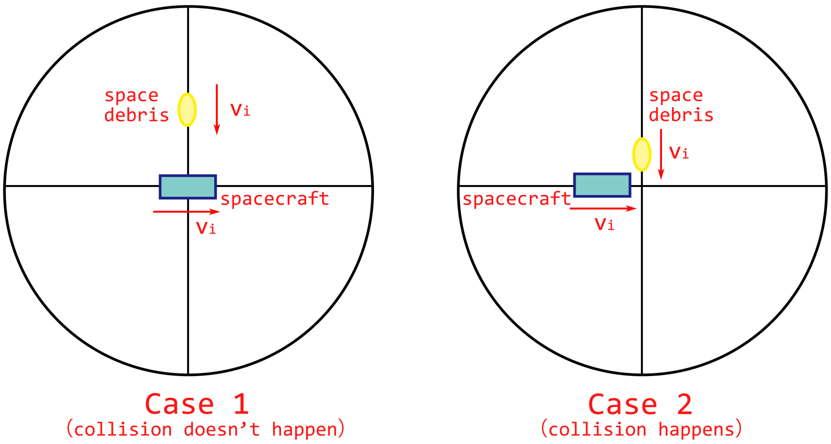

Figure 9: Two cases at a certain collision position.

Here we only need to consider the case when the collision happens. Then by the mass density model in the NASA91 space debris model, we can get the mass density ρ_i' of the space debris in different sizes. Since throughout the assessment model we set the diameters of small debris, medium debris, and large debris as 0.1[m], 1[m], and 5[m], we then can calculate the volume V_i of the space debris in different sizes. Then the mass of the debris in different sizes can be calculated.

Then since the debris orbit is perpendicular to the motional orbit of the spacecraft and the radiuses of these two orbits are both fixed at R_i, we then can get the velocity v_i corresponding to R_i, and we can further get the relative velocity of the debris with respect to the spacecraft. Here we only care about the magnitude of the relative velocity v_r:

$$v_r = \sqrt{2}v_i = \sqrt{\frac{2GM_e}{R_i}} \tag{5}$$

Then we can calculate the relative kinetic energy by the formula:

$$\begin{aligned} E_{r_i} &= \frac{1}{2}mv_r^2 \\ &= \frac{1}{2}\rho_i' V_i \frac{2GM_e}{R_i} \\ &= \frac{\pi\rho_i' D_i^3 GM_e}{6R_i} \end{aligned}$$

Since we choose the best latitude (135°) in the first step of the Model Simulation section and the debris orbit is perpendicular to the motional orbit of the spacecraft by our assumption, we then can get that the latitude of the debris orbit is 135-90=45°. With this data, we can get the spatial distribution of the space debris on a certain orbit in a certain year through the software of the ORDEM2000 space debris model.

Now we can get the E_{r_i} corresponding to the size of the debris in a certain year. Also, the number of debris in different sizes has been obtained from the ORDEM2000 software. Suppose the ratio of the number of the debris in three sizes we get is $x : y : z$, then we can get the weight distribution of the debris in three sizes. Then the final risk value s_i in a certain year can be obtained by weighted average and mathematical expectation method.

Then we first consider four collision positions, and then decide how many positions of these four collision positions are the collision-happening ones and how many are not collision-happening ones. We can then get a table of these all situations. Further consider the size allocation at four collision positions, we then conduct permutation and combination to get a list of possible situations. Calculate their risk values s_i respectively and average these values, and we can get a representative risk value in a certain year.

6.2.3 Simulation of the Risk Value Assessment

First subsititute exact values into the parameters:

$$D_1 = 0.1[\text{m}],\quad D_2 = 1[\text{m}],\quad D_3 = 5[\text{m}]R_i = 7171000[\text{m}],\quad GM_e = 3.9867{\times}10^{14}[\text{m}^3/\text{s}^2]t_k = 2016,\quad N = 3$$

Then we can get the mass density of the debris in three sizes by mass density model:

$$\rho_i' = 2.8D_i^{-0.74}[\text{g/cm}^3]\rho_1' = 0.5095[\text{g/cm}^3],\quad \rho_2' = 0.0927[\text{g/cm}^3],\quad \rho_3' = 0.0282[\text{g/cm}^3],$$

Since $E_{r_i} = \frac{\pi\rho_i' D_i^3 GM_e}{6R_i}$, we then have:

$$E_{r_1} = 1.4832\times10^{13}[\text{J}],\quad E_{r_2} = 2.6989\times10^{15}[\text{J}],\quad E_{r_3} = 1.0253\times10^{17}[\text{J}],$$

For the water jet model, the value of E_{r_i} can be directly used for further calculation. However, for the net catch model, the extent to which the spacecraft and the devices are affected by the collision is greater. Because the net is exposed to the space, and the material of the net is not as strong as the surface of the spacecraft. Therefore, here we multiply E_{r_i} by a risk constant q=1.2.

$$E_{r_1}' = 1.7798\times10^{13}[\text{J}],\quad E_{r_2}' = 3.2387\times10^{15}[\text{J}],\quad E_{r_3}' = 1.2304\times10^{17}[\text{J}],$$

Then we can obtain a group of data about the number of the debris in different sizes by the ORDEM2000 software. With this group of data we can calculate the weight distribution of the number of the debris in three sizes as follows:

/	small	medium	large
2016	92.15%	5.71%	2.14%
2017	92.18%	5.75%	2.07%
2018	92.39%	5.59%	2.02%

Table 11: Weight distribution of the number of the debris in three sizes.

Then we consider four cases that 1 or 2 or 3 or 4 positions of the four collision positions make the real collision happens. Then for each case, we have different combinations of debris in three sizes. We can see that from the following table:

Real-collision position quantity	Quantity of combinations of three sizes
1	3
2	6
3	10
4	15

Table 12: Combinations of debris in three sizes.

To make it more clearly, we take the case when real-collision position quantity is 2, and its combinations are as follows (it is expected to have 6 methods):

Methed No.	Small debris quantity	Medium debris quantity	Large debris quantity
1	2	0	0
2	0	2	0
3	0	0	2
4	1	0	1
5	0	1	1
6	1	1	0

Table 13: Combinations of debris in three sizes when real-collision position quantity is 2.

Then according to the method quantity and the exact quantity allocation of the debris in three sizes, we can design an algorithm (the code is attached in the end) which multiplies the weight of the debris in three sizes by the above factors to get a score for the risk value at a certain year. Since we take t_k=2016, and N=3, and the larger the E_r is, the smaller the risk value is, finally we can get following table:

Year	Score (water)	Score (net)
2016	-3.9684×10^{16}	-4.7620×10^{16}
2017	-3.9510×10^{16}	-4.7412×10^{16}
2018	-3.9488×10^{16}	-4.7385×10^{16}

Table 14: Risk value score of water jet model and net catch model.

6.3 Grades Standardization Model

Through the assessment of the Gain Ratio per year, we have got a group of values of x_i. In addition, we have also got a group of values of s_i through the assessment of the Risk Value. However, these two groups of data are not obtained under the same evaluation criterion. That means these two groups of data have different value distribution, and the full marks of these two groups are different. Therefore, we need to adjust the values of these two groups of data so that the evaluation criterion is standardized. To achieve this goal, we introduce a grades standardization model which is frequently applied to adjust the raw score of students in universities.

- **Definition of Raw Score and Standard Score:**

1. Raw score is "the score directly presenting on the examination papers" in universities (Zhao 1). In this problem, we take the values of x_i and s_i that we directly obtain from calculation as the raw score.
2. Standard score is the score which is obtained from "the standardized transformation of the raw score" (Zhao 1). It quantitively reflects the extent to which each value ranks in its data group. Also, we can apply the same standard average score to two groups of data. Then the data in these two different groups is comparable.

- **Calculation of the Standard Score**

1. "According to the principle of the educational statistics", the standard score and the raw score are formulated as:

$$Z = (X - A)/S \tag{6}$$

   In this formula, Z is the standard score, and X is the raw score. In addition, A is the average value of all Xs in a certain group, and S is the standard deviation of all Xs in the group.
2. In Eq. (XX), the unit of Z is related to the standard deviation. Therefore, the values of Zs may differ from positive numbers to negative numbers, which doesn't correspond to the real situation. So further adjustion is needed. Therefore, we further introduce the following formula called "T-transform" (Zhao 1):

$$T = 500 + 100Z \tag{7}$$

   In this formula, Z is the standard score we calculate by Eq. (XX), and 100 is the adjusting constant. 500 is the standard average score we set, and T is the T-standard score since it is obtained by T-transforming the original standard score.

   Then the T-standard score we get can be applied to conduct comparison between the unitized gain ratio x_i and the unitized risk value s_i.

Weights of Gain Ratio and Risk in RSD models

The illustration of the hierarchy indicates that every matrix related has the form $\begin{pmatrix} 1 & a \\ 1/a & 1 \end{pmatrix}$. The eigenvalues

$$(1 - \lambda)^2 = 1 \Rightarrow \lambda = 2$$

$CI = (\lambda - 2)(2 - 1) = 0$, so in this case any choice of a would be justified.

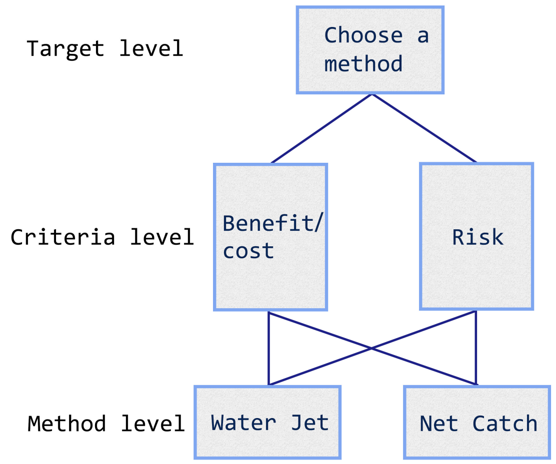

Figure 10: Level Analysis.

It turns out that the eigenvector corresponding to λ is $\begin{pmatrix} a \\ 1 \end{pmatrix}$.
Since choice of a depends on subjective feelings, we may assume a = 4/5 for judgment criteria with respect to the objective, a = 5/6 for RSD models with respect to Gain Ratio, and a = 3/2 for RSD models with respect to Risk. Hence in this simulation,

$$\begin{pmatrix} \text{Weight of water jets} \\ \text{Weight of net catching} \end{pmatrix} = \begin{pmatrix} 5/6 & 3/2 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 4/5 \\ 1 \end{pmatrix} = \begin{pmatrix} 13/6 \\ 9/5 \end{pmatrix}$$

So the weight proportion is 65/54.

6.4 Final Grades of the Assessment Model

Since we have got the data of the gain ratio and the risk value, we then standardize their values.

Gain Ratio	Risk Value
617.84	359.32
621.39	557.79
498.36	582.89
512.13	/
370.13	/
380.15	/

Table 15: Standard values of water jet model.

Gain Ratio	Risk Value
586.12	359.36
428.50	557.46
551.99	583.18
459.38	/
340.75	/
633.25	/

Table 16: Standard values of net catch model.

With two tables above, we can average each group of data and get the final grade:
Water Jet: 498.92 and 471.10
Net Catch: 493.23 and 471.27

Then we times these standard values by weight distribution 65/54. Finally we get the final grades of the Assessment Model:
Water: 486.30
Net: 483.26

What if there are actually other factors other than Grain Ratio and Risk that influence the economical opportunity?

In this case the matrix is adjusted to have a higher dimension for calculating the weight. We may need to consider the constriction from CI/RI more explicitly when assigning friction.

Strengths and Weaknesses

Strengths

- Our model of assessment is comprehensive. When using two factors, namely Gain Ratio and Risk to evaluate the economic attractivity, we not only obtain representative values by proper standardization, but also take into the weight of two factors in different RSD models into consideration. Such a treatment improves and guarantees the quality of comparisons between different RSD models.
- Our model describing the space debris distribution can tell us the mass and number distributions of debris on any specific orbits, greatly facilitating the analysis and assessment of RSD models' debris removal.

Weaknesses

- Some models' assumptions are not very proper. For example, for Water Jets model we have assumed the constant property of the mass of the space craft. However, for real-life cases the mass of the water is often not negligible compared to that of the space craft, so the actual mass of the space craft may change noticeably through the debris removal process.

- We only use two RSD models evaluated by our assessment model. It is very likely that there exists another RSD model that may have more attractive economical opportunity.

7 Future Work

- Improve the assumptions such that our RSD model becomes closer to real-life situations.
- Improve the assessment model and apply the model to other possible kinds of RSD models.

8 Conclusion

The economical performance of the alternatives removing space debris is evaluated by the combination of three mathematical models. The Removing Space Debris Model deals with the problem of how the space debris is removed. The Debris Distribution in Space Model discusses the relationship between time and the debris distribution. The Assessment Model explores the comprehensive strength of different approaches based on the risk and the ratio between benefit and cost (Gain Ratio). In this paper, we have constructed two RSD models–Water Jet Model and Net-catching Model and explored their overall scores. Given the fact that the performance of the two approaches is dependent on the debris distribution in space, which has close link to time. Therefore, the whole Assessment Model is time-dependent. The Assessment Model tells us that the Gain Ratio weighs 54.62 % and the risk accounts for 45.38 %. Besides, with the help of ORDEM2000 Model, simulation has been conducted, which tells us that the Water Jet Model scores 498.9 in Gain Ratio and 471.1 in Risk and that the Net-catching Model scores 493.2 in Gain Ratio and 471.3 in Risk. As we can see, the Water Jet Model has a relatively large advantage in Gain Ratio and the two models perform closely in risk. In conclusion, the Water Jet Model is a better alternative.

9 Executive Summary

The purpose of this report is to present our mathematical modelling which simulates various alternatives' performance in removing space debris, and the modeling results showing our preference and recommendation of which alternative is the most suitable.

Typically there are two methods of removing the space debris: either by destroying debris directly or moving the debris to outer, relatively useless orbits. Since the latter method is usually accompanied by higher energy requirements and risks, it is not suitable in our discussion based on economical opportunity. Therefore, the following models focus on removing debris by destruction. More specifically, assumption is made that the debris would be finally moved to earth atmosphere and burnt up completely.

The models now in concern are Water Jet model and Net Catching model. Intuitively speaking, Water Jet model operates in the way as follows:

- Space craft carrying the water moves to the same orbit as the one in which the target debris is moving,
- The space craft ejects water to the debris when they approach,
- The debris is pushed to the lower orbit and
- Under the influence of air resistance, the orbit of the debris gradually becomes lower and would be burned up in the earth's atmosphere.

While Net Catching model can be described as:

- Space craft carrying the net moves to the orbit lower than the one in which the target debris is moving,
- Space craft captures the debris by net when the Earth's core, space craft, and debris is in the same line,
- After capturing, the space craft and debris move together toward lower orbit, and
- Release the debris, which would be burned up in the earth's atmosphere.

Given these two options, our mathematical model is used to determine which one would be better under some criteria. Define such criteria using two factors: Gain Ratio and Risk, where Gain Ratio is the ratio of benefit and cost. The modelling operates in such a way that both Gain Ratio and Risk are first standardized under some mechanisms so that they can be compared to each other. However, the "relative importance" of these two factors may not be the same, which should also be taken into account. This is accomplished by applying analytic hierarchy process.

The standardization and weighting would eventually assign each option (Water Jet model or Net Catching model) a quantitative value which can be used to make comparisons intuitionally.

Our modelling result is derived from simulation in the condition of ORDEM 2000 model, which illustrates the debris distribution in space. Given such a condition, we can simulate the performance of Water Jet model and Net Catching model, and assess the performance quantitatively by means of the criteria just described.

The major results from the simulation conducted shows that Water Jet Model is assigned a quantitative value 498.9 for Gain Ratio, 471.1 for Risk; Net Catching Model is assigned a quantitative value 493.2 for Gain Ratio, 471.3 for Risk. The weighted value for Gain Ratio is 54.62% and for Risk is 45.38%. In conclusion from the data, Water Jet model is better than the other option.

Therefore, the conclusion is that in the similar situation, Water Jet model is more likely to perform better. In the current situation, usage of Water Jet model for debris removal is recommended.

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