

Departament of Quantitative Methods

Diversity in Extreme Learning Machine ensembles

Carlos Perales-González¹

¹PhD student from Universidad Loyola Andalucía

Thesis directors: Francisco Fernández-Navarro, David Becerra-Alonso, Mariano

Carbonero-Ruz



Overview

Introduction

Introduction

What is machine learning?

Extreme Learning Machine / Ridge classification

Ensembles

State of art

Diverse ensemble

Future work



Future work

Machine learning

Machine learning: field of computer science that uses statistical techniques to give computer systems the ability to "learn" (i.e., progressively improve performance on a specific task) with data, without being explicitly programmed [1].

Machine learning is the study of pattern recognition.

- Unsupervised: inferring a hidden structure from unlabeled data. I.e.: Clustering, autoencoders neural networks, . . .
- Supervised: learning a function that maps from features to target based on training data. I.e.: SVM, Ridge regression, decision tree, . . .
 - 1. Classification. Target is a category.
 - 2. Regression. Target is a real value.



Supervised classification

Extreme Learning Machine / Ridge classification

Algorithms minimize the error of classification. How?

- Linear methods.
- Nonlinear methods.
 - 1. Pure nonlinear methods. I.e.: Decision trees.
 - 2. Kernels. I.e.: RBF, polynomial.

How is the error function?

- Heuristic. Constrain programming, huge and hard loss functions. I.e.: genetic or greedy algorithms.
- Analytic. Simpler algorithms, based on convex optimization. I.e.: SVM. ELM.

Convex optimization is fast. But **strong assumption**: relation between features and target is convex. **Solution**: ensembles.



ELM I

Extreme Learning Machine for classification, a.k.a. Ridge classification [2], [3], [4].

$$f(\mathbf{x}) = \mathbf{h}'(\mathbf{x})\,\boldsymbol{\beta},\tag{1}$$

where

- $\mathbf{x} \in \mathbb{R}^m$ is the vector of attributes, m is the dimension of the input space.
- $\beta = (\beta_j, j = 1, \dots, J) \in \mathbb{R}^{d \times J}$ is ELM matrix.
- $\mathbf{h}: \mathbb{R}^m \to \mathbb{R}^d$ is the mapping function and d is the number of hidden nodes (the dimension of the transformed space).



Learning problem: Let us also denote **H** as $\mathbf{H} = (h'(\mathbf{x}_i), i = 1, ..., n) \in \mathbb{R}^{n \times d}$ as the transformation of the training set and $\mathbf{Y} \in \mathbb{R}^{n \times J}$ is the labels "1-of-J" encoded.

 $\min_{\boldsymbol{\beta} \in \mathbb{R}^{d \times J}} \left(\|\boldsymbol{\beta}\|^2 + C \|\mathbf{H}\boldsymbol{\beta} - \mathbf{Y}\|^2 \right), \tag{2}$

where $C \in \mathbb{R}^+$ is a cross-validated hyper-parameter.

$$\beta = \left(\frac{\mathbf{I}}{C} + \mathbf{H}'\mathbf{H}\right)^{-1} \mathbf{H}'\mathbf{Y} \tag{3}$$



It uses:

Introduction

- Matrix programming.
- Use Tikhonov regularization.

Machine learning problems are not always solvable. To avoid inverse matrix problems, regularization term appears. Different mapping functions h haven been proposed

- Single Hidden Layer ELM, or Neural ELM.
- Kernel ELM, or Kernel Ridge classification.

Proposed solution to avoid convex assumption: ensembles.



Bagging and Boosting

Bagging (bootstrap aggregating) is a learning method for generating several versions of a base learner by selecting some subsets from the training set and using these as new learning sets [5]. Random, different classifiers.

Boosting is a family of machine learning meta-algorithms which focus on combining base learners over several iterations and generate a weighted majority hypothesis [6].



•00000

Our proposal I

Explicit diverse ensembles for classification. This was sent to the International Conference on Hybrid Artificial Intelligent Systems (HAIS) 2018 and accepted.

$$\min_{\boldsymbol{\beta}^{I} \in \mathbb{R}^{d \times J}} \frac{1}{2} \left(\left\| \boldsymbol{\beta}^{I} \right\|^{2} + C \left\| \mathbf{H} \boldsymbol{\beta}^{I} - \mathbf{Y} \right\|^{2} + \left(D + \frac{n}{s} \right) \sum_{j=1}^{J} \sum_{k=1}^{I-1} \left\langle \boldsymbol{\beta}_{j}^{I}, \mathbf{u}_{j}^{k} \right\rangle^{2} \right)$$

$$(4)$$

where:

- $\mathbf{u}^k \in \mathbb{R}^{d \times J}$ is the column-by-column normalized $\boldsymbol{\beta}^k$ from the iteration k of the ensemble.
- D > 0 is a hyper-parameter like C.



000000

Hence, β_i^l could be obtained analytically as:

$$\beta_j^I = \left(\frac{\mathbf{I}}{C} + \mathbf{H}'\mathbf{H} + \frac{1}{C}\left(D + \frac{n}{s}\right)\mathbf{M}_j^I\right)^{-1}\mathbf{H}'\mathbf{Y}_j \quad j = 1, \dots, J \quad (5)$$

where \mathbf{M}_{i}^{I} is defined as

$$\mathbf{M}_{j}^{I} \equiv \sum_{k=1}^{I-1} \mathbf{u}_{j}^{k} \left(\mathbf{u}_{j}^{k} \right)^{I} \tag{6}$$



Results I

	Accuracy (Acc)				
	DELM	AELM	BRELM	NCELM	
car	0.929711	0.834618	0.901805	0.905111	
winequality-red	0.853687	0.840085	0.839670	0.837363	
ERA	0.829479	0.822201	0.828019	0.828428	
LEV	0.836345	0.786404	0.792371	0.798220	
SWD	0.787940	0.764487	0.759893	0.760442	
newthyroid	0.932035	0.817172	0.812035	0.819509	
automobile	0.867376	0.834618	0.841636	0.846499	
squash-stored	0.694286	0.751429	0.694063	0.711937	
squash-unstored	0.814286	0.830952	0.813810	0.812381	
pasture	0.833333	0.766667	0.811111	0.826667	



Results II

	Diversity (d)				
	DELM	AELM	BRELM	NCELM	
car	0.999206	0.180213	0.176621	0.181212	
winequality-red	0.926890	0.152451	0.124054	0.185804	
ERA	0.968917	0.138991	0.143748	0.156551	
LEV	0.992860	0.098013	0.089886	0.133830	
SWD	0.98095 3	0.130116	0.138222	0.137884	
newthyroid	0.886023	0.043061	0.040141	0.057340	
automobile	0.932272	0.314529	0.311612	0.317588	
squash-stored	0.668662	0.216839	0.181023	0.217834	
squash-unstored	0.568838	0.130116	0.145780	0.155101	
pasture	0.081884	0.181297	0.175300	0.187400	



- Solve convex assumption, looking for local convexities.
- Sensitivity analysis. It helps to establish a ranking of good solutions.
- Diverse solutions are looked for in a explicit way.

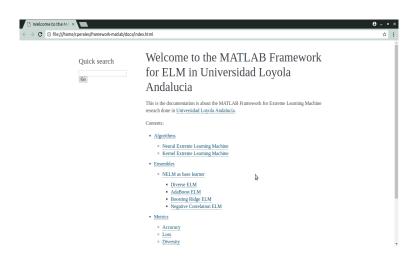


Work in progress

Improving our proposal to HAIS 2018.

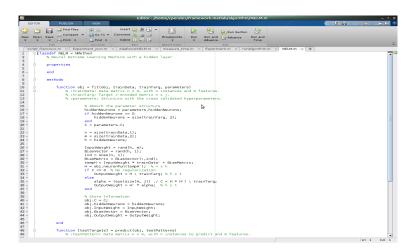
- More ensembles to compare against.
- More data sets.
- Minimizing runtime.
- Sensitivity analysis.





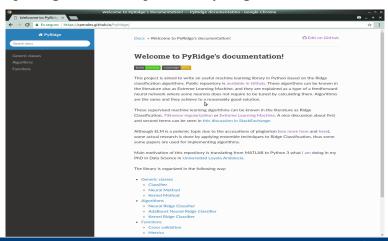


Framework in MATLAB (2)





https://github.com/cperales/PyRidge

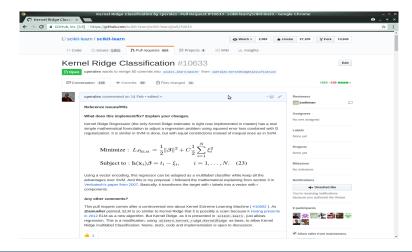




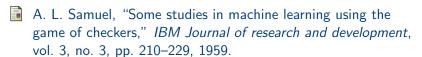
```
pyridge - [-/pruebas/pyridge] - .../pyridge/algorithm/neural.py - PyCharm Community Edition 2017.2.4
pyridge pyridge algorithm foreural.py
                                                                                                                                                                                                                                                                                                                                                          index.rst × is conf.py × is preprocess.py × is cross_val.py × is adaboost.py × is algorithm/neural.py × is algorithm/neu
                                                                                                                                                                                                                                                                                                                                                                                                                                                                        self.input_weight = np.random.rand(h, m)
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               alpha = np.eya(H.shape(0)) + \
sol*(C* np.dox(H. H.transpose())
sol*(C* np.dox(H.H.transpose())
solf.output_weight = np.dox(H.H.transpose(),
np.linalg.solve(alpha,
train
```



scikit-learn / sklearn



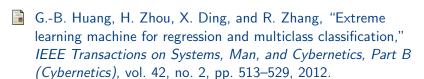




- A. E. Hoerl and R. W. Kennard, "Ridge regression: Biased estimation for nonorthogonal problems," *Technometrics*, vol. 12, no. 1, pp. 55–67, 1970.
- G.-B. Huang, Q.-Y. Zhu, and C.-K. Siew, "Extreme learning machine: theory and applications," *Neurocomputing*, vol. 70, no. 1-3, pp. 489–501, 2006.



References II



- L. Bbeiman, "Bagging Predictors," *Machine Learning*, vol. 24, pp. 123–140, 1996.
- Y. Freund and R. E. Schapire, "A Short Introduction to Boosting," *Journal of Japanese Society for Artificial Intelligence*, vol. 14, no. 5, pp. 771–780, 1999.





THANK YOU

