Date: CHAPTER 3 M T W T F S S

* Measures of Central Tendency or Averages:-

UN.Grouped Data | Grouped Data:

1- The Arithmetic Mean

$$\bar{x} = \frac{\Sigma x}{n} \qquad \bar{x} = \frac{\Sigma fx}{\Sigma f}$$

* Change of Origin & Scale Method [Coding Method]

1- Arithmetic Mean:-

$$\bar{x} = a + \frac{\Sigma u}{n} \times h \qquad \bar{x} = a + \frac{\Sigma fu}{\Sigma f} \times h \quad \Rightarrow u = \frac{x-a}{h}$$

(a: any value from x; h: class interval or width)

2- Geometric Mean:-

$$G = \text{antilog}\left[\frac{1}{n}\Sigma \log x\right] \qquad G = \text{antilog}\left[\frac{1}{\Sigma f}\Sigma f \log x\right]$$

3- Harmonic Mean:-

$$H = \frac{n}{\Sigma(1/x)} \qquad H = \frac{\Sigma f}{\Sigma f(1/x)}$$

* Median:-

$$l + \frac{h}{f}\left[\frac{n}{2} - C\right]$$

- l: lower class of median class
- h: class interval
- f: frequency of median class
- C: commulative frequency above median class
- $\frac{n}{2} = \Sigma f/2$

* Mode:-

$$l + \frac{f_m - f_1}{(f_m - f_1) + (f_m - f_2)} \times h$$

- l: lower class boundry of model class
- f_m: f of model class
- f_1: f associated with the class preceding the model class
- f_2: associated with class following the model class
- h: class interval

KARAM NOTES

Median class = $\frac{n}{2}$ = answer look into commulative frequency column.

Model class: The highest frequency in your given data.

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Median:-

It is a value which divides your data into 2 equal parts.

For Grouped data:-

$$\text{median} = l + \frac{h}{f}\left(\frac{n}{2} - c\right)$$

- l → lower class boundry of median class
- h → class Interval
- f → frequency of certain median class
- c → commulative frequency above median class.
- $n = \Sigma f$

median class = $\frac{n}{2}$ → answer look into commulative frequency coloum:-

Q-

Marks	f	class boundries	cf	
30—39	8	29.5 — 39.5	8	
40—49	87	39.5 — 49.5	95	
50—59	190	49.5 — 59.5	285 = c	
60—69	304	59.5 — 69.5	589	median class
70—79	211	69.5 — 79.5	800	
80—89	85	79.5 — 89.5	885	
90—99	20	89.5 — 99.5	905	

$\Sigma f = 905$ → no. of students:-

median class: $\frac{n}{2} = \frac{\Sigma f}{2} = \frac{905}{2} = 452.5$ & $h = \frac{\text{Range}}{\text{No. of classes}} = \frac{99-30}{7} = 9.8 \approx 10.$

$$\text{median} = 60 + \frac{10}{304}(452.5 - 285)$$

median = 65.0099 → marks

KARAM NOTES

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Mode:-

It is most popular or most frequent value of data:-

For Grouped data:-

$$l + \frac{f_m - f_1}{(f_m - f_1) + (f_m - f_2)} \times h$$

- l → lower class boundry of model class
- f_m → f of model class
- f_1 → F associated with the class preceding the model class.
- f_2 → F associated with class following model class
- h → class interval

For ungrouped data:-

1 Mode = Unimodel distribution (one mode exist)
2 Mode = Bimodel distribution (two mode exist)

Model class:-

The highest frequency in your given data.

The values we will take in Mode is from previous data.

$$l + \frac{f_m - f_1}{(f_m - f_1) + (f_m - f_2)} \times h = 59.5 + \frac{(304 - 285)}{(304 - 285) + (304 - 800)} \times 10$$

$$59.5 + \frac{19}{19 + (-496)}$$

$$59.5 + \frac{(304 - 190)}{(304 - 190) + (304 - 211)} \times 10$$

$$59.5 + \frac{114}{114 + 93} \times 10$$

$$59.5 + \frac{114}{207} \times 10$$

$$59.5 + 5.5072$$

$$65.0073 = \text{mode}$$

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Numerical relation b/w mean, median & mode.

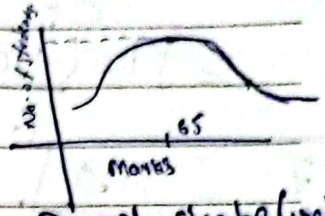
Imperical

Mode = 3 Median − 2 mean (unimodel relation)

mean > median > mode (skewed distribution)

mean < median < mode (skewed distribution)

mean = median = mode (symmetrical distribution)



Barrel shape (unimodel)

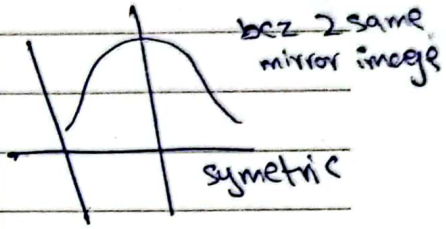
The following table shows the distribution of maximum load in short terms supported by certain cables produced by a company-

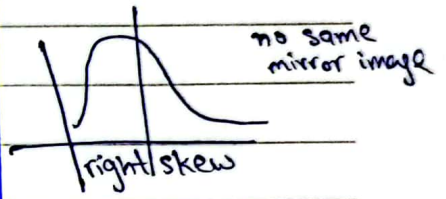
Max loads (short terms)	9.8 – 10.2	10.3 – 10.7	10.8 – 11.2	11.3 – 11.7	11.8 – 12.2	12.3 – 12.7
No. of cables (f)	7	12	17	14	6	4
class boundaries	9.75 – 10.25	10.25 – 10.75	10.75 – 11.25	11.25 – 11.75	11.75 – 12.25	12.25 – 12.75
cf	7	19	36	50	56	60

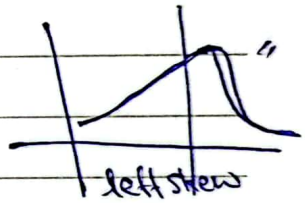
Σf = 60

d	−2	−1	0	1	2	3
fd	−14	−12	0	14	12	12

Σfd = −14 − 12 + 0 + 14 + 12 + 12 = 26 − 26 = 0







Median:-

$$l + \frac{h}{f}\left(\frac{n}{2} - c\right)$$

$$10.75 + \frac{0.5}{17}(30 - 19)$$

Median = 11.0735

Mode

$$10.75 + \frac{(17-12)}{(17-12)+(17-14)} \times 0.5$$

$$10.75 + \frac{5}{8} \times 0.5$$

Mode = 11.0625

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3.17

No.	x	log x
1	9	0.9542
2	12	1.0792
3	15	1.1761
4	15	1.1761
5	16	1.2041
6	18	1.2553
7	20	1.3010
8	20	1.3010
9	25	1.3979
10	30	1.4771

~~iii) Median:~~

$\Sigma x = 180$ $\Sigma \log x = 12.3220$

i) Arithmetic Mean ii) Geometric Mean.

$\bar{x} = \frac{\Sigma x}{n}$

$\bar{x} = \frac{180}{10}$

$\bar{x} = 18$

$G = \text{antilog}\left[\frac{1}{n}\Sigma \log x\right]$

$\text{antilog}\left[\frac{1}{10} \cdot 12.3220\right]$

$\text{antilog}(1.2322) \Rightarrow 17.0687 = G$

3.18

Family	A	B	C	D	E	F	G	H	I	J	= 10
Income (Rs) (x)	85	70	10	75	500	8	42	250	40	36	= 1116
$\Sigma \log x = 17.4313$	1.9294	1.8451	1.0000	1.8751	2.6990	0.9031	1.6232	2.3979	1.6021	1.5563	
$\frac{1}{n}(\Sigma x)$	0.3470	0.0118	0.0143	0.1000	0.0133	0.0020	0.0250	0.0238	0.0040	0.0250	0.0278

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① Arithmetic Mean :- ② Geometric Mean :- iii) Harmonic Mean

$\bar{x} = \frac{\Sigma x}{n}$ $\quad G = \text{antilog}\left[\frac{1}{n}\Sigma \log x\right]$ $\quad H = \frac{n}{\Sigma(1/x)}$

$\bar{x} = \frac{1116}{10}$ $\quad = \text{antilog}\left[\frac{1}{10}\ 17.4183\right]$ $\quad = \frac{10}{0.3470}$

$\bar{x} = 111.6$ $\quad 55.3516 \approx G$ $\quad H = 28.8157$

Geometric Mean is best average :-

3.19 :-

Rs(x)	1/x	log x
60	0.0167	1.7782
80	0.0125	1.9031
90	0.0111	1.9542
96	0.0104	1.9823
120	0.0083	2.0792
150	0.0067	2.1761
200	0.0050	2.3010
360	0.0028	2.5563
480	0.0021	2.6812
520	0.0019	2.7160
1060	0.0009	3.0253
1200	0.0008	3.0792
1450	0.0007	3.1614
2500	0.0004	3.3979
7200	0.0001	3.8573
Σ = 15566	0.0804	38.6487

$n = 15$

① Arithmetic Mean :-

$\frac{\Sigma x}{n} = \frac{15566}{15}$

$\bar{x} = 1037.7333$

② Geometric Mean.

$\text{antilog}\left[\frac{1}{n}\Sigma \log x\right]$

$\text{antilog}\left[\frac{1}{15}(38.6487)\right]$

$\text{antilog}(2.5766)$

$G = 377.2072$

③ Harmonic Mean.

$H = \frac{n}{\Sigma(1/x)}$

$H = \frac{15}{0.0804}$

$H = 186.5672$

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3.21

Piece goods:-	Price Per metre(x)	Quality
Unbleached	8.37	286
Bleached	9.50	255
Printed flags	9.16	64
Other sorts	9.84	172
Dyed in piece	13.65	165
Of dyed yarn	11.95	80

$\Sigma x = 62.4700$

(i) Simple average of column 2:- (Arithematic Mean)

$$\bar{x} = \frac{\Sigma x}{n} = \frac{62.4700}{6} \Rightarrow \boxed{10.4117 = \bar{x}}$$

3.28

b)

	x
A to B	30
B to C	60

Harmonic mean is best for correct average speed of the person

$$H = \frac{n}{\Sigma \frac{1}{x}} = \frac{2}{\frac{1}{30}+\frac{1}{60}} = \frac{2}{0.0500} = \boxed{40 \text{ miles per hour} = H}$$

(c)

n	x
A	8
B	7.5
C	5.5

$$H = \frac{n}{\Sigma(\frac{1}{x})} = \frac{3}{\frac{1}{8}+\frac{1}{7.5}+\frac{1}{5.5}} = \frac{3}{0.1250+0.1333+0.1818}$$

$$\boxed{H = 6.8164}$$

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3.29

Interval	(x) Km (speed)	time.	1/x
1	10		0.1000
2	15		0.0667
3	20		0.0500
4	25		0.0400
5	20		0.0500
6	30		0.0333
7	40		0.0250
8	50		0.0200
9	30		0.0333
10	40		0.0250
			0.4434

a) Harmonic Mean:- $H = \frac{n}{\Sigma(1/x)}$

$= \frac{10}{0.4434} \Rightarrow$ **H = 22.5530**

(b)

Person	Ride (x)
1	10
2	8
3	6

$H = \frac{3}{\Sigma f\left(\frac{1}{10}+\frac{1}{8}+\frac{1}{6}\right)} =$ **7.6596 = H**

3.31 :-

i) Geometric Mean.

$G = \text{antilog}\left[\frac{1}{\Sigma f} \Sigma f \log x\right]$

$\text{antilog}\left[\frac{1}{100}(169.1837)\right]$

$G = 49.1855$

2) Harmonic Mean

$H = \frac{\Sigma f}{\Sigma f(1/x)}$

$H = \frac{100}{(2.0714)}$

$H = 48.2765$

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Weekly Income	No. of Workers	x	1/x	f(1/x)	log x	f log x
35–39	15	$\frac{35+39}{2}=37$	0.0270	0.4050	1.5682	23.5230
40–44	13	42	0.0238	0.3094	1.6232	21.1016
45–49	17	47	0.0213	0.3621	1.6721	28.4257
50–54	29	52	0.0192	0.5568	1.7160	49.7640
55–59	11	57	0.0175	0.1925	1.7559	19.3149
60–64	10	62	0.0161	0.1610	1.7924	17.9240
65–69	5	67	0.0149	0.0745	1.8261	9.1305
Σ	100		0.1398	2.0714		169.1837

3.31

Variable	f	x	1/x	f(1/x)	log x	f log x
0–5	2	2.5	0.4000	0.8000	0.3979	0.7959
5–10	5	7.5	0.1333	0.6665	0.8751	4.3753
10–15	7	12.5	0.0800	0.5600	1.0969	7.6784
15–20	13	17.5	0.0571	0.7423	1.2430	16.1595
20–25	21	22.5	0.0444	0.9333	1.3522	28.3962
25–30	16	27.5	0.0364	0.5824	1.4393	23.0293
30–35	8	32.5	0.0308	0.2464	1.5119	12.0951
35–40	3	37.5	0.0267	0.0801	1.5740	4.7221
Σ	75		0.8087	4.6110		~~70.4905~~ 97.2525

Geometric Mean

$$G = \text{antilog}\left[\frac{1}{\Sigma f}\Sigma f \log x\right]$$

$$\text{antilog}\left[\frac{1}{75}(97.2525)\right]$$

~~G = 16.0894~~

G = 19.80

Harmonic Mean

$$H = \frac{\Sigma f}{\Sigma f(1/x)}$$

$$H = \frac{75}{4.6110}$$

H = 16.2655

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3.36 ::

Age of head	f	cf
under 25	44	44
25 & " 30	79	123
30 & " 40	152	275
40 & " 850	122	397
50 & " 60	141	538
60 & " 65	100	638
65 & " 70	58	696.
70 & " 75	32	728.
75 & " 85	28.	756

n = 756

median class = n/2 = 378.

Median: $l + \frac{h}{f}\left[\frac{n}{2} - c\right]$

$40 + 0.0820(378 - 275)$

$40 + 0.0820(103)$

$40 + 8.4460$

48.4460 years.