ECON 5213: ADVANCED ECONOMETRICS

PROF. LE WANG

Problem Set #4

Part I: Analytical Questions

Question 1. [Normal Distribution and Quantile]. Suppose that the τ^{th} quantile of the standard normal distribution is given by z_τ. How can you use this information to obtain the τ^{th} quantile of the normal distribution with mean μ and variance σ^2 without looking up any statistical tables? For example, we know that the 95^{th}quantile of the standard normal distribution is 1.644854. For a normally distributed variable with $\mu = 1, \sigma = 2$, can you immediately calculate the 95^{th} quantile with just a calculator on your smart phone?

Question 2. [Properties of Standard Normal].

(1) Show that the standard normal is symmetric around zero. Remember that from our class and Homework 3, this means that $X - \mu$ and $\mu - X$ have the same distribution.
(2) Show that the property of symmetry of tail areas indeed holds.

$$\Phi(z) = 1 - \Phi(-z)$$

Question 3. [Standard Normal to Normal]. Suppose that $Z \sim \mathcal{N}(0, 1)$, show that $X = \mu + \sigma Z$ is indeed distributed from a normal distribution with mean μ and variance σ^2. Note that the proof is similar to what we discussed in class. In fact, it is simply the reverse of the substitution that we used in class. I just want you to practice this a bit more.

Part II: Applied Questions

Do the following exercises in Stata. Turn in your answer (along with your do file, log file, and graphs). Set the seed at 123456 at the start of each problem. As we have emphasized, you should also take the initiative to learn Stata. And this homework serves as a way for me to assess your self-learning.

Question 1. [Uniform Distribution]. Since its version 14, Stata introduces two new functions for uniform random numbers: `runiform(a,b)` and `runiformint(a,b)`. Before that, we have to write our own program to generate uniformly distributed random variates over the interval (a, b). Now, pretend that such command does not exist. Write the code to simulate a sample of 1000 observations following a uniform distribution over the support $(3, 8)$ and plot the histogram for this sample.

Question 2. [Inverse Probability Transform]. As we show in class, the following theorem holds

Theorem. *Let F_X be a CDF. Let U be a random variable that is uniformly distributed. Then, $X = F_X^{-1}(U)$ is distributed from the CDF, F_X.*

Use this result, simulate a sample of 1000 observations following a logistic distribution, $F_X(x) = \frac{1}{1+\exp(-x)}$ and plot the histogram for this sample. You *cannot* use Stata's built-in random number generator programs.

Question 3. [Uniform Distributions and Discrete Variables]. In class, we showed that in order to generate a fair dice with equal probability for integers $1, 2, 3, 4, 5, 6$, we can generate a set of values, denoted by $X1$, randomly drawn from the standard uniform, and then reclassify them as follows

(1) $X = 1$ if $X1 >= 0 \& X1 <= 1/6$
(2) $X = 2$ if $X1 > 1/6 \& X1 <= 2/6$
(3) $X = 3$ if $X1 > 2/6 \& X1 <= 3/6$
(4) $X = 4$ if $X1 > 3/6 \& X1 <= 4/6$
(5) $X = 5$ if $X1 > 4/6 \& X1 <= 5/6$
(6) $X = 6$ if $X1 > 5/6$

Theoretically show that this approach indeed generate a **fair dice.**

Question 4. [Uniform Distributions and Discrete Variables]. In class, we simulated a fair coin tossing experiment. Let's say we would like to simulate an unfair coin tossing experiment with Heads being more likely ($\Pr[\text{Head}] = .7$). How will you simulate such experiment in Stata?