

# Tracking Control of Nonholonomic Mobile Robots with Velocity and Acceleration Constraints

Xiaohan Chen, Yingmin Jia, and Fumitoshi Matsuno

**Abstract**—This paper studies the tracking control of nonholonomic mobile robots with bounded velocities and accelerations. Two first-order filters driven by the tracking errors are developed to produce bounded and uniformly continuous feedback signals for the tracking controller. The feedback signals and their time derivatives are directly constrained by two filter parameters. A set of four inequality conditions on the filter parameters and a feedback parameter is derived to guarantee the satisfaction of the velocity and acceleration constraints. The system stability is proved by the Lyapunov stability theory. Simulation results are provided to verify the effectiveness of our proposed tracking controller.

**Index Terms**—Tracking control; velocity and acceleration constraints; nonholonomic mobile robots; first-order filters.

## I. INTRODUCTION

Research on tracking control of nonholonomic mobile robots extensively exists in the literature. The tracking control approaches include sliding mode control [1]-[2], linearization [3], backstepping [4]-[5], neural network-based control [6]-[7] and fuzzy control [8]-[9]. Practically, limited properties of actuators constrain the velocities and accelerations of mobile robots. For the tracking controllers without considering input constraints, the control inputs applied to physical robotic systems are directly truncated when the magnitudes of control inputs overstep the input bounds. Direct truncation of control inputs could degrade the controller performance, or even destabilize the controlled system. Therefore, taking input constraints into consideration is necessary when designing controllers.

Attention has been given to the tracking control of nonholonomic mobile robots in the presence of input constraints. In [10], input-constrained stabilization and tracking controllers were designed using the passivity and normalization techniques, and in [11] the backstepping approach was employed to design a simultaneous stabilization and tracking controller for input-bounded mobile robots. In [12], the control Lyapunov function approach was utilized to overcome the difficulties of controller design resulting from input constraints. In [13], a receding horizon tracking controller was proposed for velocity-constrained mobile robots. In [10]-[13], only the kinematics of mobile robots was considered, and correspondingly only the velocity constraints were taken into account. However, the actuators of mobile robots cannot track velocity commands changing dramatically.

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To avoid sharp changes in the velocity commands, the dynamics of mobile robots was taken into consideration where the control inputs are the motor torques. In [14], a moving horizon  $H_\infty$  tracking scheme was proposed for wheeled mobile robots in the presence of both torque saturations and external disturbance. In [15], a one-step ahead backstepping method was proposed to design the global path tracking controller for unicycle-type mobile robots with bounded torque. In [16], a new adaptive scheme was incorporated in the control design procedure to meet the challenges from torque saturation, parameter uncertainties and external disturbance. In both [15] and [16], the estimated bounds of control torques are with complicated forms in terms of design parameters, so that it is arduous to determine the design parameters for given torque bounds. In [17], the finite-time control technique was used to design the torque-constrained tracking controller for nonholonomic mobile robots. The proposed controller was in terms of parameters which should satisfy complicated inequality conditions to ensure the satisfaction of torque constraints, and the system stability was guaranteed only for the initial states located in a specific region.

It is worth noting that some commercial mobile robots only provide velocity control interface, which means that direct control on the dynamics level is unavailable. Therefore, we consider the input-constrained tracking control problem of nonholonomic mobile robots in which the linear velocity  $v$  and the angular velocity  $w$  of mobile robots are taken as control inputs. The velocities are limited by  $|v| < c_v$  and  $|w| < c_w$ , where  $c_v$  and  $c_w$  are velocity bounds. Moreover, to avoid sharp changes in the velocity commands, the accelerations are limited by  $|\dot{v}| < a_v$  and  $|\dot{w}| < a_w$ , where  $a_v$  and  $a_w$  are acceleration bounds. These acceleration constraints present more controller design challenges comparing to controller design with only velocity constraints in [10]-[13]. In this paper, two first-order filters are proposed to produce bounded and uniformly continuous feedback signals incorporated in the tracking controller. The two filters are driven by tracking errors, and are in terms of two filter parameters which directly confine the magnitudes of the feedback signals and their time derivatives. For any given velocity and acceleration bounds, a set of four inequality conditions imposed on the filter parameters and a feedback parameter are derived to guarantee the satisfaction of the velocity and acceleration constraints. Compared to the existing work, we consider both the limits of velocity and acceleration from a more practical point of view. Besides, in comparison with [15]-[17], the controller design parameters in this paper are easier to compute due to the less conservative conditions imposed

on them.

The rest of this paper is structured as follows. Section 2 states the tracking control problem of nonholonomic mobile robots subject to velocity and acceleration constraints. Section 3 presents the main results of controller design. Simulation results are provided in Section 4, and concluding remarks are given in Section 5.

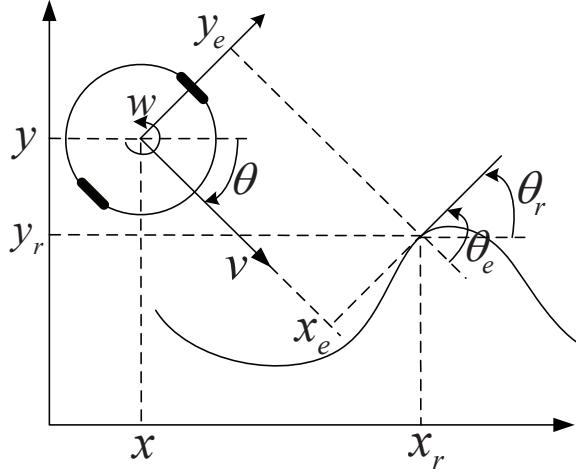


Fig. 1. Trajectory tracking configuration of a nonholonomic mobile robot.

## II. PROBLEM STATEMENT

In this paper, we consider the unicycle-type nonholonomic mobile robot with the kinematics

$$\begin{aligned}\dot{x} &= v \cos \theta \\ \dot{y} &= v \sin \theta \\ \dot{\theta} &= w\end{aligned}\quad (1)$$

where  $(x, y, \theta)$  are the position and orientation of the mobile robot, and  $v, w$  are the linear and angular velocities, respectively (see Fig. 1). Given the facts that some commercial wheeled mobile robots only provide velocity control interface and the robot motors cannot tracking steep time-varying velocity commands, we propose two assumptions on the velocity and acceleration constraints.

*Assumption 1:* The mobile robot is subject to the velocity constraints

$$|v| \leq c_v, |w| \leq c_w \quad (2)$$

and the acceleration constraints

$$|\dot{v}| \leq a_v, |\dot{w}| \leq a_w \quad (3)$$

where  $c_v, c_w, a_v$  and  $a_w$  are positive constants.

*Assumption 2:* The reference trajectory for the mobile robot satisfies the kinematics

$$\begin{aligned}\dot{x}_r &= v_r \cos \theta_r \\ \dot{y}_r &= v_r \sin \theta_r \\ \dot{\theta}_r &= w_r,\end{aligned}\quad (4)$$

with the velocity constraints

$$\begin{aligned}0 < v_r &\leq \bar{v}_r < c_v \\ |w_r| &\leq \bar{w}_r < c_w\end{aligned}\quad (5)$$

and the acceleration constraints

$$\begin{aligned}|\dot{v}_r| &\leq \bar{\dot{v}}_r < a_v \\ |\dot{w}_r| &\leq \bar{\dot{w}}_r < a_w\end{aligned}\quad (6)$$

where  $(x_r, y_r, \theta_r, v_r, w_r)$  are the desired values for  $(x, y, \theta, v, w)$ , and  $\bar{v}_r, \bar{w}_r, \bar{\dot{v}}_r, \bar{\dot{w}}_r$  are positive constants.

*Remark 1:* The velocity constraints (5) and the acceleration constraints (6) imposed on the reference trajectory are reasonable, because the mobile robot is impossible to track a trajectory beyond its maneuverability.

As shown in Fig. 1, the tracking errors are defined in the local frame of the mobile robot as

$$\begin{bmatrix} x_e \\ y_e \\ \theta_e \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_r - x \\ y_r - y \\ \theta_r - \theta \end{bmatrix}. \quad (7)$$

Differentiating both sides of (7) yields the tracking error system

$$\begin{aligned}\dot{x}_e &= v_r \cos \theta_e - v + w y_e \\ \dot{y}_e &= v_r \sin \theta_e - w x_e \\ \dot{\theta}_e &= w_r - w.\end{aligned}\quad (8)$$

We aim at finding the control inputs  $v$  and  $w$  for the mobile robot under the velocity constraints (2) and acceleration constraints (3), such that the tracking errors  $x_e, y_e$  and  $\theta_e$  will converge to zero.

## III. CONTROLLER DESIGN

*Lemma 1:* If a bounded scalar function  $z(t)$  satisfies  $z \rightarrow 0$  as  $t \rightarrow \infty$  and  $\dot{z} = -z + f(t)$ , where  $f(t)$  is bounded and uniformly continuous, then  $\dot{z} \rightarrow 0$  and  $f(t) \rightarrow 0$  as  $t \rightarrow \infty$ .

*Proof:* Since  $z$  and  $f(t)$  are bounded,  $\dot{z} = -z + f(t)$  is bounded. Then  $z$  is uniformly continuous. As  $f(t)$  is uniformly continuous,  $\dot{z} = -z + f(t)$  is also uniformly continuous. Since  $\int_0^\infty \dot{z} dt = z(\infty) - z(0) = -z(0)$ , the Barbalat's lemma [18] ensures  $\dot{z} \rightarrow 0$  as  $t \rightarrow \infty$ . As a consequence,  $f(t) \rightarrow 0$  as  $t \rightarrow \infty$ . ■

We design the controller

$$\begin{aligned}v &= v_r \cos \theta_e + \phi_1 \\ w &= w_r + \frac{k v_r y_e}{\sqrt{1 + x_e^2 + y_e^2}} + \phi_2\end{aligned}\quad (9)$$

where  $k$  is a positive feedback parameter, and  $\phi_1, \phi_2$  are feedback functions generated from the first-order filters

$$\begin{aligned}\dot{\phi}_1 &= -\phi_1 + \frac{\alpha_1 x_e}{\sqrt{1 + x_e^2 + y_e^2}} \\ \dot{\phi}_2 &= -\phi_2 + \alpha_2 \sin \theta_e\end{aligned}\quad (10)$$

where  $\alpha_1, \alpha_2$  are positive filter parameters, and  $\phi_1(0) = \phi_2(0) = 0$ . It is easy to obtain

$$\begin{aligned}|\phi_1| &\leq \alpha_1 \\ |\phi_2| &\leq \alpha_2\end{aligned}\quad (11)$$

and subsequently

$$\begin{aligned}|\dot{\phi}_1| &\leq |\phi_1| + \left| \frac{\alpha_1 x_e}{\sqrt{1 + x_e^2 + y_e^2}} \right| \leq 2\alpha_1 \\ |\dot{\phi}_2| &\leq |\phi_2| + |\alpha_2 \sin \theta_e| \leq 2\alpha_2.\end{aligned}\quad (12)$$

Thus,  $\phi_1$  and  $\phi_2$  are bounded and uniformly continuous. The Lyapunov function is chosen as

$$V(t) = \sqrt{1+x_e^2+y_e^2} - 1 + \frac{1-\cos\theta_e}{k} + \frac{\phi_1^2}{2\alpha_1} + \frac{\phi_2^2}{2k\alpha_2}. \quad (13)$$

Now we state the main results.

**Theorem 1:** Consider the tracking error system (8) under the Assumption 1-2. If the controller (9) is applied with the parameters  $k$ ,  $\alpha_1$  and  $\alpha_2$  satisfying

$$\begin{cases} \bar{v}_r + \alpha_1 \leq c_v \\ \bar{w}_r + k\bar{v}_r + \alpha_2 \leq c_w \\ \bar{v}_r + k\bar{v}_r^2 + \alpha_2\bar{v}_r + 2\alpha_1 \leq a_v \\ \bar{w}_r + k(\bar{v}_r + \bar{v}_r^2 + \bar{v}_r c_w) + k\bar{v}_r\alpha_1/2 + 2\alpha_2 \leq a_w \end{cases} \quad (14)$$

then the control inputs generated from (9) satisfy the velocity constraints (2) and the acceleration constraints (3), and  $x_e$ ,  $y_e$  and  $\sin\theta_e$  will converge to zero. Further if  $V(0) < 2/k$ ,  $\cos\theta_e \rightarrow 1$  as  $t \rightarrow \infty$ .

*Proof:* With (8)-(10), we get the derivative of  $V$  as

$$\begin{aligned} \dot{V} &= \frac{x_e\dot{x}_e + y_e\dot{y}_e}{\sqrt{1+x_e^2+y_e^2}} + \frac{\dot{\theta}_e \sin\theta_e}{k} + \frac{\phi_1\dot{\phi}_1}{\alpha_1} + \frac{\phi_2\dot{\phi}_2}{k\alpha_2} \\ &= -\frac{\phi_1^2}{\alpha_1} - \frac{\phi_2^2}{k\alpha_2}. \end{aligned} \quad (15)$$

Since  $\phi_1$ ,  $\phi_2$ ,  $\dot{\phi}_1$  and  $\dot{\phi}_2$  are bounded (see (11) and (12)),  $\dot{V}$  and  $\ddot{V} = -2\phi_1\dot{\phi}_1/\alpha_1 - 2\phi_2\dot{\phi}_2/(k\alpha_2)$  are bounded. Therefore,  $\dot{V}$  is uniformly continuous. As  $V$  is non-increasing and  $V \geq 0$ ,  $V$  has a finite limit. Then  $\int_0^\infty \dot{V}dt = V(\infty) - V(0)$  exists and is finite. Applying the *Barbalat's lemma* [18] ensures that  $\dot{V} \rightarrow 0$  as  $t \rightarrow \infty$ , i.e.,  $\phi_1 \rightarrow 0$  and  $\phi_2 \rightarrow 0$  as  $t \rightarrow \infty$ . Since  $V(t) \leq V(0)$ ,  $x_e$  and  $y_e$  are bounded. In view of (5), (8), (9) and (11),  $v$ ,  $w$ ,  $\dot{x}_e$ ,  $\dot{y}_e$  and  $\dot{\theta}_e$  are also bounded. Subsequently,  $\frac{x_e}{\sqrt{1+x_e^2+y_e^2}}$ ,  $\frac{y_e}{\sqrt{1+x_e^2+y_e^2}}$  and  $\sin\theta_e$  are bounded and uniformly continuous. In view of (10),  $x_e$  and  $\sin\theta_e$  will converge to zero according to Lemma 1.

From (8) and (9), we get

$$\dot{\theta}_e = -\frac{kv_r y_e}{\sqrt{1+x_e^2+y_e^2}} - \phi_2. \quad (16)$$

We can obtain that  $\dot{\theta}_e$  is uniformly continuous as  $\frac{kv_r y_e}{\sqrt{1+x_e^2+y_e^2}}$  and  $\phi_2$  are uniformly continuous. Since  $\sin\theta_e \rightarrow 0$  as  $t \rightarrow \infty$ ,  $\theta_e$  has a finite limit. Then  $\int_0^\infty \dot{\theta}_e = \theta_e(\infty) - \theta_e(0)$  is finite. Applying the *Barbalat's lemma* [18] gives that  $\dot{\theta}_e \rightarrow 0$ , i.e.,  $\frac{kv_r y_e}{\sqrt{1+x_e^2+y_e^2}} + \phi_2 \rightarrow 0$ . Since we have proven that  $\phi_2 \rightarrow 0$  and assume that  $v_r > 0$  (see Assumption 2),  $y_e \rightarrow 0$  as  $t \rightarrow \infty$ .

As we have only shown  $\sin\theta_e \rightarrow 0$ ,  $\cos\theta_e$  could converge to  $-1$  or  $1$ . In fact, for  $\cos\theta_e(0) \neq -1$  we always can choose  $k$  small enough to guarantee  $V(0) < 2/k$  so that  $\cos\theta_e$  will converge to  $1$ . Otherwise, since  $V(t)$  is non-increasing,  $\cos\theta \rightarrow -1$  leads to a contradiction that  $V(\infty) = 2/k > V(0)$ .

Now, we estimate the bounds of the velocities and the accelerations of the mobile robot driven by the controller (9). It is clear from (5), (9) and (11) that

$$\begin{aligned} |v| &\leq |v_r \cos\theta_e| + |\phi_1| \leq \bar{v}_r + \alpha_1 \\ |w| &\leq |w_r| + |kv_r y_e / \sqrt{1+x_e^2+y_e^2}| + |\phi_2| \leq \bar{w}_r + k\bar{v}_r + \alpha_2. \end{aligned} \quad (17)$$

From (5), (8), (9) and (11), we have

$$\begin{aligned} |\dot{\theta}_e| &= \left| \frac{kv_r y_e}{\sqrt{1+x_e^2+y_e^2}} + \phi_2 \right| \leq \left| \frac{kv_r y_e}{\sqrt{1+x_e^2+y_e^2}} \right| + |\phi_2| \\ &\leq k\bar{v}_r + \alpha_2 \end{aligned} \quad (18)$$

and

$$\begin{aligned} &\left| d \left( \frac{y_e}{\sqrt{1+x_e^2+y_e^2}} \right) / dt \right| \\ &= \left| \frac{v_r \sin\theta_e (1+x_e^2) - wx_e (1+x_e^2+y_e^2) + x_e y_e \phi_1}{(1+x_e^2+y_e^2)^{3/2}} \right| \\ &\leq \left| \frac{v_r \sin\theta_e}{\sqrt{1+x_e^2+y_e^2}} \right| + \left| \frac{wx_e}{\sqrt{1+x_e^2+y_e^2}} \right| + \\ &\quad \left| \frac{x_e y_e \phi_1}{(1+x_e^2+y_e^2)^{3/2}} \right| \\ &\leq \bar{v}_r + c_w + \frac{\alpha_1}{2}. \end{aligned} \quad (19)$$

According to (9), the derivatives of  $v$  and  $w$  are

$$\begin{aligned} \dot{v} &= \dot{v}_r \cos\theta_e - v_r \dot{\theta}_e \sin\theta_e + \dot{\phi}_1 \\ \dot{w} &= \dot{w}_r + \frac{k\dot{v}_r y_e}{\sqrt{1+x_e^2+y_e^2}} + kv_r d \left( \frac{y_e}{\sqrt{1+x_e^2+y_e^2}} \right) / dt + \dot{\phi}_2. \end{aligned} \quad (20)$$

Then we can obtain from (5), (6), (12) and (18)-(20) that

$$\begin{aligned} |\dot{v}| &\leq |\dot{v}_r \cos\theta_e| + |v_r \dot{\theta}_e \sin\theta_e| + |\dot{\phi}_1| \\ &\leq \bar{\dot{v}}_r + \bar{v}_r (k\bar{v}_r + \alpha_2) + 2\alpha_1 \\ &= \bar{\dot{v}}_r + k\bar{v}_r^2 + \bar{v}_r \alpha_2 + 2\alpha_1 \\ |\dot{w}| &\leq |\dot{w}_r| + \left| \frac{k\dot{v}_r y_e}{\sqrt{1+x_e^2+y_e^2}} \right| + \\ &\quad \left| kv_r d \left( \frac{y_e}{\sqrt{1+x_e^2+y_e^2}} \right) / dt \right| + |\dot{\phi}_2| \\ &\leq \bar{\dot{w}}_r + k\bar{\dot{v}}_r + k\bar{v}_r (\bar{v}_r + c_w + \alpha_1/2) + 2\alpha_2 \\ &= \bar{\dot{w}}_r + k(\bar{v}_r + \bar{v}_r^2 + \bar{v}_r c_w) + k\bar{v}_r \alpha_1/2 + 2\alpha_2. \end{aligned} \quad (21)$$

Therefore, in view of (17) and (21), if (14) is met, the velocity constraints (2) and the acceleration constraints (3) are satisfied. ■

**Remark 2:** The tracking errors are filtered by (10) to produce the bounded and uniformly continuous feedback functions  $\phi_1$  and  $\phi_2$ . Therefore, the control inputs generated from the controller (9) are velocity-bounded and acceleration-bounded, which are welcome to practical systems.

**Remark 3:** According to (5) and (6), there always exist feasible  $k$ ,  $\alpha_1$  and  $\alpha_2$  such that (14) and  $V(0) < 2/k$  (for  $\cos\theta_e(0) \neq -1$ ) are satisfied. Large values of  $\alpha_1$  and  $\alpha_2$  make  $\phi_1$  and  $\phi_2$  react fast to the tracking errors so that the tracking convergence will be accelerated. Besides, increasing  $k$  yields the growth of control force to speed up the tracking convergence.

#### IV. SIMULATION RESULTS

The velocity and acceleration constraints of the mobile robot are set as  $c_v = 1.5\text{m/s}$ ,  $c_w = 2\text{rad/s}$ ,  $a_v = 1.5\text{m/s}^2$  and  $a_w = 2\text{rad/s}^2$ . The reference trajectory is

$$\begin{aligned} x_r &= 5 \sin\left(\frac{2\pi}{60}t\right) \\ y_r &= 5 \sin\left(\frac{4\pi}{60}t\right). \end{aligned} \quad (22)$$

The controller (9) is applied to drive the mobile robot with  $k = 0.1$ ,  $\alpha_1 = 0.3$  and  $\alpha_2 = 0.48$  satisfying (14). The initial pose of the mobile robot is  $(x_0, y_0, \theta_0) = (-4\text{m}, 2\text{m}, 0)$ . In order to test the robustness of the proposed controller, the position and orientation measurements are disturbed by the random noises uniformly distributed in  $[-0.03\text{m}, 0.03\text{m}]$  and  $[-\pi/36, \pi/36]$ , respectively.

Fig. 2 shows that the mobile robot gradually converges to the reference trajectory and finally moves along it. Fig. 3 demonstrates that the Lyapunov function  $V$  decreases to zero and the tracking errors  $x_e$ ,  $y_e$  and  $\theta_e$  converge to small neighbors of zero. Fig. 4 gives the results of control inputs. We can observe that the linear velocity  $v$  and the angular velocity  $w$  are bounded by (2), and  $v_a(v_a = \dot{v})$ ,  $w_a(w_a = \dot{w})$  are bounded by (3). The simulation results also show the robustness of our proposed controller against measurement noises.

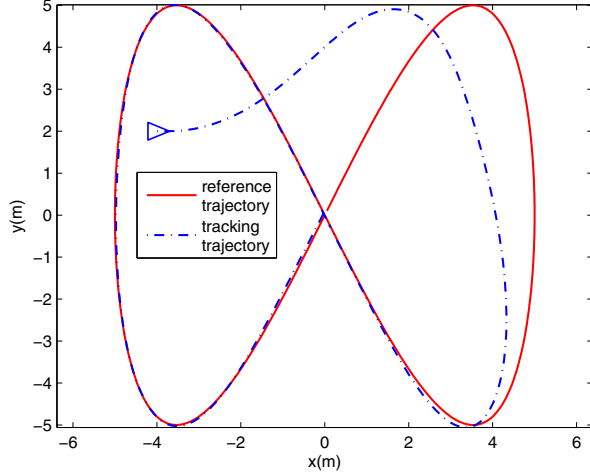


Fig. 2. Reference trajectory and tracking trajectory.

#### V. CONCLUSIONS

We addressed the tracking control problem of nonholonomic mobile robots subject to velocity and acceleration constraints. The main contribution of this work is the design of two first-order filters to produce bounded and uniformly continuous feedback signals for the tracking controller. Any given velocity and acceleration constraints can be satisfied by designing the filter parameters and the feedback parameter satisfying a set of four inequality conditions. The simulation

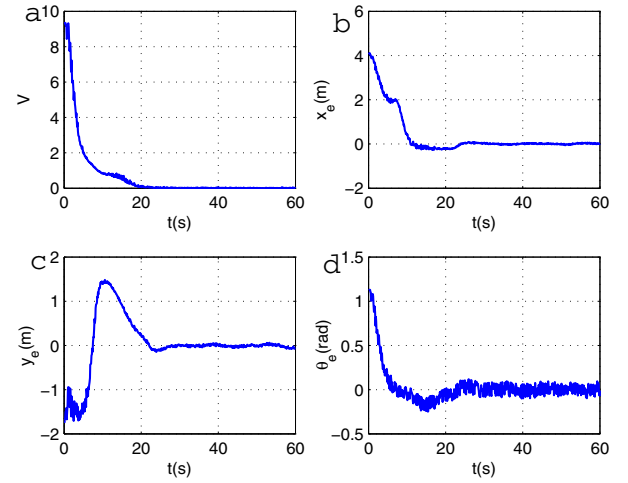


Fig. 3. (a) Lyapunov function  $V$ . (b) Tracking error  $x_e$ . (c) Tracking error  $y_e$ . (d) Tracking error  $\theta_e$ .

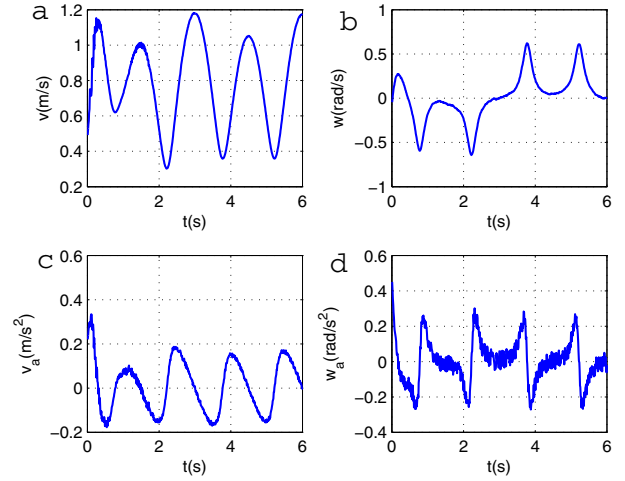


Fig. 4. (a) Linear velocity  $v$ . (b) Angular velocity  $w$ . (c) Linear acceleration  $v_a(v_a = \dot{v})$ . (d) Angular acceleration  $w_a(w_a = \dot{w})$ .

results confirmed the effectiveness of our proposed tracking controller.

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