P3: MPC with EKF estimation for a pendulum

In model predictive control (MPC) we repeatedly solve an optimal control problem given the current state of the system. In practice, this full state is often not directly measurable and we require a state-estimation algorithm to recover it from the available measurements of the system. Furthermore, we often face the challenge that the dynamic model contains unknown and uncertain parameters. Especially if these parameters are time-varying, we cannot simply identify them prior to the control task. Instead, we must simultaneously estimate states and parameters. In this project, we design an extended Kalman filter (EKF) to estimate the states and uncertain parameters of the dynamic system and design an MPC algorithm to control the system. For this investigation, we consider the cart-pole system shown in Figure 1.

In this system a freely rotating pendulum of length l_p and mass m_p is linked to a cart of mass m_c. We

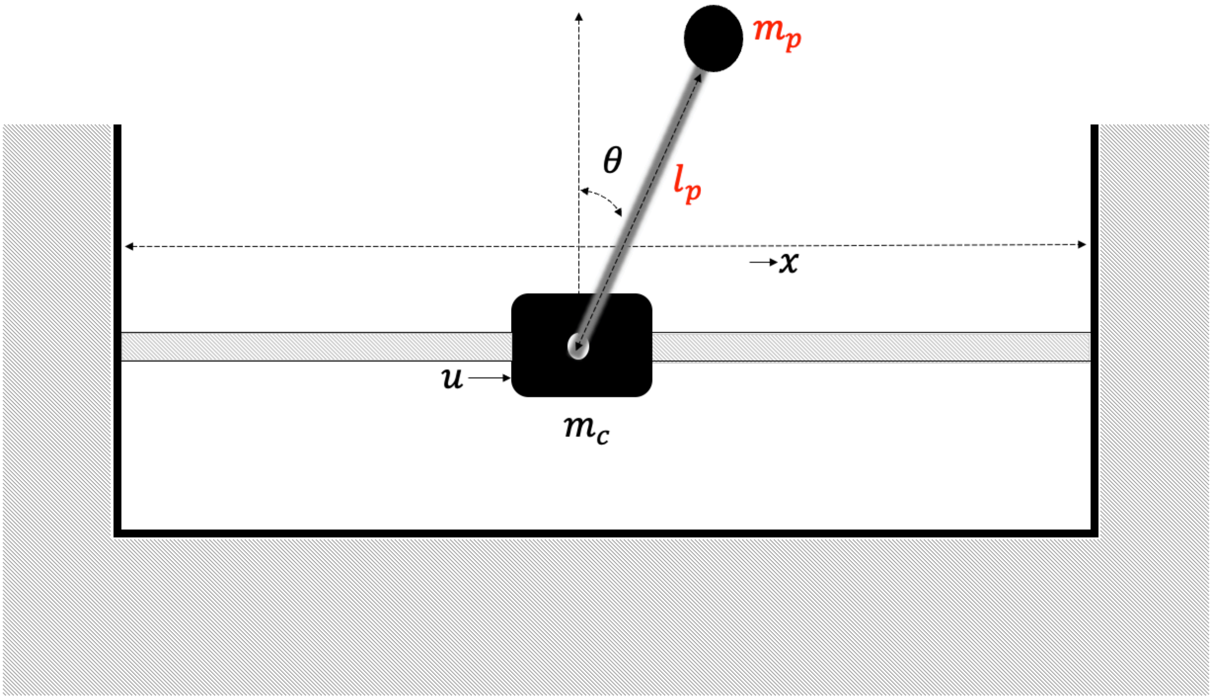

Figure 1: Friction-less Cart-Pole

denote with θ the angle of the pendulum (with respect to the upright position) and $\dot\theta$ describes its angular velocity. The carts position and velocity are denoted with x and $\dot x$, respectively. We assume that the pendulum and the cart move frictionless. The horizontal motion of the cart is controlled by a control input u . The objective of the controller is to swing up and balance the pendulum in the upright position. For parameter estimation, we will assume that the mass and length of the pendulum are unknown (l_p and m_p). The system dynamics are given below with the unknown parameters highlighted in red [1], [2].

$$\ddot x = \frac{u + m_p l_p (\sin\theta)\dot\theta^2 - m_p g \cos\theta \sin\theta}{m_c + m_p - m_p \cos^2\theta}, \tag{1}$$

$$\ddot\theta = \frac{u\cos\theta + (m_c + m_p) g \sin\theta + m_p l_p (\cos\theta\sin\theta)\dot\theta^2}{m_p l_p \cos^2\theta - (m_c + m_p) l_p}. \tag{2}$$

The fixed system parameters are given in Table 1. To highlight the importance of the unknown parameters, we showcase in Figure 2 two system responses with different values for m_p and l_p.

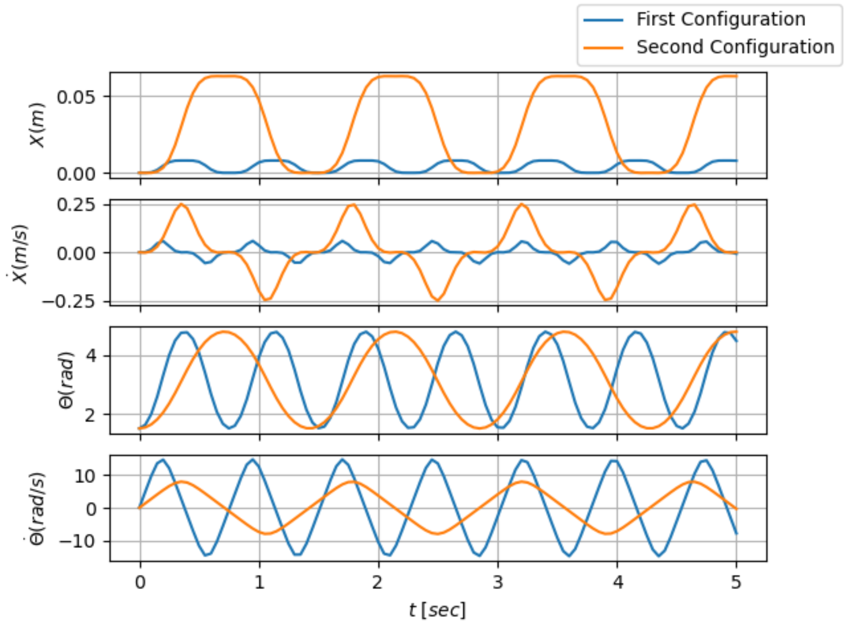

Figure 2: Two system responses (configurations) with different values of m_p and l_p. In both configurations, the pendulum was displaced to θ = 1.5 and allowed to oscillate autonomously. For the first configuration m_p and l_p are set to 0.23 kg and 0.36 m, and for the second 0.1 kg and 0.1 m, respectively.

Table 1: Fixed system parameters and initial guesses for estimated parameters.

Parameter	Plant	observer model
initial state	$x = 0.5, \dot{x} = 0, \theta$ = 3.1 , $\dot{\theta} = 0$	$x = -0.5, \dot{x} = 0, \theta$ = 2.8 , $\dot{\theta} = 0$
Pendulum Mass m_p	0.23 kg (*fixed*)	0.1 kg (*initial estimate*)
Pendulum Length l_p	0.36 m (*fixed*)	0.1 m (*initial estimate*)
Cart Mass m_c	2.4 kg	2.4 kg
Gravity	9.81 m/s^2	9.81 m/s^2

Mandatory Tasks

The following tasks **have to be completed** in order to pass the project.

- **Implement an EKF** for state and parameter estimation (**without MPC controller**).
  - Reformulate your model such that the uncertain parameters are states.
  - Discretize your system to use a discrete-time EKF.
  - Compare estimated vs. true states and parameters.
  - Test your estimator with and without process and measurement noise.
- **Implement the MPC** controller (**without EKF**).
  - Discretize the system using orthogonal collocation on finite elements.
  - The controller should be able to swing up and balance the pendulum at any given position (x_f) of the cart.
  - Assume full state-feedback and knowledge of the pendulum mass and length.

- **Combine the EKF and MPC in a loop**.
    - Estimate states and parameters with the EKF and use this information for the MPC controller.
    - You will need to excite the system in order to have a reasonable estimation. This can be done by alternating through different set points . This is not the only way though.
    - You want to both control and have an accurate parameter estimation.

Additional tasks

Below are **suggested** additional tasks to obtain an excellent grads for the project. We want to emphasize that students are encouraged to come up with their own ideas for additional investigations and not all of the suggestions below must be included for an excellent grade.

- Implement a continuous-time EKF with discrete measurements [1].
- Incorporate obstacle avoidance (constraining the pole movement in parts of the state space) through your optimization problem.
- Compute, visualize and analyze the standard deviation for your estimated states and parameters.

Deliverables

The following materials have to be submitted in order to pass the semester project:

- **Recorded final presentation** (video screencast). The presentation must be **5-7 minutes** (for the entire group) and the file should not exceed **200 mb**. Highlight on the slides which group member(s) are responsible.
- **Written report** to present and discuss the obtained results. You must use the **supplied template on Moodle** (Tex or Word) and write no more than **3-4 pages** (for the entire group). Highlight which group member worked on which section.
- **Source code** of your project. Please ensure that the code is executable and optionally add a short explanation of the structure (readme). **You must write your own Python code using only the packages provided in the PAS Anaconda environment file**[2]. Please ask your supervisor if you wish to use additional packages.

Please ensure that all formal conditions (e.g. page limits, highlight responsible author) are satisfied, as we will deduct points for significant violations. Deadline for the submission is 27.01.2023. Please submit all deliverables via moodle.

Supervisor

Please address questions to:

Name	Contact
Omar Sayed	omar.sayed@tu-dortmund.de

[1] https://en.wikipedia.org/wiki/Extended_Kalman_filter#Discrete-time_measurements
[2] https://moodle.tu-dortmund.de/mod/resource/view.php?id=1272938

References

[1] Reza Olfati-Saber. "Nonlinear Control of Underactuated Mechanical Systems with Application to Robotics and Aerospace Vehicles". PhD thesis. Massachusetts Institute of Technology, 2001.

[2] Elisa Sara Varghese, Anju K. Vincent, and V. Bagyaveereswaran. "Optimal Control of Inverted Pendulum System Using PID Controller, LQR and MPC". In: ***IOP Conference Series: Materials Science and Engineering***. Vol. 263. IOP Publishing, 2017.