DC motor with Torque control

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1 Introduction

DC machines were historically the preferred choice for industrial applications requiring variable speed control due to their straightforward torque and flux regulation via armature and field current adjustments. Their ability to achieve *four-quadrant operation* (forward/reverse motoring and braking) with minimal control complexity made them indispensable for applications demanding high-speed precision or rapid dynamic responses.

However, DC motors rely on commutators and brushes, which introduce significant maintenance and safety challenges. Modern AC drives now dominate most industrial applications, but DC motors retain niche relevance in specialized systems. Below are the *four primary DC motor types*, each with distinct advantages and limitations:

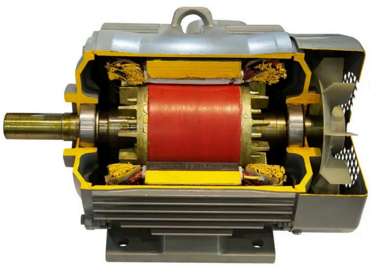

Figure 1: DC motor

1.1 Types of DC Motors

1. Separately Excited DC Motor

- **Pros**:
  - Independent Control of Torque and Flux
  - Wide Speed Range
  - Speed direction can be reversed by reversing armature voltage, enabling energy-efficient braking.
- **Cons**:
  - Higher Cost and Complexity
  - Maintenance Challenges
  - Inefficiency at Low Speeds

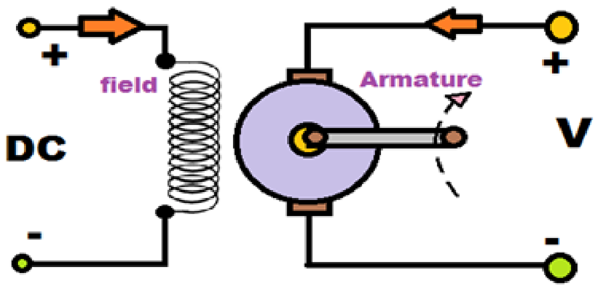

Figure 2: permanent Dc motor

2. **Series DC Motor**
    - **Pros**:
        - Extremely high starting torque (up to 500% of rated torque), suited for traction systems like locomotives.
        - Simple construction with series-connected armature and field windings.
    - **Cons**:
        - Poor speed regulation; speed drops significantly under load.
        - Cannot operate safely without a load due to runaway speed risks.

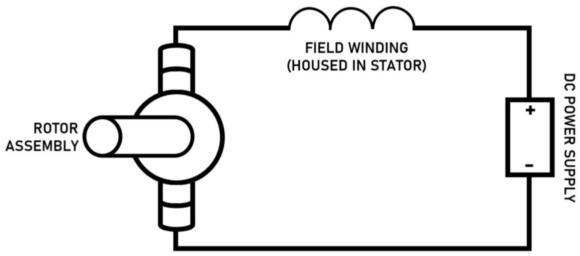

Figure 3: series DC motor

3. **Shunt DC Motor**
    - **Pros**:
        - Near-constant speed under varying loads, ideal for lathes and conveyor belts.
        - Cost-effective for applications requiring steady operation, such as pumps and fans.
    - **Cons**:
        - Low starting torque compared to series motors.
        - Inefficient at low speeds and bulky compared to AC alternatives.

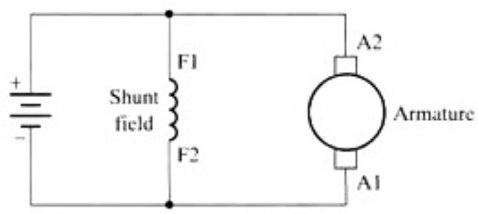

Figure 4: shunt Dc motor

2 general Idea:

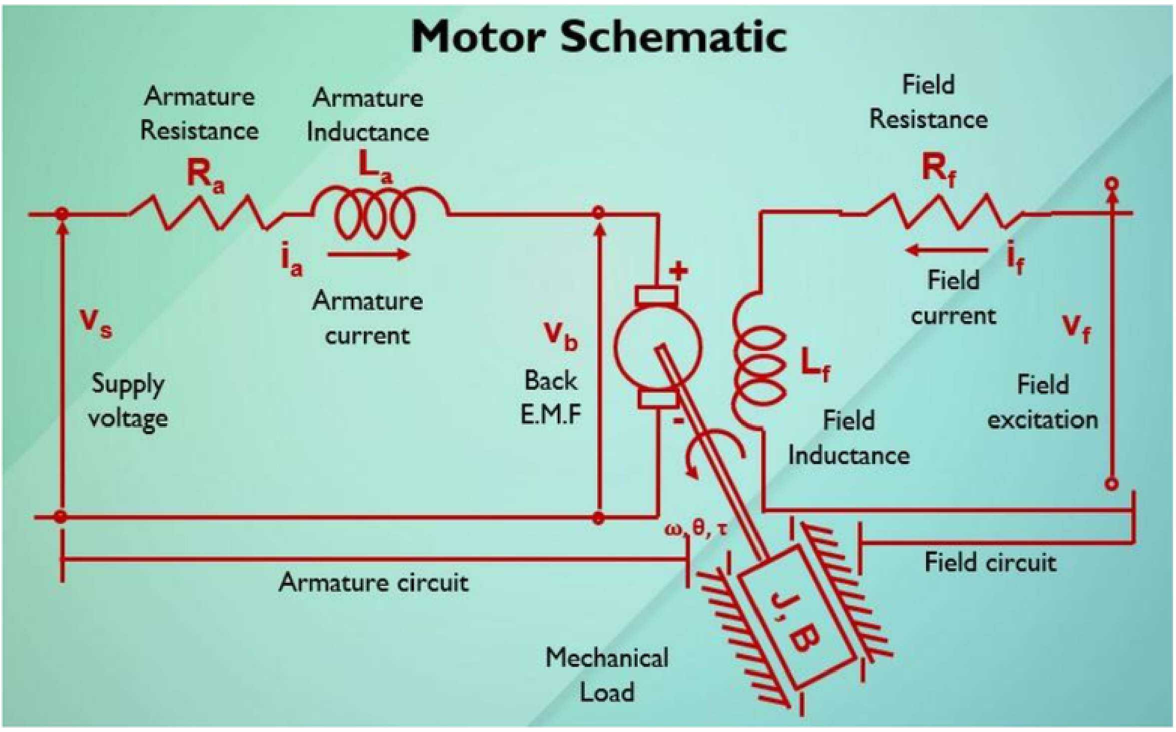

Figure 5: Dc motor schematic

DC motor dynamics are governed by electrical and mechanical equations. The torque T_e is proportional to armature current i_a, expressed as:

$$T_M = K_f \cdot i_a \tag{1}$$

where K_f is the torque constant. The back-emf voltage v_b relates to angular velocity ω as:

$$e_a = K_v \cdot \omega \tag{2}$$

These equations form the basis of the motor's state-space model, which can be derived using Kirchhoff's laws and Newtonian mechanics.
generally to achieve torque control we need only to adjust armature currant (i_a) as it shown in equation ref1 torque of motor is directly proportional to armature current(i_a)

3 Objectives:

- Develop a Detailed Electrical Model of the DC Motor
- Design Electrical Control for Torque Regulation
- Examine the Impact of changing Electrical Parameters

4 Expectations:

- Accurate Electrical Representation
- Effective Current and Voltage Control
- Clearly explain the electrical principles behind the DC motor's operation and torque control

5 methods:

in This project we will focus on analysis of torque control methods for shunt DC motors. There are three control methods :

1. Field Resistance Control: Adjusting the field resistance R_f to change field flux
2. Armature Voltage Control: Adjusting the voltage V_A applied to the armature
3. Armature Resistance Control: Inserting additional resistor R_{add} in series with the armature circuit

The analysis focuses on the torque equation:

$$T = \frac{k \cdot \phi \cdot v}{R_a} - \frac{k^2 \cdot \phi}{R_a} \cdot \omega \tag{3}$$

6 Mathematical Derivation

6.1 Basic Shunt DC Motor Equations

6.1.1 Torque Equation

The fundamental torque equation for a DC motor is:

$$T = K_T \cdot I_a \cdot \Phi \tag{4}$$

Where:

- T is the developed torque (N¬m)
- K_T is the torque constant
- I_a is the armature current (A)
- Φ is the field flux (Wb)

6.1.2 Armature Circuit Equation

For the armature circuit:

$$V_a = E_a + I_a \cdot R_a \tag{5}$$

Where:

- V_a is the applied armature voltage (V)
- E_a is the back EMF (V)
- I_a is the armature current (A)
- R_a is the armature resistance (Œ©)

6.1.3 Back EMF Equation

The back EMF is proportional to the product of field flux and angular velocity:

$$E_a = K_e \cdot \Phi \cdot \omega \tag{6}$$

Where:

- E_a is the back EMF (V)
- K_e is the voltage constant
- Φ is the field flux (Wb)
- ω is the angular velocity (rad/s)

6.1.4 Field Circuit Equation

For the field circuit in a shunt DC motor:

$$I_f = \frac{V_f}{R_f} \tag{7}$$

Where:

- I_f is the field current (A)
- V_f is the field voltage (V)
- R_f is the field resistance (Œ©)

6.1.5 Field Flux Relationship

The field flux is proportional to the field current:

$$\Phi = K_f \cdot I_f \tag{8}$$

Where:

- Φ is the field flux (Wb)
- K_f is a constant
- I_f is the field current (A)

6.2 Derivation of the Torque Equation

Starting with the basic equations, we can derive the torque equation used in this project:

From the armature circuit equation:

$$V_a = E_a + I_a \cdot R_a \tag{9}$$

Substituting the back EMF equation:

$$V_a = K_e \cdot \Phi \cdot \omega + I_a \cdot R_a \tag{10}$$

Solving for armature current:

$$I_a = \frac{V_a - K_e \cdot \Phi \cdot \omega}{R_a} \tag{11}$$

Now, substituting this into the torque equation:

$$T = K_T \cdot I_a \cdot \Phi = K_T \cdot \Phi \cdot \frac{V_a - K_e \cdot \Phi \cdot \omega}{R_a} \tag{12}$$

Expanding:

$$T = \frac{K_T \cdot \Phi \cdot V_a}{R_a} - \frac{K_T \cdot K_e \cdot \Phi^2 \cdot \omega}{R_a} \tag{13}$$

For DC motors, the torque constant K_T and the voltage constant K_e are equal when using consistent units. Let's denote this common constant as k:

$$T = \frac{k \cdot \Phi \cdot V_a}{R_a} - \frac{k^2 \cdot \Phi^2 \cdot \omega}{R_a} \tag{14}$$

This is the torque equation used in this project:

$$T = \frac{k \cdot \Phi \cdot v}{R_a} - \frac{k^2 \cdot \Phi^2}{R_a} \cdot \omega \tag{15}$$

7 Torque Control Methods

7.1 Field Resistance Control

In field resistance control, we adjust the field resistance R_f to change the field flux Φ.

From the field circuit equation:

$$I_f = \frac{V_f}{R_f} \tag{16}$$

And the field flux relationship:

$$\Phi = K_f \cdot I_f = K_f \cdot \frac{V_f}{R_f} \tag{17}$$

Substituting into the torque equation:

$$T = \frac{k \cdot K_f \cdot \frac{V_f}{R_f} \cdot V_a}{R_a} - \frac{k^2 \cdot (K_f \cdot \frac{V_f}{R_f})^2 \cdot \omega}{R_a} \tag{18}$$

Simplifying:

$$T = \frac{k \cdot K_f \cdot V_f \cdot V_a}{R_f \cdot R_a} - \frac{k^2 \cdot K_f^2 \cdot V_f^2 \cdot \omega}{R_f^2 \cdot R_a} \tag{19}$$

This shows that increasing R_f will decrease the torque, and the effect is more significant at higher speeds.

7.2 Armature Voltage Control

In armature voltage control, we adjust the armature voltage V_a while keeping the field circuit constant.

The torque equation becomes:

$$T = \frac{k \cdot \Phi \cdot V_a}{R_a} - \frac{k^2 \cdot \Phi^2 \cdot \omega}{R_a} \tag{20}$$

Since Φ is constant, the torque is directly proportional to V_a, and the speed-dependent term remains unchanged.

7.3 Armature Resistance Control

In armature resistance control, we add an external resistance R_{add} in series with the armature circuit.

The total armature resistance becomes:

$$R_{total} = R_a + R_{add} \tag{21}$$

The torque equation becomes:

$$T = \frac{k \cdot \Phi \cdot V_a}{R_a + R_{add}} - \frac{k^2 \cdot \Phi^2 \cdot \omega}{R_a + R_{add}} \tag{22}$$

Increasing R_{add} will decrease the torque at all speeds, but it also increases power loss in the resistor.

8 Power Loss Analysis

8.1 Power Loss in Field Resistance Control

The power loss in the field circuit is:

$$P_{loss,f} = I_f^2 \cdot R_f = \frac{V_f^2}{R_f} \tag{23}$$

8.2 Power Loss in Armature Resistance Control

The power loss in the additional armature resistance is:

$$P_{loss,add} = I_a^2 \cdot R_{add} \tag{24}$$

Where:

$$I_a = \frac{V_a - k \cdot \Phi \cdot \omega}{R_a + R_{add}} \tag{25}$$

This power loss represents wasted energy and reduces the overall efficiency of the motor.

9 Hand manual solution:

9.1 Field Resistance Control

Assumptions:

- Armature resistance, $R_a = 0.5\ \Omega$
- Machine constant, $k = 0.5$
- Supply voltage, $V = 220$ V
- Angular speed, $\omega = [0 : 10 : 200]$ rad/s
- Field resistance values, $R_{f,\text{values}} = [240,\ 288,\ 360,\ 480]\ \Omega$

For $R_f = 240\ \Omega$:

- Field current:

$$I_f = \frac{V}{R_f} = \frac{220}{240} = 0.916 \text{ A}$$

- Flux per pole:

$$\phi = kI_f = 0.5 \times 0.916 = 0.458$$

- Torque (T):

$$T = \frac{k\phi V}{R_a} - \frac{(k\phi)^2}{R_a}$$

Substituting the values:

$$T = \frac{0.5 \times 0.458 \times 220}{0.5} - \frac{(0.5 \times 0.458)^2}{0.5} \times 0$$

$$T = \frac{50.38}{0.5} - \frac{0.0524}{0.5} \times 0$$

$$T = 100.76 - 0 = 100.76 N.m$$

9.2 Armature Voltage Control

Assumptions:

- Armature resistance: $R_a = 0.5\ \Omega$
- Machine constant: $k = 0.5$
- Field resistance: $R_f = 240\ \Omega$
- Angular speed: $\omega = [0 : 10 : 200]$ rad/s
- Armature voltage values: $V_{a,\text{values}} = [220,\ 176,\ 132,\ 88]$ V

Calculation for $V_a = 132$ V, $\omega = 10$ rad/s:

- **Field current:**

$$I_f = \frac{V}{R_f} = \frac{220}{240} = 0.916 \text{ A}$$

- **Flux per pole:**

$$\phi = kI_f = 0.5 \times 0.916 = 0.458$$

- **Torque (T):**

  The torque equation is:

$$T = \frac{k\phi V_a}{R_a} - \frac{(k\phi)^2}{R_a} \cdot \omega$$

  Substituting the given values:

$$T = \frac{0.5 \times 0.458 \times 132}{0.5} - \frac{(0.5 \times 0.458)^2}{0.5} \times 10$$

$$T = \frac{30.228}{0.5} - \frac{0.0524}{0.5} \times 10$$

$$T = 60.456 - 1.048 = 59.408 \text{ N} \cdot \text{m}$$

  Result: $T \approx 59.4 \text{ N} \cdot \text{m}$

9.3 Armature Resistance Control

Assumptions:

- Armature resistance: $R_a = 0.5\ \Omega$
- Additional armature resistance: $R_{add,\text{values}} = [0,\ 0.5,\ 1,\ 2]\ \Omega$
- Total armature resistance: $R_{total} = R_a + R_{add}$
- Field resistance: $R_f = 240\ \Omega$
- Machine constant: $k = 0.5$
- Armature voltage: $V_a = 220$ V
- Field voltage: $V_f = 220$ V
- Angular speed: $\omega = [0 : 10 : 200]$ rad/s

Calculation for $R_{add} = 2\ \Omega$, $\omega = 200$ rad/s:

- **Total armature resistance:**

$$R_{total} = R_a + R_{add} = 0.5 + 2 = 2.5\ \Omega$$

- **Field current:**

$$I_f = \frac{V_f}{R_f} = \frac{220}{240} = 0.916 \text{ A}$$

- **Flux per pole:**

$$\phi = kI_f = 0.5 \times 0.916 = 0.458$$

- **Torque (T):**

$$T = \frac{k\phi V_a}{R_{total}} - \frac{(k\phi)^2}{R_{total}} \cdot \omega$$

Substituting the values:

$$T = \frac{0.5 \times 0.458 \times 220}{2.5} - \frac{(0.5 \times 0.458)^2}{2.5} \times 200$$

$$T = \frac{50.38}{2.5} - \frac{0.0524}{2.5} \times 200$$

$$T = 20.152 - 4.192 = 15.96 \text{ N} \cdot \text{m}$$

Result: $T \approx 16.0 \text{ N} \cdot \text{m}$

10 MATLAB code:

10.1 Field Resistance Control

```
% Field Resistance Control with Torque, Power Loss, and Efficiency
  Plots

clear all; close all; clc;

%% Motor Parameters
Ra = 0.5;               % Armature resistance (ohms)
Rf_nominal = 240;       % Nominal field resistance (ohms)
k = 0.5;                % Motor constant
Va = 220;               % Armature voltage (V)
Vf = 220;               % Field voltage (V)

%% Field Resistance Variation
Rf_values = [Rf_nominal, Rf_nominal*1.2, Rf_nominal*1.5, Rf_nominal*2];
num_Rf = length(Rf_values);

%% Speed Range
omega = 0:10:200;      % Angular velocity range (rad/s)
num_omega = length(omega);

%% Initialize Arrays
torque = zeros(num_omega, num_Rf);
power_loss = zeros(num_omega, num_Rf);
efficiency = zeros(num_omega, num_Rf);  % Efficiency matrix

%% Calculate Field Current and Flux for Each Rf Value
phi_values = zeros(num_Rf, 1);
for i = 1:num_Rf
    If = Vf / Rf_values(i);                 % Field current
    phi_values(i) = k * If;                 % Field flux
end

%% Calculate Torque, Power Loss, and Efficiency
for j = 1:num_Rf
    phi = phi_values(j);
    for i = 1:num_omega
```

```
        w = omega(i);

        % Torque Equation
        torque(i,j) = (k * phi * Va / Ra) - (k^2 * phi^2 * w / Ra);

        % Armature Current
        Ia = (Va - k * phi * w) / Ra;

        % Power Loss
        power_loss(i,j) = Ia^2 * Ra;

        % Efficiency
        P_out = torque(i,j) * w;
        P_in = Va * Ia;
        efficiency(i,j) = (P_out / P_in) * 100;  % in percent
    end
end

%% Plot Torque vs Speed
figure;
for j = 1:num_Rf
    plot(omega, torque(:,j), 'LineWidth', 1.5);
    hold on;
end
grid on;
title('Torque vs Speed for Different Field Resistances');
xlabel('Angular Velocity (rad/s)');
ylabel('Torque (N m)');
legend_str = cell(num_Rf,1);
for j = 1:num_Rf
    legend_str{j} = ['Rf = ' num2str(Rf_values(j)) ' \Omega'];
end
legend(legend_str);
hold off;

%% Plot Power Loss vs Speed
figure;
for j = 1:num_Rf
    plot(omega, power_loss(:,j), 'LineWidth', 1.5);
    hold on;
end
grid on;
title('Power Loss vs Speed for Different Field Resistances');
xlabel('Angular Velocity (rad/s)');
ylabel('Power Loss (W)');
legend(legend_str);
hold off;

%% Plot Efficiency vs Speed
figure;
for j = 1:num_Rf
    plot(omega, efficiency(:,j), 'LineWidth', 1.5);
    hold on;
end
grid on;
title('Efficiency vs Speed for Different Field Resistances');
xlabel('Angular Velocity (rad/s)');
ylabel('Efficiency (%)');
```

```
legend(legend_str);
hold off;

%% Print Values at 200 rad/s
fprintf('At 200 rad/s:\n');
for j = 1:num_Rf
    fprintf('Rf = %d ohms: Torque = %.2f N m , Power Loss = %.2f W,
    Efficiency = %.2f%%\n',...
        Rf_values(j), torque(end,j), power_loss(end,j), efficiency(end,
    j));
end
```

Listing 1: Field Resistance Control Implementation

```
    Efficiency at 200 rad/s:
Rf = 240 ohms: 20.83%
Rf = 288 ohms: 17.36%
Rf = 360 ohms: 13.89%
Rf = 480 ohms: 10.42%
```

Listing 2: Field Resistance control efficency

10.2 Matlab results:

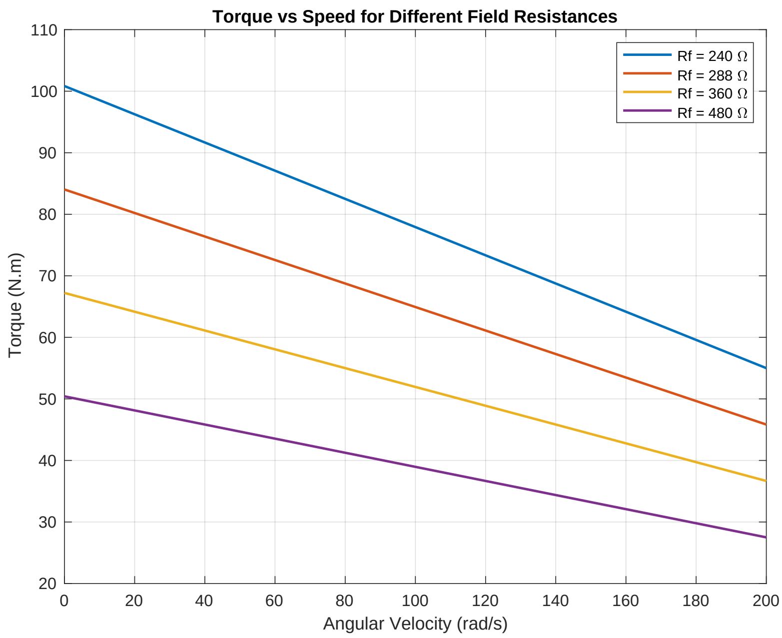

Figure 6: Field Resistance Control results

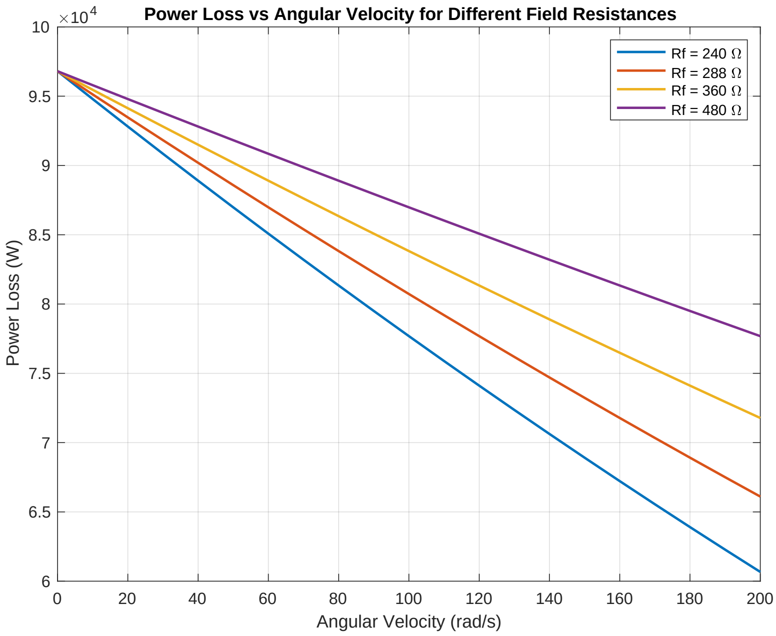

Figure 7: power loss

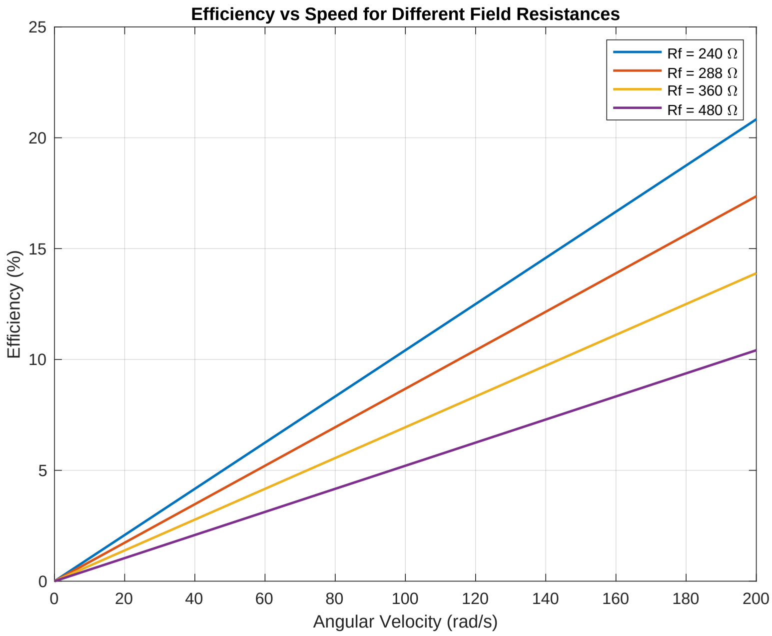

Figure 8: Efficiency

10.3 Armature Voltage Control

```
% Armature Voltage Control with Torque, Power Loss, and Efficiency
    Plots

clear all; close all; clc;

%% Motor Parameters
Ra = 0.5;              % Armature resistance (Ohms)
Rf = 240;              % Field resistance (Ohms)
k = 0.5;               % Motor constant
```

```
Va_nominal = 220;     % Nominal armature voltage (V)
Vf = 220;             % Field voltage (V)

%% Armature Voltage Variation
Va_values = [Va_nominal, Va_nominal*0.8, Va_nominal*0.6, Va_nominal
    *0.4];
num_Va = length(Va_values);

%% Speed Range
omega = 0:10:200;     % Angular velocity range (rad/s)
num_omega = length(omega);

%% Constant Field Flux
If = Vf / Rf;
phi = k * If;

%% Initialize Arrays
torque = zeros(num_omega, num_Va);
power_loss = zeros(num_omega, num_Va);
efficiency = zeros(num_omega, num_Va);

%% Calculate Torque, Power Loss, and Efficiency
for j = 1:num_Va
    Va = Va_values(j);
    for i = 1:num_omega
        w = omega(i);

        % Torque
        T = (k * phi * Va / Ra) - (k^2 * phi^2 * w / Ra);
        torque(i,j) = T;

        % Armature Current
        Ia = (Va - k * phi * w) / Ra;

        % Power Loss
        power_loss(i,j) = Ia^2 * Ra;

        % Efficiency
        P_out = T * w;
        P_in = Va * Ia;
        efficiency(i,j) = (P_out / P_in) * 100;
    end
end

%% Plot Torque vs Speed
figure;
for j = 1:num_Va
    plot(omega, torque(:,j), 'LineWidth', 1.5);
    hold on;
end
grid on;
title('Torque vs Speed for Different Armature Voltages');
xlabel('Angular Velocity (rad/s)');
ylabel('Torque (N m)');
legend_str = cell(num_Va,1);
for j = 1:num_Va
    legend_str{j} = ['Va = ' num2str(Va_values(j)) ' V'];
end
```

```
legend(legend_str);
hold off;

%% Plot Power Loss vs Speed
figure;
for j = 1:num_Va
    plot(omega, power_loss(:,j), 'LineWidth', 1.5);
    hold on;
end
grid on;
title('Power Loss vs Speed for Different Armature Voltages');
xlabel('Angular Velocity (rad/s)');
ylabel('Power Loss (W)');
legend(legend_str);
hold off;

%% Plot Efficiency vs Speed
figure;
for j = 1:num_Va
    plot(omega, efficiency(:,j), 'LineWidth', 1.5);
    hold on;
end
grid on;
title('Efficiency vs Speed for Different Armature Voltages');
xlabel('Angular Velocity (rad/s)');
ylabel('Efficiency (%)');
legend(legend_str);
hold off;

%% Print Values at 200 rad/s
fprintf('At 200 rad/s:\n');
for j = 1:num_Va
    fprintf('Va = %d V: Torque = %.2f N m, Power Loss = %.2f W, Efficiency = %.2f%%\n',...
        Va_values(j), torque(end,j), power_loss(end,j), efficiency(end,j));
end
```

Listing 3: Armature Voltage Control Implementation

```
    At 200 rad/s:
Va = 220 V: Torque = 79.83 N m, Power Loss = 60668.06 W, Efficiency = 20.83%
Va = 176 V: Torque = 59.66 N m, Power Loss = 33886.72 W, Efficiency = 26.04%
Va = 132 V: Torque = 39.49 N m, Power Loss = 14849.39 W, Efficiency = 34.72%
Va = 88 V: Torque = 19.33 N m, Power Loss = 3556.06 W, Efficiency = 52.08%
```

Listing 4: Armature Voltage Control efficency

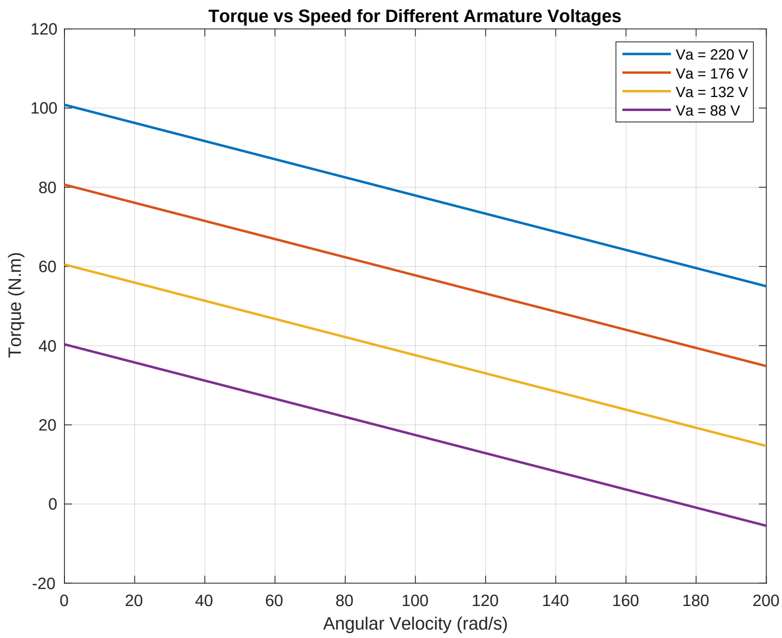

Figure 9: Armature Voltage Control results

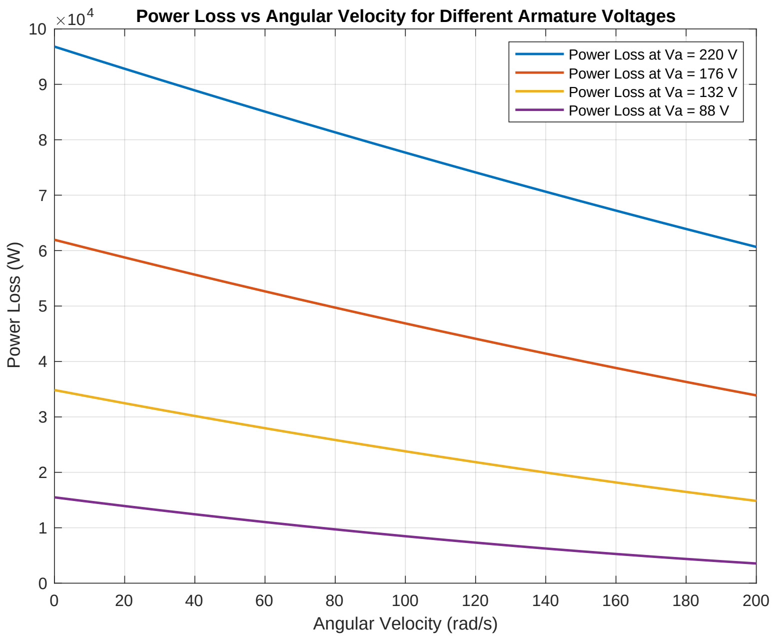

Figure 10: power loss

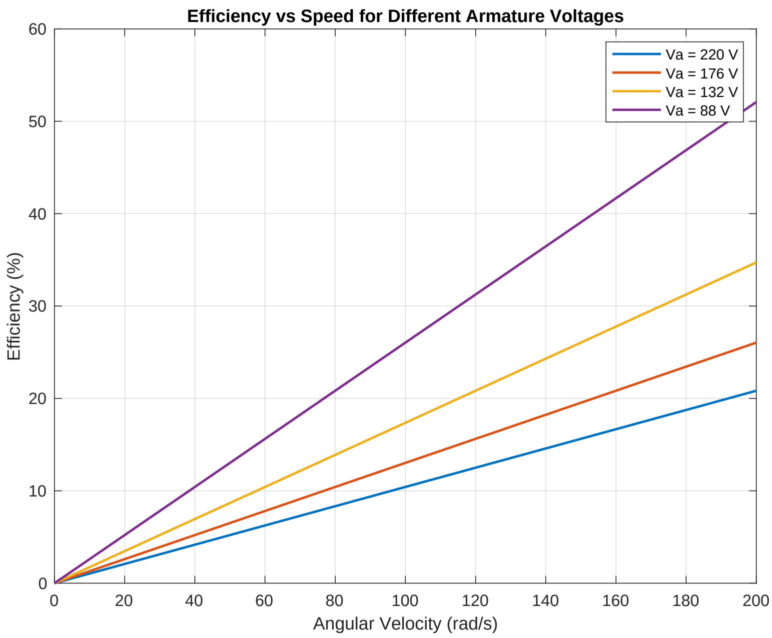

Figure 11: Efficiency

10.4 Armature Resistance Control

```
% Armature Resistance Control with Efficiency, Torque, and Power Loss
    Plots

clear all; close all; clc;

%% Motor Parameters
Ra = 0.5;              % Armature resistance (Ohms)
Rf = 240;              % Field resistance (Ohms)
k = 0.5;               % Motor constant
```

```
Va = 220;               % Armature voltage (V)
Vf = 220;               % Field voltage (V)

%% Additional Resistance Values
Radd_values = [0, 0.5, 1, 2];  % Additional resistances (Ohms)
num_Radd = length(Radd_values);

%% Speed Range
omega = 0:10:200;     % Angular velocity range (rad/s)
num_omega = length(omega);

%% Constant Field Flux
If = Vf / Rf;
phi = k * If;

%% Initialize Arrays
torque = zeros(num_omega, num_Radd);
power_loss = zeros(num_omega, num_Radd);  % Power loss in resistor
efficiency = zeros(num_omega, num_Radd);

%% Calculate Torque, Power Loss, and Efficiency
for j = 1:num_Radd
    R_total = Ra + Radd_values(j);
    for i = 1:num_omega
        w = omega(i);

        % Torque
        T = (k * phi * Va / R_total) - (k^2 * phi * w / R_total);
        torque(i,j) = T;

        % Armature Current
        Ia = (Va - k * phi * w) / R_total;

        % Power Loss in additional resistor
        power_loss(i,j) = Ia^2 * Radd_values(j);

        % Power
        P_out = T * w;
        P_in = Va * Ia;

        % Efficiency
        efficiency(i,j) = (P_out / P_in) * 100;
    end
end

%% Plot Torque vs Speed
figure;
for j = 1:num_Radd
    plot(omega, torque(:,j), 'LineWidth', 1.5);
    hold on;
end
grid on;
title('Torque vs Speed for Different Additional Resistances');
xlabel('Angular Velocity (rad/s)');
ylabel('Torque (N m)');
legend_str = cell(num_Radd,1);
for j = 1:num_Radd
    legend_str{j} = ['Radd = ' num2str(Radd_values(j)) ' \Omega'];
```

```
end
legend(legend_str);
hold off;

%% Plot Power Loss vs Speed
figure;
for j = 1:num_Radd
    plot(omega, power_loss(:,j), 'LineWidth', 1.5);
    hold on;
end
grid on;
title('Power Loss vs Speed for Different Additional Resistances');
xlabel('Angular Velocity (rad/s)');
ylabel('Power Loss (W)');
legend(legend_str);
hold off;

%% Plot Efficiency vs Speed
figure;
for j = 1:num_Radd
    plot(omega, efficiency(:,j), 'LineWidth', 1.5);
    hold on;
end
grid on;
title('Efficiency vs Speed for Different Additional Resistances');
xlabel('Angular Velocity (rad/s)');
ylabel('Efficiency (%)');
legend(legend_str);
hold off;

%% Print Efficiency at 200 rad/s
fprintf('Efficiency at 200 rad/s:\n');
for j = 1:num_Radd
    fprintf('Radd = %g ohms: %.2f%%\n', Radd_values(j), efficiency(end,
  j));
end
```

Listing 5: Armature Resistance Control Implementation

```
    Efficiency at 200 rad/s:
Radd = 0 ohms: 14.35%
Radd = 0.5 ohms: 14.35%
Radd = 1 ohms: 14.35%
Radd = 2 ohms: 14.35%
```

Listing 6: Armature Resistance Control efficiency

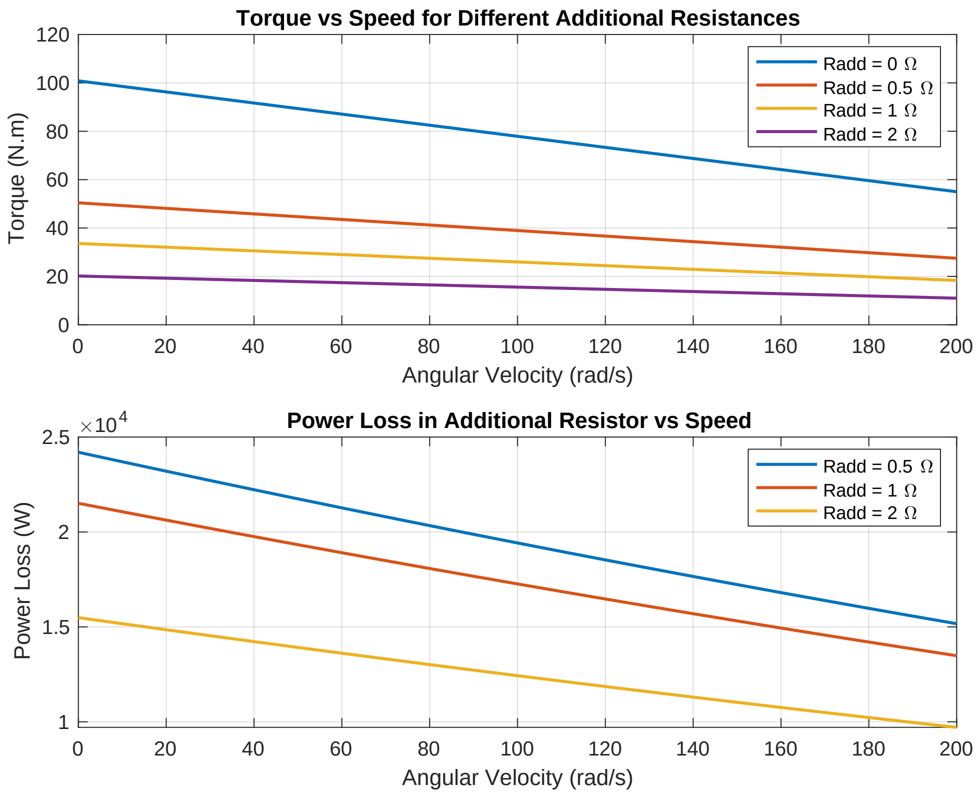

Figure 12: Armature Resistance Control

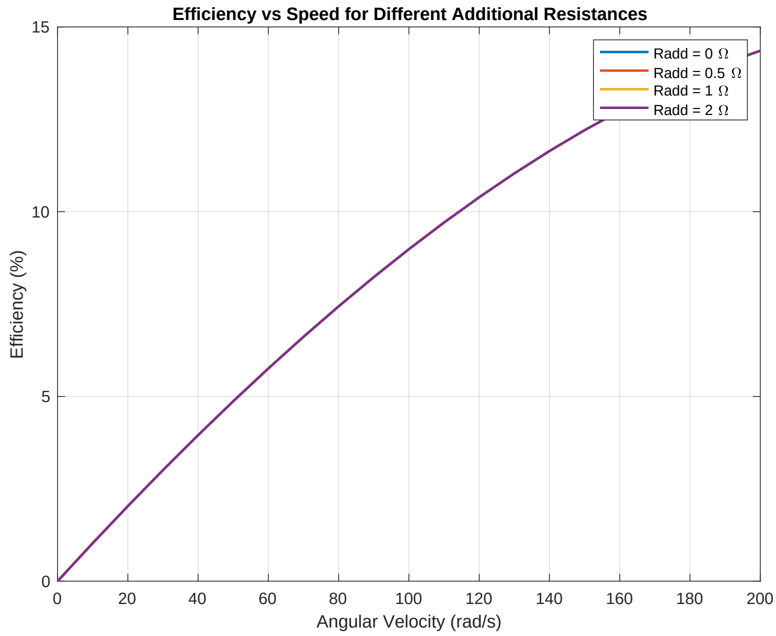

Figure 13: Efficiency

11 Discussion of Results

11.1 Comparison of Torque Control Methods

The mathematical analysis and simulation results for the three torque control methods reveal significant differences in their performance characteristics, efficiency, and applicability. These methods—field resistance control, armature voltage control, and armature resistance control—each present distinct torque-speed relationships and efficiency profiles.

11.1.1 Field Resistance Control

Field resistance control operates by adjusting the field resistance R_f to modify field flux Φ. The mathematical relationship governing this method is:

$$T = \frac{kK_f V_f V_a}{R_f R_a} - \frac{k^2 K_f^2 V_f^2 \omega}{R_f^2 R_a} \tag{26}$$

The simulation results demonstrate that increasing the field resistance R_f leads to a significant reduction in torque capability, particularly at higher speeds. At 200 rad/s, the efficiency decreases from 20.83% at $R_f = 240\Omega$ to merely 10.42% at $R_f = 480\Omega$. This efficiency reduction is directly attributable to increased losses in the field circuit, calculated as:

$$P_{loss,f} = \frac{V_f^2}{R_f} \tag{27}$$

The torque-speed curves for different field resistances illustrate the inverse relationship between field resistance and torque production capability. This method is primarily suited for operation above rated speed, where reduced torque requirements are acceptable.

11.1.2 Armature Voltage Control

For armature voltage control, with constant field flux, the torque equation simplifies to:

$$T = \frac{k\Phi V_a}{R_a} - \frac{k^2 \Phi^2 \omega}{R_a} \tag{28}$$

The simulation results reveal a direct proportionality between armature voltage and torque. At 200 rad/s, reducing armature voltage from 220V to 88V decreases torque from 79.83 Nm to 19.33 Nm, while simultaneously increasing efficiency from 20.83% to 52.08%. This counterintuitive efficiency improvement occurs because the power loss, given by $I_a^2 R_a$ where $I_a = \frac{V_a - k\Phi\omega}{R_a}$, decreases more rapidly than the output power as voltage is reduced.

The torque-speed curves demonstrate linear characteristics, making this method highly predictable and controllable for below-rated speed operation.

11.1.3 Armature Resistance Control

In armature resistance control, the torque equation becomes:

$$T = \frac{k\Phi V_a}{R_a + R_{add}} - \frac{k^2 \Phi^2 \omega}{R_a + R_{add}} \tag{29}$$

The simulation results show that increasing additional resistance R_{add} reduces torque across all speed ranges. However, the efficiency remains consistently low at approximately 14.35% regardless of the additional resistance value. This is due to the substantial power loss in the additional resistor, calculated as:

$$P_{loss,add} = I_a^2 R_{add} \tag{30}$$

The torque-speed curves for different additional resistances demonstrate a consistent reduction in torque capability as resistance increases, without the efficiency benefits observed in other methods.

11.2 Comparative Analysis

The comparative analysis of the three methods reveals that:

- Armature voltage control provides the highest efficiency (up to 52.08% at lower voltages) and most linear control characteristics, making it optimal for applications requiring precise torque control below rated speed.
- Field resistance control offers moderate efficiency (10.42% to 20.83%) and is suitable for above-rated speed operation, despite reduced torque capability.
- Armature resistance control consistently provides the lowest efficiency (approximately 14.35%) regardless of resistance values, making it the least energy-efficient option.

The power loss versus speed curves further illustrate that armature voltage control exhibits the lowest losses at reduced voltage settings, while armature resistance control incurs significant additional losses due to the external resistor.

12 Conclusion

12.1 Field Resistance Control

Field resistance control modifies the field flux by adjusting the field circuit resistance. This method presents particular characteristics that define its applicability in DC motor control systems.

12.1.1 Advantages

- Enables operation at speeds above rated speed by reducing field flux
- Simple implementation with basic circuit elements
- Provides effective speed adjustment at the higher end of the speed range
- No additional power components required in the armature circuit

12.1.2 Disadvantages

- Significantly reduces torque capability at higher speeds
- Decreasing efficiency as field resistance increases
- Field circuit losses increase with field weakening
- Limited torque response due to field circuit time constant
- Not suitable for applications requiring high torque at high speeds

12.2 Armature Voltage Control

Armature voltage control directly modulates the armature circuit voltage while maintaining constant field flux, resulting in predictable and efficient torque control.

12.2.1 Advantages

- Maintains good efficiency, especially at reduced voltage levels
- Provides linear torque-speed characteristics for predictable control
- Well-suited for speed control below rated speed
- Direct and proportional relationship between voltage and torque
- No additional resistive losses in the control circuit

12.2.2 Disadvantages

- Limited to operation below rated speed
- Requires more complex power electronic circuits for voltage control
- Higher cost due to power electronic requirements
- Potential for increased harmonics with certain voltage control methods
- Less suitable for applications requiring operation across wide speed ranges

12.3 Armature Resistance Control

Armature resistance control inserts additional resistance in series with the armature circuit, providing simple but inefficient torque control.

12.3.1 Advantages

- Extremely simple implementation with basic resistive elements
- Low initial cost compared to electronic control methods
- Robust operation in harsh environments
- No specialized electronic components required
- Suitable for applications where simplicity outweighs efficiency concerns

12.3.2 Disadvantages

- Poor efficiency due to significant power dissipation in additional resistors
- Consistent power loss regardless of the specific resistance value
- High thermal management requirements for the additional resistors
- Energy inefficient, leading to higher operating costs
- Reduced torque capability across all speed ranges

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