ECON 5213: ADVANCED ECONOMETRICS

PROF. LE WANG

Pratice Questions

Question 1. [Independence]. As discussed in class, when $Y = \{0, 1\}$ and $X = \{0, 1\}$, zero risk difference ($\Pr[Y = 1|X = 1] = \Pr[Y = 1|X = 0]$) implies independence. Using the definition of independence, show this is indeed true.

Question 2. [Partial Identification and the Law of Total Probability]. Again, we do not know the wages for women who do not work. In other words, we can only observe wages for those who do work $S = 1$ and have the knowledge of conditional distribution of wages for this subgroup

$$F(y \mid S = 1)$$

Bound the distribution of wages, $F(y)$ assuming negative selection (i.e., the distribution of wages for those who do not work is **better** than that for those who work.).

Question 3. [Conditional CDF and Conditional Expectation]. Show the following is true.

$$F_{\{Y|X\}}(y|x) = \Pr[Y \leq y \mid X] = \mathbb{E}[\mathbb{I}[Y \leq y] \mid X]$$

Question 4. [Conditional Expectation and Conditional Distribution for discrete variables]. Suppose that our decision to go to college is determined by the following equation:

$$College_i = \mathbb{I}[x_i'\lambda - v_i \geq 0]$$

where $X \in R^k$ is a set of observable characterisitcs and $v = S(X'\theta)v^*$ (v^* is a homoskedastic error term independent of X and distributed from the CDF $F(\cdot)$); $\mathbb{I}[\cdot]$ is an indiator function, equal to one if the argument is satisfied and zero otherwise.

You can think of the argument in the choice equation as a reduced-form approximation to the difference between wages for college graduates and wages for non-college graduates. This formulation is consitent with the traditional Roy model, which is an important workhorse for our understanding of selection and much of the modern econometrics. Show the following is true:

(1) $\mathbb{E}[College|X] = \Pr[College = 1|X]$

(2) $\Pr[College = 1|X] = F\left[\frac{X'\lambda}{S(X'\theta)}\right] \equiv m(X)$. This result is used for constructing an *internal* instrumental variable for endogenous binary variables.

(3) $\mathbb{E}[College \mid X] = \mathbb{E}[College \mid m(X)]$. **Hint:** This is a result of the general law of iterated expectation, together with the fact that $\mathbb{E}[y|x] = \mathbb{E}[y|x,w]$ where $w = f(x)$; $f(\cdot)$ can be any arbitrary function. The intuition for this latter fact is simple: once we know x, we will definitely know w, and thus adding w to the information set does not add any new information.

Question 5. [ANOVA Theorem]. A final property of the CEF, closely related to both the decomposition and prediction properties, is the analysis of variance (ANOVA) theorem.

$$Var(y) = Var(\mathbb{E}[y|x]) + \mathbb{E}[Var(y|x)]$$

where $Var(y|x)$ is conditional variance of y. Show this indeed holds. **Hint:** You can use the same trick that we used to show the decomposition property in class.