



Figure 2: B+ Tree of order  $d = 4$  and height  $h = 2$ .

When answering the following questions, be sure to follow the procedures described in class and in your textbook. You can make the following assumptions:

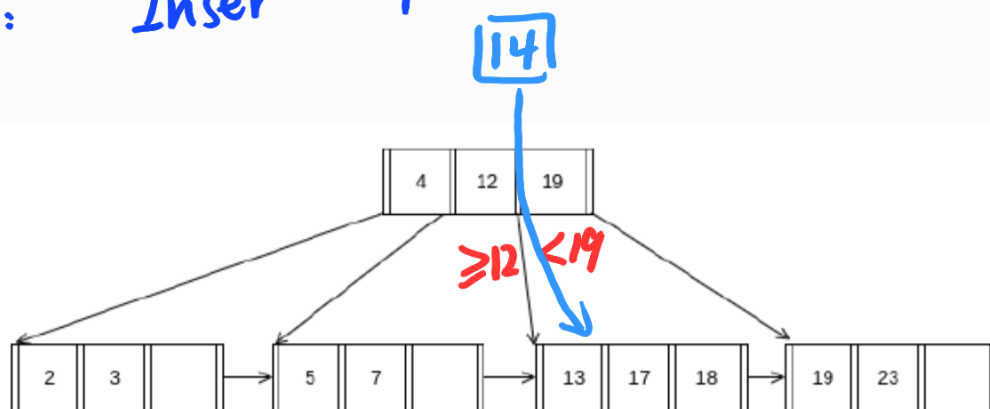
- A left pointer in an internal node guides towards keys  $<$  than its corresponding key, while a right pointer guides towards keys  $\geq$ .
- A leaf node underflows when the number of **keys** goes below  $\lceil \frac{d-1}{2} \rceil$ .
- An internal node underflows when the number of **pointers** goes below  $\lceil \frac{d}{2} \rceil$ .

$$\lceil \frac{d-1}{2} \rceil = \lceil \frac{3}{2} \rceil = 2 \Rightarrow \text{叶子节点关键字} < 2 \text{ 时下溢}$$

$$\lceil \frac{d}{2} \rceil = \lceil \frac{4}{2} \rceil = 2 \Rightarrow \text{非叶节点指针} < 2 \text{ 时下溢}$$

$$d = 4 \Rightarrow \text{B+ 树一个节点引出的最大路径数为4, 则节点key数} \leq 3$$

(a): Insert 14 to B+ tree



$$\text{key num} = 4 > 3$$

13 | 14 | 17 | 19

split

merge

4 | 12 | 17 | 19



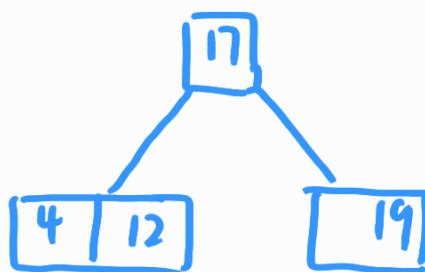
13 | 14



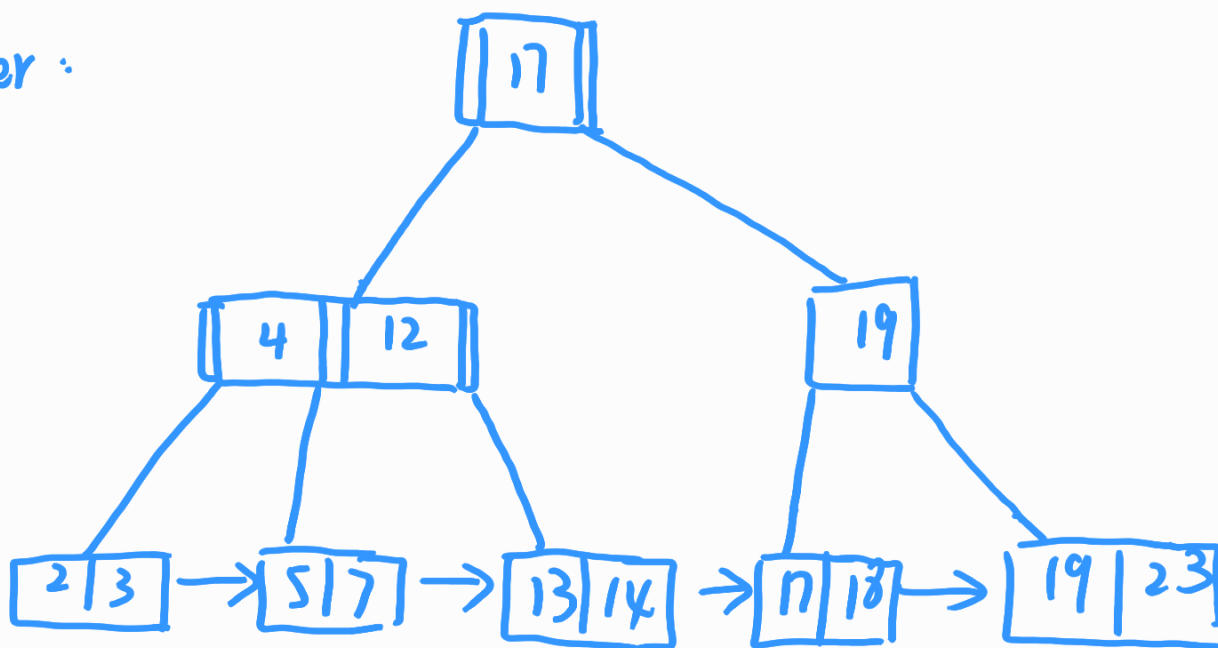
17 | 18

4 | 12 | 17 | 19

root split

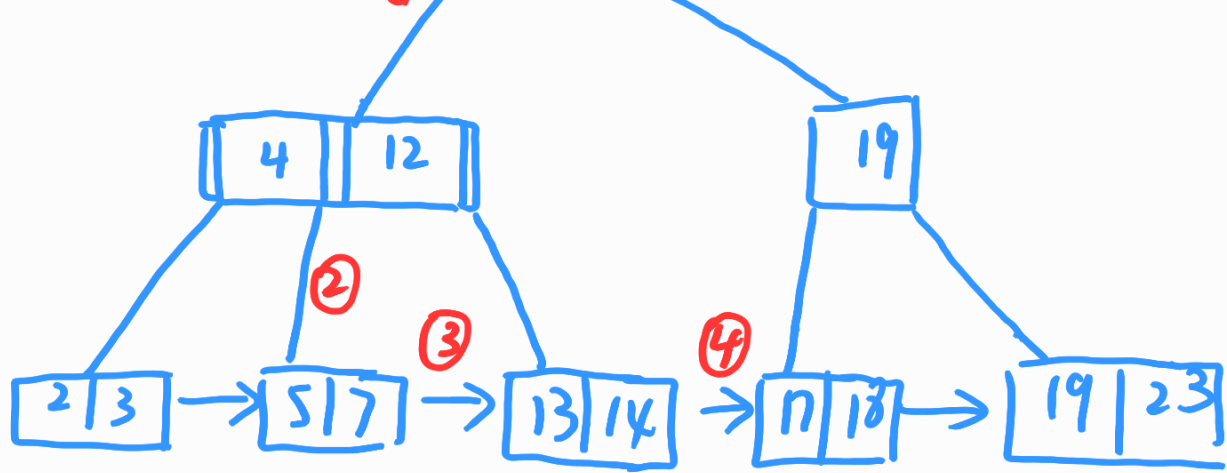


Answer :



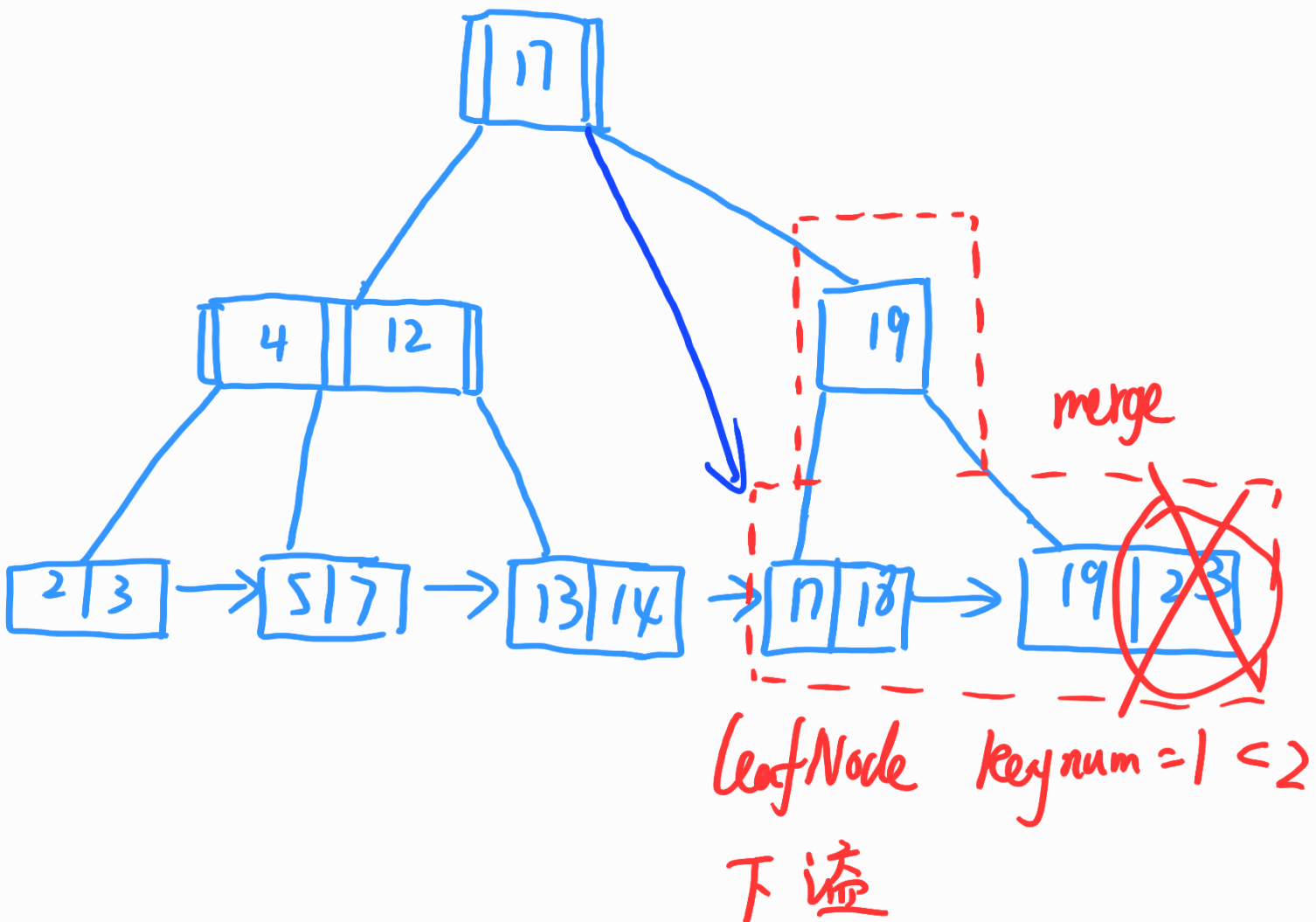
(b) find all keys in [5, 15]

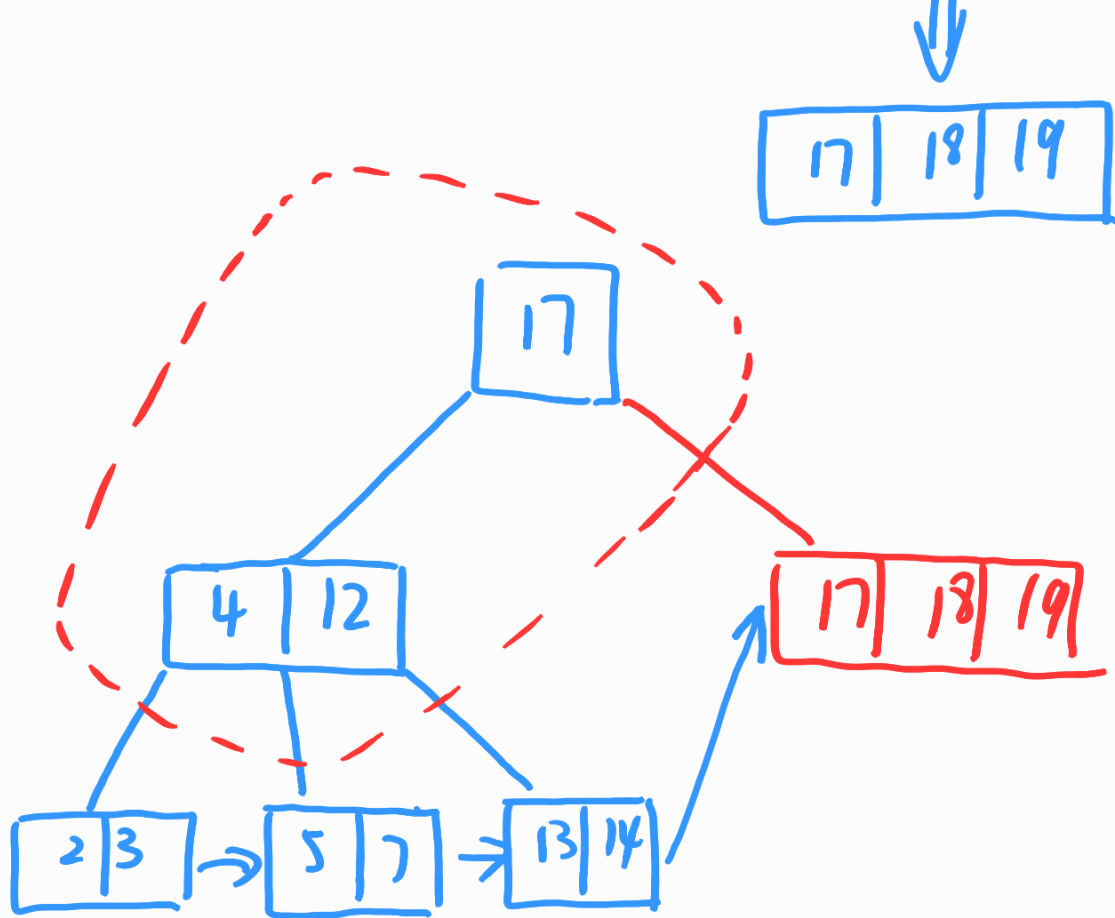




Answer : 4

(c) Then delete 23

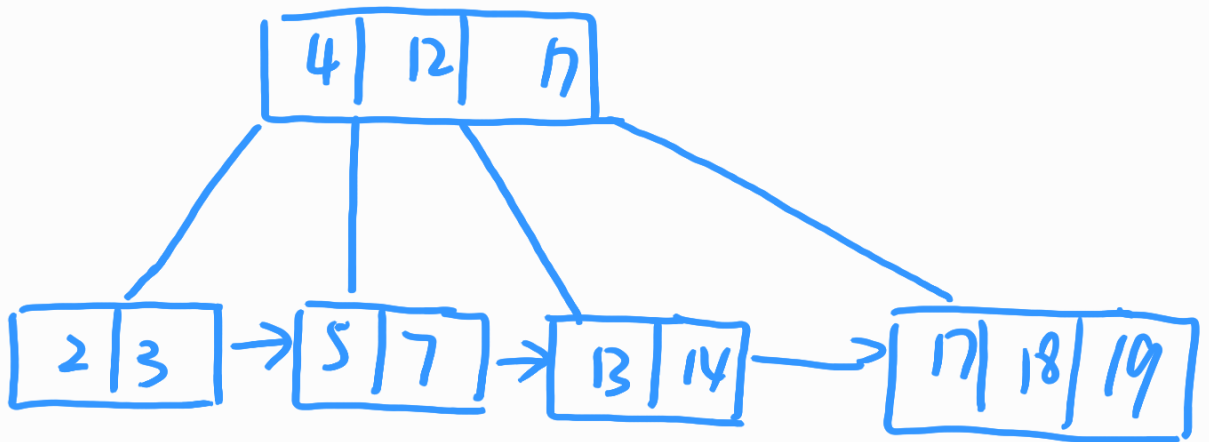




⚠ B类树要求 leaf node 在同一层

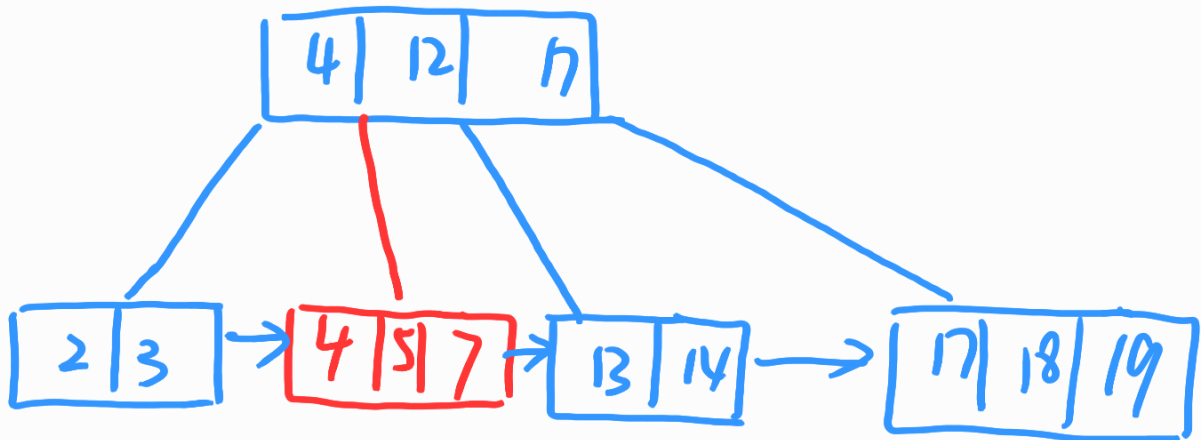
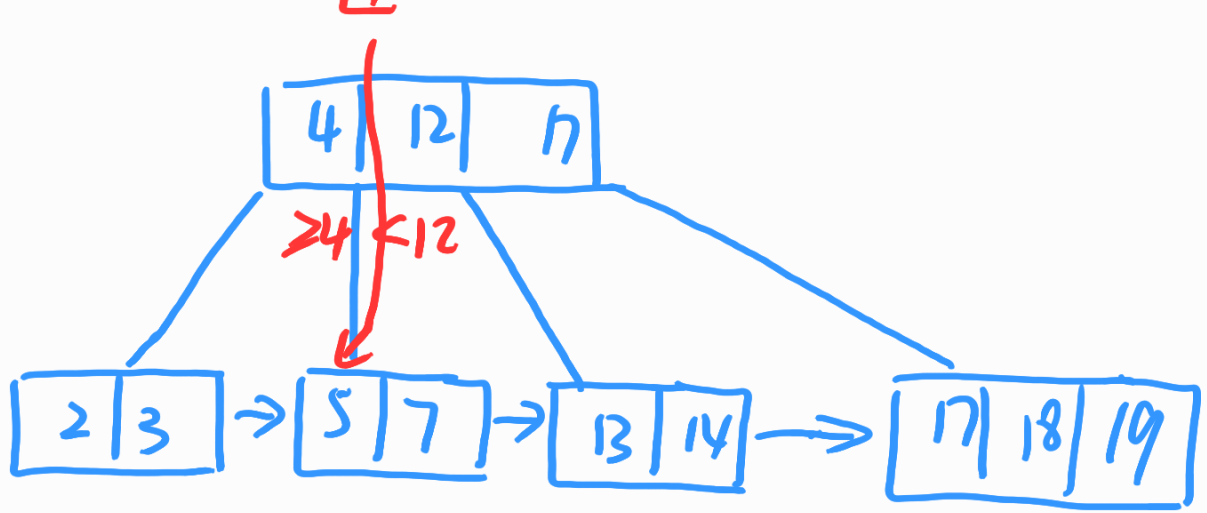


Answer

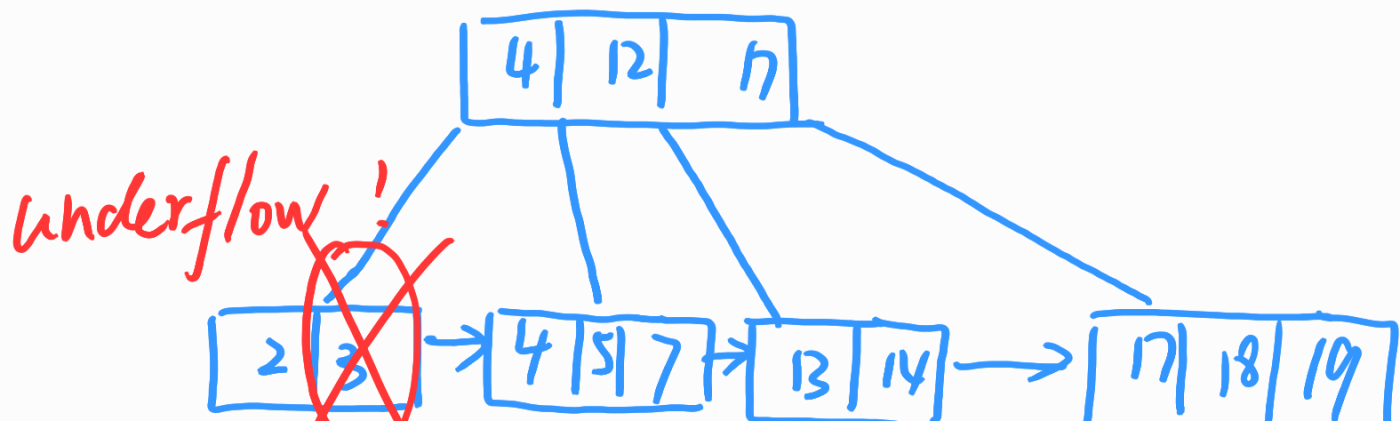


(d) Finally insert 4 and delete 3

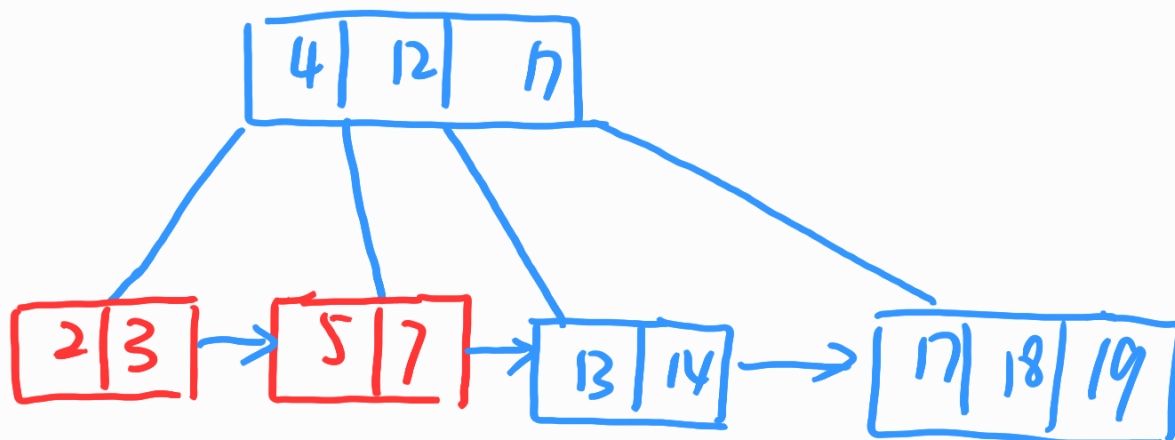
insert 4: [4]



delete 3 :



抢 右邻结点 key num = 3 > 2



Answer :

