

# HMMA 307 : Advanced Linear Modeling

## Chapter 4 : ANOVA 2 Factors

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[https://github.com/WalidKandouci/HMMA307\\_Modeles\\_Lineaires\\_Avances\\_Cours\\_5](https://github.com/WalidKandouci/HMMA307_Modeles_Lineaires_Avances_Cours_5)

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# ANOVA 2 Factors

► Factor 1: I Classes

► Factor 2: J Classes

Example: Judges tasting wines

	Wine 1	Wine 2	Wine 3
Judge 1	[6,7,8]	[1,2,3,5]	[1,3]
Judge 2	[3,8,9]	[1]	[1,2,3]
Judge 3	[5,7,8]	[2,5]	[1,3]

# Model

Our model:

$$y_{i,j,k} \sim \mathcal{N}(\mu_{i,j}, \sigma^2)$$

- ▶  $y$  are iid
- ▶  $k = 1, \dots, n_j$
- ▶  $\forall i = 1, \dots, I$
- ▶  $\forall j = 1, \dots, J$

Cases:

- ▶  $y_{1,1,1} = 6$
- ▶  $y_{1,1,2} = 7$
- ▶  $y_{1,1,3} = 8$
- ▶  $y_{2,2,1} = 1$

# Model

$$y_{i,j,k} = \mu + \alpha_i + \beta_j + \epsilon_{i,j}$$

- ▶  $\mu$ : Global effect
- ▶  $\alpha_i$ : First factor effect
- ▶  $\epsilon_{i,j}$ : Second factor effect

$$\text{cov}(\epsilon_{i,j,k}, \epsilon_{i',j',k'}) = \sigma^2 \delta_{i,i'} \delta_{j,j'} \delta_{k,k'}$$

# Balanced experiment plan

$$\forall i = 1, \dots, I, \forall j = 1, \dots, J: n_{i,j} = k$$
$$n = IJK$$

- $n_{i,j}$ : observations for each  $(i, j)$

# Design matrix

Our design matrix:

$$X = [\mathcal{I}_n, \mathbb{I}_{C_1}, \dots, \mathbb{I}_{C_I} \mathbb{I}_{D_1}, \dots, \mathbb{I}_{D_J}]$$

- ▶  $\mathbb{I}_{C_I}$ : Indicators of modalities of the first factors
- ▶  $\mathbb{I}_{D_J}$ : Indicators of modalities of the second factors

# Design matrix

$$X = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix} \quad y = \begin{bmatrix} 6 \\ 7 \\ 8 \\ 1 \\ 2 \\ 3 \\ 1 \\ \cdot \\ \cdot \\ \cdot \end{bmatrix}$$

- ▶ 27 rows
- ▶ Columns of X:  $(J_1, J_2, J_3, V_1, V_2, V_3)$