DMS

Week : 1

Data : 16-3-2022

- **Logic**
  - Solution of Problem
  - Reason behind of Problem Solving
  - Convince other to solve / Solution
  - Convince someone on a decision
- **Propositional Logic**
  - Validation of Solution Logic
  - Yes or No / True or False

Example:

$x + 2 = 4$ $\quad\quad$ $2+2=4$ ✓

If $x = 2$ then true $\quad\quad$ $2+3 = 4$ ×

- Time is not
- Declarative Statement (Discription)
- Facts
- Variables $(p, q, r, s, \ldots)$ can be use as Propositional assignment.

$p \rightarrow$ This is discrete mathmatics class.

Propositional Logic Operators

(1) Negation $(\neg)$

- $(\neg)$ is used for negation / NOT operator
- $\neg p \Rightarrow$ Not p
- Outcome can be change with time but it will remain proposition.

$p \rightarrow$ True	$p \rightarrow$ False
$\neg p \rightarrow$ False	$\neg p \rightarrow$ True

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Propositional Logic Operator

(2) Conjunction (^)

- (^) is used as AND operator
- If both variables are true then result is true else always false.

q : True
p : True

$\boxed{p \&\& q}$ → In programming.

p	q	p^q
True	True	True
True	False	False
False	True	False
False	False	False

(3) Disjunction (v)

- (v) is used as OR operator
- If both variables are false the result is false else always true

$\boxed{p \| q}$ → In programming

p	q	p‖q
True	True	True
True	False	True
False	True	True
False	False	False

(4) Implication $(\rightarrow)$

- $(\rightarrow)$ is used as cause or effect
- Value of Second variable depends upon the value of first.

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

(5) BiImpliment $(\leftrightarrow)$

- $(\leftrightarrow)$ is used a BiImplication
- Value of both variables depends on each other.
- Expression : $p \rightarrow q \wedge q \rightarrow p$

p	q	$p \rightarrow q$	$q \rightarrow p$	$q \leftrightarrow p$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	F	F	F

$$q \leftrightarrow p = q \wedge p$$

Additional Operation Condition

(1) ~~Converse~~ Inverse

$\neg p \rightarrow \neg q$

p	q	$\neg p$	$\neg q$	$\neg p \rightarrow \neg q$
T	T	F	F	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

(2) **Inverse** Converse

$q \rightarrow p$

(3) **Contrapositive**

$\neg q \rightarrow \neg p$

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Tautology

- Returns always TRUE value
- Many combinations that always returns TRUE

p	$\neg p$	$p \vee \neg p$
T	F	T
F	T	T

Prediction

- Check the condition weather the value is TRUE or FALSE

Example

$x > 5$

$P(x): x > 5$

IF $p(3)$ then FALSE

$p(10)$ then TRUE

$y = x + 5$

$P(x,y) : y = x + 5$

$P(5,10) : 10 = 5+5$

TRUE

$P(2,5) : 5 = 2+5$

FALSE

TRUE is the values of of x and y satisfy the equation

Operation Presidence

1. $\neg$
2. $\wedge$
3. $\vee$
4. $\rightarrow$
5. $\leftrightarrow$

Logical Equivalance

Proof: $\neg(p \wedge q) \equiv \neg p \vee \neg q$

p	$\neg p$	q	$\neg q$	$p \wedge q$	$\neg(p \wedge q)$	$\neg p \vee \neg q$
T	F	T	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	T	F	T	F	T	T

Quantifier

(i) $\forall$ (For all)

$P(x)$: $x$student Enroll in DMS has passed PF

$\forall x\, P(x) \Rightarrow$ True

$P(x_1) \wedge P(x_2) \wedge P(x_3) \wedge \dots P(x_n)$

All values must TRUE for condition

(ii) $\exists$ (There exist any)

$Q(x)$: Student x got A+ in PF

$\exists x\, Q(x) \Rightarrow$ True

$Q(x_1) \vee Q(x_2) \vee Q(x_3) \dots \wedge Q(x_n)$

Any value must true for condition

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Examples and Uses :

Domain = { SE 22 }

$P(x)$: " x is enrolled in DMS "

$\forall \rightarrow$ For all

$\exists \rightarrow$ For anyone

$Q(x)$: " x is enrolled in OOP "

- $\forall x \, (P(x) \wedge Q(x))$

→ To get enroll in OOP and DMS it is compulsory to pass PF

$P(x)$ = x enroll in OOP
$Q(x)$ = x enroll in DMS
$R(x)$ = x has passed PF

① $R(x) \rightarrow (P(x) \wedge Q(x))$

② $S(x,y)$: Student in course x and y

$x \rightarrow$ OOP and $y \rightarrow$ DMS

$$\begin{cases} R(x) \rightarrow S(x,y) \\ R(x) \rightarrow S(OOP, DMS) \end{cases}$$

$z \rightarrow$ certain Student

$R(x) \rightarrow (z, OOP, DMS)$

Nested Quantifiers

$P(x,y)$: $x+y = 10$ Domain = $\{1, 2, \dots 10\}$

① $\forall x \exists y\, P(x,y)$ — All the values of x, only value of y.
$P(1,9)$: True — if x ay and y
$P(2,8)$: True satisfy the equation
⋮
$P(10,7)$: False — if x and y do not satisfy the equations

$x+y = y+x$
② $\forall x \forall y\, P(x,y)$: — All the values of x and
③ $\forall x \exists y\, P(x,y)$: — All the values of x any / only y

④ $P(x,y)$: " x student is enroll in y course. "

- $\forall x \forall y\, P(x,y)$: All student are enroll in all course
- $\exists x \exists y\, P(x,y)$: Any Student is enroll in any course

Date : 31-3-2022

Set Theory

- Collection of well-defined data
- Must be distinct
- Any element must not repeat

– Roster Method
$S = \{2, 4, 6, 8\}$
– Set builder notation
$S = \{x \mid 0 < x < 10$ and $x \% 2 = 0\}$
$S = \{$Set of all first 10 Even Numbers$\}$

Subset

- If a set is a part of another set

$E = \{\text{set of Even numbers}\}$
$N = \{\text{set of Natural numbers}\}$

- E is subset of N $\boxed{E \subseteq N}$

$S = \{2, 4, 6, 8\}$
$T = \{1, 2, 3, \ldots 10\}$

- S is subset of T $\boxed{S \subseteq T}$
- All the element of subset must be in actual set.

① $P(x) = \{x \in E \longrightarrow x \in N\}$
② $P(x) = \{x \in S \longrightarrow x \in T\}$

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Proper Set

$A \subset B$

$A = \{1, 2, 3, 4, 5\}$
$B = \{1, 2, 3, 4, 5, 6\}$

→ A is proper set of B

$\forall x (x \in A \rightarrow x \in B\} \wedge \exists x (x \in B \rightarrow x \notin A\}$

- Every element of A must be in set B except one element of B
- Every Proper set is a Subset

Equal Set

- Every element of set A must be in set B

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$A = \{1, 3, 4\}$
$B = \{1, 3, 4\}$
→ A and B ~~are~~ have same element so they are equal set

$\forall x (x \in A \leftrightarrow x \in B)$
$\forall x (x \in A \rightarrow x \in B) \wedge \forall x (x \in B \rightarrow x \in A)$

Empty set

- Representation : $\{ \}$, $\emptyset$
  $\{\emptyset\}$ → This is not empty set. There exist only null element in this set

Set Cardinality.

- Cardinality → number of element in set

$A = \{1, 2, 3, \dots 10\}$. Carbinality $= 10 = |A|$
$B = \{1, 2, 3, 5, \dots 19\}$ Cardinality $= 10 = |B|$

Set Interval

[ ] → closed Interval e.g [a, z] → closed (Included)
( ] → open Interval e.g (a, z] → open (not Included)

Subset Theorm

- Subset associated with a non-empty set 2^n
- Set of 1 element has 2 subset

$A = \{a\} \Rightarrow$ Subset $= \{\phi\}, \{a\}$

Power Set

- Set of all the possible subset

Set Operations

(i) Union $(\cup)$ e.g $A = \{1, 2, 3\}, B = \{4, 5\}, A \cup B = \{1, 2, 3, 4, 5\}$
(ii) Intersection $(\cap)$ e.g $A = \{1, 2, 3, 4\}, B = \{4, 5\}, A \cap B = \{4\}$
(iii) Difference
(iv) Compliment
(v) Cartesion Product

- Cardinality of unknown elements of set (union)

Let

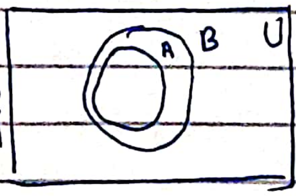
$A = N$ and $B = M$ where N and M are finite numbers

$|A \cup B| = \cancel{N+M}\ |A| + |B| - |A \cap B|$

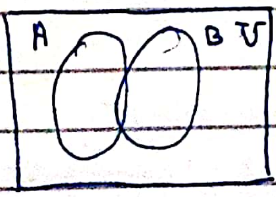
$\boxed{|A \cup B| = |A| + |B| - |A \cap B|}$ → number of elements of union of two sets

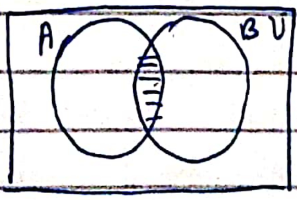
Van Diagram

- $A \subseteq B$ $A = \{a, b, c\}$ $B = \{a, b, c, d, e, f\}$



- $A \cup B$





- $A \cap B$

$A - B \Rightarrow A / B = \{g, h\}$

$\exists x (x \in A \longrightarrow x \notin B)$

Disjoint Set

- Two sets are said to be disjoint sets if they have no element in common.

Cartesian Product

$A = \{a_1, a_2, a_3, \ldots a_n\}$
$B = \{b_1, b_2, b_3, \ldots b_m\}$
$A \times B = \{(a_i, b_i)\}$ $a_i \in A$, $b_i \in B \wedge i = \{1, 2, 3, \ldots n\}$

$A = \{1, 2, 3\}$, $B = \{a, b\}$, $A \times B = ?$
$A \times B = \{(1, a)(1, b)(2, a)(2, b)(3, a)(3, b)\}$

- $A \times B \rightarrow$ number of pairs of cartesian product is equal to $|A| \times |B|$
- $A \times B \neq B \times A$
- Relation between (b)
  $\{(1, b)(2, b)(3, b)\}$

Date : 7-4-2022

Single Term Set

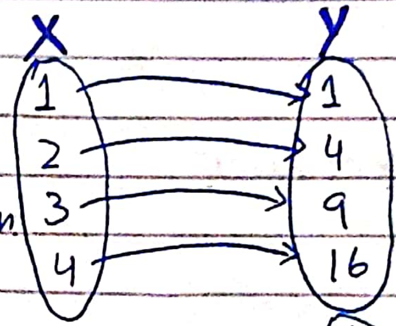
- A set having only 1 element
- $A = \{x\}$

Function

$f : A \rightarrow B$

↑ Domain ↑ Co-domain

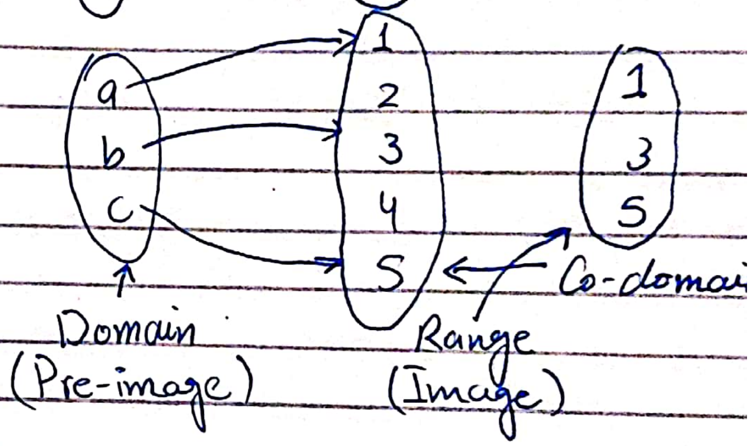
- Range is subset of Co-domain



$f(a) = 1$

$f(b) = 3$

$f(c) = 5$



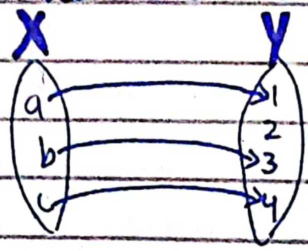
- 1 is a image of a
- a is the pre-image of 1
- 3 is the image of b
- b is the pre-image of 3
- 5 is the image of c
- c is the pre-image of 5

II Type of Functions

(i) One-To-One (Injective)
(ii) Onto (Subrjective)
(iii) One-to-One Correspondance (Bijective)

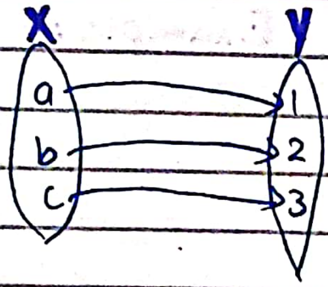
(i) One-to-One Function

- If every pre-image has a unique image then the function is called 1-to-1
- The size of Range must be equal to domain or greater then domain.



(ii) Onto Function

- Co-domain and range must be equal



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Sequence

- Finite Series

$a_m, a_{m+1}, a_{m+2}, \dots a_n$

term

Index Subscript

final term

- finite Series must have final term

Finding term from explicit formula

$a_k = \dfrac{k}{k+1}$ → 1st 6 terms: $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{6}{7}, \frac{7}{8}$

Index	1	2	3	4	5	6
	1	$-\frac{1}{4}$	$\frac{1}{9}$	$-\frac{1}{16}$	$\frac{1}{25}$	$-\frac{1}{36}$

$\Rightarrow a_k = \frac{(-1)^{k+1}}{k^2}$

Index	1	2	3	4	5	6
	2	6	12	20	30	42

$\Rightarrow a_w = w(w+1)$

Arithmatic Sequence

A sequence in which each term is achieve by adding a constant term in ~~period~~ previous one.

$1, 2, 3, 4, 5, 6, \ldots$

$a, a+d, a+2d, a+3d, \ldots$

General term

first term $\leftarrow a_n = a + (n-1)d$ difference

$\downarrow$

Index

Example:

$4, 1, -2, \ldots$ which term is -77

$a = 4 \qquad d = 1 - 4 = -3$

$a_n = a + (n-1)d$

$-77 = 4 + (n-1)(-3)$

$-77 = 4 - 3n + 3$

$-77 = 7 - 3n$

$-77 - 7 = -3n$

$-84 = -3n$

$\boxed{n = 28}$

Example: $S = 3, 9, 15, 21, \ldots$

$d = 9 - 3 = 6$

$a_{20} = a + (20-1)d$

$a_{20} = 3 + (\cancel{20}\ 19)(6)$

$a_{20} = 3 + 114 = 117$

Example:

Find the 36th term of AP whose 3rd term is 7 and 8th term is 17

$a_3 = 17$ and $a_8 = 17$

$a_{36} = a + (36-1)d$

$a_{36} = 3 + (36-1)2$

$a_{36} = 3 + 35 \times 2 = 73$

Geometric Sequence

A sequence in which each term ~~is~~ except its first term is achieved by multiplying preceding term with a constant.

General formula

$a_n = ar^{n-1}$ for all integers $n \geq 1$ $\boxed{r = \frac{a_n}{a_{n-1}}}$

Example:

Find the 8th term of Sequence :

4, 12, 36, 108, ...

$r = \frac{12}{4} = 3$

$a_8 = 4(3)^{8-1} = 4 \times 3^7 = 4(2187) = 8748$

Example:

which term of GP is 1/8 if 1st term is 4 and $r = \frac{1}{2}$

$a_n = ar^{n-1}$

$\frac{1}{8} = 4\left(\frac{1}{2}\right)^{n-1}$

$\frac{1}{8 \times 4} = \left(\frac{1}{2}\right)^{n-1}$

$\frac{1}{32} = \left(\frac{1}{2}\right)^{n-1}$

$\left(\frac{1}{2}\right)^5 = \left(\frac{1}{2}\right)^{n-1}$

$5 = n-1 \Rightarrow \boxed{n=6}$

Example:

Write the GP with the term who 2nd term is 9 and 4th term is 1.

$a_4 = 1 \rightarrow 1 = ar^{4-1} \rightarrow 1 = ar^3$

$a_2 = 9 \rightarrow 9 = ar^{2-1} \rightarrow 9 = ar$

$\frac{1}{9} = \frac{\cancel{a}r^{\cancel{3}}}{\cancel{a}\cancel{r}} \Rightarrow r^2 = \frac{1}{9} \Rightarrow \boxed{r = \frac{1}{3}}$

$9 = a\left(\frac{1}{3}\right) \Rightarrow a = \frac{1}{27}$

Series

- Addition of Sequence

e.g. $2+4+8+16+32+\dots$ or $\sum_{i=1}^{5} 2^i = 2^1+2^2+2^3+2^4+2^5$

Sigma

Examples:

1) $\sum_{i=1}^{4} (2i-1) = (2(1)-1)+(2(2)-1)+(2(3)-1)+(2(4)-1)$
$= 1+3+5+7$

2) $\sum_{i=0}^{5} \frac{(-1)^i}{i+1} = \frac{(-1)^0}{0+1} + \frac{(-1)^1}{1+1} + \frac{(-1)^2}{2+1} + \frac{(-1)^3}{3+1} + \frac{(-1)^4}{4+1} + \frac{(-1)^5}{5+1}$

$= \frac{1}{1} + \frac{-1}{2} + \frac{1}{3} + \frac{-1}{4} + \frac{1}{5} + \frac{-1}{6}$

$= 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6}$

Sigma Rules:

① $\sum_{i=m}^{n} c.a_i = c\sum_{i=m}^{n} a_i$

② $\sum_{k=m}^{n} (a_k+b_k) = \sum_{k=m}^{n} a_k + \sum_{k=m}^{n} b_k$

Arithmatic Series

The sum of terms of AP in arithmatic series.

General Formula

$S_n = \frac{n}{2}[2a+(n-1)d]$

Date : 14-4-22

Arithmetic Series

$S = \frac{n}{2}[2a+(n-1)d]$ ——— (i)

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$S = \frac{n}{2}[a_i + a_n]$ ——— (ii)

Example:

1. $S_{10} = ?$

$3, 9, 15, 21$

$a = 3, \quad d = 6, \quad n = 10$

$S_{10} = \frac{10}{2}[2(3) + (10-1)(6)]$

$S_{10} = 5[6 + 9(6)] = 5[6 + 54] = 5 \times 60 = 300$

Geometric Series

$S = \frac{a(r^n - 1)}{r - 1}$ $r \rightarrow$ Common Ratio

Example:

1. $4 + 12 + 36 + 108 + \ldots$

$a = 4, \quad n = 6, \quad r = 3$

$S = \frac{4(3^6 - 1)}{3 - 1} = \frac{4(3^6 - 1)}{2} = 1456$

2. $\frac{9}{4} + \frac{3}{2} + 1 + \frac{2}{3}$

$a = \frac{9}{4}, \quad r = \frac{2}{3}, \quad n = 6$

$S = \frac{\frac{9}{4}\left(\left(\frac{2}{3}\right)^6 - 1\right)}{\frac{2}{3} - 1} = \frac{\frac{9}{4}\left(\frac{64}{729} - 1\right)}{\frac{2-3}{3}} = \frac{\frac{9}{4}\left(\frac{64 - 729}{729}\right)}{-\frac{1}{3}}$

$= \frac{\frac{9}{4}(-63)}{-\frac{1}{3}} = \frac{665}{36}$

Principal of Mathematical Induction

$$1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2} \quad \text{for } n \geq 1$$

(1) Basic Step

Prove for $n = 1$

$$1 = \frac{n(n+1)}{2} = \frac{1(1+1)}{2} = \frac{2}{2} = 1$$

(2) Inductive Step

Let it be true for $n = k \geq 1$

$$1 + 2 + 3 + \dots + k = \frac{k(k+1)}{2}$$

Now we have to show it is true for $n = k+1$

$$1 + 2 + 3 + \dots + k + (k+1) = \frac{k(k+1)}{2} + (k+1)$$

R.H.S :

$$= \frac{k(k+1)}{2} + (k+1)$$

$$= \frac{k(k+1) + 2(k+1)}{2}$$

$$= \frac{k(k+1) + 2k + 2}{2}$$

$$= \frac{k(k+1) + 2(k+1)}{2}$$

$$= \frac{(k+1)(k+2)}{2} = \frac{(k+1)(k+1+1)}{2}$$

Hence it is proved.

- Principle of Mathematical Induction is a method of prove.

Date : 20-4-22

Prove that $\sum_{i=0}^{n} r^i = \frac{r^{n+1}-1}{r-1}$ for $n \geq 0$

① Let $P(n)$ be the property

Basic Step

Let it be true for $n=0$

$$r^0 = \frac{r^{0+1}-1}{r-1} = \frac{\cancel{r-1}}{\cancel{r-1}}$$

$$1 = 1$$

Inductive Step

Let $P(k)$ be true for some $n=k \geq 0$

$$\sum_{i=0}^{k} r^i = \frac{r^{k+1}-1}{r-1}$$

We have to prove that $P(k+1)$ is true.

$$\sum_{i=0}^{k} r^i + r^{k+1} = \frac{r^{k+1}-1}{r-1} + r^{k+1}$$

$$\sum_{i=0}^{k+1} r^i = \frac{r^{k+1}-1+(r-1)(r^{k+1})}{(r-1)} - r^{k+1}$$

$$= \frac{\cancel{r^{k+1}} - 1 + r.r^{k+1} - \cancel{r^{k+1}}}{r-1}$$

$$\sum_{i=0}^{k+1} r^i = \frac{r^{k+2}-1}{\cancel{r-1}} \quad \frac{r^{k+1+1}-1}{r-1}$$

So it is Proved that $P(k+1)$ is also true which complete the proof.

① Prove using mathematical induction

$1+3+5+\dots+(2n-1) = n^2$ for $n \geq 1$

Basic Step

Let it is true for $n=1$

$1 = (1)^2$

$1 = 1$ Proved

Induction Step

Let it be true for $n=k$

$1+3+5+\dots+(2k-1) = k^2$

We have to ~~prove~~ prove it for $(k+1)$

~~$1+3+5+\dots+(2(k+1)-1) = k+1$~~

$1+3+5+\dots+(2k-1)+[2(k+1)-1] = (k)^2+[2(k+1)-1]^2$

$= k^2+[2k+2-1]^2$

$= k^2+2k+1$

$1+3+5+\dots+(2k-1)+[2(k+1)-1] = (k+1)^2$

2) Prove using mathematical induction

$1+6+11+\dots+5n-4 = \frac{n(5n-3)}{2}$

Basic Step

Let it be true for $n=1$

$1 = \frac{5-3}{2} = \frac{2}{2} = 1$

Proved

Inductive Step

Let it be true for $n=k$

$1+6+11+\dots+5k-4 = \frac{k(5k-3)}{2}$

We have to prove it for $(k+1)$

$1+6+11+\dots+(5k-4)+5(k+1)-4 = \frac{k(5k-3)}{2}+5(k+1)-4$

$= \frac{5k^2-3k}{2}+5k^2+5-4$

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$$= \frac{5k^2 - 3k}{2} + 5k + 1$$

$$= \frac{5k^2 - 3k + 10k + 2}{2} = \frac{5k^2 + 7k + 2}{2}$$

$$= \frac{5k^2 + 5k + 2k + 2}{2} = \frac{5k(k+1) + 2(k+1)}{2}$$

$$= \frac{(5k+2)(k+1)}{2}$$

① For all integers $n \geq 0$ prove that $2^{2n} - 1$ is divided by. 3

Basic

Let $P(n) = 2^{2n} - 1$ is divided by 3

Show that $P(0) = 2^{2(0)} - 1 = 2^0 - 1 = 0$

$\Rightarrow$ $3.0 = 0$ so $P(0)$ is proved

Inductive

Let $P(k) = 2^{2k} - 1$ is divided by 3

Show that $P(k+1)$ is divided by 3

let

$2^{2k} - 1$ is divided by 3

$$\boxed{2^{2k} - 1 = 3r}$$

$$2^{2(k+1)} - 1 = 2^{2k+2} - 1 = 2^{2k} . 2^2 - 1$$

$$2^{2k} . 4 - 1 = 2^{2k}(3+1) - 1 = 3.2^{2k} + 2^{2k} - 1$$

$$3.2^{2k} + 3r = 3(2^{2k} + r)$$

Well Ordering Principle

Every non-empty set S - of non-negative contains a least element.

$S = \{\underset{=}{2}, 4, 6, 8, 10\}$

Fibonnacci Sequence

- $F_n = F_{n-1} + F_{n-2}$
- Recursive Sequence
- Every element of sequence is sum of last two terms.

e.g 0 1 1 2 3 5 8 13 ...

→ Initial Condition

$F_0 = 0$ and $F_1 = 1$

e.g

$C_k = C_{k-1} + kC_{k-2} + 1$ for $k \geq 2$ and $C_0 = 1, C_2 = 2$

$C_3 = C_{3-1} + 3C_{3-2} + 1$

$C_3 = C_2 + 3C_1 + 1 = 2 + 3 + 1 = 6$

$C_4 = C_{4-1} + 4C_{4-2} + 1$

$= C_3 + 4C_2 + 1 = 6 + 8 + 1 = 15$

$C_5 = C_{5-1} + kC_{5-2} + 1$

$= C_4 + kC_3 + 1 = 15 + 5(6) + 1 = 15 + 30 + 1 = 46$

Date: 21-4-22

Find 1st four terms of following Recursive Relation.

$a_k = 2a_{k-1} + k$ for all $k \geq 2$

$a_1 = 1$

$\cancel{a_2 = 2(2)}$

$a_2 = 2a_{2-1} + 2 = 2(1) + 2 = 4$

$\cancel{a_3 = 3a_{3-1} + 3 = 3(4) + 3 = 15}$

$\cancel{a_4 = 4a_{4-1} + 3 = 4(15) + 3 =}$

$a_3 = 2a_{3-1} + 2 = 2a_2 + 2 = 2(4) + 3 =$

a_4

$a_1 = 1$

$a_2 = 2a_{2-1} + 2 = 2(1) + 2 = 4$

$a_3 = 2a_{3-1} + 3 = 2(4) + 3 = 11$

$a_4 = 2a_{4-1} + 4 = 2(11) + 4 = 26$

Does an Explict formula Satisfy a Recursive Relation (RR)

Let $a_n = 3n + 1$ for $n \geq 0$

Show that it satisfy RR

$a_k = a_{k-1} + 3$ for $k \geq 1$

$a_k = 3k + 1$ | $a_{k-1} = 3(k-1) + 1$

$= 3k - 3 + 1$

$a_{k-1} = 3k - 2$

R.H.S:

$= a_{k-1} + 3$

$= 3k - 2 + 3 = 3k + 1 =$ L.H.S

Let $b_n = 4^n$ for $n \geq 0$

Show that it satisfy RR

$b_n = 4b_{k-1}$ for $k \geq 1$

$b_k = 4^k$ | $b_{k-1} = \cancel{4^{k-1}}$ $\quad 4.4^{k-1}$

$= \cancel{4^k . 4^{-1}}$ $\quad 4^{k-1+1}$

$= 4^k$

Let $t_n = 2 + n$

RR $\rightarrow$ $t_k = 2t_{k-1} - t_{k-2}$ for $k \geq 1$

$t_k = 2 + k$ | $2t_{k-1} = 2 + (k-1) = k + 1$ $\cancel{2t_{k-1}}$

$t_{k-2} = \cancel{2t_{k-1}} - \cancel{t_{k-2}}$ $\quad 2 + k - 2 = k$

R.H.S: $t_k = 2[k+1] - k = 2k + 2 - k = k + 2$

$\boxed{t_k = 2 + k}$

$T_k = 2T_{k-1} + 1$

$T_1 = 1$, $T_2 = 3$ and $T_3 = 7$

$T_3 = 2T_{3-1} + 1 = 2T_2 + 1 = 2(3) + 1 = 7$

$T_4 = 2T_{4-1} + 1 = 2T_3 + 1 = 2(7) + 1 = 15$

$T_5 = 2T_{5-1} + 1 = 2T_4 + 1 = 2(15) + 1 = 31$

$T_6 = 2T_{6-1} + 1 = 2T_5 + 1 = 2(31) + 1 = 63$

Date: 27-4-2022

Solve Recurance and Find Closed form / Explicit

· **Iterative Method**

$a_0, a_1, a_3 \dots$

$a_k = a_{k-1} + 2$

$a_0 = 1 \longrightarrow a_0 = 1 + 2 \times 0$

Guess explicit formula

$a_1 = 1 + 2 = 3 \rightarrow a_1 = 1 + 2 \times 1$

$a_2 = 3 + 2 = 5 \rightarrow a_2 = 1 + 2 \times 2$

$a_3 = 5 + 2 = 7 \rightarrow a_3 = 1 + 2 \times 3$

$a_4 = 7 + 2 = 9 \rightarrow a_4 = 1 + 2 \times 4$

Closed form:

$$\boxed{a_n = 1 + 2n}$$

· Let $a_k = r a_{k-1}$ for $k \geq 1$

$\boxed{a_0 = a}$

$a_0 = a$

$a_1 = ar$

$a_2 = r[ar] = ar^2$

$a_3 = r[ar^2] = ar^3$

$a_4 = r[ar^3] = ar^4$

Closed form: $\boxed{a_n = ar^n}$

· Let $c_k = 3c_{k-1} + 1$ for $k \geq 2$

$c_1 = 1$

$c_2 = 3 + 1 = 4$

$c_3 = 12 + 1 = 13$

$c_4 = 39 + 1 = 40$

$c_5 = 120 + 1 = 121$

Closed form:

$$\boxed{c_n = 3^{n-1} + 3^{n-2} + \dots 3 + 1}$$

Example 1:

$d_0 = 170$ //Add daily 2 products

$d_1 = 170+2 = 172 \rightarrow d_1 = 170+2$

$d_2 = 172+2 = 174 \rightarrow d_2 = (170+2)+2$

$d_3 = 174+2 = 176 \rightarrow d_3 = \boxed{(170+2+2)+2}$

$d_4 = 176+2 = 178 \rightarrow d_4 = \boxed{(170+2+2+2)+2}$

Closed form: $\underbrace{\boxed{d_k = d_{k-1} + 2}}_{\text{Recursion Equ.}} \Rightarrow \boxed{d_k = 170 + 2k}$

$d_{30} = ?$ $d_{30} = 170 + 2 \times 30 = 170 + 60 = 230$

Example 2:

$d_0 = 3$ mins $= 180$ sec

$d_{14} = ?$

$d_1 = 180 - 3 = 177$

$d_2 = 177 - 3 = 174$

$d_3 = 174 - 3 = 171$

Closed form: $\underbrace{\boxed{d_k = d_{k-1} - 3}}_{\text{Recursion Equ.}} \Rightarrow \boxed{d_k = 180 - 3n}$

$d_{14} = 180 - 3(14) = 180 - 42 = \cancel{138}\ 138$

Tower of Hanoi

$T_1 = 1 \rightarrow T_1 = 1$

$T_2 = 3 \rightarrow T_2 = 2+1$

$T_3 = 7 \rightarrow T_3 = 2(2+1)+1 = 2^2+2+1$

$T_4 = \quad \rightarrow T_4 = 2(2^2+2+1)+1 = 2^3+2^2+2+1$

Recursion Equation

~~Explicit formula~~: $T_n = 2^{n-1} + 2^{n-2} + \dots 2+1$

Modular Arithmatic

~~Congruence~~

$2\overline{)19}$
18
1

$\Longrightarrow 19 \equiv \underbrace{1 \text{ Mod } 2}_{\text{Congruence}} \Longrightarrow 2 \mid 19-1$

Congruent

$Z \rightarrow$ Set of Integerst, no fraction or decimal

$-\infty \leftrightarrow 0 \leftrightarrow +\infty$

$-5 \equiv 2 \text{ Mod } 7 \rightarrow 7\overline{)-5} \rightarrow 7 \mid -5-2$

7
2

$7 \mid -7$

$-13 \equiv 1 \text{ Mod } 7 \rightarrow 7\overline{)-13} \rightarrow 7 \mid -13-1$

14

Two integer 'a' and 'b' Mod 'n' are congruvent to each other 'mod n' if their remainder 'mod n' is same.

Date: 28-4-2022

Z mod 2

$29 \equiv 1 \text{ Mod } 2$
$30 \equiv 0 \text{ Mod } 2$
$31 \equiv 1 \text{ Mod } 2$
$32 \equiv 0 \text{ Mod } 2$

Even Integers

$Z_2 = \{0, 1\}$ → Odd Integers

Infinity Many → Even Integer

Z mod 3

$30 \equiv 0 \text{ Mod } 3$
$31 \equiv 1 \text{ Mod } 3$

$32 \equiv 2 \text{ Mod } 3$

$33 \equiv 0 \text{ Mod } 3$

$Z_3 = \underbrace{\{0, 1, 2\}}$ $\qquad$ $Z_5 = \{0, 1, 2, 3, 4\}$

Infinitly many. $\qquad$ Group

$\boxed{Z_n = \{0, 1, 2, \dots n-1\}}$

Group

Modular Addition

$Z_5 = \{0, 1, 2, 3, 4\}$

$3 \text{Mod} 5 + 4 \text{Mod} 5 = 7 \text{Mod} 5 \equiv 2 \text{Mod} 5$

$3 \text{Mod} 5 + 2 \text{Mod} 5 + 1 \text{Mod} 5 = 6 \text{Mod} 5 \equiv 1 \text{Mod} 5$

Set Z_5 is closed under Adition

Modular Subtraction

$3 \text{Mod} 5 - 4 \text{Mod} 5 = -1 \text{Mod} 5 \equiv 4 \text{Mod} 5$

$2 \text{Mod} 5 - 4 \text{Mod} 5 = -2 \text{Mod} 5 \equiv 3 \text{Mod} 5$

Set Z_5 is closed under Subtraction

Encreption

$Z_{26} = \{0, 1, 2, \dots 26\}$

F A N

$k = 17$

$y = x + k \text{Mod} 26 \implies y_1 = x_1 + k \text{Mod } 26 = 5 + 17 \text{Mod}$

$= 22 \text{Mod } 26 = W$

$y_2 = x_2 + k \bmod 26 = 0 + 17 \bmod 26 = 17 \bmod 26 = R$

$y_3 = x_3 + k \bmod 26 = 13 + 17 \bmod 26 = 30 \bmod 26$

$= 4 \bmod 26$

FAN → WRE

Decryption

WRE → 17

22 17 4

$x = y - k \bmod 26$

$x_1 = y_1 - k \bmod 26 = 22 - 17 \bmod 26 = 5 \bmod 26 = F$

$x_2 = y_2 - k \bmod 26 = 17 - 17 \bmod 26 = 0 \bmod 26 = A$

$x_3 = y_3 - k \bmod 26 = 4 - 17 \bmod 26 = -13 \bmod 26 = 13 \bmod 26$

$= N$

WRE → FAN

FAST

K = 20

A	F	K	P	U	Z
0	5	10	15	20	25

~~Encrption~~ Encryption

F = 5 , A = 0 , S = 18 , T = 20

$y = x + k \bmod 26 = 5$

$y_1 = x_1 + k \bmod 26 = \cancel{0}\,5 + 20 \bmod 26 = 25 \bmod 26 = Z$

$y_2 = x_2 + k \bmod 26 = \cancel{18}\,0 + 20 \bmod 26 = 20 \bmod 26 = U$

$y_3 = x_3 + k \bmod 26 = \cancel{2}\,18 + 20 \bmod 26 = 38 \bmod 26 = \cancel{12 \bmod 26 = 0}$

$y_4 = x_4 + k \bmod 26 = 19\cancel{20} + 20 \bmod 26 = \cancel{40}\,39 \bmod 26 = 15\cancel{\bmod 26} = P$

$38 \bmod 26 = 14 \bmod 26 = M$

~~ZUOP~~

$39 \bmod 26 = 15 \bmod 26 = N$ ~~26~~

26   26

14   15

12   13

$\boxed{\text{ZUMN}}$

Decryption

$x = y - k \bmod 26$

$x_1 = y_1 - k \text{ Mod } 26 =$
$x_2 = y_2 - k \text{ Mod } 26 =$
$x_3 = y_3 - k \text{ Mod } 26 =$
$x_4 = y_4 - k \text{ Mod } 26 =$