Power Electronics Project

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Contents

Abstract

This report presents the analysis, modeling, and simulation of a single-phase inverter using MATLAB/Simulink. The inverter is designed to operate with a DC input voltage of 20 V and an R–L load consisting of 34 Ω resistance and 33 mH inductance at a 50 Hz output frequency. Multiple modulation strategies are implemented and evaluated, including square wave conduction, quasi-square wave operation with third-harmonic elimination, selective harmonic elimination (SHE) targeting the removal of the 3rd and 5th harmonics, and both bipolar and unipolar PWM techniques. Additional analysis is performed to determine the minimum switching frequency modulation index that satisfies the requirements of $V_{1,\text{rms}} = \frac{16}{\sqrt{2}}$ and a total harmonic distortion (THD) of 10% or less. A multilevel inverter configuration is also developed as an extended task.

For each modulation technique, waveforms of gate signals, load voltage, load current, and supply current are obtained and analyzed. The results are compared in terms of harmonic distortion and output waveform quality, and a summary table is provided to present the performance of all techniques. The study demonstrates the effectiveness of optimized switching strategies in improving inverter output quality while satisfying power quality constraints.

1. Introduction

Power electronic inverters play a critical role in modern energy conversion systems, enabling the transformation of DC power into AC power for a wide range of applications including renewable energy systems, industrial drives, and consumer electronics. The performance of an inverter is largely determined by the switching strategy used to control the output waveform, as different modulation techniques yield different levels of harmonic distortion, switching losses, and output quality.

This project focuses on the simulation and analysis of a single-phase inverter using MATLAB/Simulink, with the objective of studying how various modulation methods influence inverter performance. The inverter under investigation operates with a DC input of 20 V and supplies an R–L load designed to emulate practical electrical characteristics. Several modulation techniques are explored, beginning with basic square-wave conduction and progressing toward more advanced approaches such as quasi-square wave generation, selective harmonic elimination (SHE), and high-frequency bipolar and unipolar pulse-width modulation (PWM). Each technique is assessed based on its ability to reduce undesirable harmonic components, achieve the required fundamental voltage level, and maintain acceptable THD values.

Furthermore, the study examines the minimum necessary switching frequency for

PWM strategies to meet specified performance criteria. A multilevel inverter topology is also implemented as a bonus enhancement to investigate the benefits of increased voltage levels on harmonic performance.

Through waveform analysis, THD measurements, and systematic comparison across modulation methods, this project provides a comprehensive understanding of single-phase inverter behavior and highlights the trade-offs associated with different switching techniques. The results support the selection of modulation strategies that enhance power quality and optimize inverter efficiency.

2. Square wave conduction

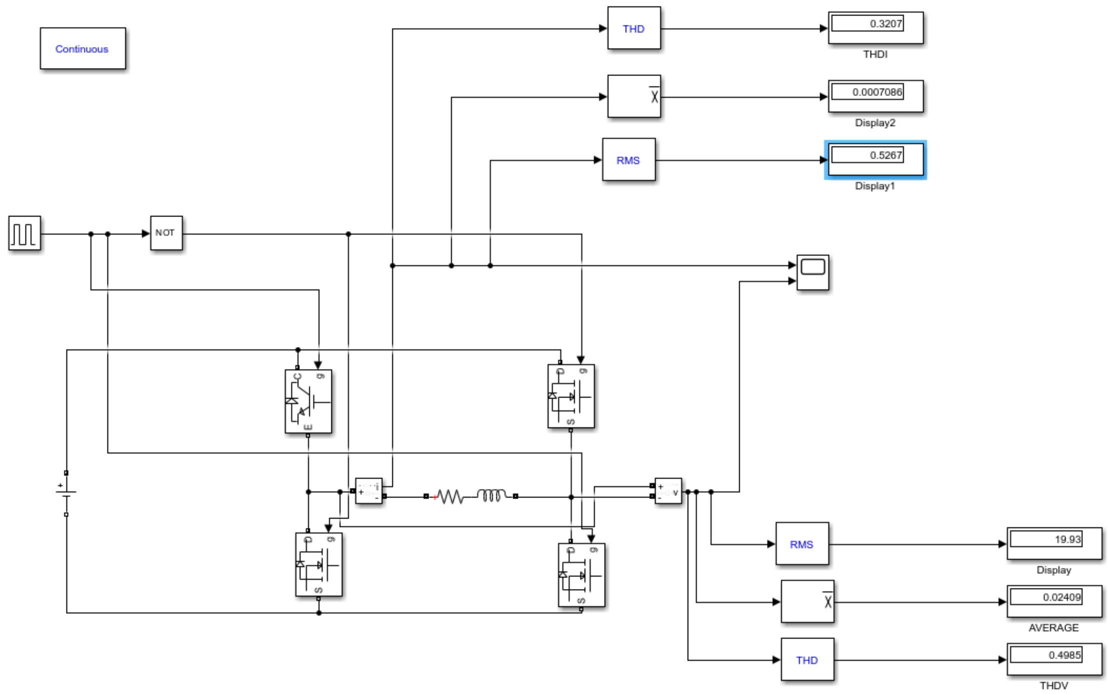

Figure 1: Square wave model

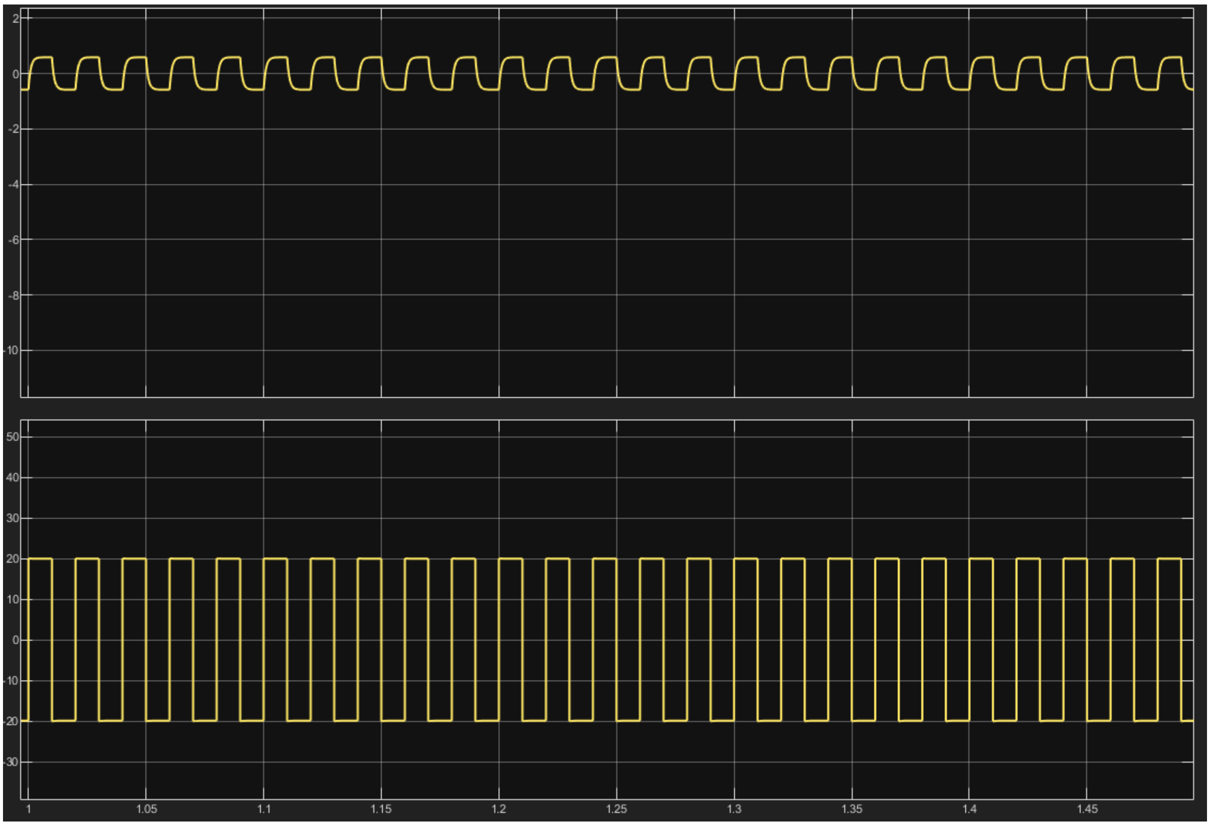

Figure 2: wave forms

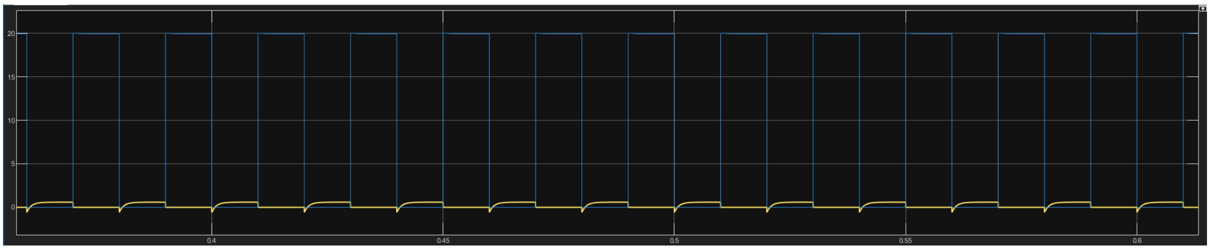

Figure 3: s1,s2

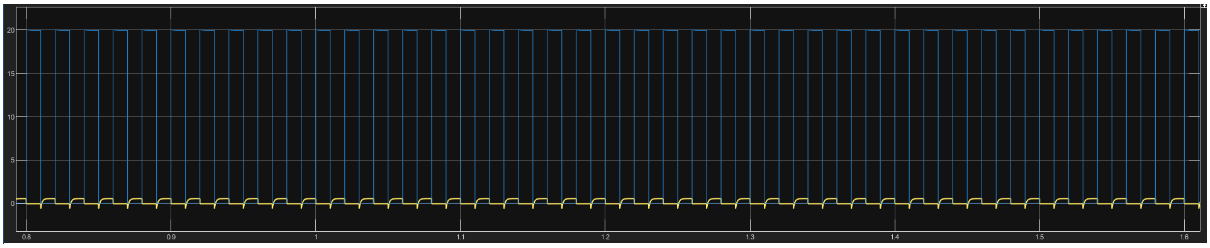

Figure 4: s3,s4

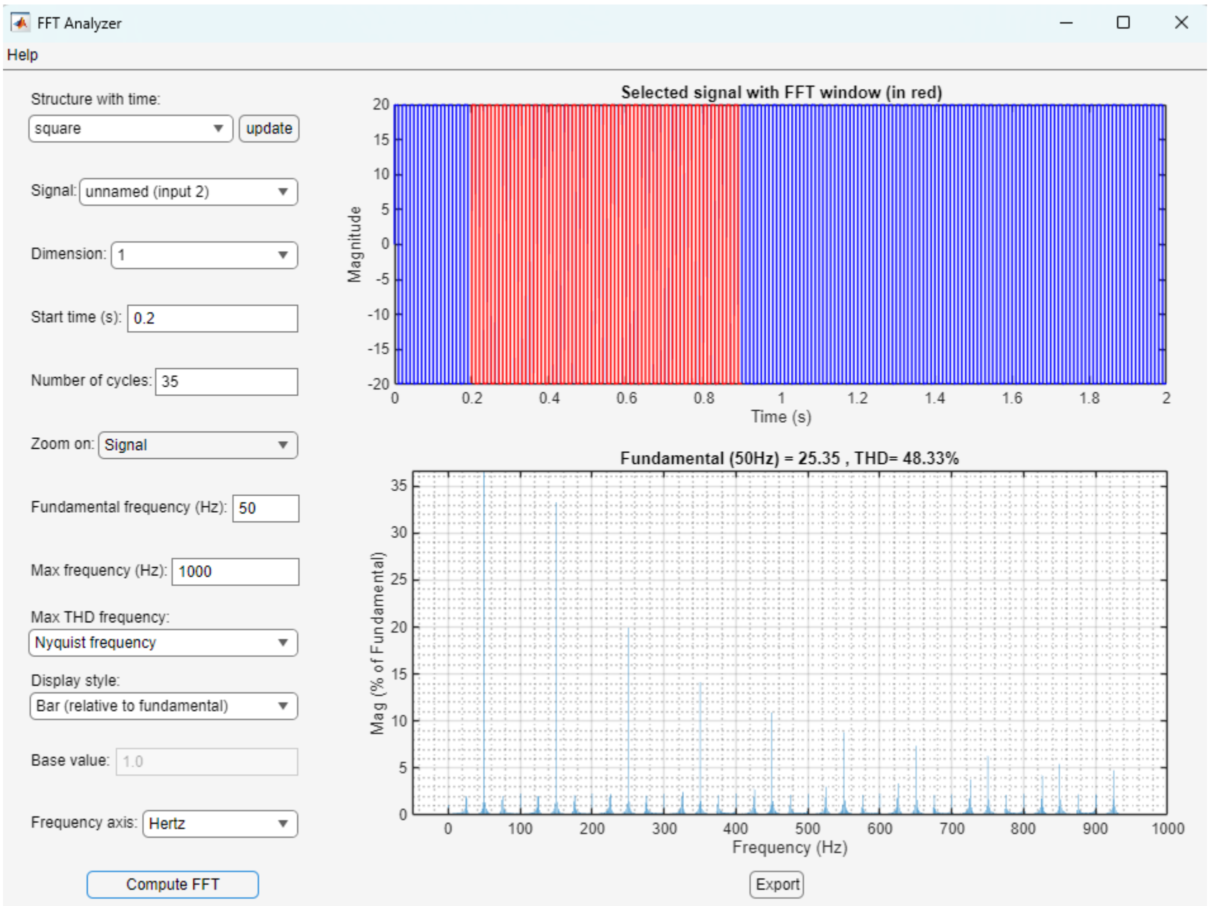

Figure 5: FFT voltage

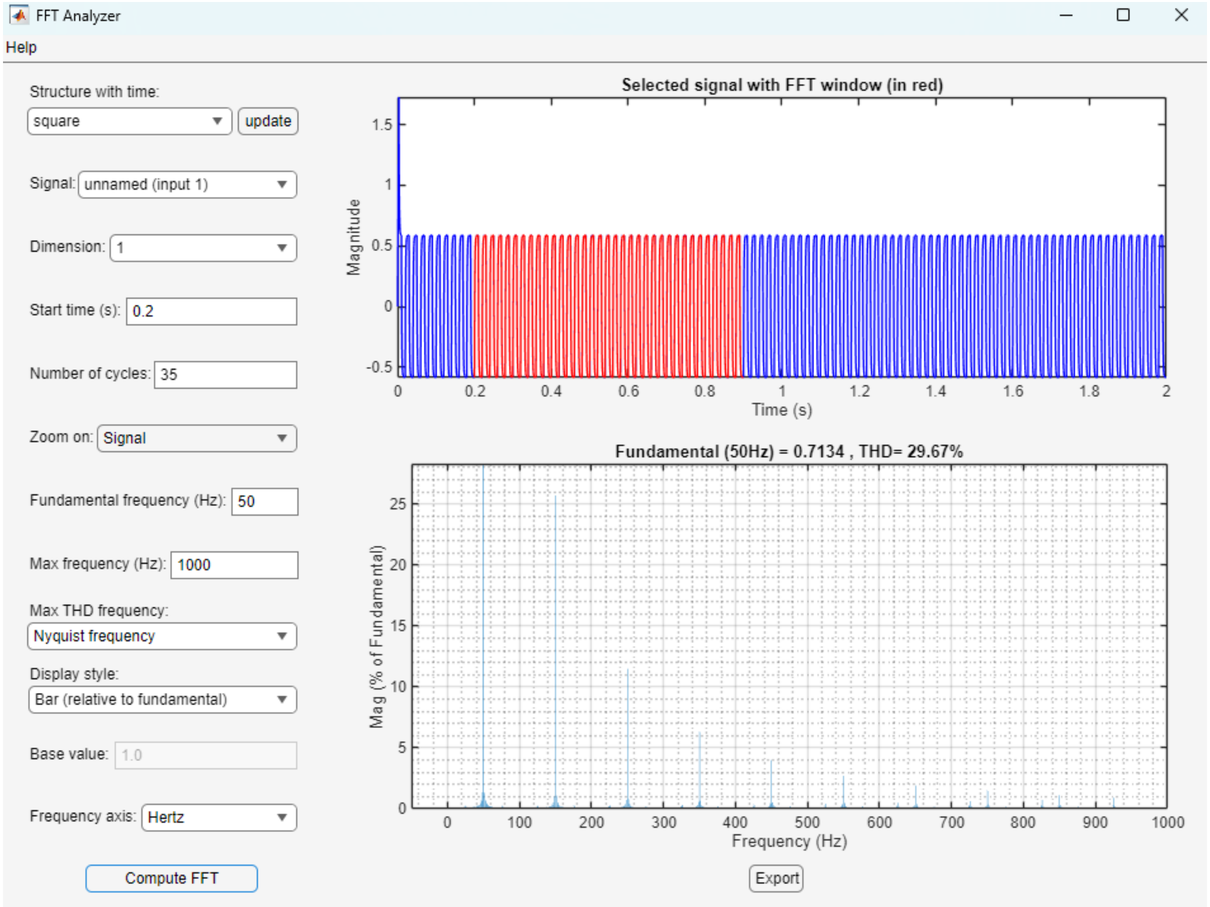

Figure 6: FFT current

3. Quasi-square wave output that eliminates the third harmonic:

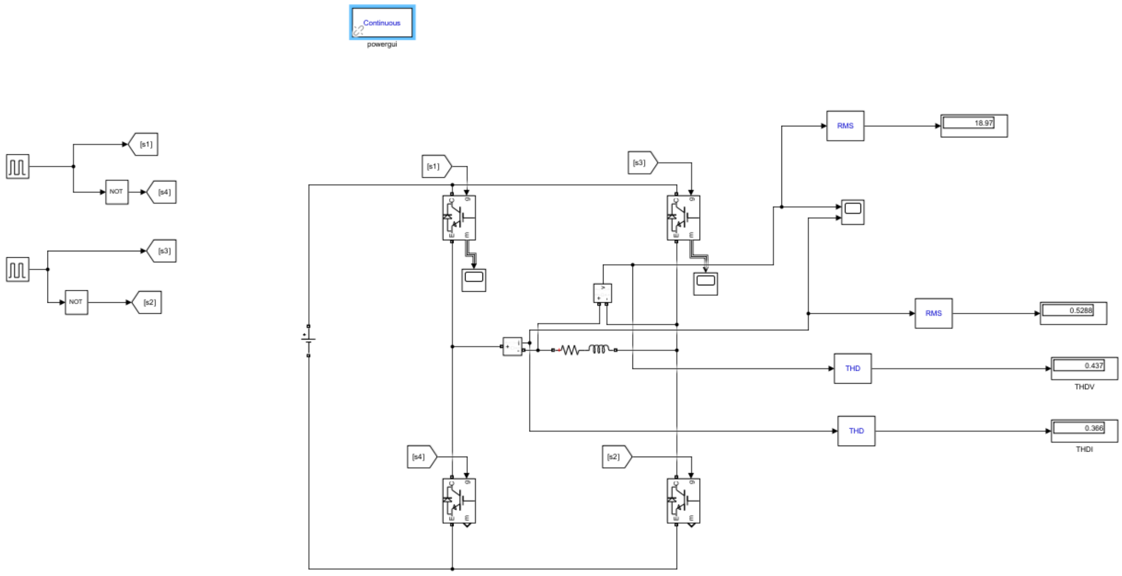

Figure 7: Quasi-square wave model

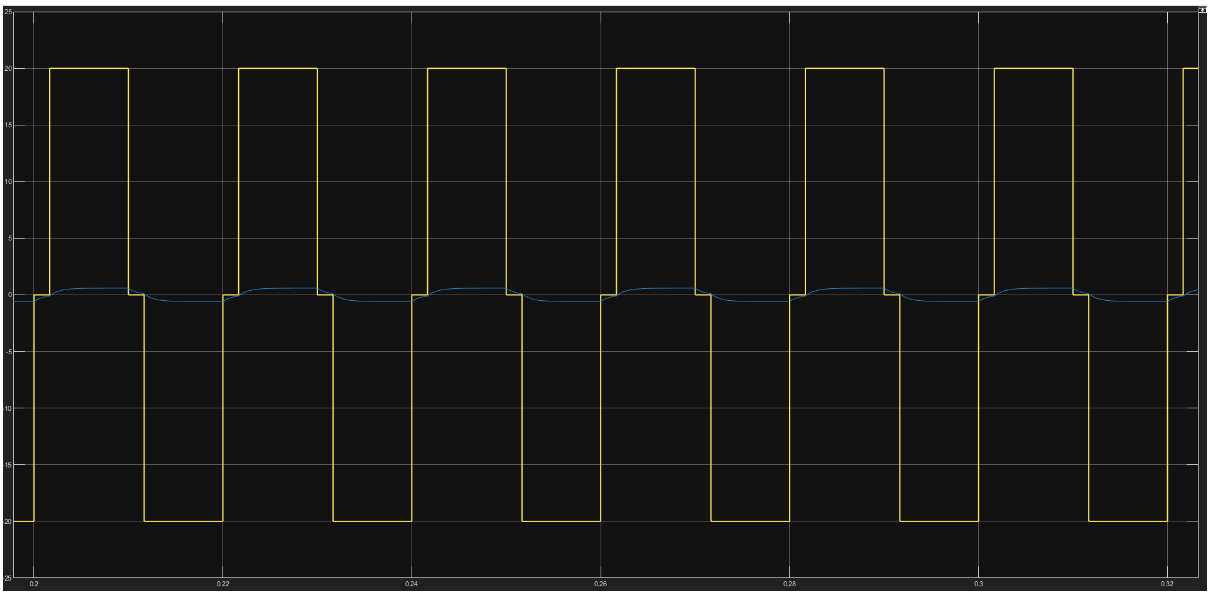

Figure 8: Quasi-square wave wave forms

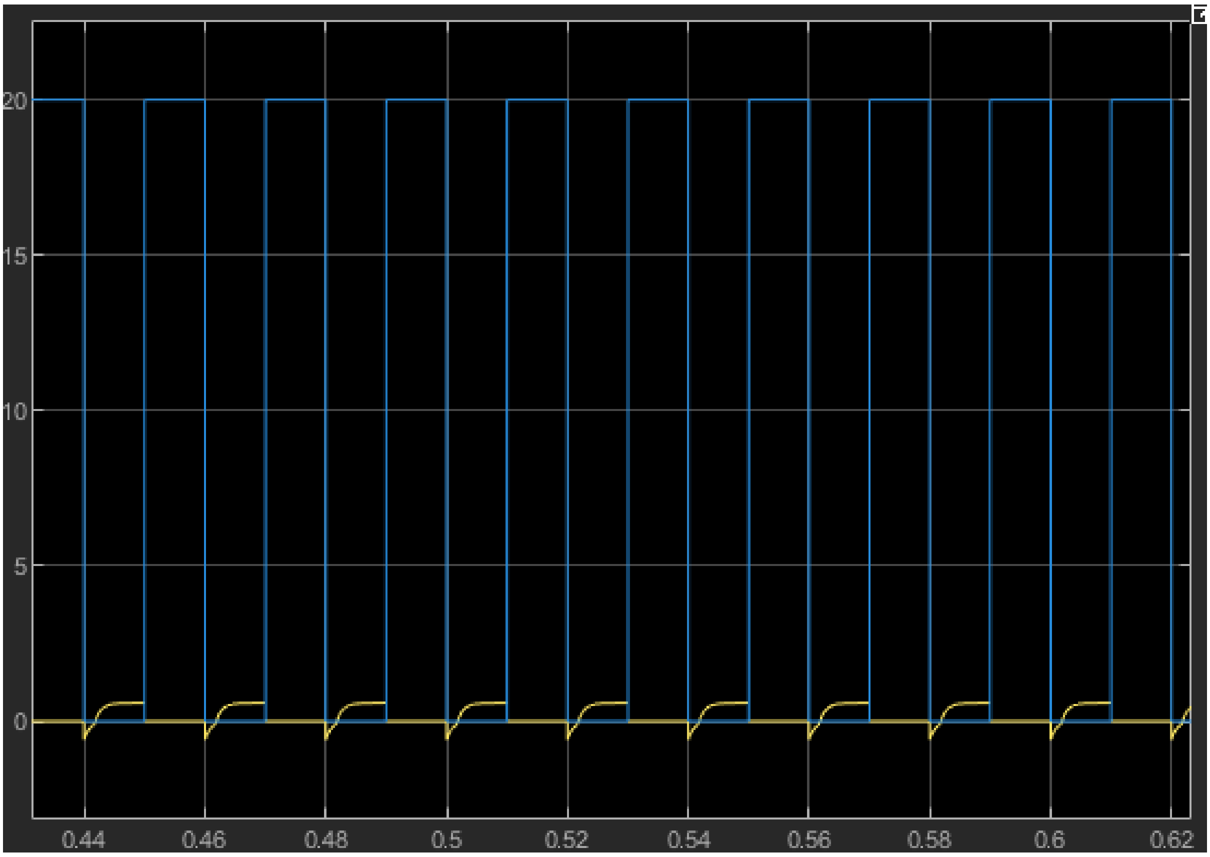

Figure 9: s1,s2

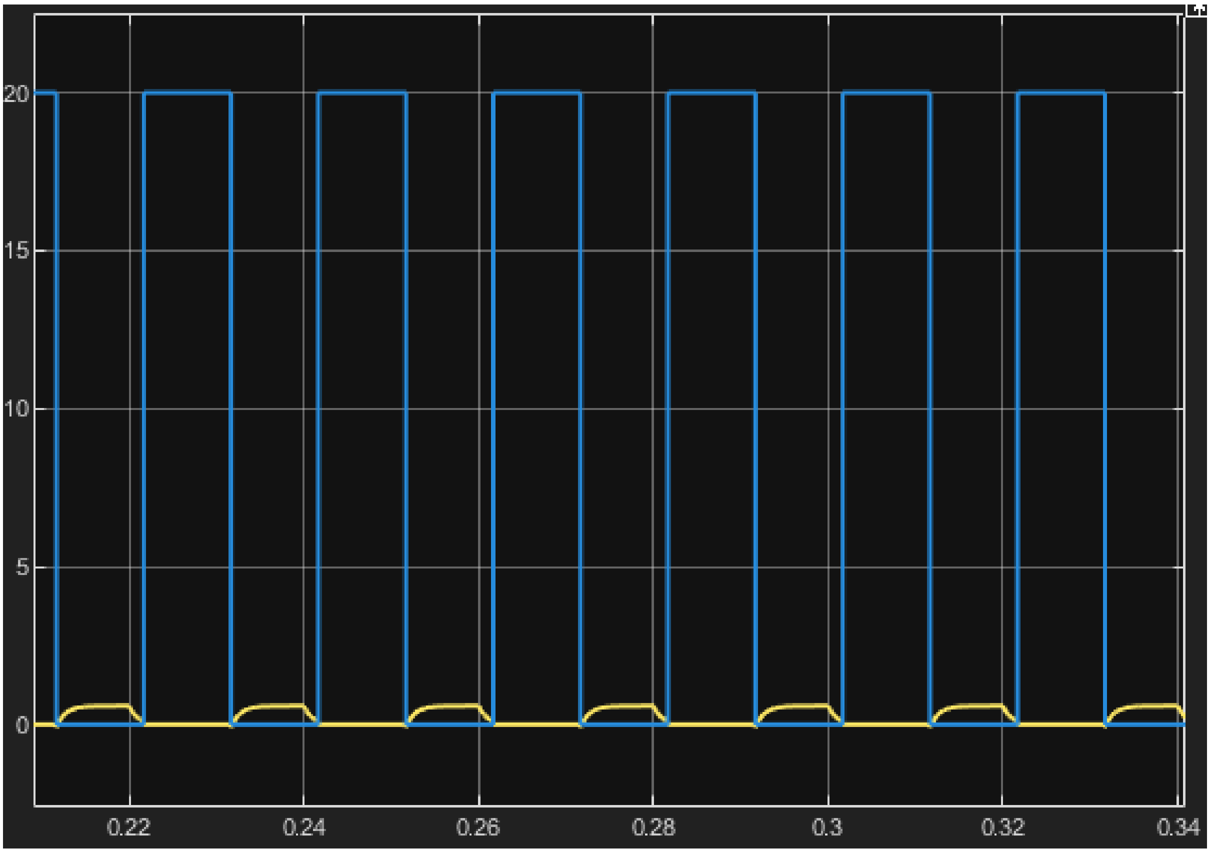

Figure 10: s3,s4

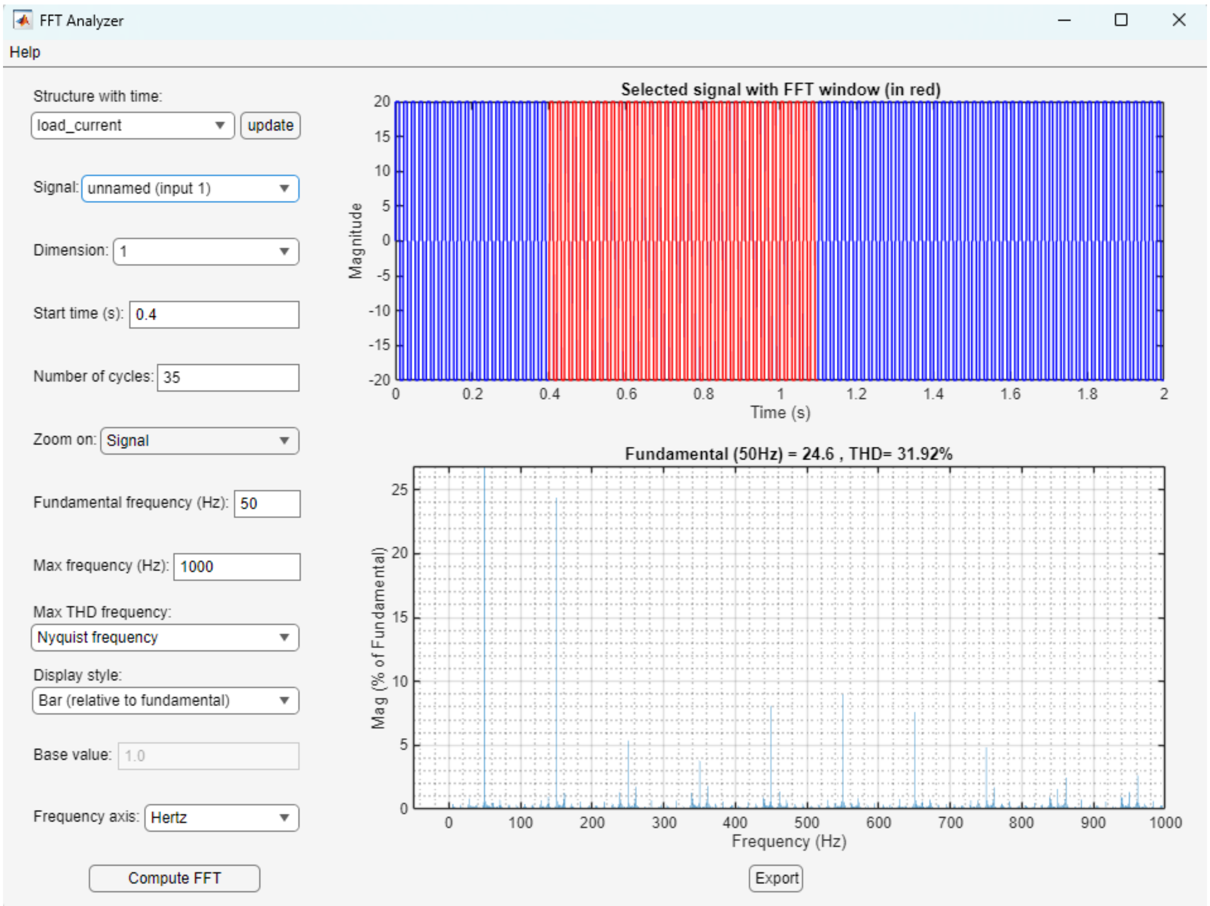

Figure 11: FFT voltage

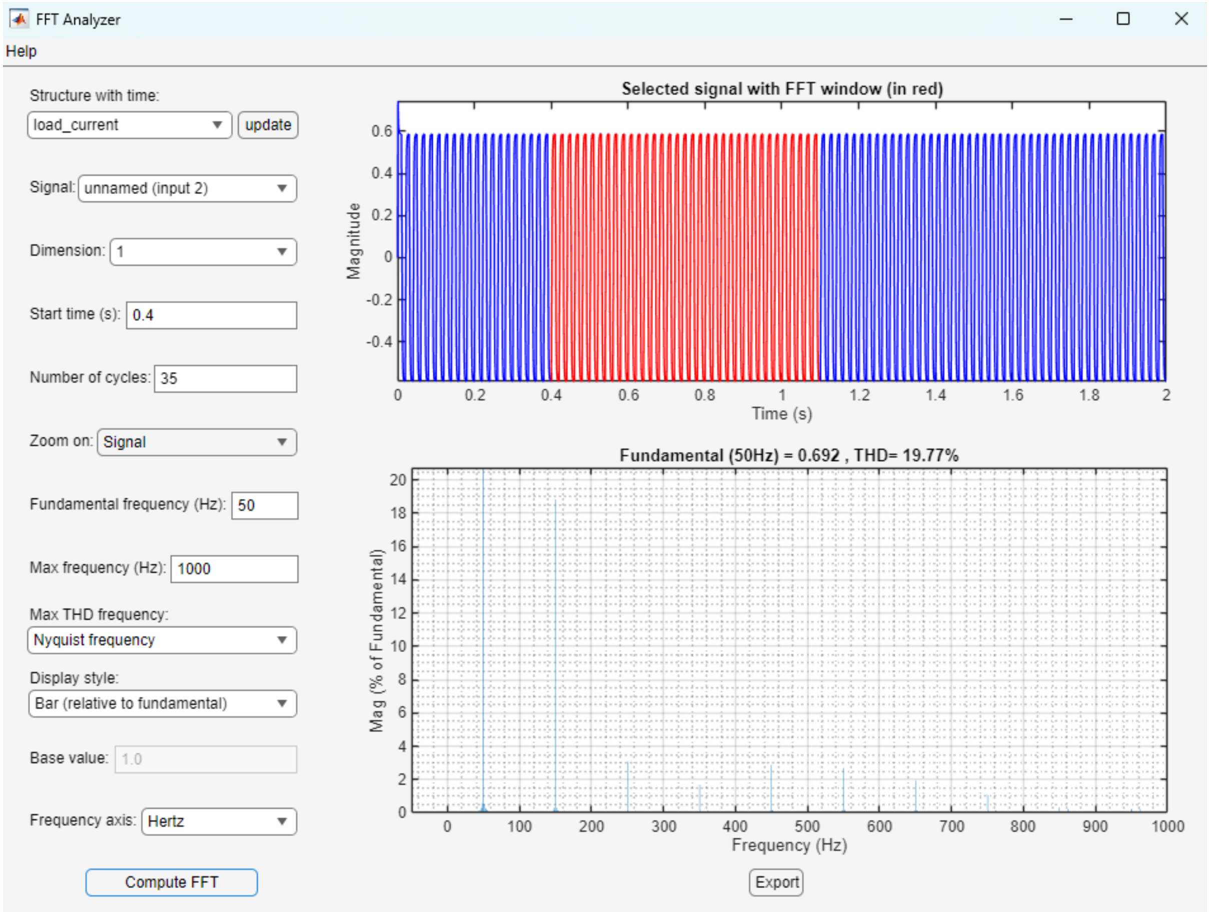

Figure 12: FFT current

4. Selected Harmonic Elimination (SHE)

In this part of the project, the selected harmonic elimination (SHE) technique is applied to the single-phase full-bridge inverter in order to cancel the 3rd and 5th voltage harmonics while maintaining a prescribed fundamental component. The inverter output is assumed to have half-wave and quarter-wave symmetry, so only odd sine harmonics are present. The waveform in each quarter cycle is defined by three switching angles $0 < \alpha_1 < \alpha_2 < \alpha_3 < 90^\circ$.

Using Fourier series, the nth harmonic coefficient of the line voltage is written as

$$b_n = \frac{4V_{dc}}{n\pi}\left[\cos(n\alpha_1) - \cos(n\alpha_2) + \cos(n\alpha_3)\right].$$

The SHE problem is formulated by imposing three conditions on these coefficients. First, the fundamental harmonic equation:

$$\cos(\alpha_1) - \cos(\alpha_2) + \cos(\alpha_3) = M,$$

where M corresponds to the desired fundamental RMS voltage

$$V_{1,\mathrm{rms}} = \frac{16}{\sqrt{2}}\ \mathrm{V}.$$

Next, the cancellation equations for the 3rd and 5th harmonics:

$$\cos(3\alpha_1) - \cos(3\alpha_2) + \cos(3\alpha_3) = 0,$$

$$\cos(5\alpha_1) - \cos(5\alpha_2) + \cos(5\alpha_3) = 0.$$

These three nonlinear equations are solved numerically in MATLAB to obtain the switching angles:

$$\alpha_1 \approx 31.42^\circ, \quad \alpha_2 \approx 54.57^\circ, \quad \alpha_3 \approx 69.23^\circ.$$

These angles are then converted into time instants for a 50 Hz period and used to construct the gating patterns in Simulink.

4.1 MATLAB Implementation of SHE Switching Angle Calculation

The following MATLAB script implements the Newton–Raphson method to determine the switching angles α_1, α_2, and α_3 that satisfy the Selective Harmonic Elimination (SHE) equations for eliminating the 3rd and 5th harmonics while achieving the required fundamental component.

Listing 1: MATLAB Code Used to Solve the SHE Nonlinear Equations

```
% Given parameters
V_DC = 20;
V1 = 16;
target = pi * V1 / (4 * V_DC); % Target for fundamental

% Initial guesses for alpha1, alpha2, alpha3
alpha = [pi/6; pi/3; pi/2];

% System of nonlinear equations
F = @(alpha) [
    cos(alpha(1)) - cos(alpha(2)) + cos(alpha(3)) - target;
    cos(3*alpha(1)) - cos(3*alpha(2)) + cos(3*alpha(3));
    cos(5*alpha(1)) - cos(5*alpha(2)) + cos(5*alpha(3))
];

```

```
% Jacobian matrix
J = @(alpha) [
    -sin(alpha(1))      sin(alpha(2))      -sin(alpha(3));
    -3*sin(3*alpha(1))  3*sin(3*alpha(2))  -3*sin(3*alpha(3));
    -5*sin(5*alpha(1))  5*sin(5*alpha(2))  -5*sin(5*alpha(3))
];

% Newton-Raphson method
tolerance = 1e-6;
max_iterations = 100;

for iter = 1:max_iterations
    F_val = F(alpha);
    J_val = J(alpha);

    delta = -J_val \ F_val;
    alpha = alpha + delta;

    if norm(delta) < tolerance
        fprintf('Converged in %d iterations.\n', iter);
        break;
    end
end

% Display results
disp('Alpha values (radians):');
disp(alpha);

alpha_deg = rad2deg(alpha);
disp('Alpha values (degrees):');
disp(alpha_deg);

% Convert degree angles into switching instants for 50 Hz
alpha1 = 31.4202 * 0.02 / 360;
alpha2 = 54.5694 * 0.02 / 360;
alpha3 = 69.2269 * 0.02 / 360;

disp(alpha1);
disp(alpha2);
```

```
disp(alpha3);
```

4.2 Numerical Results

The Newton–Raphson algorithm successfully converged to the correct switching angles within five iterations. The obtained values in both radians and degrees are summarized in the following output.

MATLAB Solver Output

Converged in 5 iterations.
Alpha values (radians): 0.5484 0.9524 1.2082
Alpha values (degrees): 31.4202 54.5694 69.2269
Switching instants (seconds): 0.0017 0.0030 0.0038

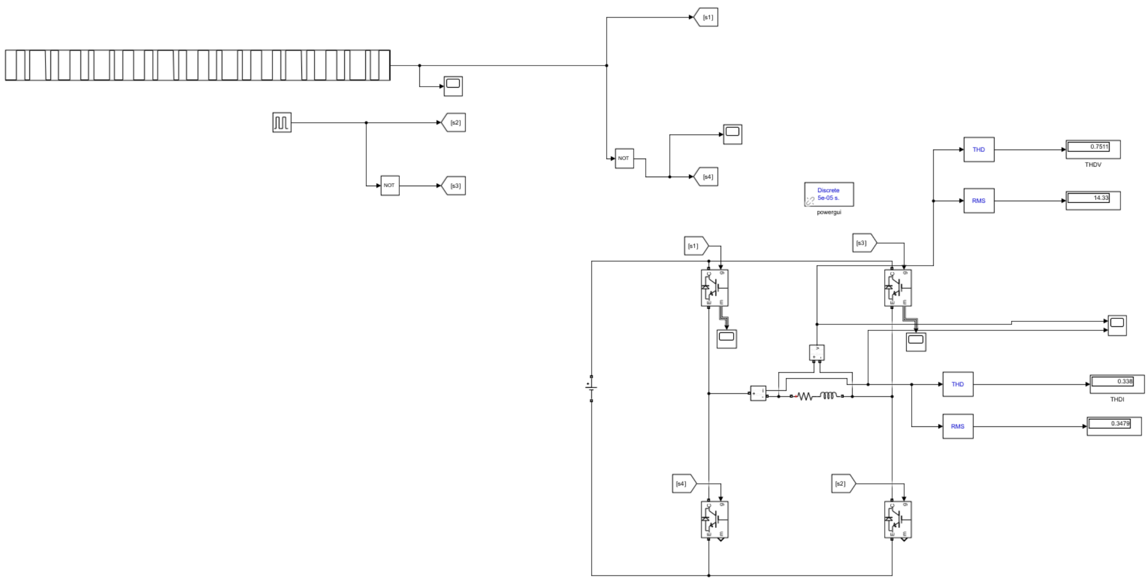

Figure 13: SHE MODEL

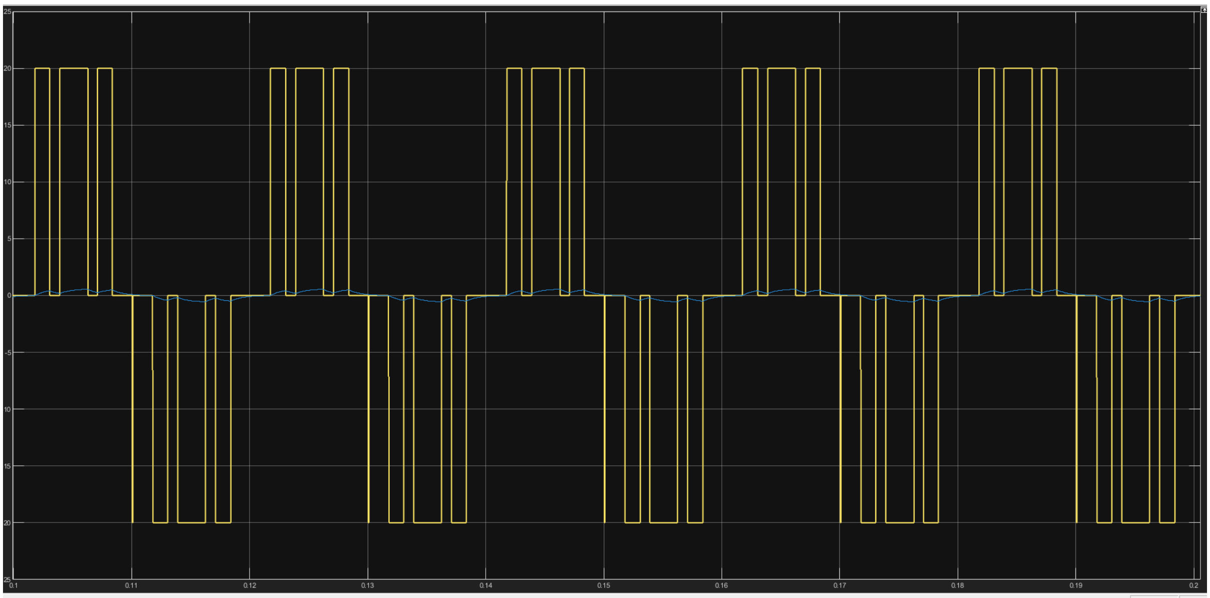

Figure 14: SHE WAVEFORMS

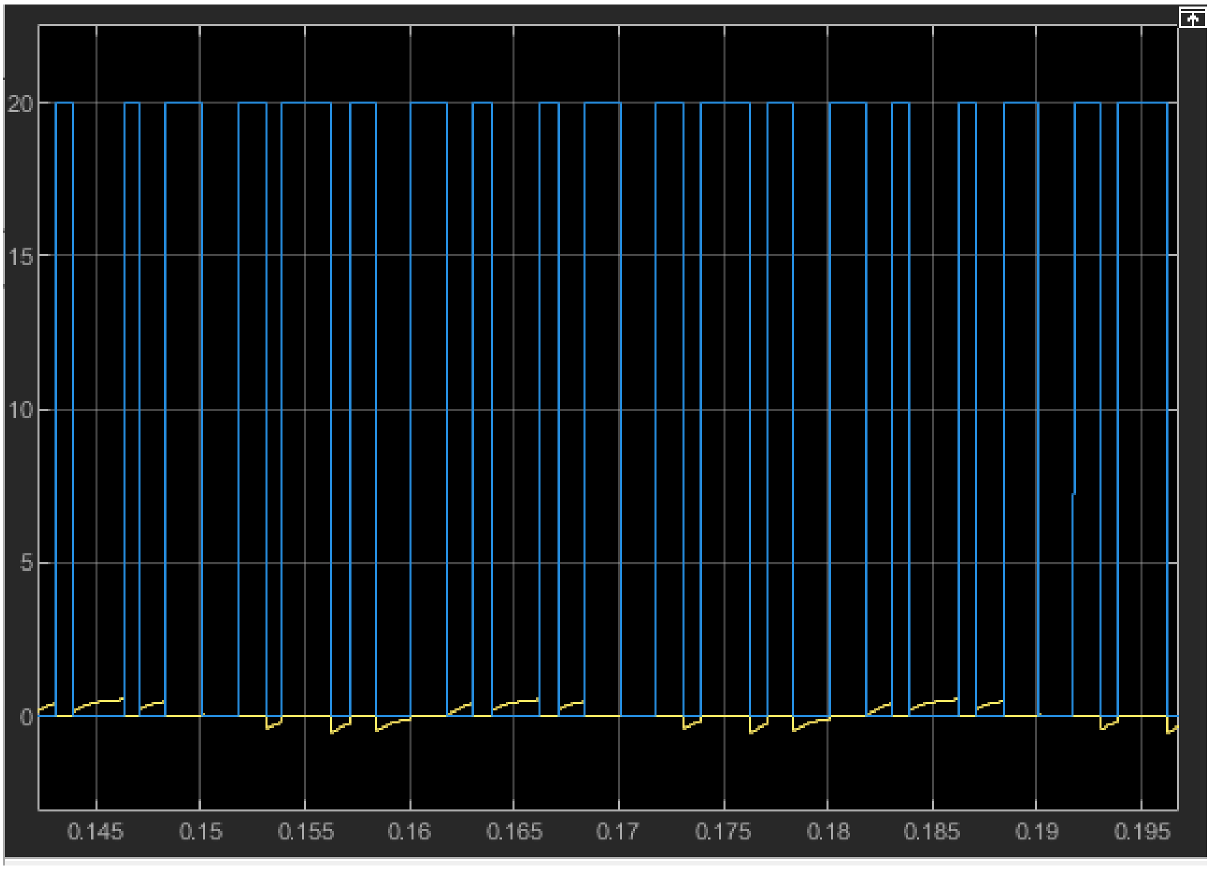

Figure 15: s1,s2

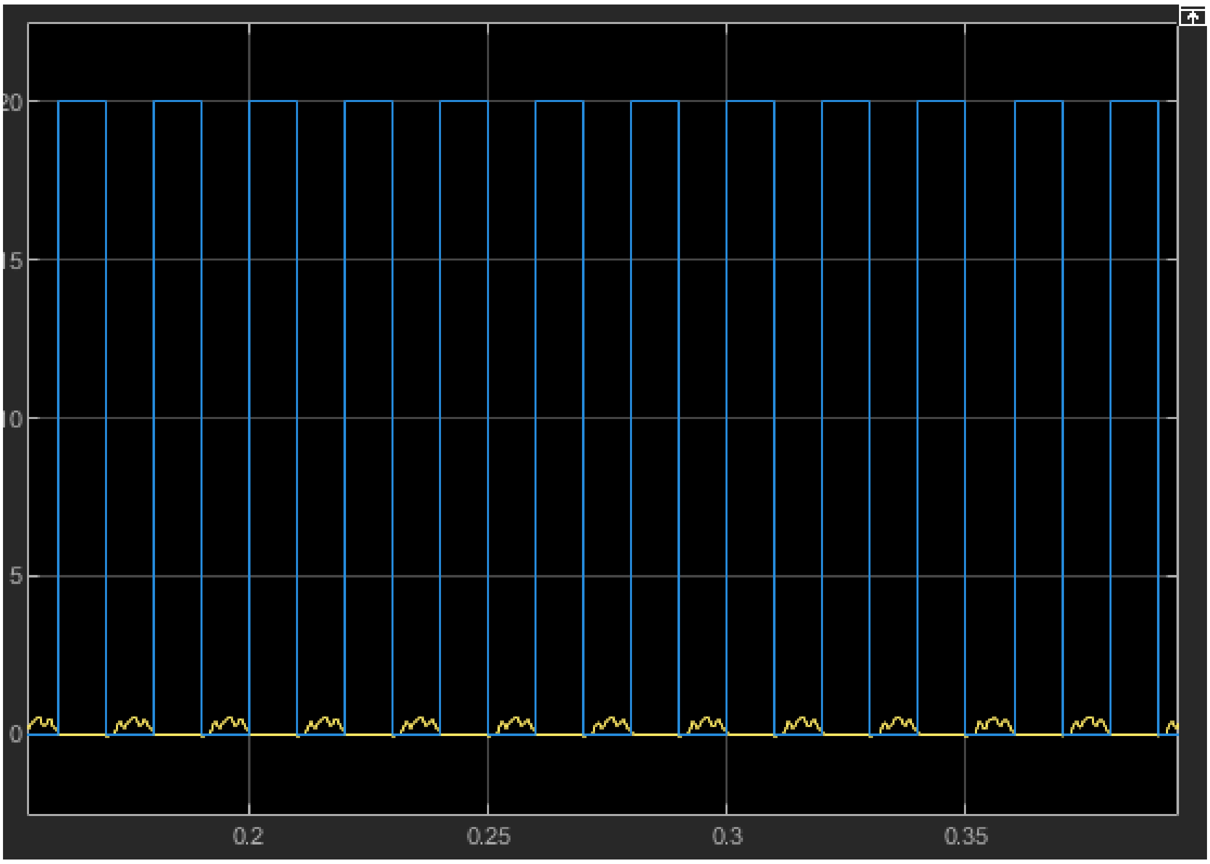

Figure 16: s3,s4

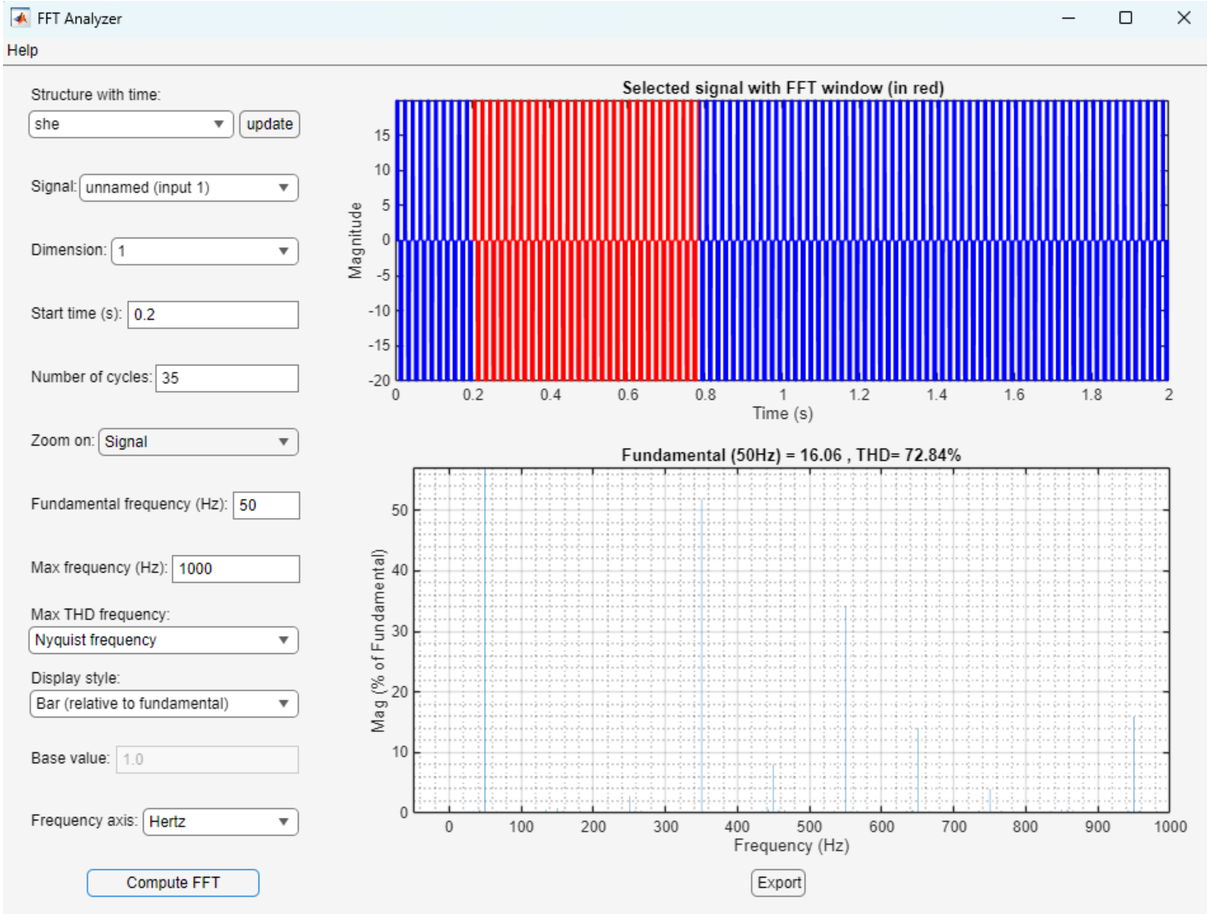

Figure 17: FFT voltage

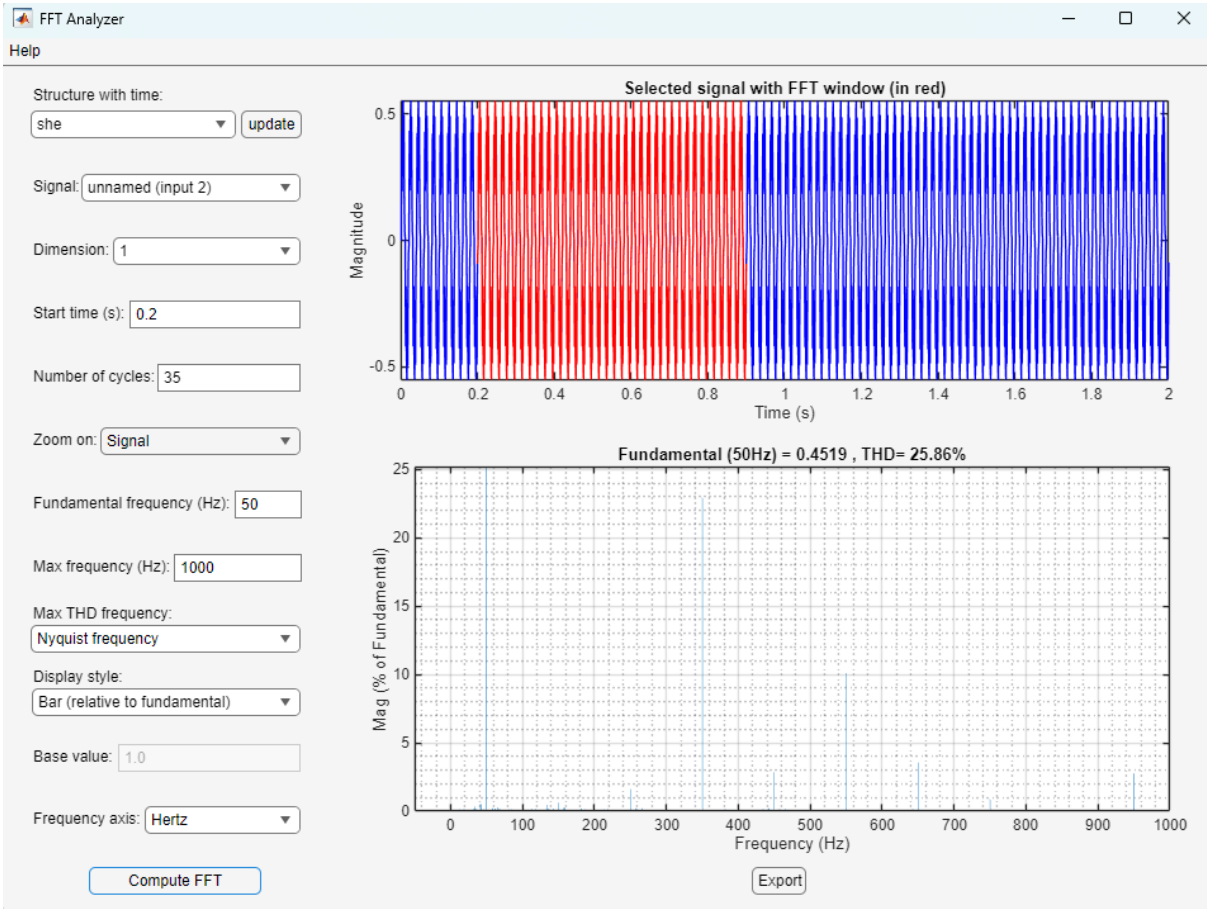

Figure 18: FFT current

5. Bipolar and Unipolar PWM Modulation Techniques

Pulse Width Modulation (PWM) is a fundamental control strategy in power electronic converters, allowing accurate regulation of the output voltage, waveform shape, and fundamental frequency. Among the various PWM approaches, the **bipolar** and **unipolar** modulation schemes are widely implemented in single-phase inverters due to their simplicity and effectiveness.

5.1 Unipolar PWM

In **unipolar PWM**, the inverter output voltage transitions among three distinct levels: $+V_{dc}$, 0, and $-V_{dc}$. This is achieved by modulating the switching devices such that each half-bridge leg is controlled independently. The presence of the intermediate zero level reduces switching losses and significantly suppresses low-order harmonics. Consequently, unipolar PWM generally provides improved waveform quality and lower total harmonic distortion (THD), making it suitable for applications requiring high efficiency and reduced electromagnetic interference.

5.2 Bipolar PWM Simulation Overview

In bipolar PWM, a low-frequency sinusoidal reference and a high-frequency triangular carrier are compared to generate the gate pulses. In this project, the reference is a 50 Hz sine wave and the carrier is a 10 kHz triangular wave, giving a frequency modulation ratio

$$m_f = \frac{f_{\text{carrier}}}{f_{\text{reference}}} = \frac{10{,}000}{50} = 200.$$

$$f_{\text{carrier}} = 200 \times 50 = 10{,}000 \text{ Hz}.$$

At every instant, the comparator checks whether the sine reference is greater or smaller than the triangular carrier:

- If $v_{\text{ref}}(t) > v_{\text{tri}}(t)$:
  - Switches S_1 and S_4 are turned ON,
  - Switches S_2 and S_3 are turned OFF,
  - The output line voltage is $v_{AB} = +V_{dc}$.
- If $v_{\text{ref}}(t) < v_{\text{tri}}(t)$:
  - Switches S_2 and S_3 are turned ON,
  - Switches S_1 and S_4 are turned OFF,
  - The output line voltage is $v_{AB} = -V_{dc}$.

Thus, the line voltage v_{AB} switches directly between $+V_{dc}$ and $-V_{dc}$ at the carrier frequency, while its average value over each carrier period follows the 50 Hz sinusoidal reference.

5.2.1 Bipolar PWM Simulation:

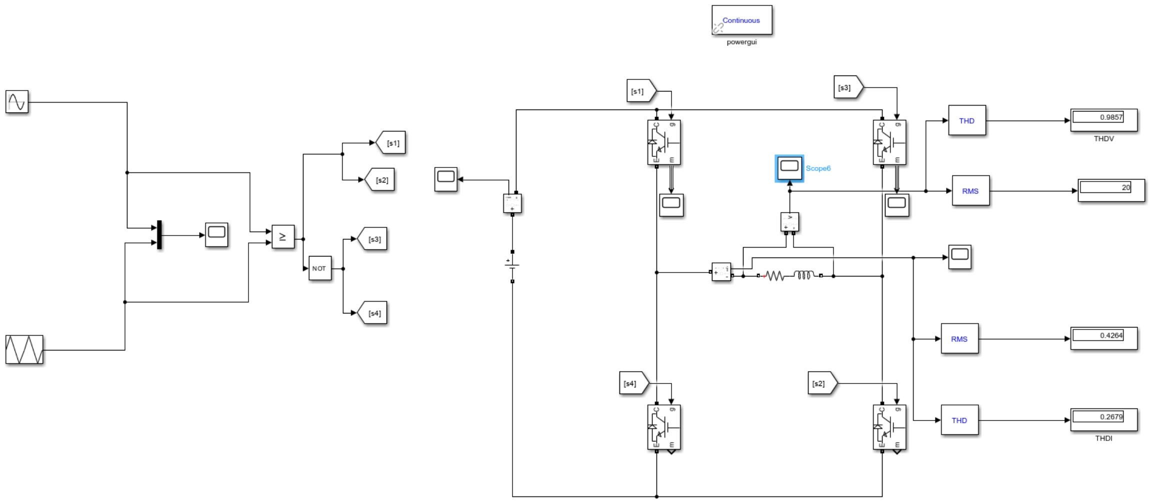

Figure 19: BIPOLAR MODEL

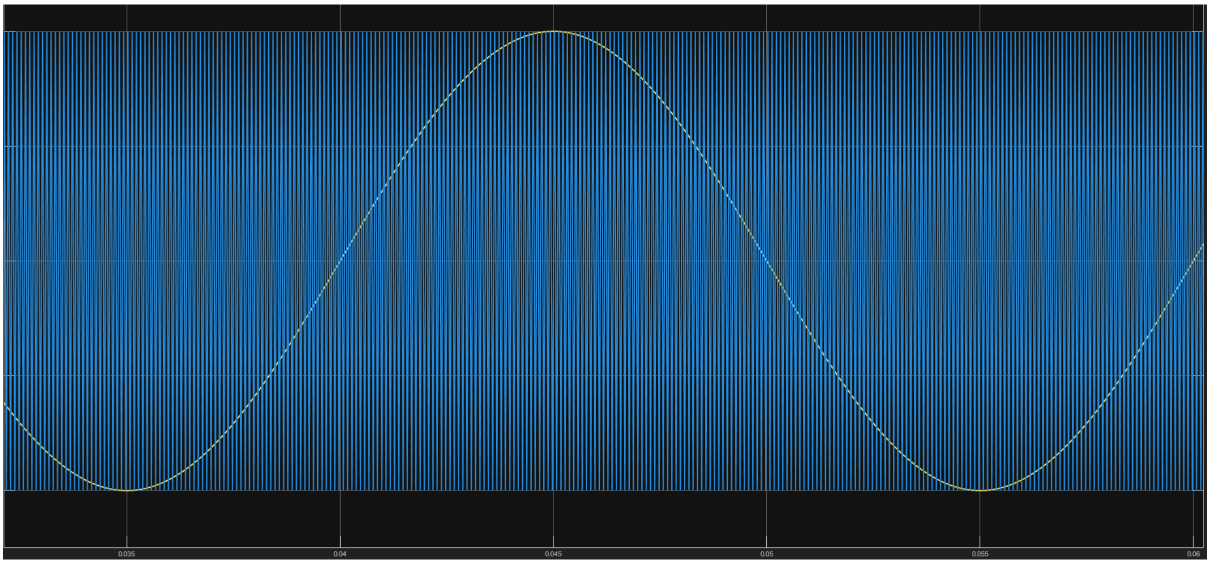

Figure 20: triangular wave & sine wave

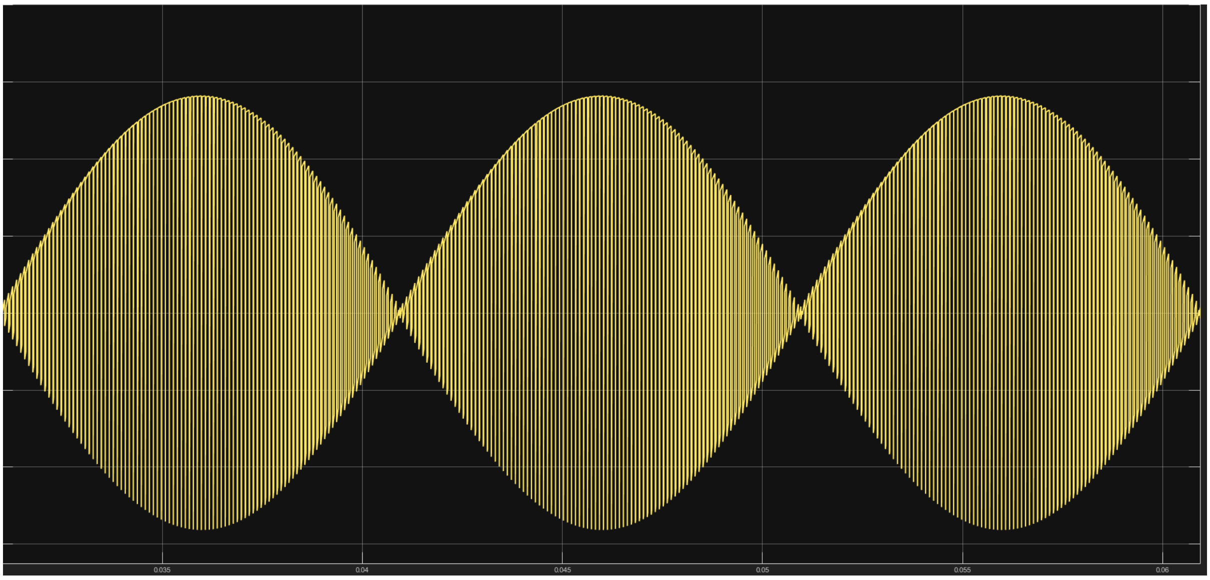

Figure 21: input current

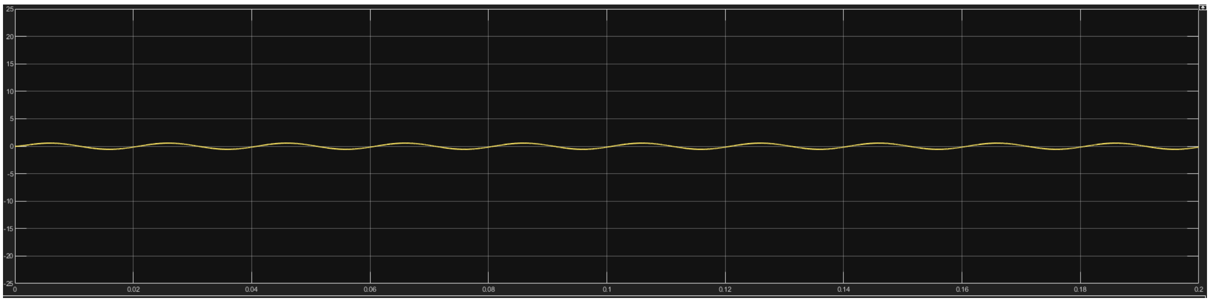

Figure 22: output current

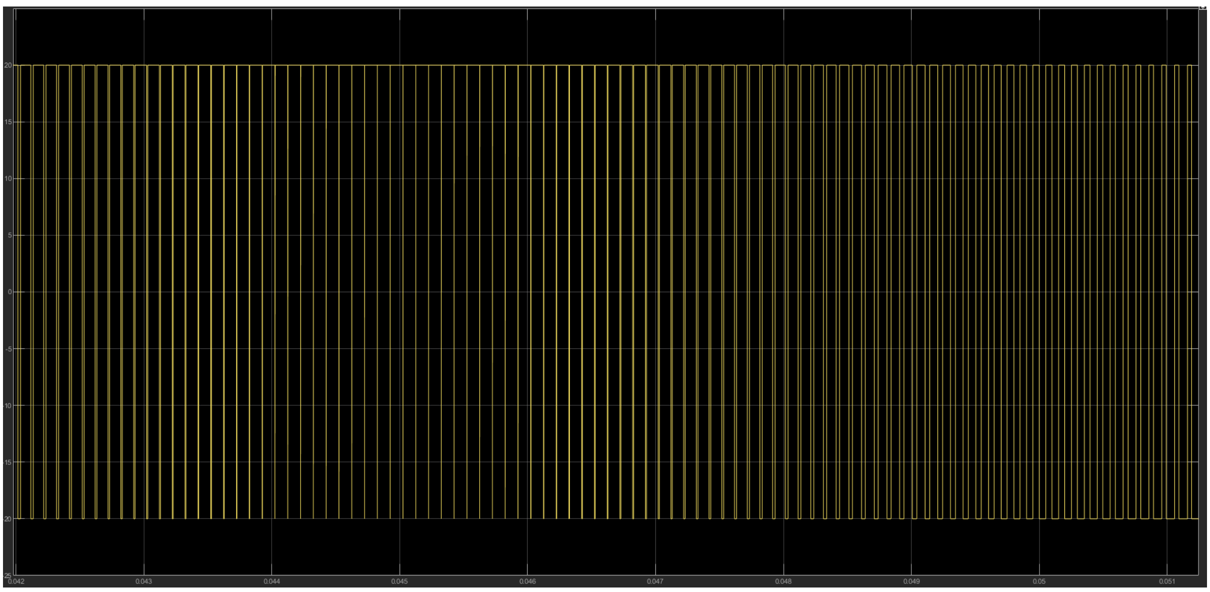

Figure 23: output voltage

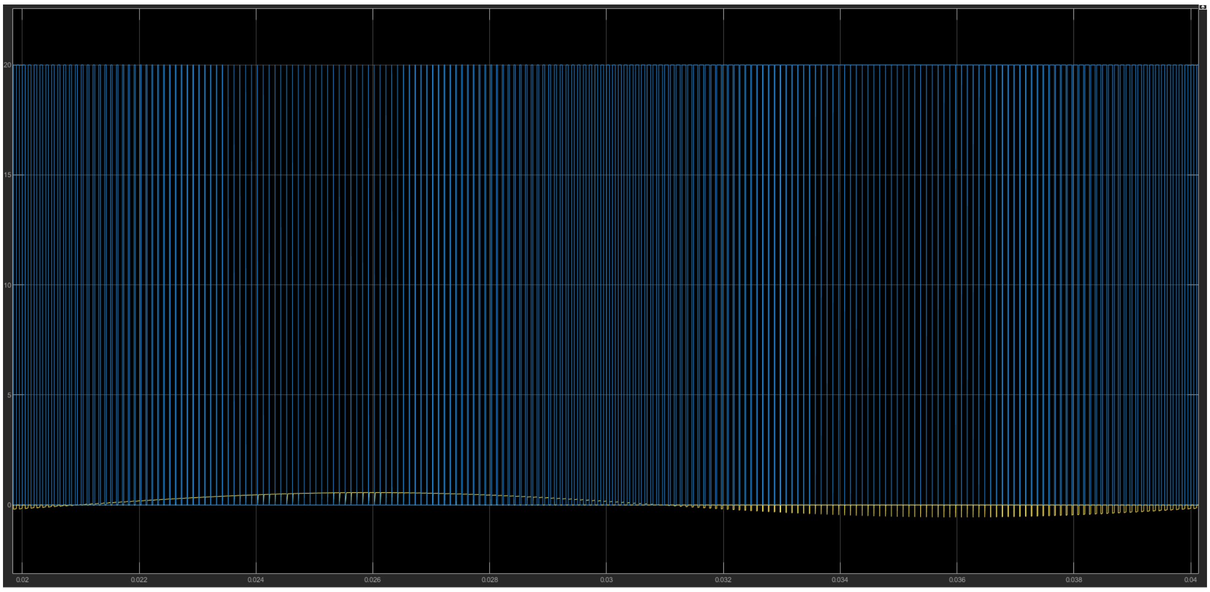

Figure 24: s1,s2

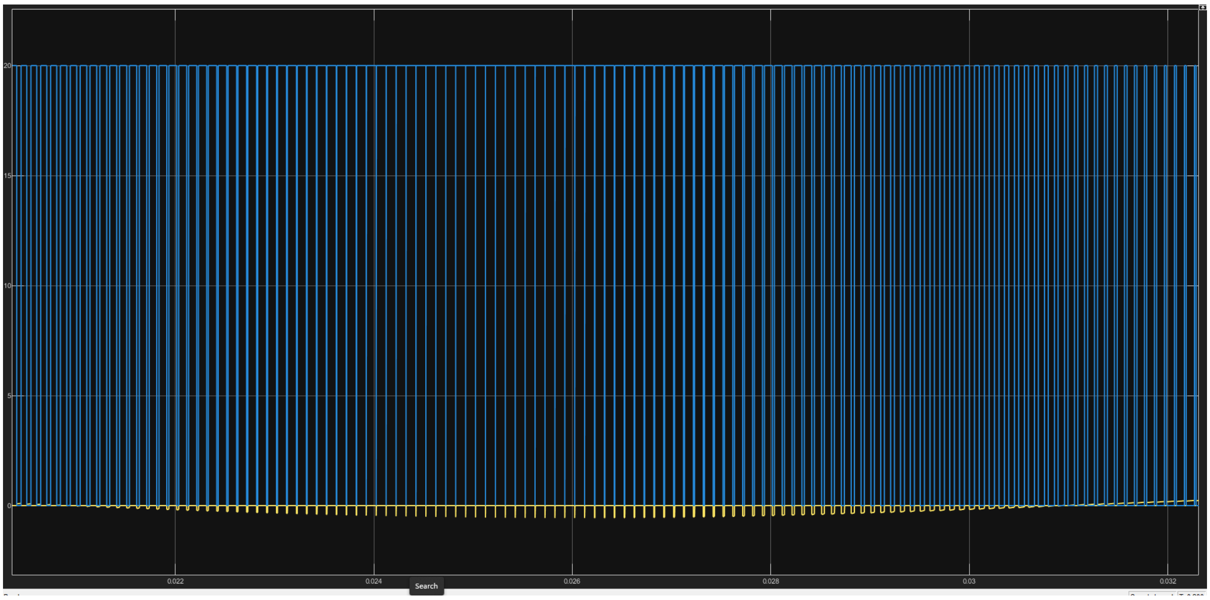

Figure 25: s3,s4

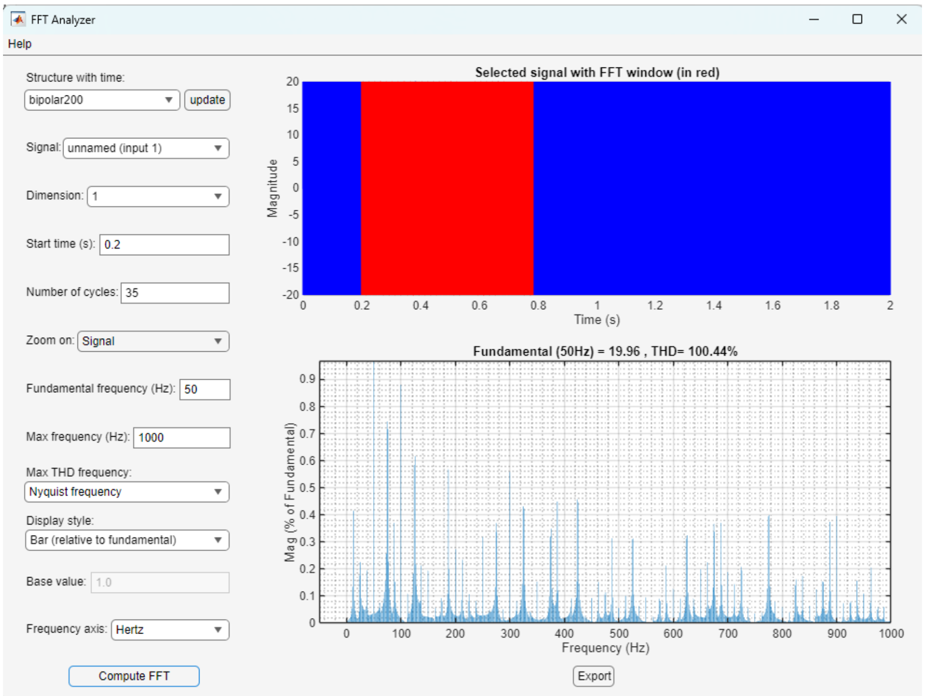

Figure 26: FFT voltage

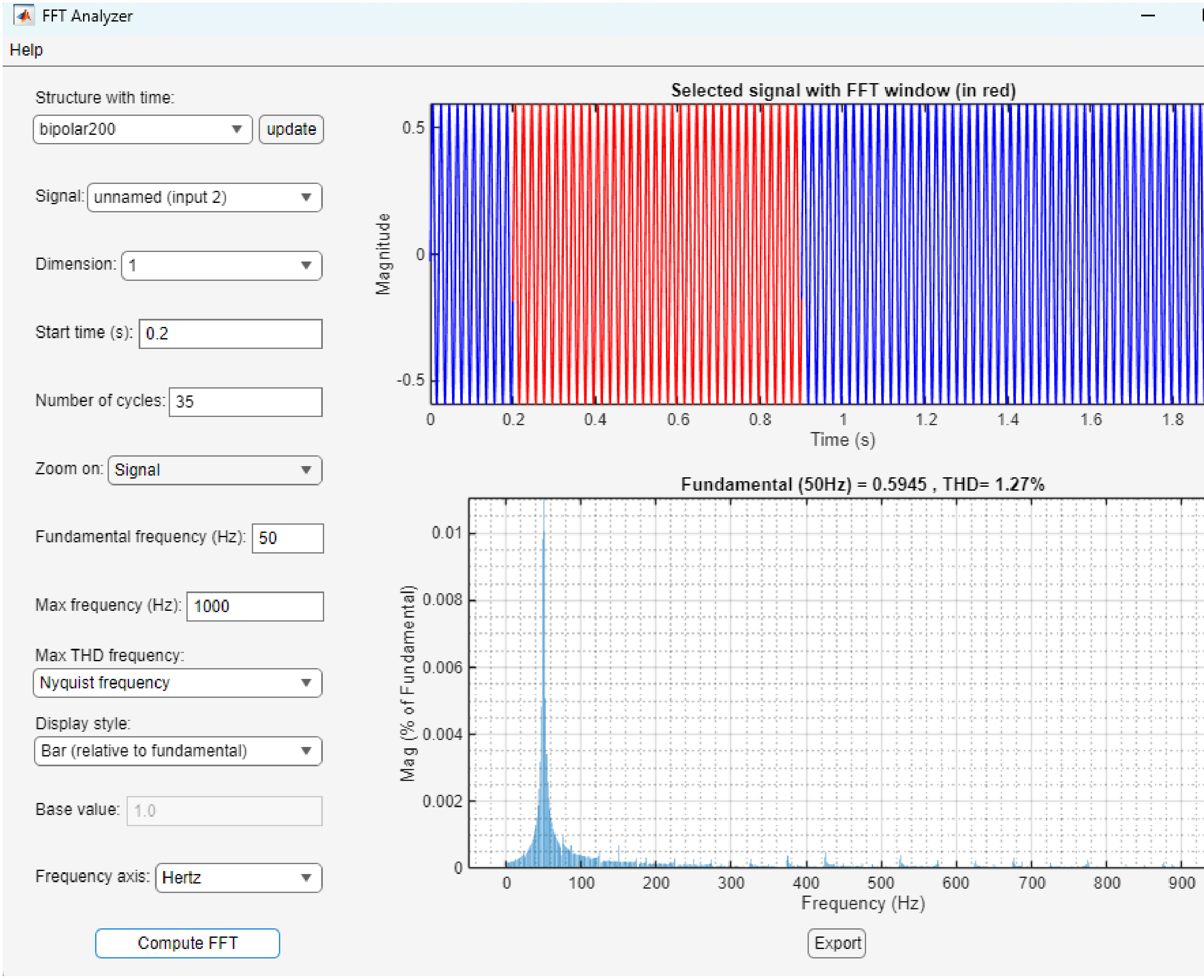

Figure 27: FFT current

5.3 Unipolar PWM Simulation Overview

In unipolar PWM, each leg of the full-bridge inverter is modulated independently using a sinusoidal reference compared with a high-frequency triangular carrier. In this project, two 50 Hz sinusoidal references are used: $v_{\text{ref,A}}(t)$ for the A-leg and $v_{\text{ref,B}}(t) = -v_{\text{ref,A}}(t)$ for the B-leg. Both are compared with the same 10 kHz triangular carrier $v_{\text{tri}}(t)$, so that the frequency modulation ratio remains

$$m_f = \frac{f_{\text{carrier}}}{f_{\text{reference}}} = \frac{10{,}000}{50} = 200.$$

For the A-leg (switches S_1 and S_2):

- If $v_{\text{ref,A}}(t) > v_{\text{tri}}(t)$:

– S_1 is ON, S_2 is OFF,

– The pole voltage is $v_{AN} = +V_{dc}$.

- If $v_{\text{ref,A}}(t) < v_{\text{tri}}(t)$:

  – S_1 is OFF, S_2 is ON,

  – The pole voltage is $v_{AN} = 0$.

For the B-leg (switches S_3 and S_4) using the inverted reference $v_{\text{ref,B}}(t) = -v_{\text{ref,A}}(t)$:

- If $v_{\text{ref,B}}(t) > v_{\text{tri}}(t)$:

  – S_3 is ON, S_4 is OFF,

  – The pole voltage is $v_{BN} = +V_{dc}$.

- If $v_{\text{ref,B}}(t) < v_{\text{tri}}(t)$:

  – S_3 is OFF, S_4 is ON,

  – The pole voltage is $v_{BN} = 0$.

The line voltage is obtained as $v_{AB} = v_{AN} - v_{BN}$, which now takes three levels $(+V_{dc}, 0, -V_{dc})$. Because each leg commutates between 0 and $+V_{dc}$ and the effective switching in v_{AB} occurs at twice the carrier frequency, unipolar PWM reduces low-order harmonics and typically yields lower THD than bipolar PWM.

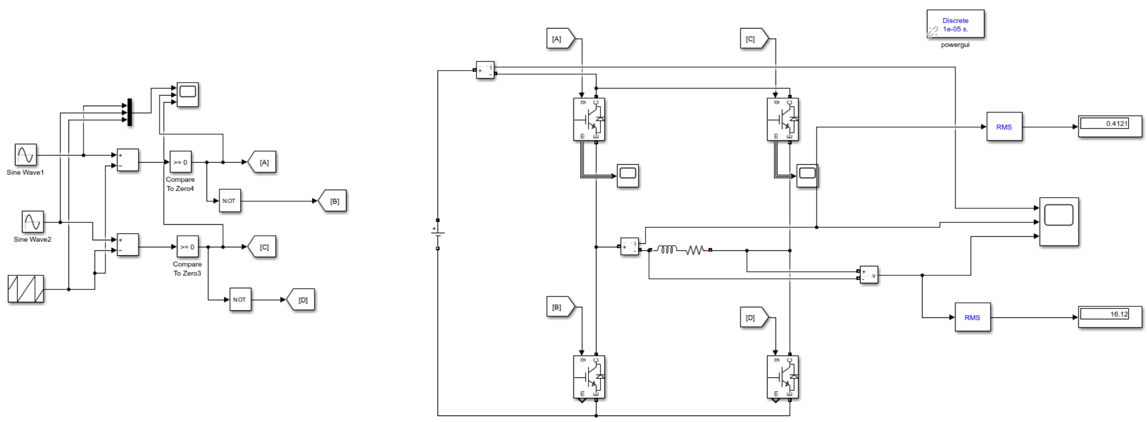

Figure 28: Unipolar MODEL

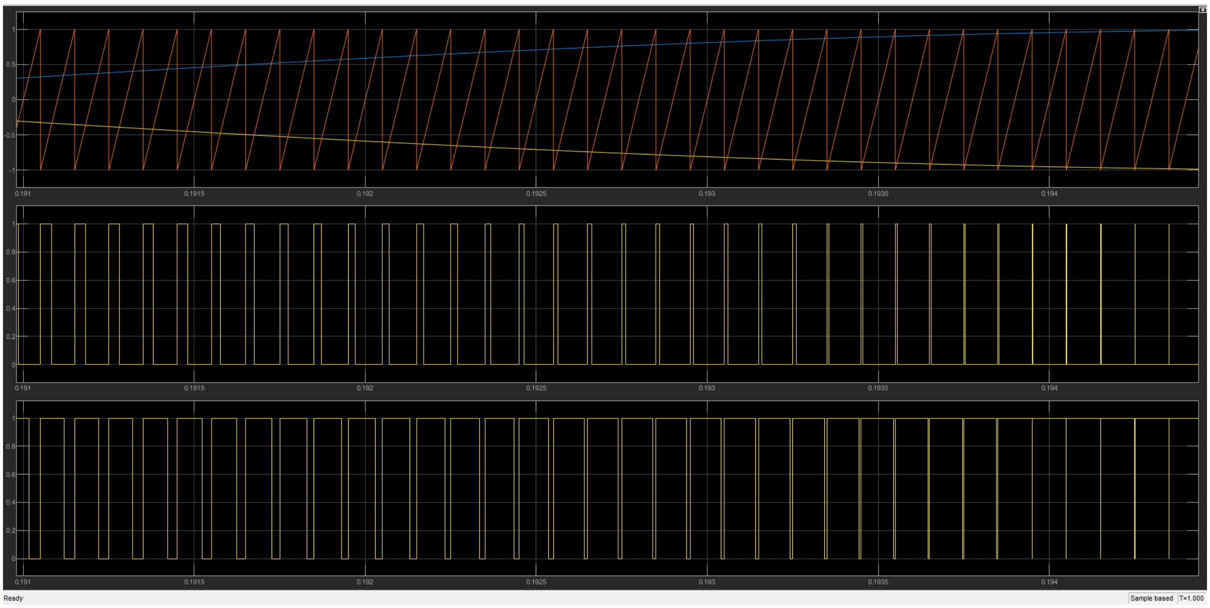

Figure 29: INPUT WAVES

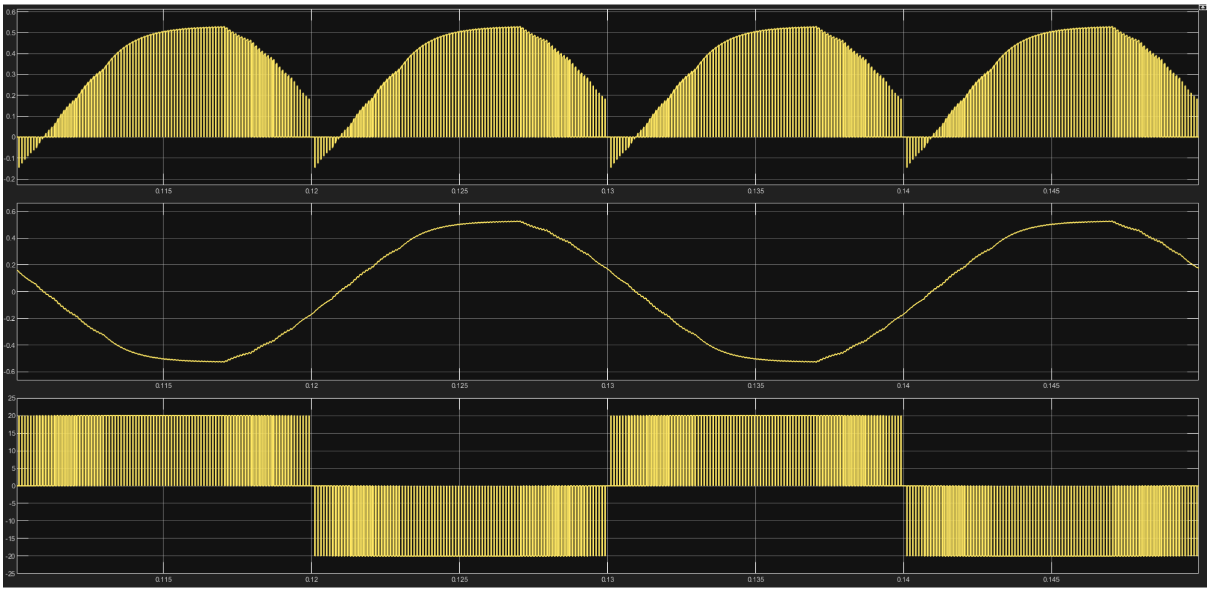

Figure 30: Unipolar WAVEFORMS

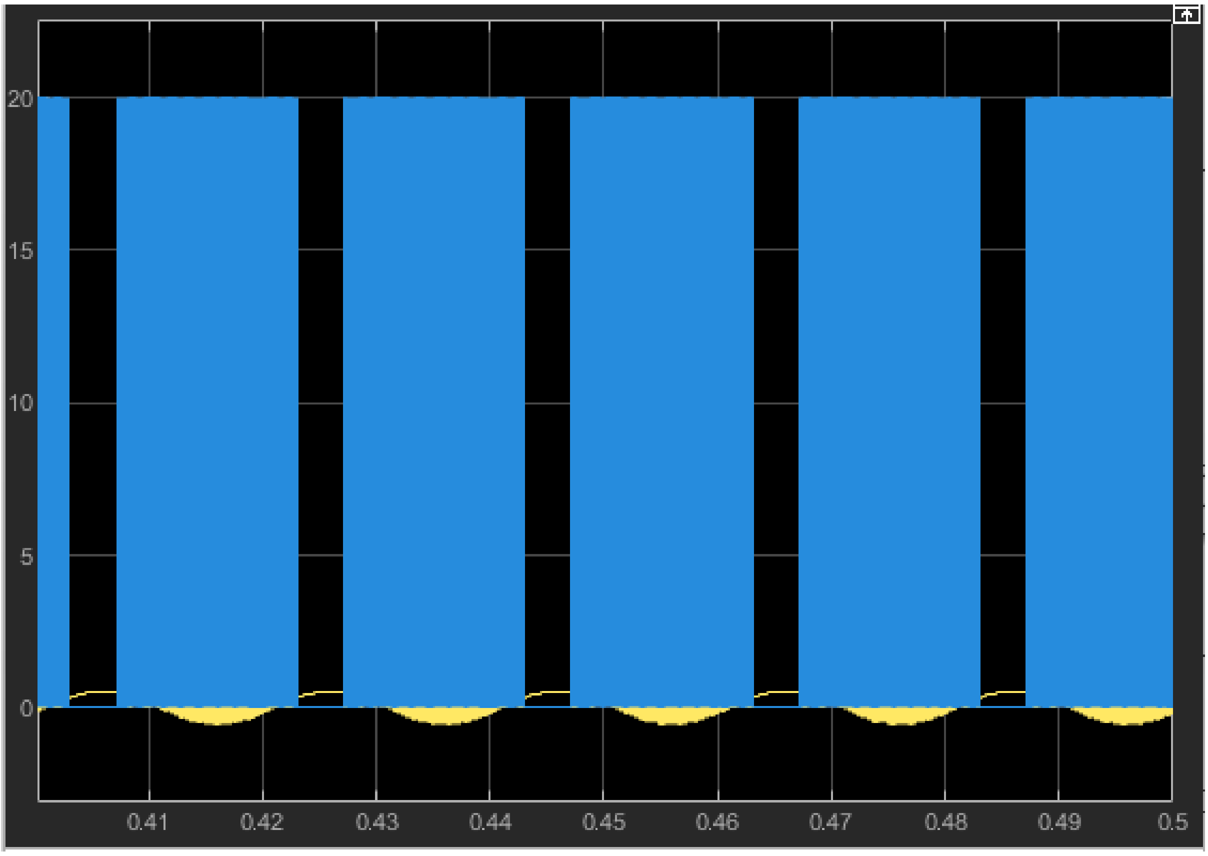

Figure 31: S1,S4

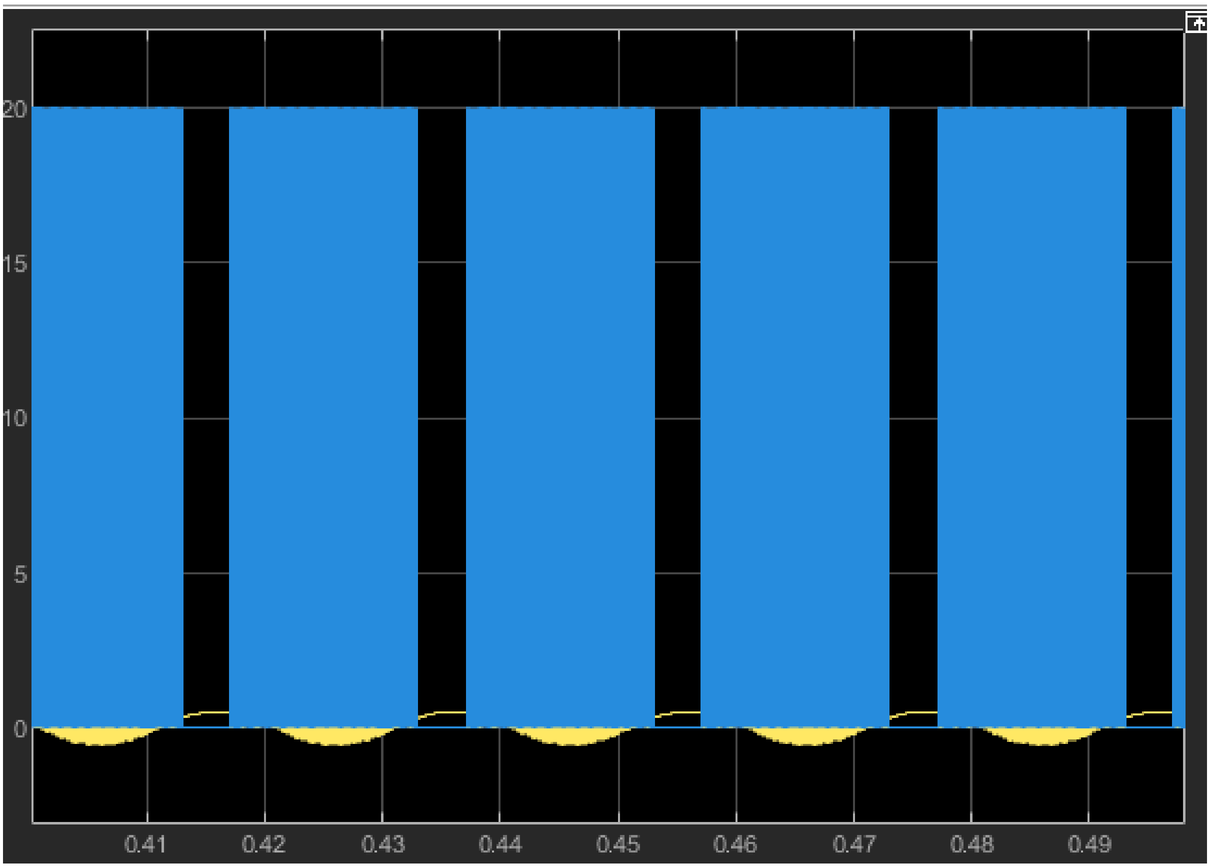

Figure 32: S2, S3

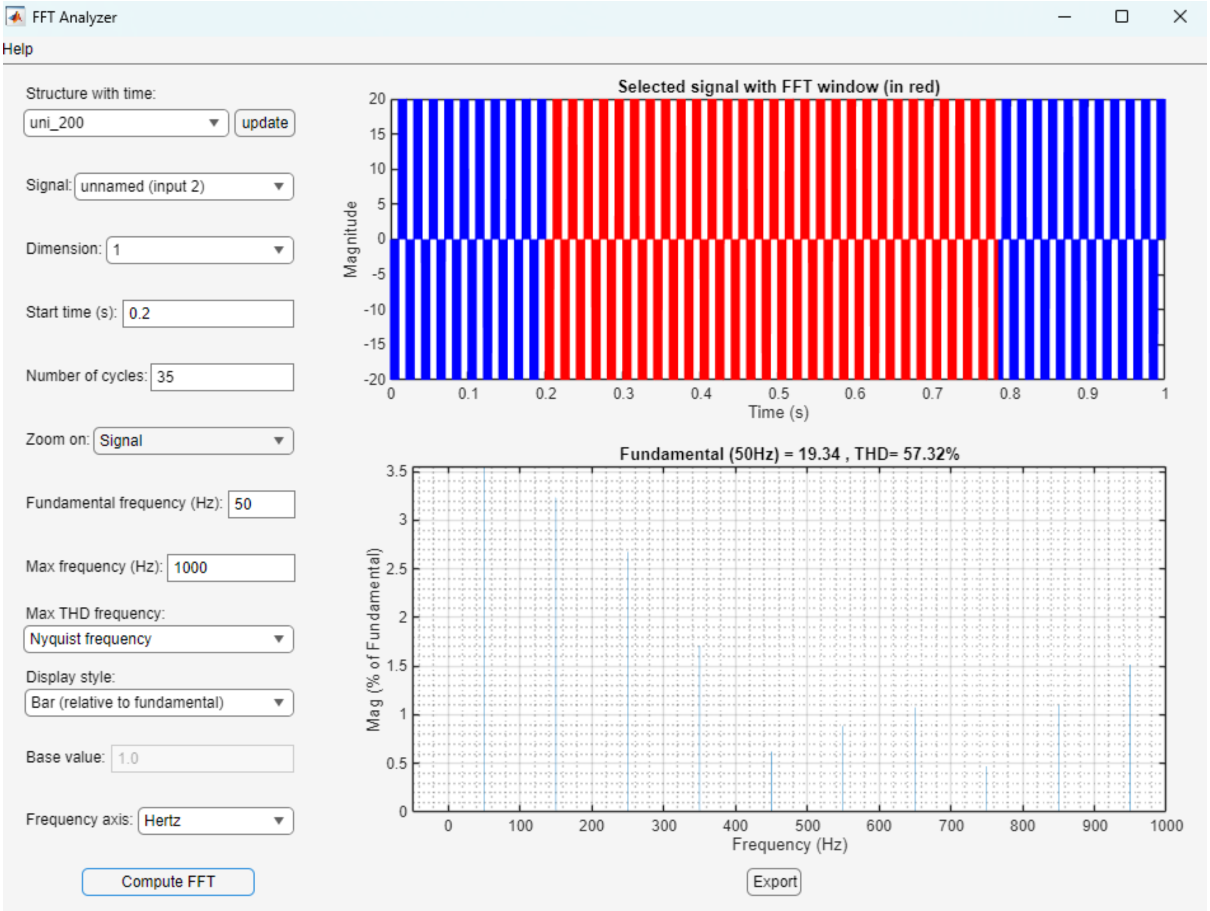

Figure 33: FFT voltage

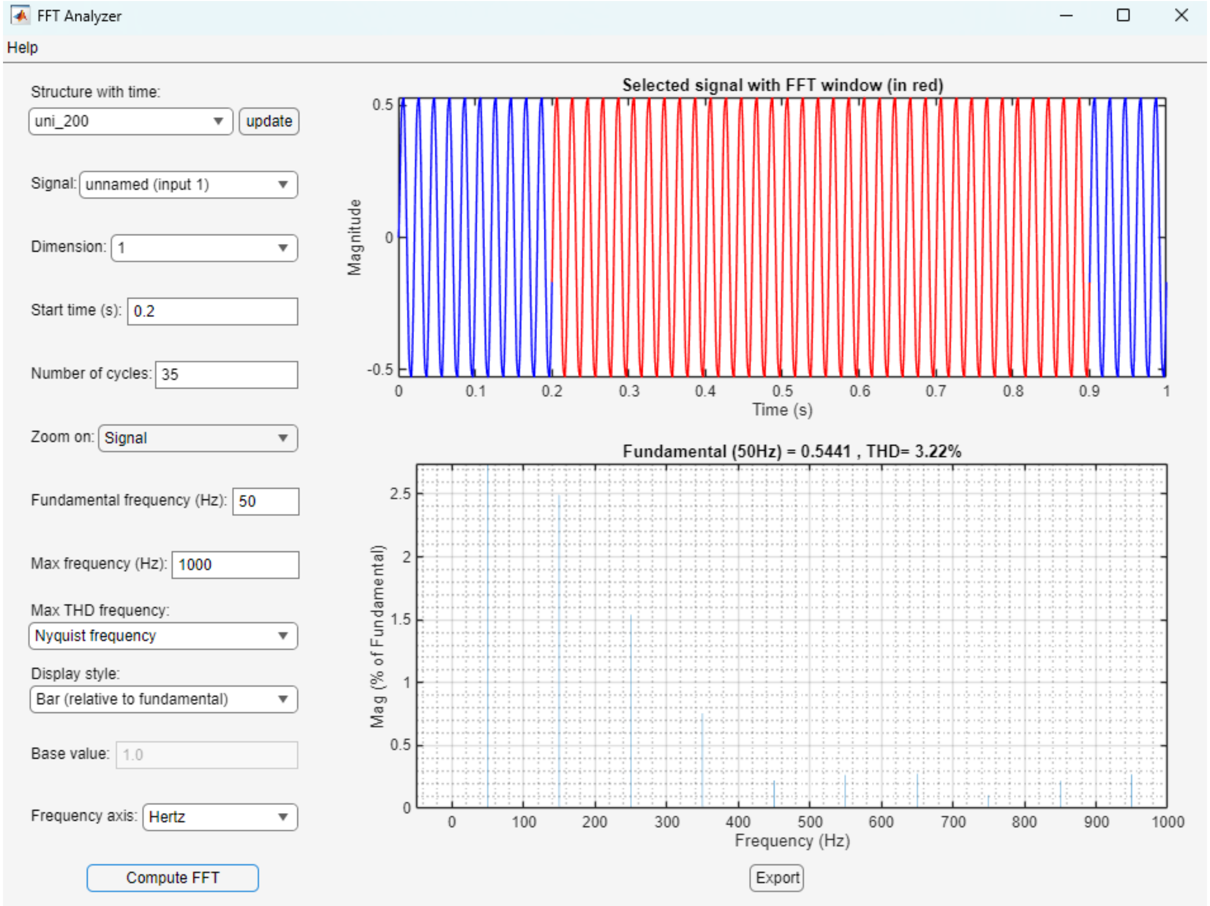

Figure 34: FFT current

6. Bipolar and Unipolar PWM find the minimum value of mf

In this point, we are required to determine the minimum value of the frequency modulation ratio m_f for both bipolar and unipolar PWM such that the output satisfies

$$V_{1,\mathrm{rms}} = \frac{16}{\sqrt{2}} \quad \text{and} \quad \mathrm{THD}_{\mathrm{current}} \leq 10\%.$$

To achieve this, Python and MATLAB scripts were developed to search for the smallest m_f that meets these conditions. The inverter is supplied by a DC source of $V_{dc} = 20$ V and feeds an R–L load with $R = 34\ \Omega$ and $L = 33$ mH. The output frequency is fixed at $f_{\mathrm{out}} = 50$ Hz, with corresponding angular frequency $\omega_0 = 100\pi$ rad/s.

In the code, the load impedance at the fundamental is first computed, and the fundamental current and its RMS value are obtained from the specified fundamental voltage. Then, to estimate the current THD, the dominant voltage harmonics are modelled using predefined normalized Fourier coefficients and modulation index values taken from Table 8–3 (Normalized Fourier Coefficients for Bipolar PWM) and Table 8–4 (Fourier Series

Quantities for the PWM Inverter) in *Power Electronics* by D. Hart. For each candidate m_f, the script converts these voltage harmonics into current harmonics through the load impedance, evaluates the RMS sum of the selected harmonics, and calculates the resulting current THD. The minimum m_f that satisfies both the fundamental RMS constraint and THD $\leq 10\%$ is then identified separately for the bipolar and unipolar PWM schemes.

6.1 unipolar calculations:

Listing 2: MATLAB Code Used to design of unipolar

```
clear; clc;

% Constants
Vdc       = 20;
R         = 34;
L         = 0.033;
f_out     = 50;
omega_0   = 2*pi*f_out;
V1_target = 16;       % fundamental amplitude
THD_limit = 10;       % percent

ma_1 = 0.8;                              %#ok<NASGU>  % not used in this code
fourier_coeffs = [0.31 0.31 0.14 0.14];

% Fundamental component
V1 = V1_target;
Z1 = sqrt(R^2 + (omega_0*L)^2);
I1_calculated       = V1 / Z1;
I1_calculated_rms   = I1_calculated / sqrt(2);

% Search variables
found  = false;
max_mf = 5000;
min_mf = NaN;

for mf = 1:max_mf
    harmonics = [2*mf + 1, 2*mf - 1, 2*mf + 3, 2*mf - 3];

    I_harmonics_rms = zeros(1, numel(harmonics));
```

```matlab

    for k = 1:numel(harmonics)
        n    = harmonics(k);
        ma_n = fourier_coeffs(k);
        Vn   = ma_n * Vdc;
        Zn   = sqrt(R^2 + (n*omega_0*L)^2);
        In   = Vn / Zn;                   % amplitude
        I_harmonics_rms(k) = In / sqrt(2);
    end

    THD = (sqrt(sum(I_harmonics_rms.^2)) / I1_calculated_rms) * 100;

    if THD <= THD_limit
        min_mf = mf;
        found  = true;
        break;
    end
end

% Output
if found
    fprintf('Minimum mf = %d, THD = %.2f%%\n', min_mf, THD);
else
    fprintf('No value of mf found that satisfies the THD limit.\n');
end
```

MATLAB Solver Output

Minimum mf = 11, THD = 9.33%

$$f_{\text{carrier}} = 11 \times 50 = 550 \text{ Hz}.$$

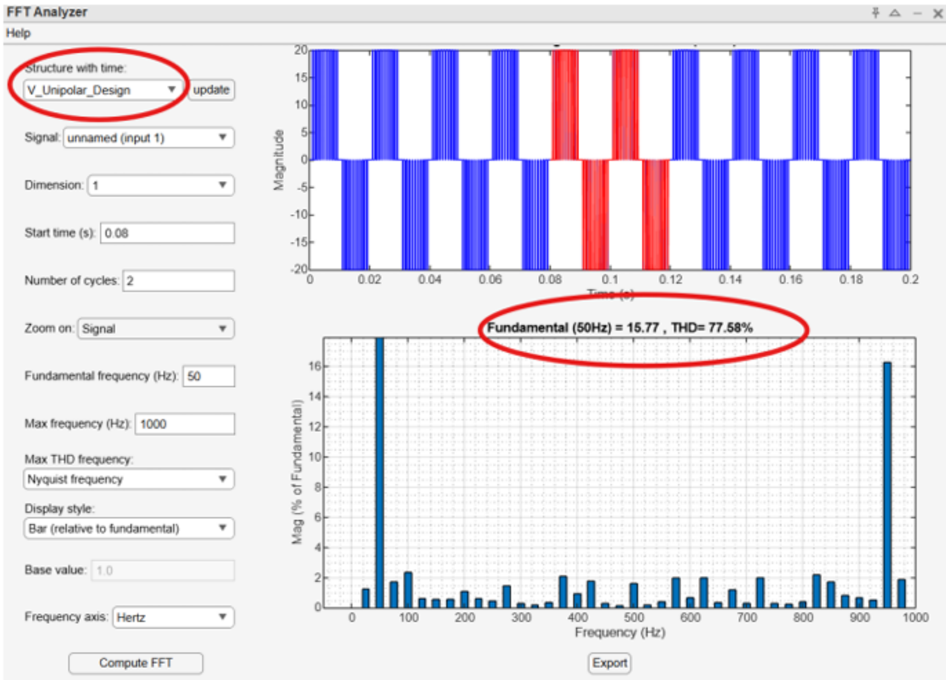

Figure 35: FFT voltage

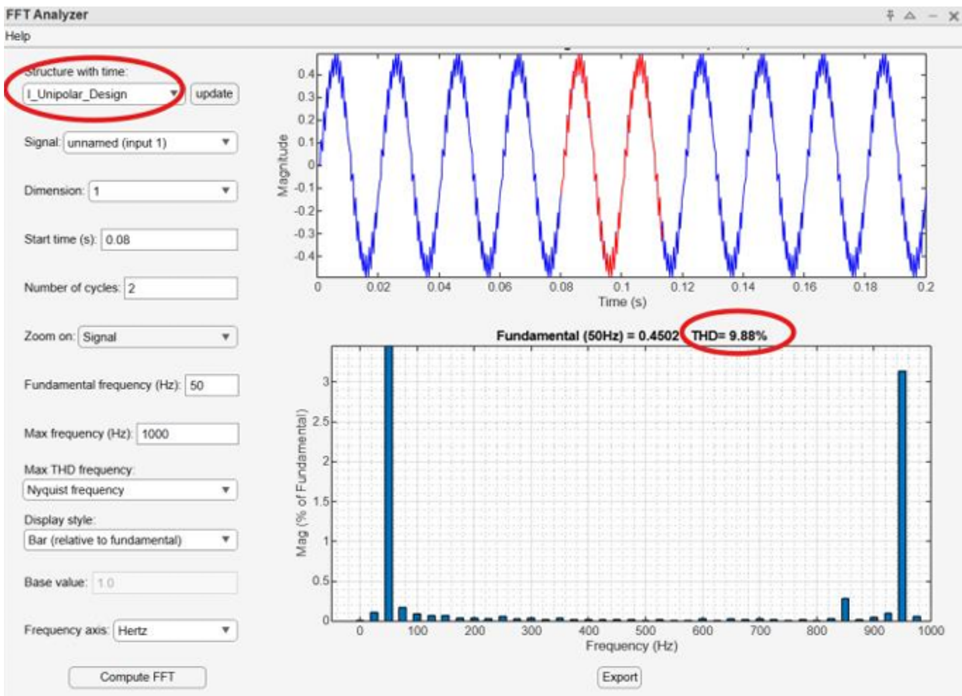

Figure 36: FFT curret

6.2 Bipolar calculations:

Listing 3: MATLAB Code Used to design of Bipolar

```
clear; clc;
```

```

% Constants
Vdc       = 20;
R         = 34;
L         = 0.033;
f_out     = 50;
omega_0   = 2*pi*f_out;
V1_target = 16;      % fundamental amplitude
THD_limit = 10;      % percent

ma_1 = 0.8;                          %#ok<NASGU>  % not used explicitly
fourier_coeffs = [0.82 0.22 0.22];

% Fundamental component
V1 = V1_target;
Z1 = sqrt(R^2 + (omega_0*L)^2);
I1_calculated     = V1 / Z1;
I1_calculated_rms = I1_calculated / sqrt(2);

% Search variables
found  = false;
max_mf = 5000;
min_mf = NaN;

for mf = 1:max_mf
    harmonics = [mf, mf + 2, mf - 2];   % sideband orders
    I_harmonics_rms = zeros(1, numel(harmonics));

    for k = 1:numel(harmonics)
        n    = harmonics(k);
        ma_n = fourier_coeffs(k);
        Vn   = ma_n * Vdc;
        Zn   = sqrt(R^2 + (n*omega_0*L)^2);
        In   = Vn / Zn;                    % amplitude
        I_harmonics_rms(k) = In / sqrt(2);
    end

```

```
    THD = (sqrt(sum(I_harmonics_rms.^2)) / I1_calculated_rms) *
        100;

    if THD <= THD_limit
        min_mf = mf;
        found  = true;
        break;
    end
end

if found
    fprintf('Minimum mf = %d, THD = %.2f%%%%\n', min_mf, THD);
else
    fprintf('No value of mf found that satisfies the THD limit.\n
        ');
end
```

MATLAB Solver Output

Minimum mf = 38, THD = 9.86%

$$f_{\text{carrier}} = 38 \times 50 = 1900 \text{ Hz}.$$

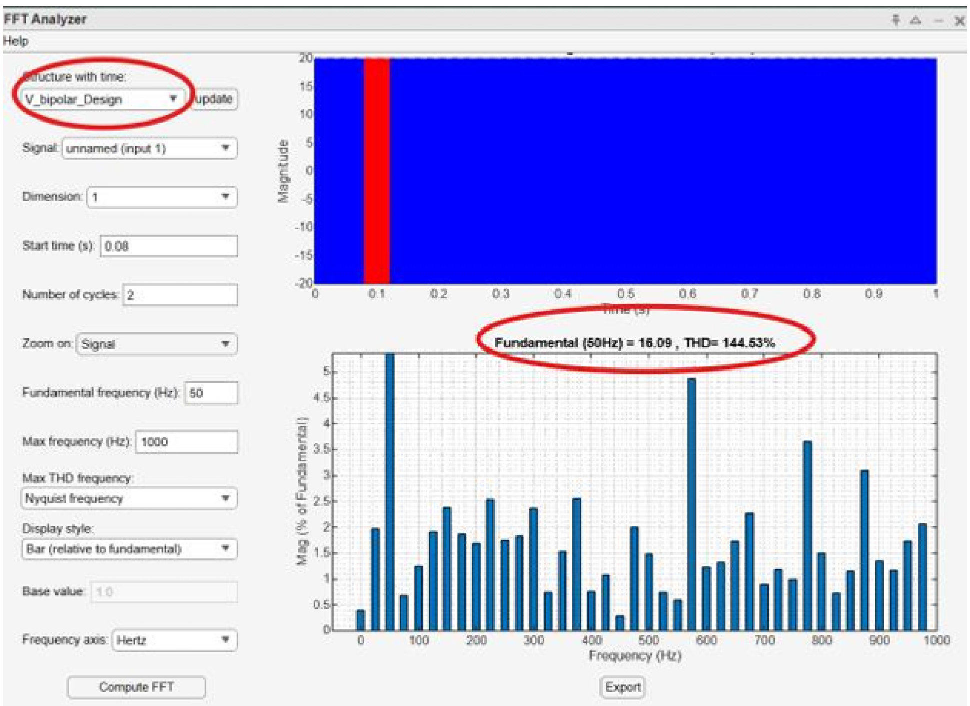

Figure 37: FFT voltage

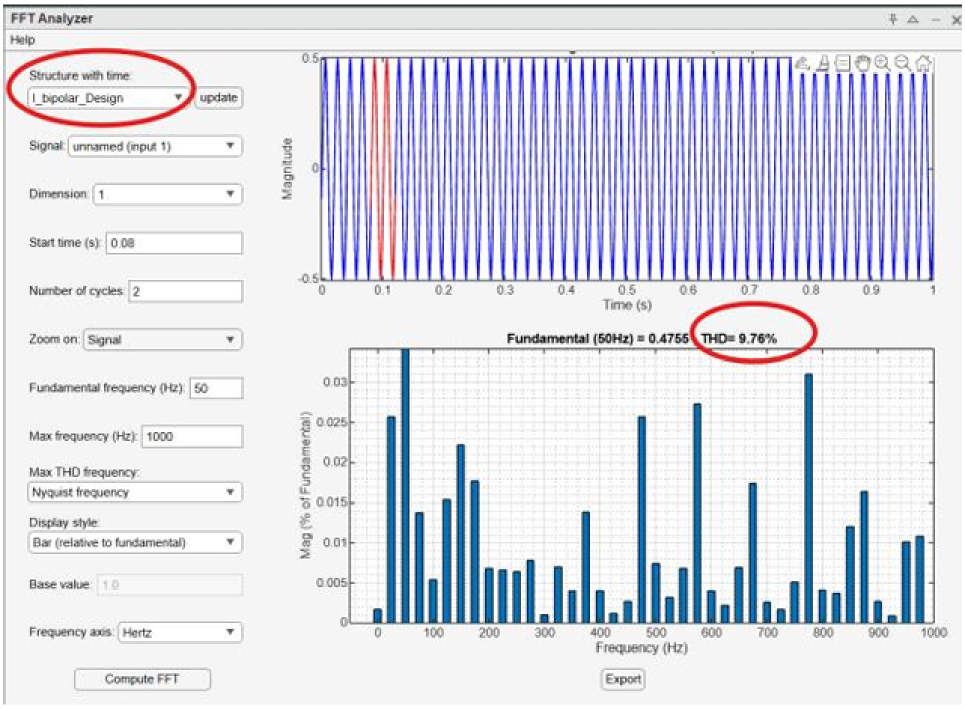

Figure 38: FFT current

7. Multi-level inverter

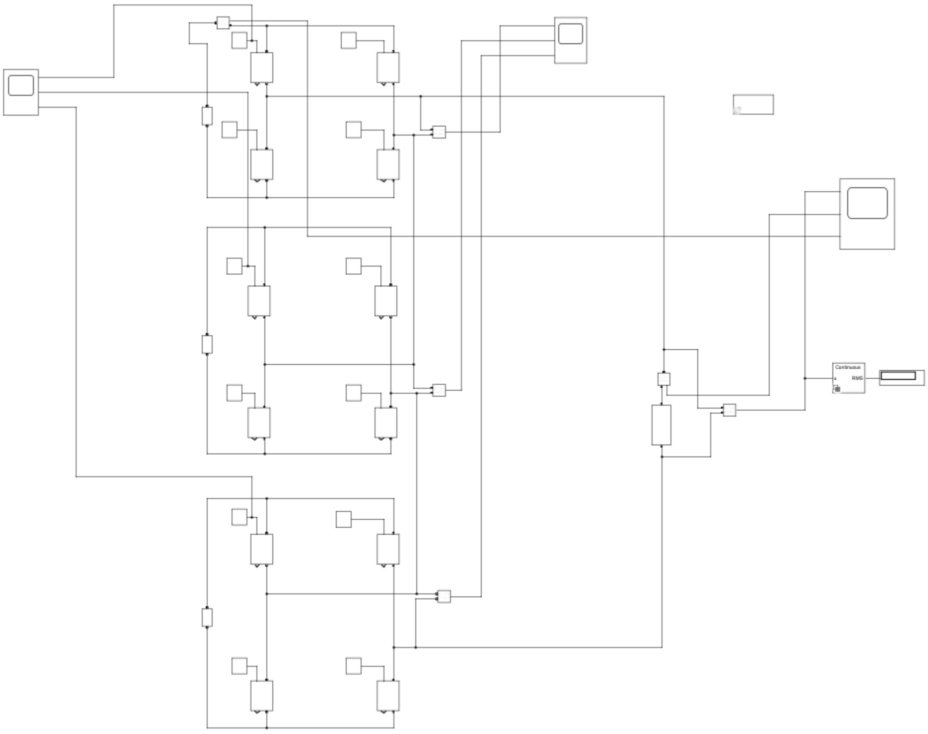

Figure 39: multi level model

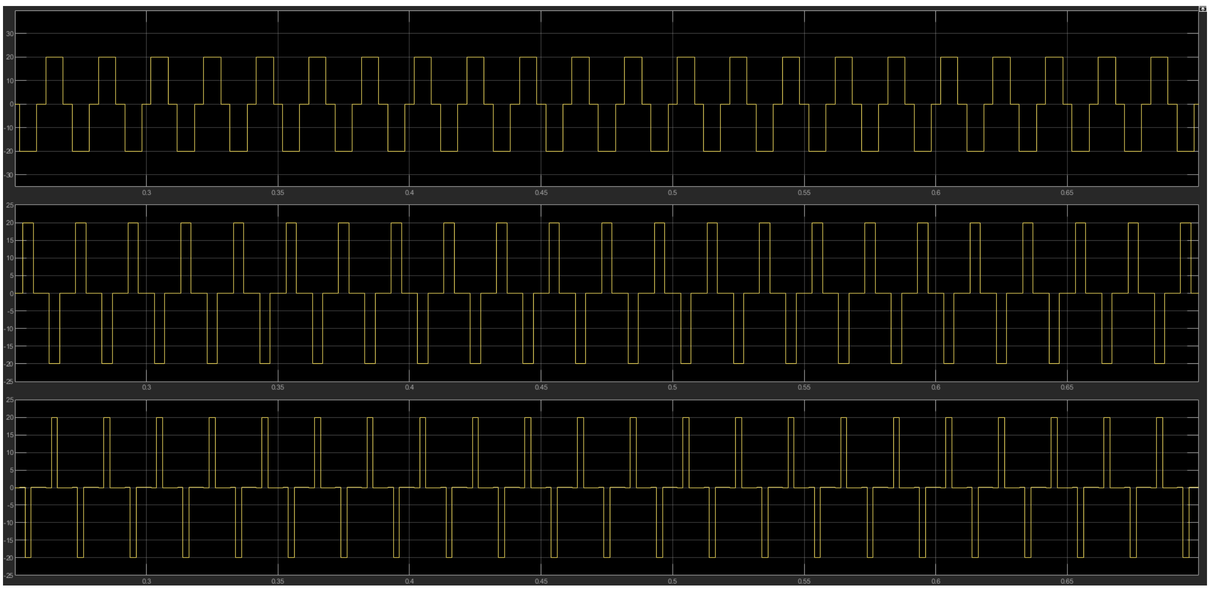

Figure 40: voltage scope

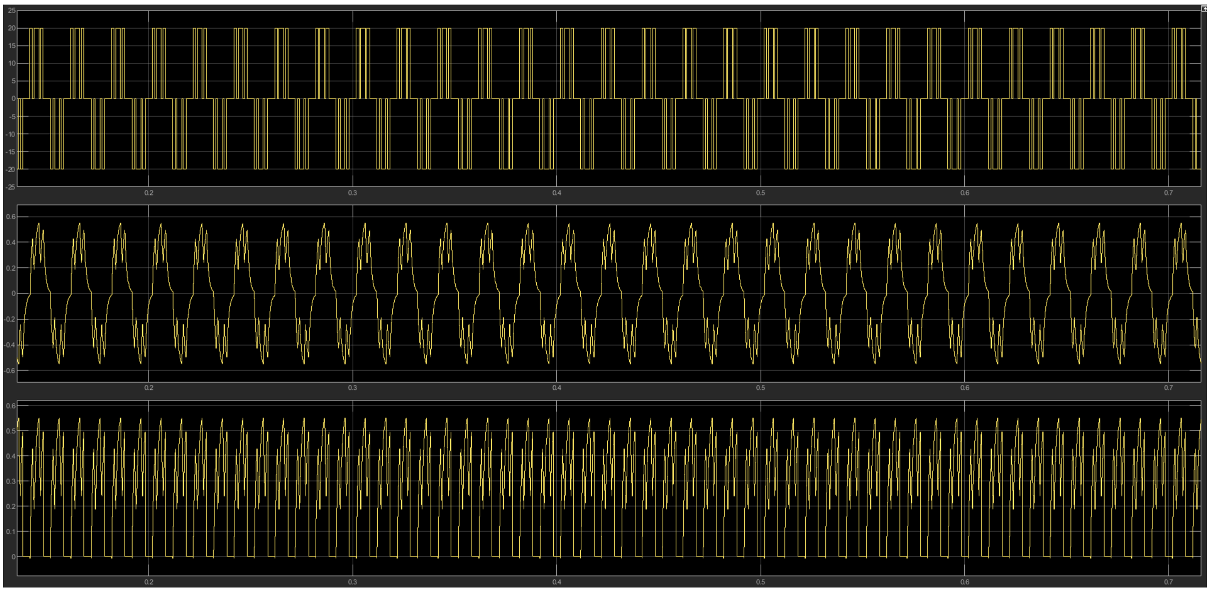

Figure 41: voltage& cuurent scope

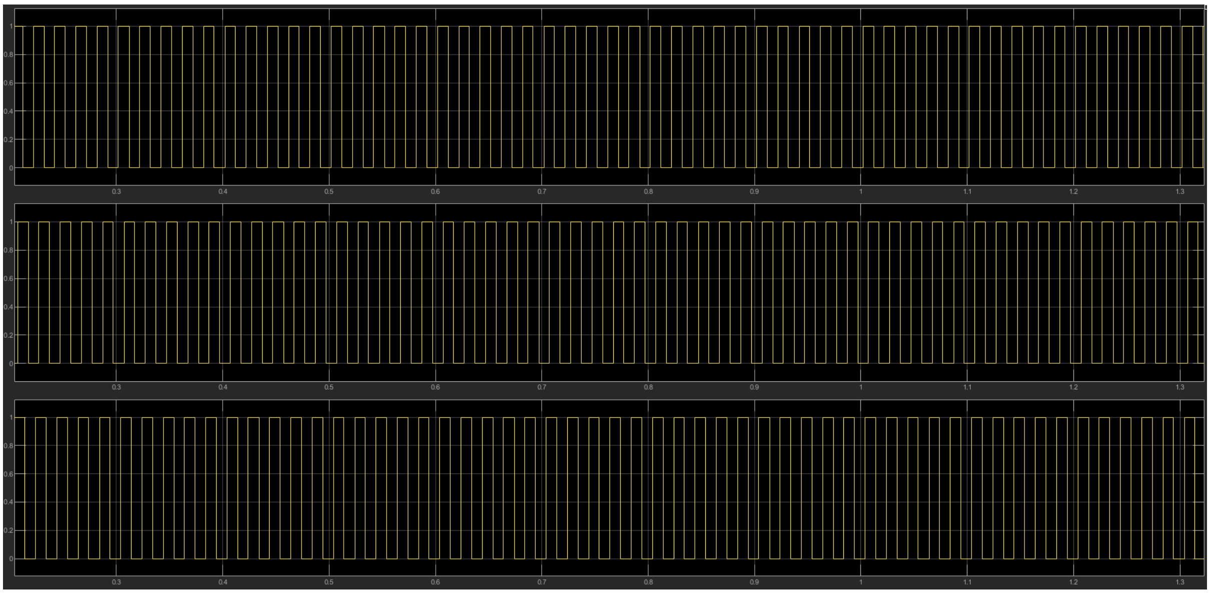

Figure 42: PWM SCOPE

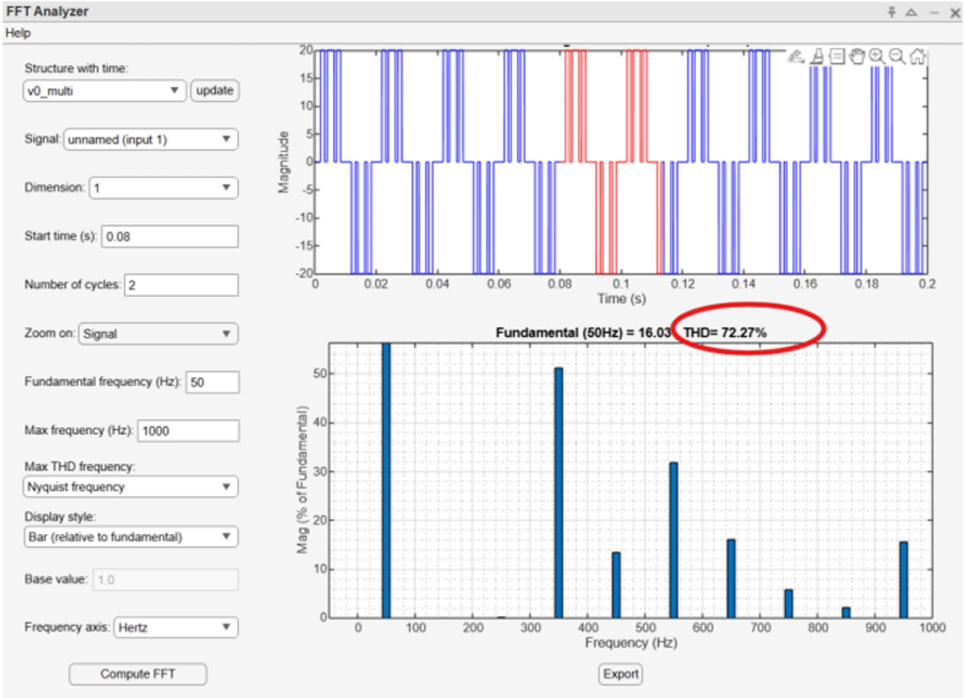

Figure 43: fft voltage

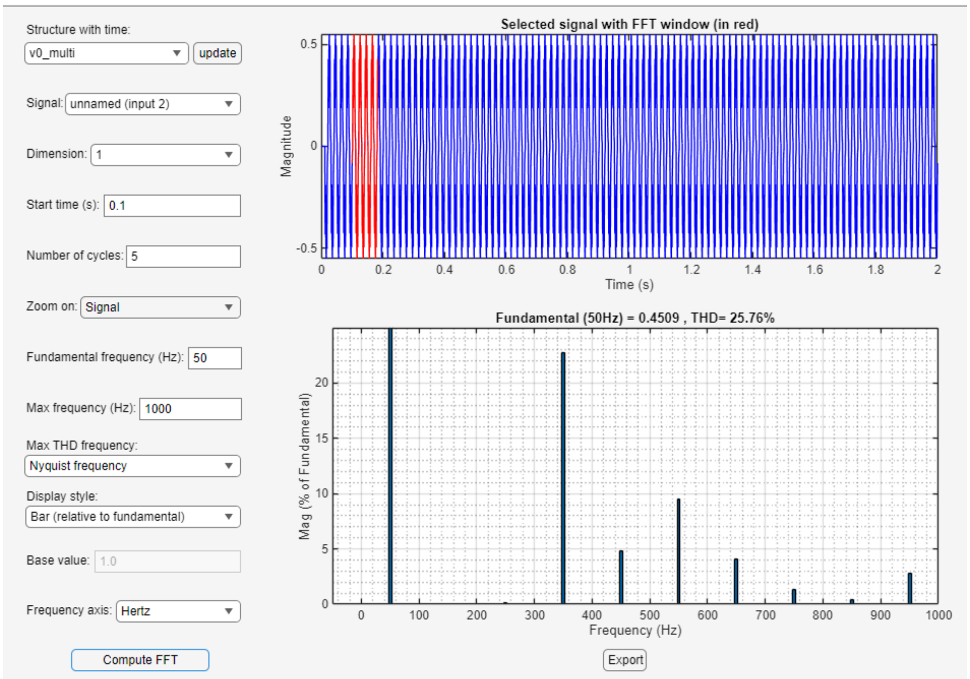

Figure 44: fft current

Table 1: Comparison of Modulation Techniques Based on Fundamental Voltage and Harmonic Distortion

Modulation Technique	$\boldsymbol{V_1}$ (V)	$\mathbf{THD}_{\boldsymbol{i}}$ (%)	$\mathbf{THD}_{\boldsymbol{v}}$ (%)
Square Wave	25.35	29.02	48.43
Quasi Square Wave	24.6	19.77	31.92
Selected Harmonic Elimination	16.06	25.86	72.84
Multilevel Inverter	16.03	25.76	72.27
Bipolar PWM ($m_f = 200$)	19.98	1.27	100.18
Unipolar PWM ($m_f = 200$)	19.34	3.22	57.132
Bipolar PWM (Design)	16.09	9.76	144.53
Unipolar PWM (Design)	15.77	9.88	77.58