Erreur Cartographie: Proposition de plan de travail

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Contents

1 Model

1.1 Sets

We can think about defining three sets:

- $\mathcal{V}$: set of vehicles to route (could be a set of integers that are the ids of the vehicles)
- $\mathcal{I}$: set of nodes for the road network, obviously for each vehicle, the starting node and destination node are in $\mathcal{I}$
- $\mathcal{E}$: set of edges, which is the road network. Basically, $\mathcal{E}$ is set of some couples (i, j) for $i, j \in \mathcal{I}$

1.2 Parameters

The following parameters will be used to parametrize the model (we can perform some variations on them, according the type on inputs we are given)

- $\mathcal{O}_v$: Origin node for vehicle v
- $\mathcal{D}_v$: Destination node for vehicle v
- $\mathcal{T}_v^O$: Origin time for vehicle v (earliest time v can depart from $\mathcal{O}_v$)
- $\mathcal{T}_v^D$: Destination time for vehicle v (latest time v can arrive from $\mathcal{D}_v$)
- $\mathcal{C}_{i,j}$: Cost of the edge $(i, j) \in \mathcal{E}$
- $\mathcal{T}_{i,j}$: Time to take the edge $(i, j) \in \mathcal{E}$
- $\mathcal{M}_{i,j}$: Minimum time to travel from node i to node j.
- Q: Maximum number of vehicles of a platoon.
- η: Saving factor.
- β: Transit delay factor.
- $B_{v,i}$: An indicator such that for each $v \in \mathcal{V}$: $B_{v,i} = \begin{cases} 1 & \text{if } i = \mathcal{O}_v \\ -1 & \text{if } i = \mathcal{D}_v \\ 0 & \text{otherwise} \end{cases}$

1.3 Decision variables

The following are the decision variables for the model:

- $f_{v,i,j}$: binary variable takes 1 if v travels on the edge $(i, j) \in \mathcal{E}$ and 0 otherwise.

- $q_{v,w,i,j}$: binary variable takes 1 if v follows w on the edge $(i,j) \in \mathcal{E}$ and 0 otherwise.
- $e_{v,i,j}$: real positive variable that is the time v enters the edge $(i,j) \in \mathcal{E}$.
- $w_{v,i}$: real positive variable that is the time v waits at $i \in \mathcal{I}$

1.4 Objective function

Given the information that vehicle travels at the same speed, the objective function is:

$$\sum_{v,i,j} \mathcal{C}_{i,j}(f_{v,i,j} - \eta \sum_{w} q_{v,w,i,j}) + \beta w_{v,i}$$

1.5 Constraints

1.5.1 Nodes outflows and inflows

Node outflows must equal inflows.

$$\sum_{j:(i,j)\in\mathcal{E}} f_{v,i,j} = \sum_{j:(j,i)\in\mathcal{E}} f_{v,j,i} + B_{v,i} \quad \forall v \in \mathcal{V}, i \in \mathcal{V}$$

1.5.2 Platooning Time condition

When platooning, the times vehicles enter an edge must be equal.

$$-M(1-q_{v,w,i,j}) \leq e_{v,i,j} - e_{w,i,j} \leq M(1-q_{v,w,i,j}) \quad \forall (v,w) \in \mathcal{V}, (i,j) \in \mathcal{E}, v > w$$

1.5.3 Platooning Unique leader for each vehicle

A vehicle can follow at most one other vehicle in its platoon

$$\sum_{v \in \mathcal{E}} q_{v,w,i,j} \leq 1 \quad \forall w \in \mathcal{V}, \forall (i,j) \in \mathcal{E}$$

1.5.4 Platoon does not exceed Q

The number of vehicles in a platoon cannot exceed Q.

$$\sum_{u} q_{u,v,i,j} \leq (Q-1)\left(1 - \sum_{w} q_{v,w,i,j}\right) \quad \forall v \in V, (i,j) \in E$$

1.5.5 Vehicles on same platoon should traverse same edge

Platooning requires both vehicles to traverse the edge

$$2q_{v,w,i,j} \leq f_{v,i,j} + f_{w,i,j} \quad \forall v, w \in \mathcal{V}, (i,j) \in \mathcal{E}$$

1.5.6 Vehicles on same platoon should traverse same edge

Platooning requires both vehicles to traverse the edge

$$2q_{v,w,i,j} \leq f_{v,i,j} + f_{w,i,j} \quad \forall v, w \in \mathcal{V}, (i,j) \in \mathcal{E}$$

1.5.7 Time v enters its first edge

$$-M\left(1 - f_{v,\mathcal{O}_v,j}\right) \leq e_{v,\mathcal{O}_v,j} - \mathcal{T}_v^O - w_{v,O_v} \leq M\left(1 - f_{v,\mathcal{O}_v,j}\right) \quad \forall v \in \mathcal{V}, j \in \mathcal{I}$$

1.5.8 $\mathcal{T}_v^D$

$\mathcal{T}_v^D$ is the final enter time plus the time required to travel the final edge plus waiting at the end

$$-M\left(1 - f_{v,i,\mathcal{D}_v}\right) \leq \mathcal{T}_v^D - e_{v,i,\mathcal{D}_v} - w_{v,\mathcal{D}_v} - T_{i,\mathcal{D}_v} f_{v,i,\mathcal{D}_v} \leq M\left(1 - f_{v,i,\mathcal{D}_v}\right) \quad \forall v \in \mathcal{V}, i \in \mathcal{T}$$

1.5.9 Intermediate enter times

Intermediate enter times are equal plus the travel and waiting times

$$\begin{aligned} -M\left(2 - f_{v,i,j} - f_{v,k,i}\right) \leq e_{v,i,j} - e_{v,k,i} - w_{v,i} - T_{k,i} f_{v,k,i} \leq M\left(2 - f_{v,i,j} - f_{v,k,i}\right) \\ \forall v \in \mathcal{V}, (i,j), (j,k) \in \mathcal{E}, \mathcal{D}_v \neq i \neq \mathcal{O}_v \end{aligned}$$

1.5.10 Enter time no flow is zero

If there is no flow, the enter time can not be nonzero.

$$e_{v,i,j} \leq M f_{v,i,j} \quad \forall v \in V, (i,j) \in \mathcal{E}$$

1.5.11 Wait time no flow is zero

$$w_{v,i} \leq M \left(\sum_{i,j} f_{v,i,j} + f_{v,j,i} \right) \forall v \in \mathcal{V}, i \in \mathcal{I}$$

The Big-M can be chosen to tighten the constraints, one valid value for M is the following one:

$$M = \max_v \left\{ \mathcal{T}_v^D \right\} - \min_v \left\{ \mathcal{T}_v^O \right\}$$