Wali Muhammad

2021 - SE - 39

APS Assignment

Chapter #6

Question #1

Solution:

(a)

The sample space S is required in Question statement which is given below:

$$S = \{ (1,1), (1,2), (1,3) (1,4) (1,5), (1,6)$$
$$(2,1), (2,2), (2,3) (2,4) (2,5), (2,6)$$
$$(3,1), (3,2), (3,3) (3,4), (3,5), (3,6)$$
$$(4,1), (4,2), (4,3) (4,4), (4,5), (4,6)$$
$$(5,1), (5,2), (5,3) (5,4), (5,5), (5,6)$$
$$(6,1), (6,2), (6,3) (6,4), (6,5), (6,6) \}$$

The two events are:

$$A = \{(1,1), (1,2), (1,3), (2,1), (2,2), (3,1)\}$$

$$B = \{(1,6), (2,6), (3,6), (4,6), (5,6), (6,6), (6,1)$$
$$(6,2), (6,3), (6,4), (6,5)\}$$

(b)

Using the sample space given in (a), we see that the two events A and B consists

A - B

of the following elements:

$A = \{(1,2),(1,4),(1,6),(2,1),(2,3),(2,5),(3,2),(3,4)$
$(3,6),(4,1),(4,3),(4,5),(5,2),(5,4),(5,6),(6,1)$
$(6,3),(6,5)\}$

$B = \{(1,3),(2,3),(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)$
$(4,3),(5,3),(6,3)\}$

The complements of A and B are

$\bar{B} = \{(1,1),(2,1),(4,1),(5,1),(6,1),(1,2),(2,2),(4,2),$
$(5,2),(6,2),(1,4),(2,4),(4,4),(5,4),(6,4),(1,5),$
$(2,5),(4,5),(5,5),(6,5),(1,6),(2,6),(4,6),(5,6)$
$(6,6)\}$

$\bar{A} = \{(1,1),(1,3),(1,5),(2,2),(2,4),(2,6),(3,1)$
$(3,3),(3,5),(4,2),(4,4),(4,6),(5,1),(5,3),$
$(5,5),(6,2),(6,4),(6,6)\}$

$A \cup B$ = The union of A and B consists of those elements which belong to A or B or both

$A \cup B = \{(1,2),(1,4),(1,6),(2,1),(2,3),(2,5),(3,2),(3,4)$
$(3,6),(4,1),(4,3),(4,5),(5,2),(5,4),(5,6),(6,1)$
$(6,3),(6,5),(1,3),(3,1),(3,3),(3,5),(5,3)\}$

Thus $A \cup B$ contain 23 elements

$A \cap B$ = The ~~inse~~ intresection of A and B consists of those elements which belongs to both A and B. Thus

$A \cap B = \{(2,3),(3,2),(3,4),(4,3),(3,6),(6,3)\}$

$n(A \cap B) = 6$

Scanned with CamScanner

$A-B = A\cap\bar{B}$ = The set of all elements of A which do not belong to B

$= \{(1,2),(1,4),(1,6),(2,1),(2,5),(4,1),(4,5),(5,2)$
$(5,4),(5,6),(6,1),(6,5)\}$

$n(A-B) = 12$

$(A\cap\bar{B})\cup\bar{A}$ = The union of $(A\cap\bar{B})$ and $\bar{A}$ consists of those elements which belongs to $(A\cap\bar{B})$ or $\bar{A}$ or both.

$= \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1)$
$(2,2),(2,4),(2,5),(2,6),(3,1),(3,3),(3,5)$
$(4,1),(4,2),(4,4),(4,5),(4,6),(5,1),(5,2)$
$(5,3),(5,4),(5,5),(5,6),(6,1),(6,2),(6,4),$
$(6,5),(6,6)\}$

$n((A\cap\bar{B})\cup\bar{A}) = 30$

Question #2

Solution:

(a)

Let A be the event that one card drawn is a king and other is a Queen. Then

A contains $\binom{4}{1}\binom{4}{1} = 16$ sample points

$$P(A) = \frac{n(A)}{n(S)} = \frac{16}{1326} = \frac{8}{663}$$

(b)

$\therefore$ S consists of $\binom{8}{5} = 56$ sample spoints

A can choose one joker and 4 other cards in $\binom{1}{1}\binom{7}{4}$ ie. 35 ways.

$$P(A \text{ has the Joker}) = \frac{35}{56} = \frac{5}{8}$$

Chapter # 8

Question # 1

Solution:

Let X denote the number of correct answers. Then

$$P(5 \le X \le 10) = \sum_{x=5}^{10} \binom{15}{x} (0.25)^x (0.75)^{15-x}$$

$$= \sum_{x=5}^{10} b(x; 15, 0.25)$$

$$= \sum_{x=0}^{10} b(x; 15, 0.25) - \sum_{x=0}^{4} b(x; 15, 0.25)$$

$= 0.9999 - 0.6865$ (From binomial tables)

$= 0.3134$

Question # 2

Solution:

$N = 10$ cans to draw from

$n = 5$, the number of cans to be drawn

$k = 5$ of the 10 cans are tomatoes

Thus the Hypergeometric distribution is

$$h(x : 10, 5, 5) = \frac{\binom{5}{x}\binom{5}{5-x}}{\binom{10}{5}}$$

Hence the probability that all contains ~~tom~~tomatoes, i.e. $x = 5$ is

$$h(5; 10, 5, 5) = \frac{\binom{5}{0}\binom{5}{0}}{\binom{10}{5}} = 0.003968$$

Let A denote the event that 3 or more cans contains tomatoes. Then

$$P(A) = \sum_{x \geq 3}^{5} \binom{5}{x}\binom{5}{5-x} \div \binom{10}{5}$$

$$= \left[\binom{5}{3}\binom{5}{2} + \binom{5}{4}\binom{5}{1} + \binom{5}{5}\binom{5}{0}\right] \div \binom{10}{5}$$

$$= (100 + 25 + 1) \div 252 = 0.5$$

Question # 3

Solution:

$$P(X = x) = \frac{e^{-1}(1)^x}{x!}$$

~~Hence the desi~~ (a)

The mean of the Poisson distribution is denoted by λ and is equal to the variance, which also denoted by λ. The relationship between the Binomial and Poisson distribution is that they both represents the member of successor in a given number of trials. The difference is that the binomial distribution has fixed (distribution has an) fixed number of trials, while the poisson distribution has an infinite number of trials.

(b)

Binomial

$$P(X=x) = \binom{n}{x} p^x (1-p)^{n-x}$$

Poisson

$$P(X=x) = \frac{e^{-1}(1)^x}{x!}$$

Hence the desired probability is

$$P(X=2) = \frac{e^{-1}(1)^2}{2!} = \frac{e^{-1}}{2!} = \frac{1}{2e}$$

$$= \frac{1}{2(2.7183)} = \frac{1}{5.4366}$$

$$P(X=2) = 0.18939$$

Scanned with CamScanner

Chapter # 09

Question # 01

Solution:

(a)

With $\mu = 40$ and $\sigma = 8$, we therefore change x values into the corresponding z values by

$$Z = \frac{X - 40}{8}$$

Thus at $x = 35$, $z = \frac{35-40}{8} = -0.625$

and at $x = 46$, $z = \frac{46-40}{8} = 0.75$

These values are indicated in the figure and the needed areas are shaded.

Now,

$$P(X \leq 35) = P(Z \leq -0.625)$$
$$= 0.5 - P(-0.625 \leq Z \leq 0)$$
$$= 0.5 - 0.2340 = 0.2660$$

This represents $100,000 \times 0.2660 = 26,600$ pairs of stockings.

Again $P(X \geq 46) = P(Z \geq 0.75)$
$$= 0.5 - P(0 \leq Z \leq 0.75)$$
$$= 0.5 - 0.2734 = 0.2266.$$

This represents $100,000 \times 0.2266 = 22,660$ pairs of stocking.

(i) $P(X > 17) = P\left(\frac{X-12}{2} > \frac{17-12}{2}\right)$

$= P(Z > 2.5) = 0.5 - P(0 < Z < 2.5)$

$= 0.5 - 0.4938 = 0.0062$

The number of days when he takes longer then 17 minutes $= 0.0062 \times 365$
$= 2$ days

(ii) $P(X < 10) = P\left(\frac{X-12}{2} < \frac{10-12}{2}\right)$

$= P(Z < -1) = 0.5 - P(-1 < Z < 0)$

$= 0.5 - 0.3413 = 0.1587$

The number of days when he takes less than 10 minutes $= 0.1587 \times 365 = 58$ days.

(iii) $P(9 < X < 13) = P\left(\frac{9-12}{2} < \frac{X-12}{2} < \frac{13-12}{2}\right)$

$= P(-1.5 < Z < 0.5)$

$= P(-1.5 < Z < 0) + P(0 < Z < 0.5)$

$= 0.4332 + 0.1915$

$= 0.6247$

Thus the number of days when he takes between 9 and 13 minutes = 0.6247 X 365 ≈ 228 days.

Question# 2

Solution:

$$E(Y) = 5E(X)+10 = 5(10)+10 = 60$$

and $Var(Y) = 25\,Var(X) = 25(25) = 625$

Hence Y is $N(60, 625)$

We draw the normal curve sketch showing x and z values and the desired area for each part. With $\mu = 60$ and $\sigma = \sqrt{625} = 25$, we have

$$z = \frac{y-60}{25}$$

(i) At $y = 54$,

$$z = \frac{54-60}{25} = -0.24$$

Therefore using area table, we get

$$P(Y \le 54) = P(Z \le -0.24)$$
$$= 0.5 - P(-0.24 \le Z \le 0)$$
$$= 0.5 - 0.0948$$
$$= 0.4052$$

Scanned with CamScanner

(ii) At $y = 68$

$$z = \frac{68 - 60}{25} = 0.32$$

Therefore using area table

$$P(Y \geq 68) = P(Z \geq 0.32)$$
$$= P(0 \leq Z < \infty) - P(0 \leq Z < 0.32)$$
$$= 0.5 - 0.1255$$
$$= 0.3745$$

(iii) we have for $y = 52$

$$z = \frac{\cancel{67} - 52 - 60}{25} = -0.32$$

and for $y = 67$

$$z = \frac{67 - 60}{25} = 0.28$$

Using area table, we threafore get

$$P(52 \leq Y \leq 67) = P(-0.32 \leq Z \leq 0.28)$$
$$= P(-0.32 \leq Z \leq 0) + P(0 \leq Z \leq 0.28)$$
$$= 0.1255 + 0.1103$$
$$= 0.2358$$

Chapter # 10

Question # 1

Solution:

$$b = \frac{n\sum XY - (\sum X)(\sum Y)}{n\sum X^2 - (\sum X)^2} = \frac{\sum XY - n\bar{X}\bar{Y}}{\sum X^2 - n\bar{X}^2}$$

and

$$a = \bar{Y} - b\bar{X}$$

(a)

Now $b = \frac{1000 - (10)(10)(20)}{2000 - (10)(10)^2} = \frac{-1000}{1000} = -1$

and

$$a = 20 - (-1)(10) = 30$$

Hence the desired estimated regression equation is

$$\hat{Y} = 30 - X$$

(d)

$$b = \frac{(10)(130628) - (1710)(760)}{(10)(293162) - (1710)^2}$$

$$= \frac{1306280 - 1299600}{2931620 - 2924100} = \frac{6686}{7523} = 0.8887$$

and

$$a = 76 - (0.8887)(171) = 76 - 151.97 = -75.97$$

Thus the desired estimated regression line is

$$\hat{Y} = -75.97 + 0.89X$$

Question # 2

Solution:

(b)

Computation of the co-efficient of correlation for the given sample values.

$$r = \frac{\sum XY - n\bar{X}\bar{Y}}{\sqrt{[\sum X^2 - n\bar{X}^2][\sum Y^2 - n\bar{Y}^2]}} = \frac{404 - (20)(2)(8)}{\sqrt{[180 - 20(2)^2][1424 - 20(8)^2]}}$$

$$r = \frac{404 - 320}{\sqrt{(100)(144)}} = 0.70$$

Scanned with CamScanner